EFFECT OF INITIAL CONDITIONS ON SELF-SIMILAR TURBULENT MIXING

David L Youngs

AWE, Aldermaston, UK

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For self-similar turbulent mixing:-

Rayleigh-Taylor mixing:-

bubble penetration $h_1 = \alpha A g t^2$

Kelvin-Helmholtz mixing (time-evolving shear layer):-

mixing zone width = c Vt

Richtmyer-Meshkov mixing (post shock behaviour):-

mixing zone width $\propto t^p$, where p is a fractional power

If loss of memory of the initial conditions occurs then for a given density ratio α, c, p should be universal constants.

Is complete loss of memory of initial conditions likely to occur in actual experiments? According to the 3D simulations described in this talk the answer is probably not.

ENGINEERING MODELS (e.g. bouyancy-drag model, k-ε model, 2-fluid model)

3D Large Eddy Simulation is impractical for complex engineering applications. Hence engineering models are commonly used and these contain “model constants”.

These models characterise turbulence by a few variables (turbulence KE, length scale, velocity separation…) - not by a spectrum. Hence assumptions about self-similar behaviour are made.

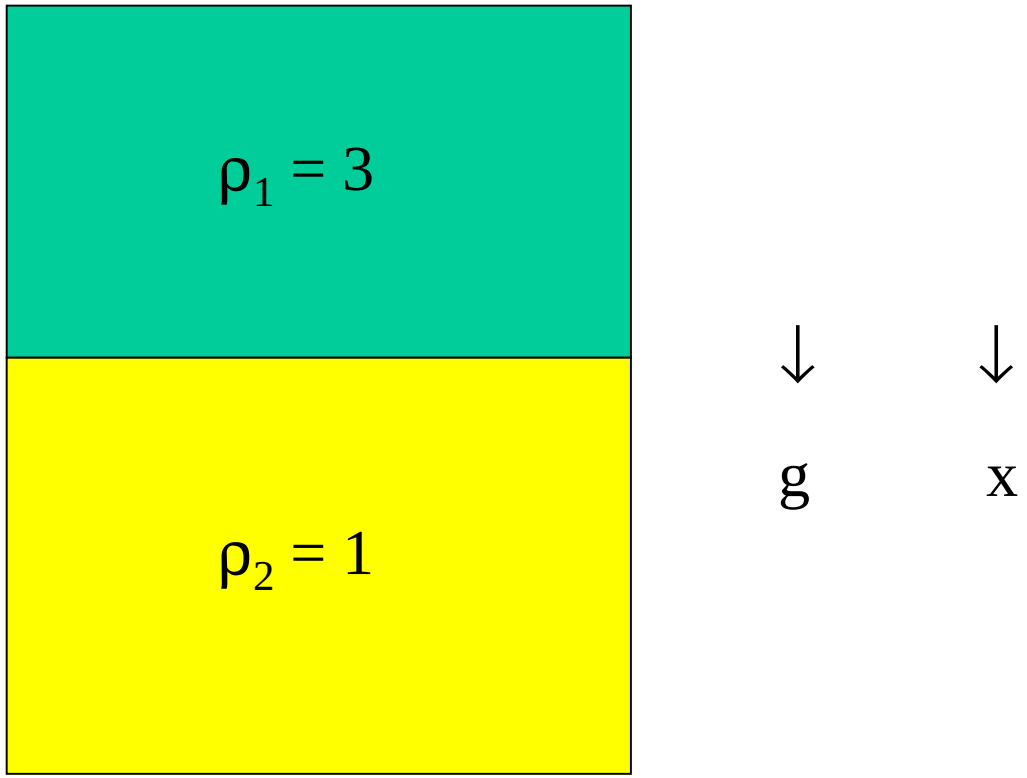
Model constants are usually chosen to match the three simple cases for self-similar RT, KH and RM mixing. If complete loss of memory of initial conditions does not occur then the engineering models are more approximate that previously assumed.

3D NUMERICAL SIMULATIONS

The three simple cases for RT, KH and RM mixing are calculated using the TURMOIL3D code.

- Simple compressible 3D hydrocode. Perfect gas EoS. No interface tracking. Calculates mixing of *miscible* fluids.
- Lagrangian phase + remap (advection phase)
- 3rd order monotonic van Leer advection
- Semi-Lagrangian option (x-direction mesh moves with the mean x-velocity). Used for the RM case.
- MILES technique

Rayleigh-Taylor Mixing



Zoning: 720 x 600 x 600 (720 meshes in the direction of gravity)
Computational region : 1.45H x H x H

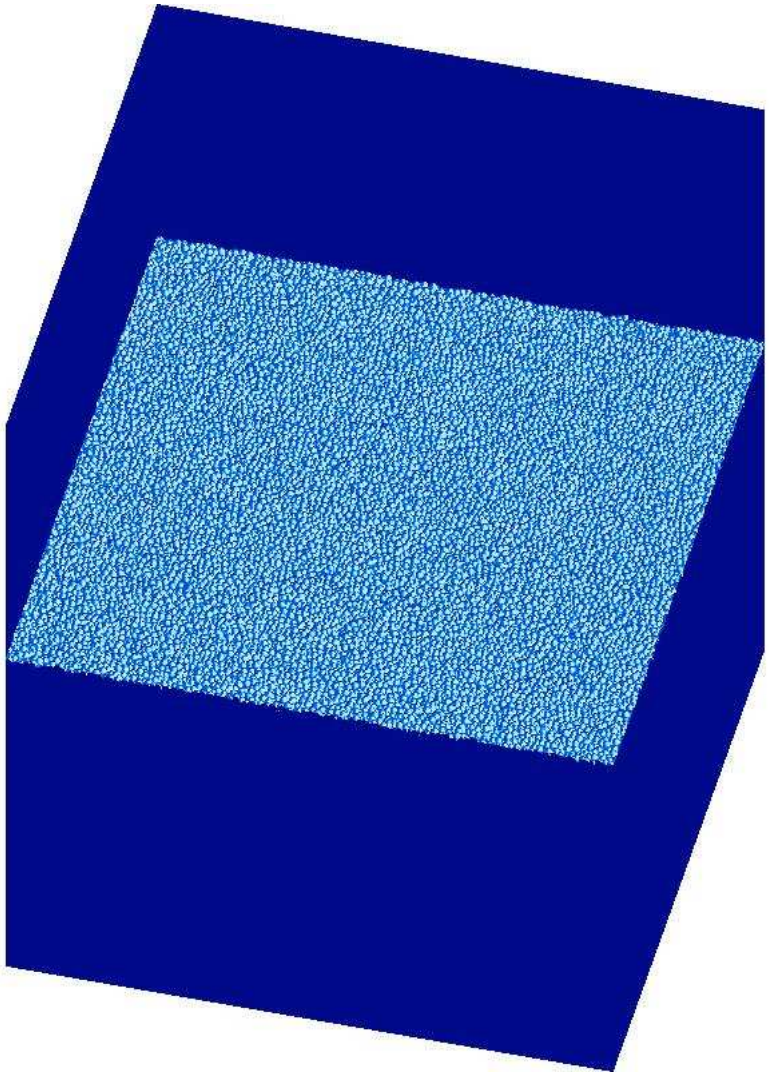
Initial sound speed high enough to give near incompressible mixing.

Initial amplitude perturbations : random combination of Fourier modes:

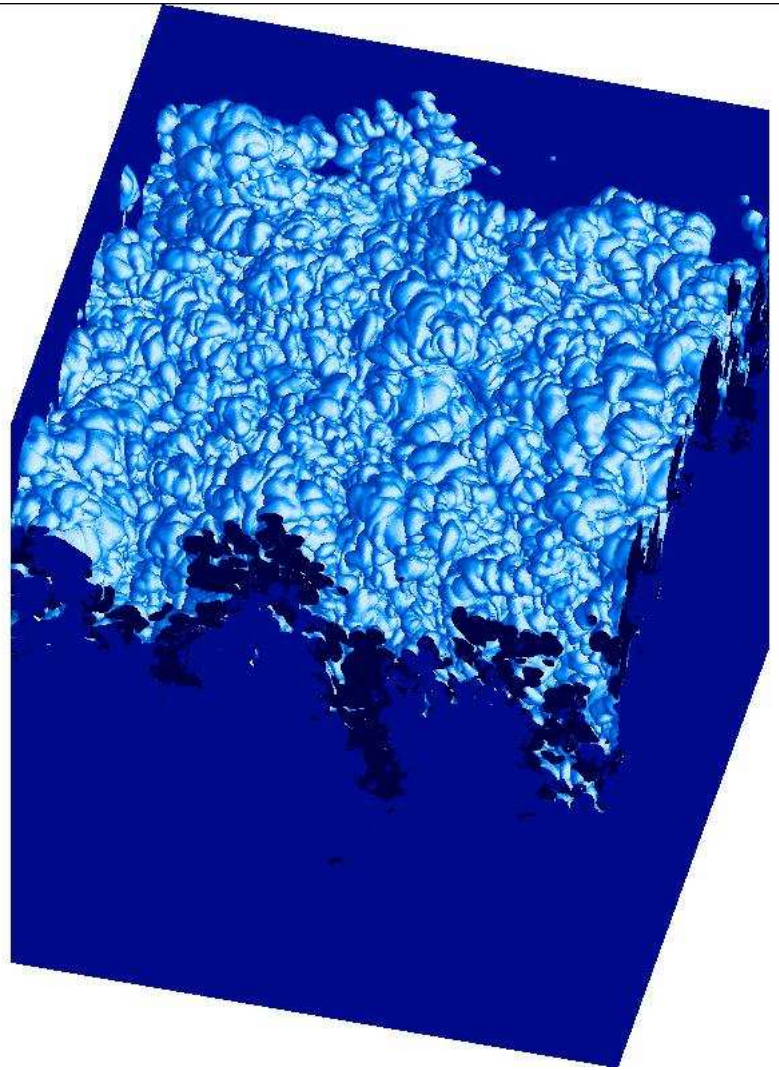
wavelengths: $4\,\Delta x$ to $8\,\Delta x$
amplitude s.d. : $0.04\,\Delta x$

Dominant length scale increases by mode coupling - expect loss of memory of the initial conditions to occur $\Rightarrow$ gt^2 growth.

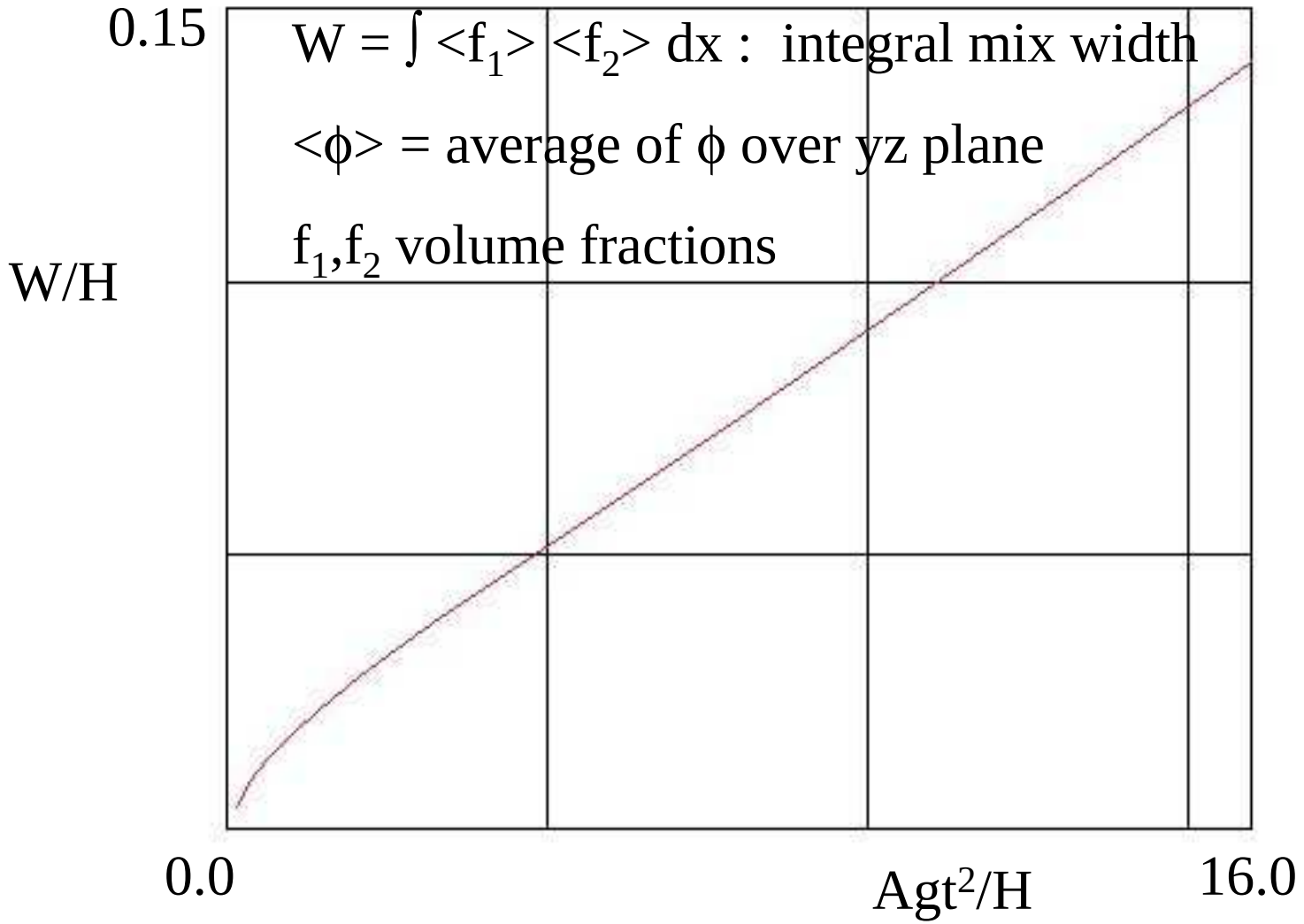
Isosurfaces for f_1 =0.99



t = 0.4



t = 4.0



$h_1 \sim 3.3\ W \Rightarrow\ \alpha = 0.027$ significantly less than most observed values

Key integral properties are examined which should approach constant values in the self-similar regime:-

(a) $$\frac{D}{P} = \frac{\text{KE dissipated}}{\text{Loss of potential energy}}$$

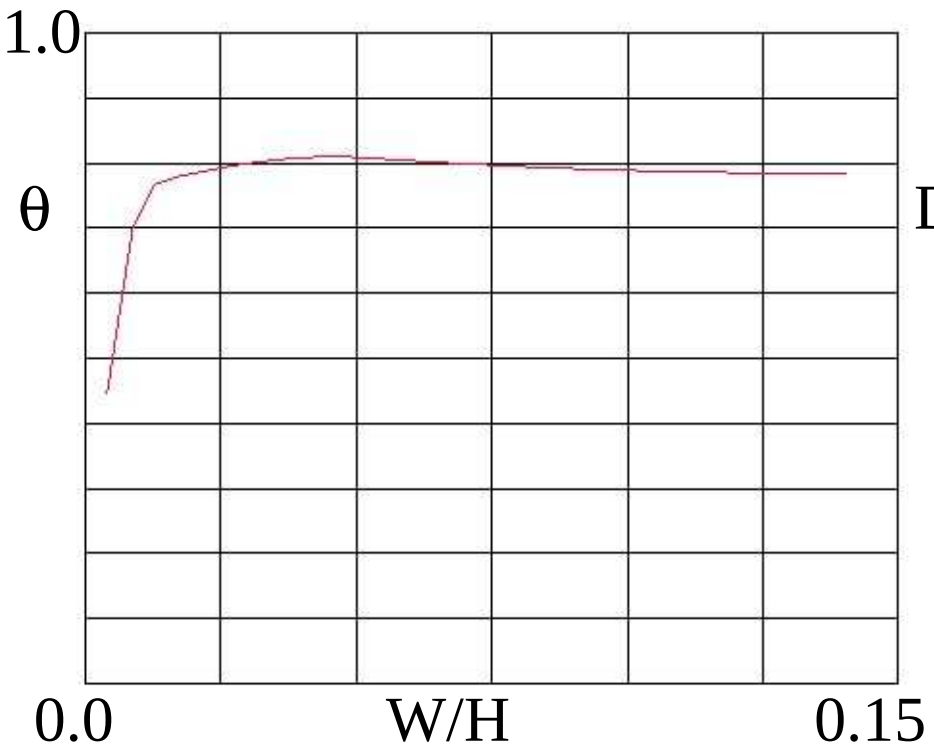
(b) Molecular mixing fraction

$$\theta = \int \overline{f_1 f_2}\, dx \Big/ \int \bar{f}_1 \cdot \bar{f}_2\, dx$$

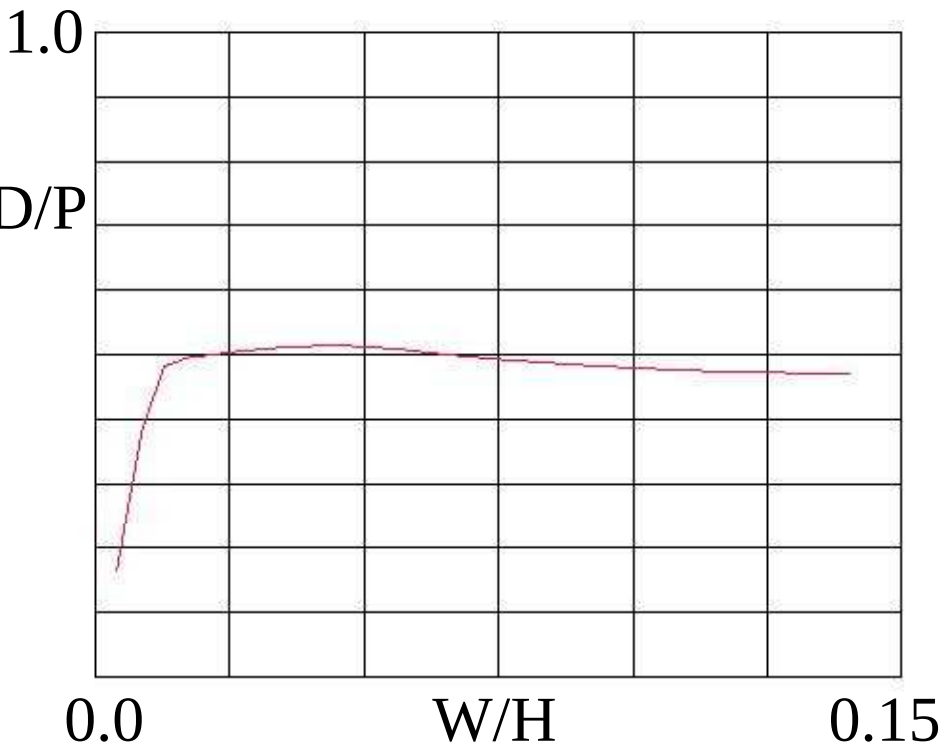
$$\overline{f_1 f_2} = \bar{f}_1 \cdot \bar{f}_2 - \sigma^2$$

$$\text{where } \sigma = \sqrt{\overline{\left(f_1 - \bar{f}_1\right)^2}}$$

$$= \text{concentration fluctuation}$$



Molecular mixing fraction



KE dissipation fraction

Self - similar mixing is seen for a large range - increase in mixing zone width by a factor 5.

However, $\alpha \sim 0.03$ is significantly less than most observed values (0.05 - 0.07). Note: $\alpha = 0.04$ for experiments of Kucherenko et al. IWPCTM8 for gaseous mixing.

Similar low calculated values of α reported by

The Alpha-Group collaboration. Dimonte, Youngs et al. Physics of Fluids 2004. Andy Cook, this workshop.

Higher experimental values of α likely to be due to non-ideal initial conditions. (suppression of fine-scale structure by surface tension may also have had an effect in experiments with *immiscible* fluids)

ENHANCED SELF-SIMILAR MIXING

Add long wavelength initial perturbations with amplitude ∝ wavelength, as suggested by Inogamov IWPCTM3 (1991)

In terms of power spectra

$$\sigma^2 = \int P(k)\,dk$$

$$a_\lambda = \left\{ \int_{2\pi/\lambda}^{\infty} P(k)\,dk \right\}^{\frac{1}{2}} = \varepsilon\lambda \qquad \left(\lambda = \frac{2\pi}{k}\right)$$

$$\Rightarrow P(k) \sim C/k^3$$

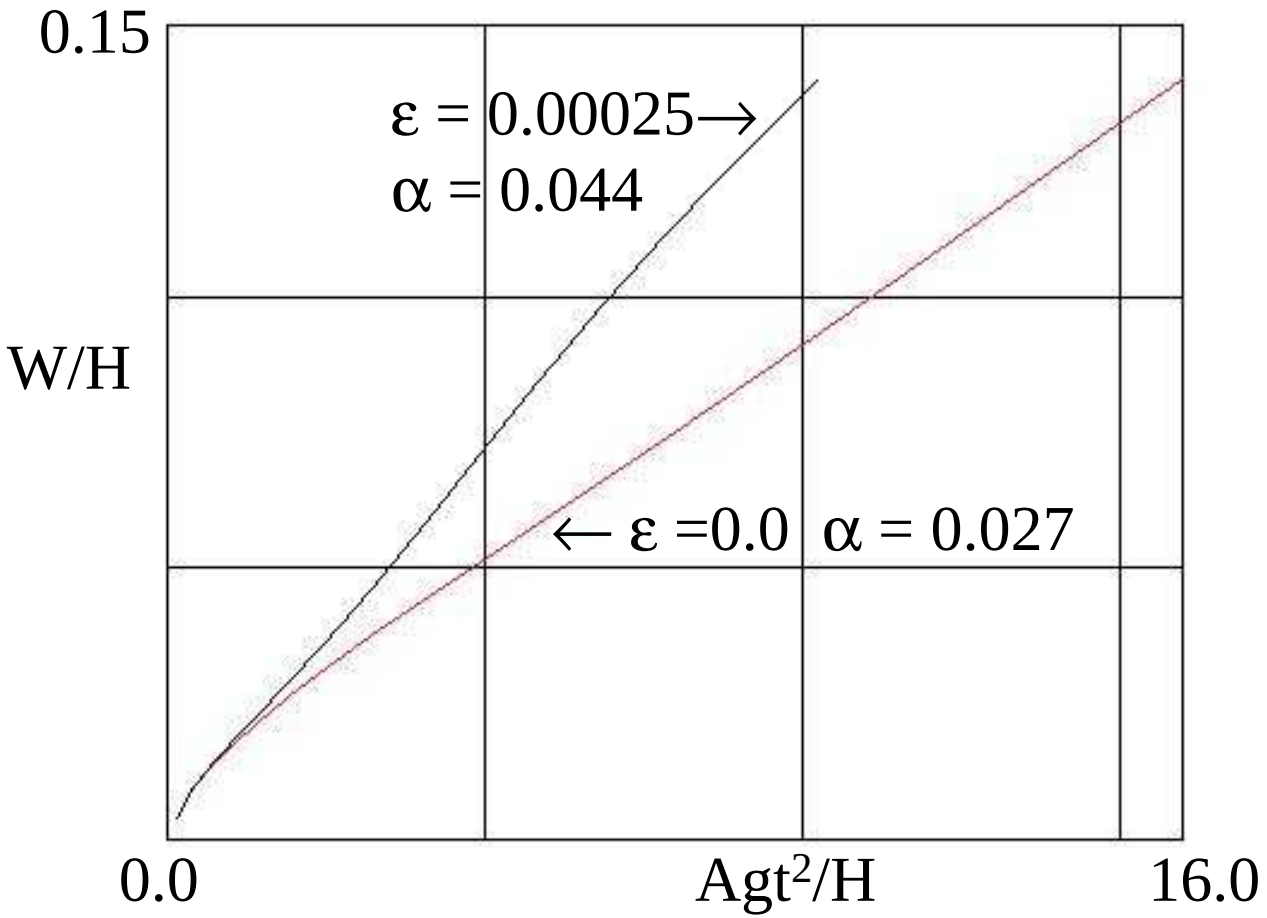
Assume $$P(k) = \begin{cases} C/k^3 & \lambda < \lambda_{max} \\ o & \text{otherwise} \end{cases}$$

where λ_{max} is of order of the size of the experimental apparatus.

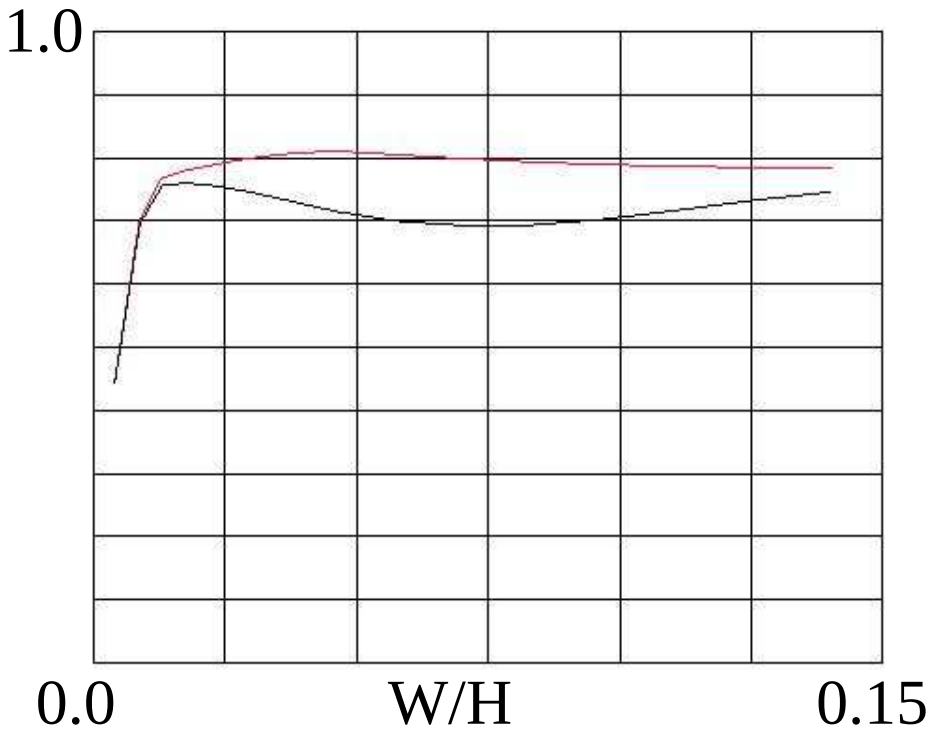
C is a dimensionless quantity. If the turbulence length scale remains less than λ_{max} then the value of λ_{max} should not affect the results. Dimensional reasoning then implies:-

$$h_1 = \alpha(\varepsilon)\, Agt^2$$

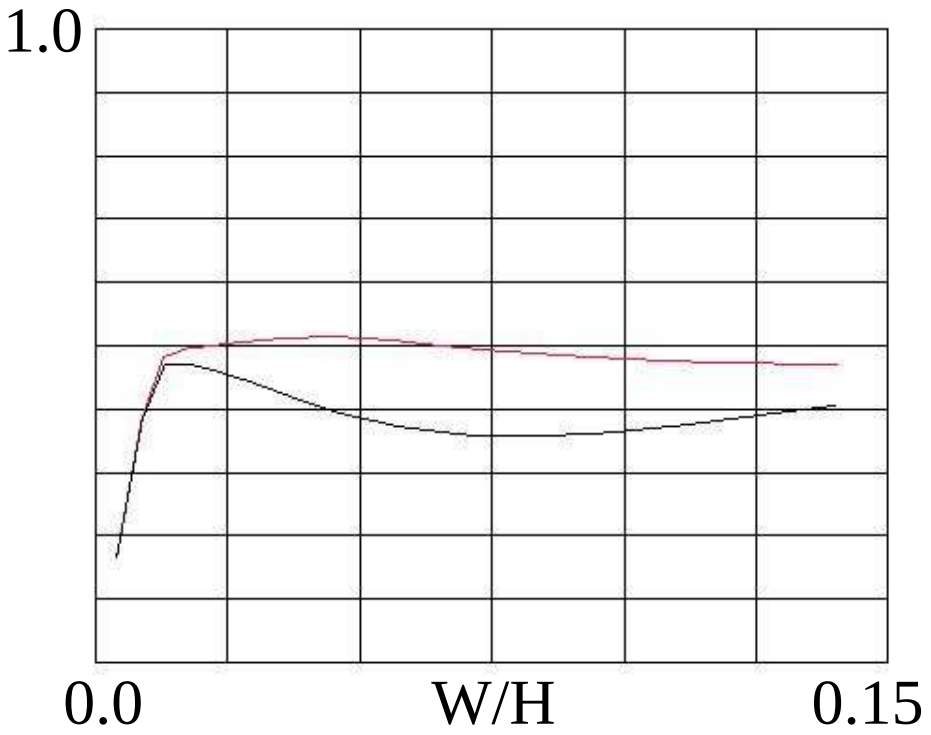
Enhanced growth rate results



Note: both calculations include the short wavelength random perturbation.



Molecular mixing fraction



KE dissipation fraction

—— ε = 0.0

—— ε = 0.00025

Enhanced rate of self-similar mixing ($\alpha = 0.05$) is achieved by adding very low amplitude long wavelength perturbations (wavelengths up to ~ 1/2 the size of the experimental domain)

It is suggested that this is the explanation for many of the higher observed growth rates.

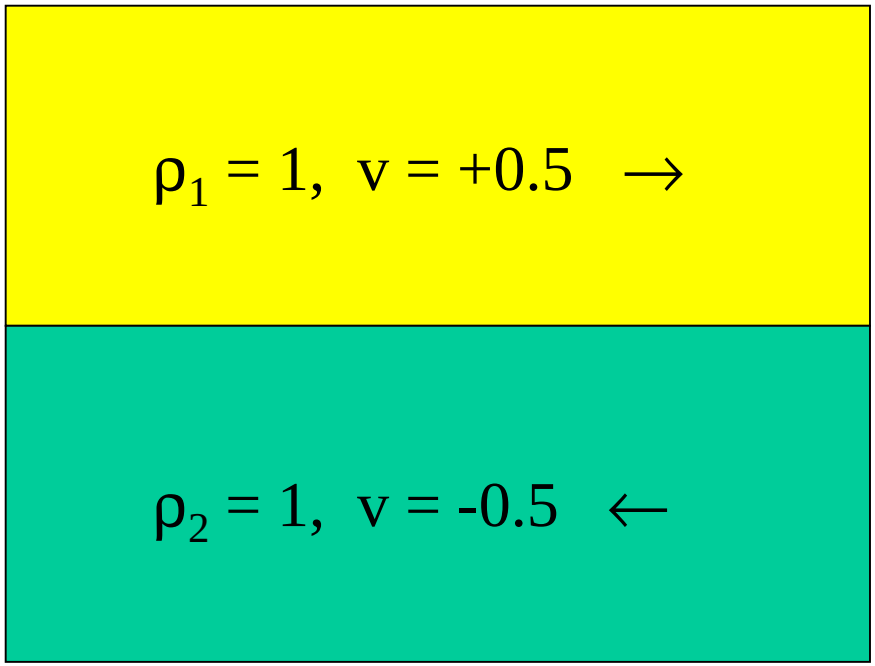
True enhanced self-similar mixing if $P(k) \sim 1/k^3$

For typical surface finishes $P(k) \sim 1/k^2$ - expect a small deviation from gt^2 growth in real problems.

Weak dependence on initial conditions expected.

KELVIN - HELMHOLTZ MIXING

Free shear layer experiments show linear growth rate but the growth rate coefficient varies from experiment to experiment. Is the behaviour analagous to self - similar RT mixing?
We consider the time-evolving shear layer: -



Computational Region

Box size 2H x 4H x H (same aspect ratio as used by Pantano and Sarkar JFM, 2002)

$0 < x < 1.0$ 600 meshes (mixing direction)

$0 < y < 4.0$ 1200 meshes (shear direction)

$0 < z < 0.5$ 300 meshes

V/c = (velocity difference) / (sound speed) = 0.2
$\Rightarrow$ near-incompressible mixing.

INITIAL PERTURBATIONS (GROWTH BY MODE COUPLING)

Single mode surface perturbations:

Initial Amplitude perturbation: $\eta = a \cosh(nt) \exp\{i(k_y y + k_z z)\}$

$$n^2 = \tfrac{1}{4} k_y^2 V^2 \quad , \quad \text{at } t = o \;\; |\eta| = a \;\; |\dot{\eta}| = o$$

Initial velocity perturbation: $\eta = a \sinh(nt) \exp\{i(k_y y + k_z z)\}$

$$\text{at} \;\; t = o \quad |\eta| = o \quad |\dot{\eta}| = na$$

Random perturbation used:

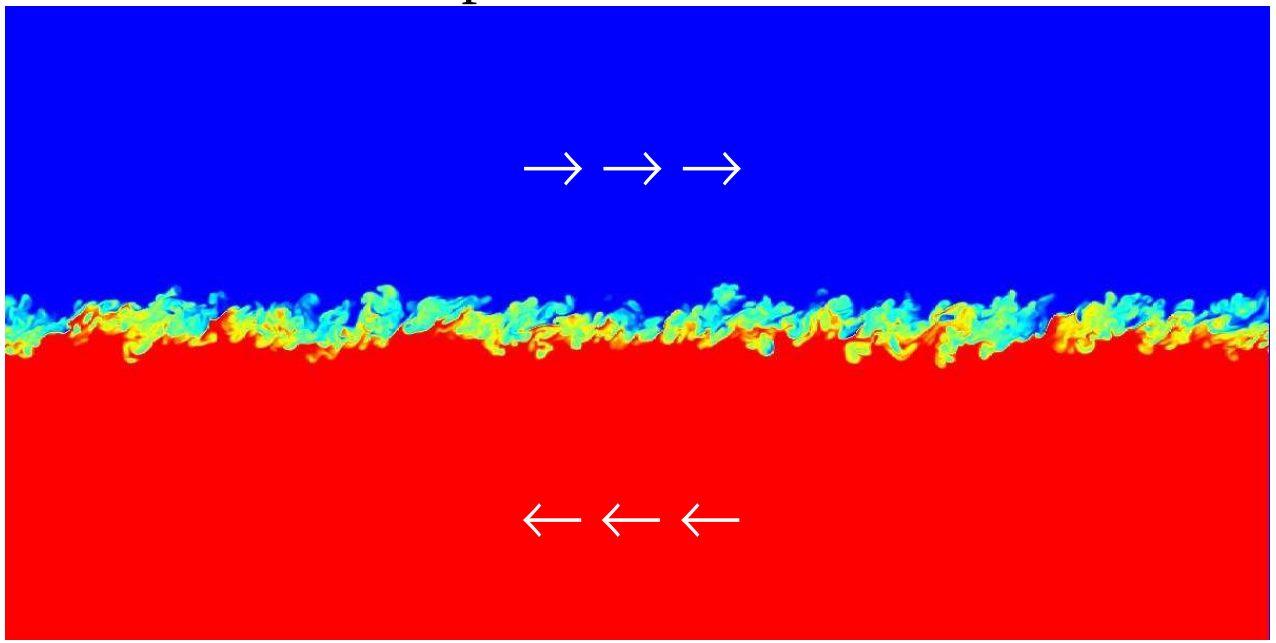
Define random initial amplitude perturbation

$4 \Delta x < \lambda < 8 \Delta x$, s.d = $0.08 \Delta x$

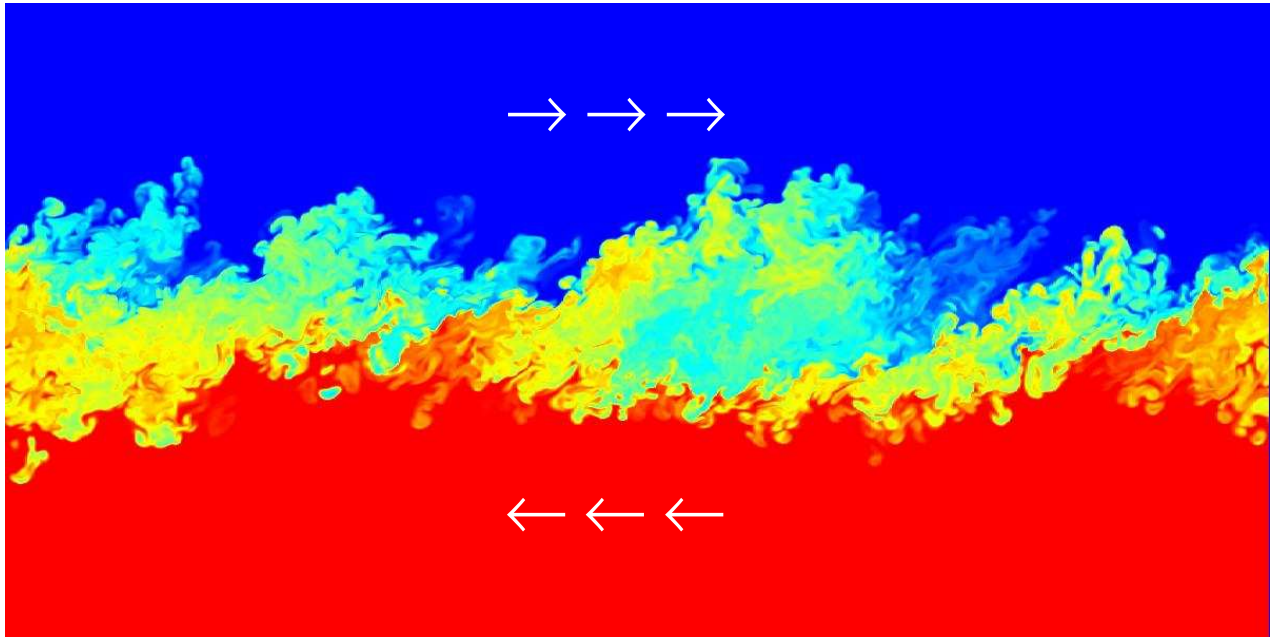
Convert to equivalent initial velocity perturbation and calculate the vector potential $\underline{A}$

Then $\underline{u}$ = curl $\underline{A}$ - divergence - free initial velocity field

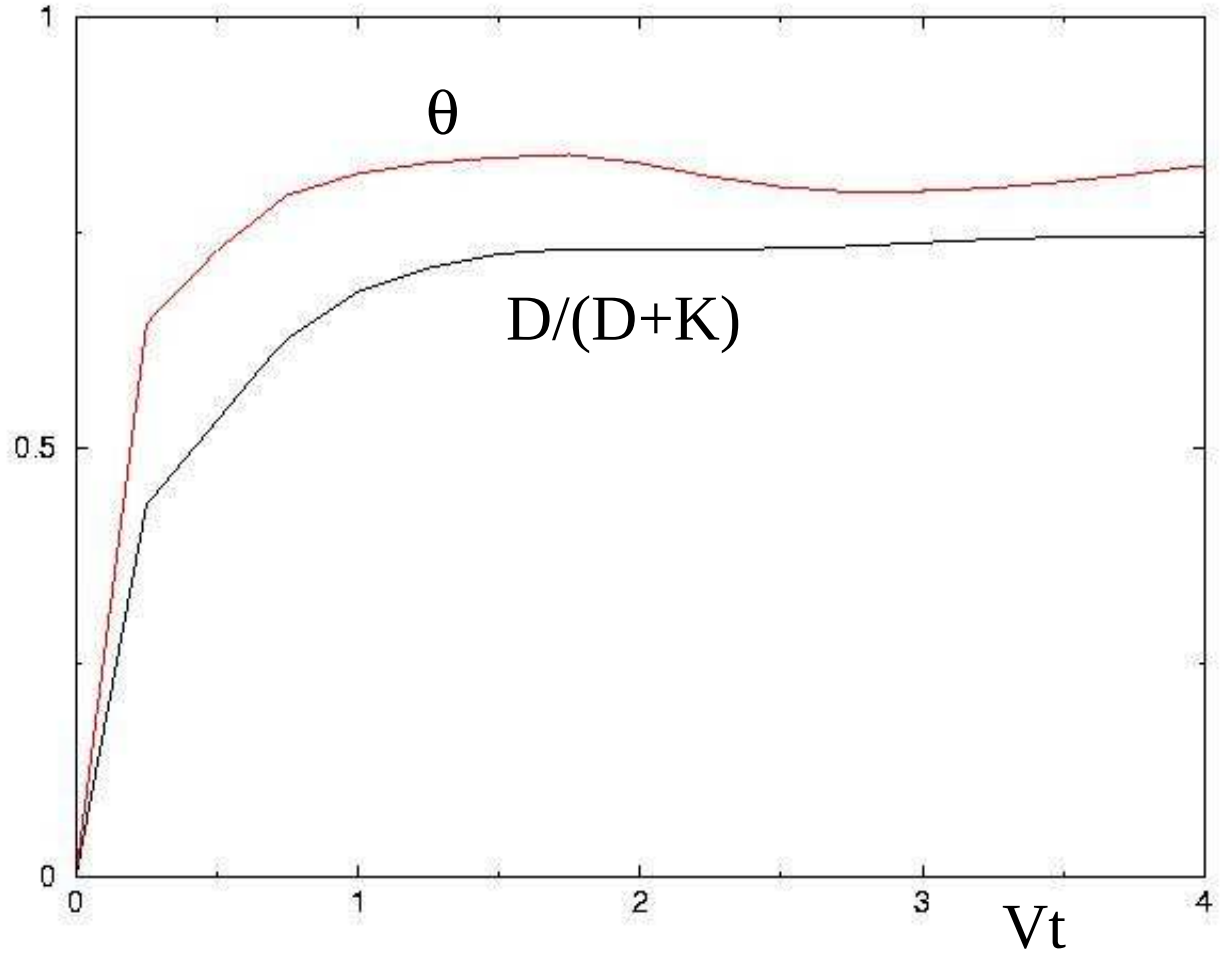
Volume fraction plots



t = 0.75



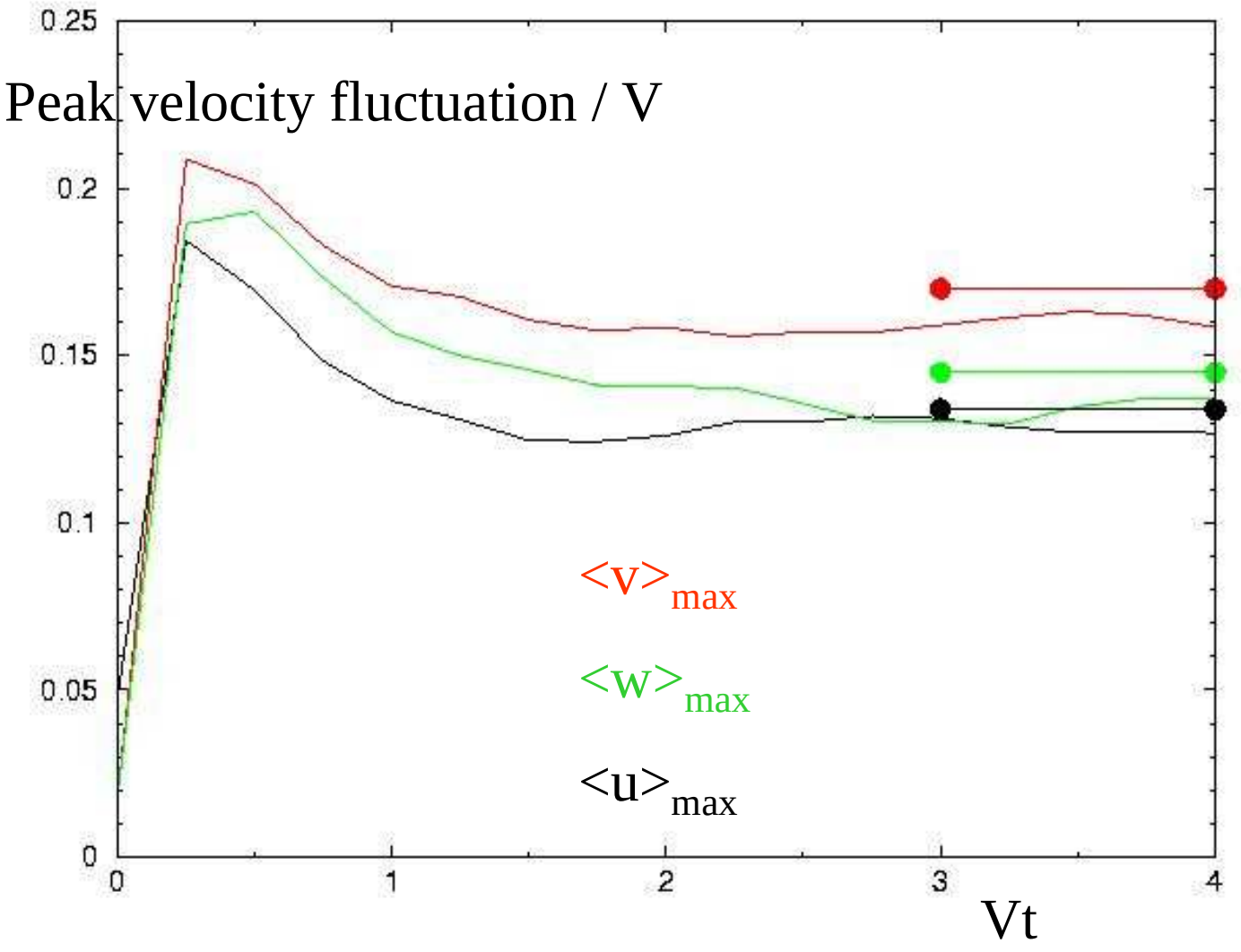
t = 4.0



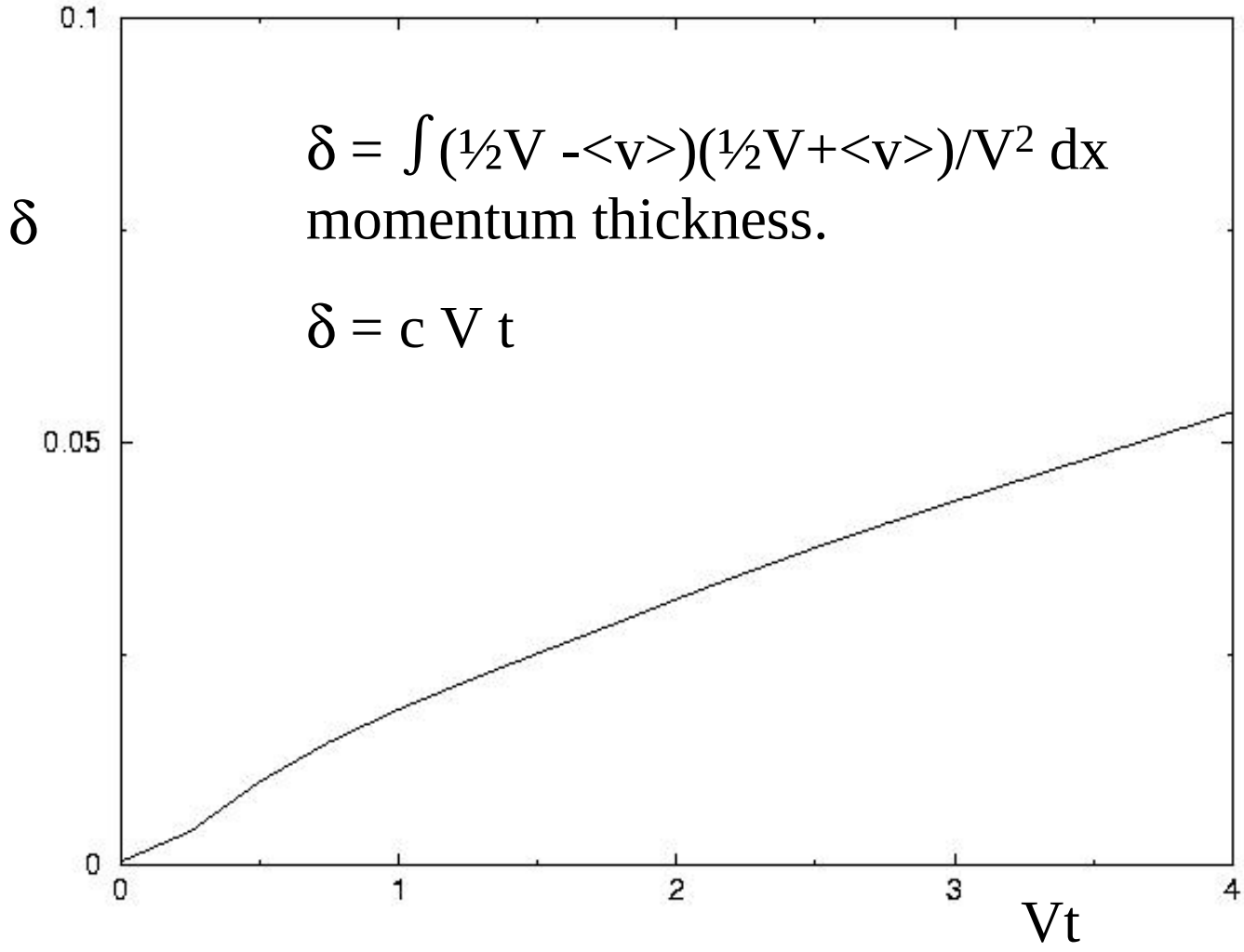
θ: Molecular mixing fraction

D: KE dissipated

K: Turbulence KE



● ● ● Pantano and Sarkar DNS (with broadband initial perturbations)



Typical experimental value (Pantano and Sarkar): c = 0.016

This simulation (Vt = 2.0 to 4.0): c = 0.011

KELVIN-HELMHOLTZ SUMMARY

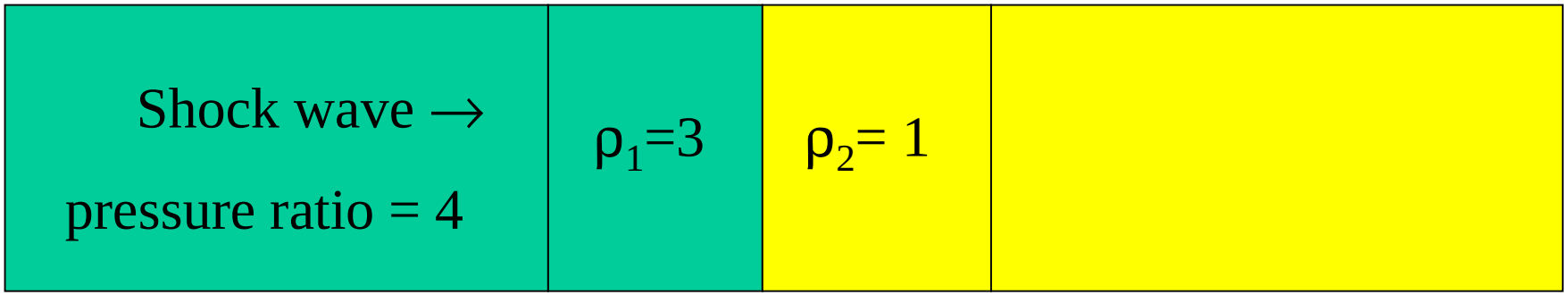
• 3D simulation of growth by mode coupling shows growth rate less than typical experimental values (c = 0.011 vs c = 0.016)

• situation appears to be similar to the RT case

• higher resolution simulations needed to confirm that the self-similar state is accurately modelled.

• plan to perform further 3D simulation with $1/k^3$ random long wavelength perturbations added and make a detailed comparison with experimental data(Pantano and Sarkar :- broadband initial perturbations needed to match observed growth rates).

RICHTMYER-MESHKOV MIXING



1D Lagrangian region 3D zoning 1D Lagrangian region

Semi-Lagrangian calculation.

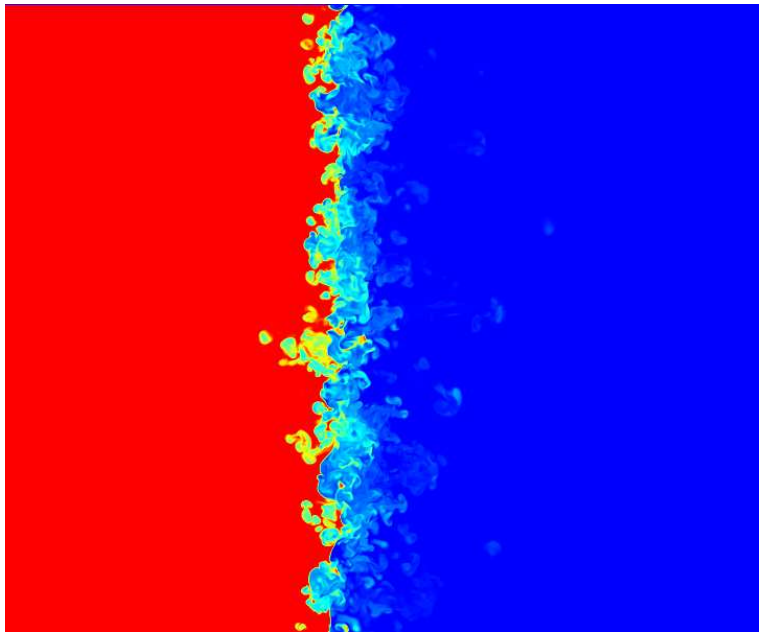
Zoning in 3D region: 720 x 600 x 600 meshes

Chosen to give $\Delta x = \Delta y = \Delta z$ after shock passage (3D box size = 1.2H x H x H)

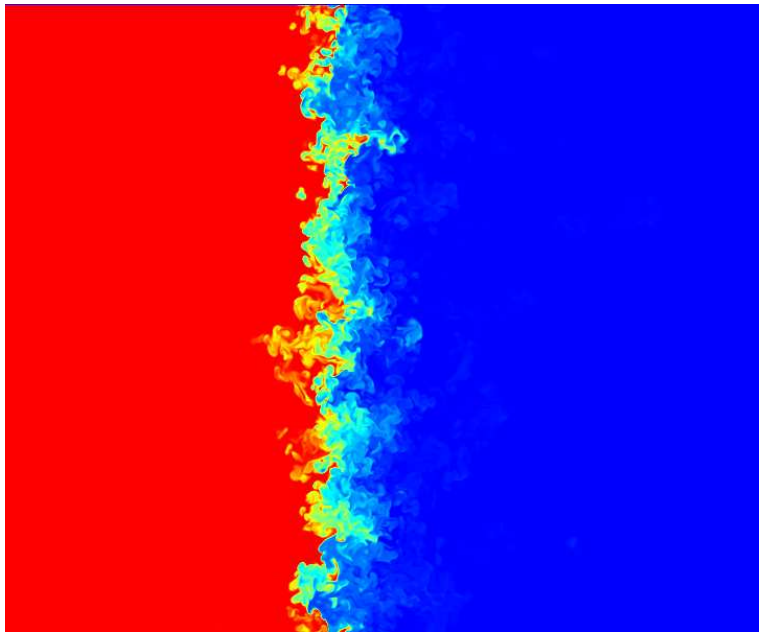
First calculation: resolved short wavelength random perturbations (growth by mode coupling)

λ_{min}= 16 Δx λ_{max}= 32 Δx s.d = 0.1 λ_{min}

Volume fraction plots



t = 6.5



t = 16.5

Time measured from shock arrival at interface

SIMPLE MODEL

Simple model for self-similar post-shock mixing zone growth

W = mixing zone width
V = mix velocity

Kinetic energy: $K \sim \frac{1}{4}(\rho_1 + \rho_2) W V^2$

KE dissipation ~ c K V/W

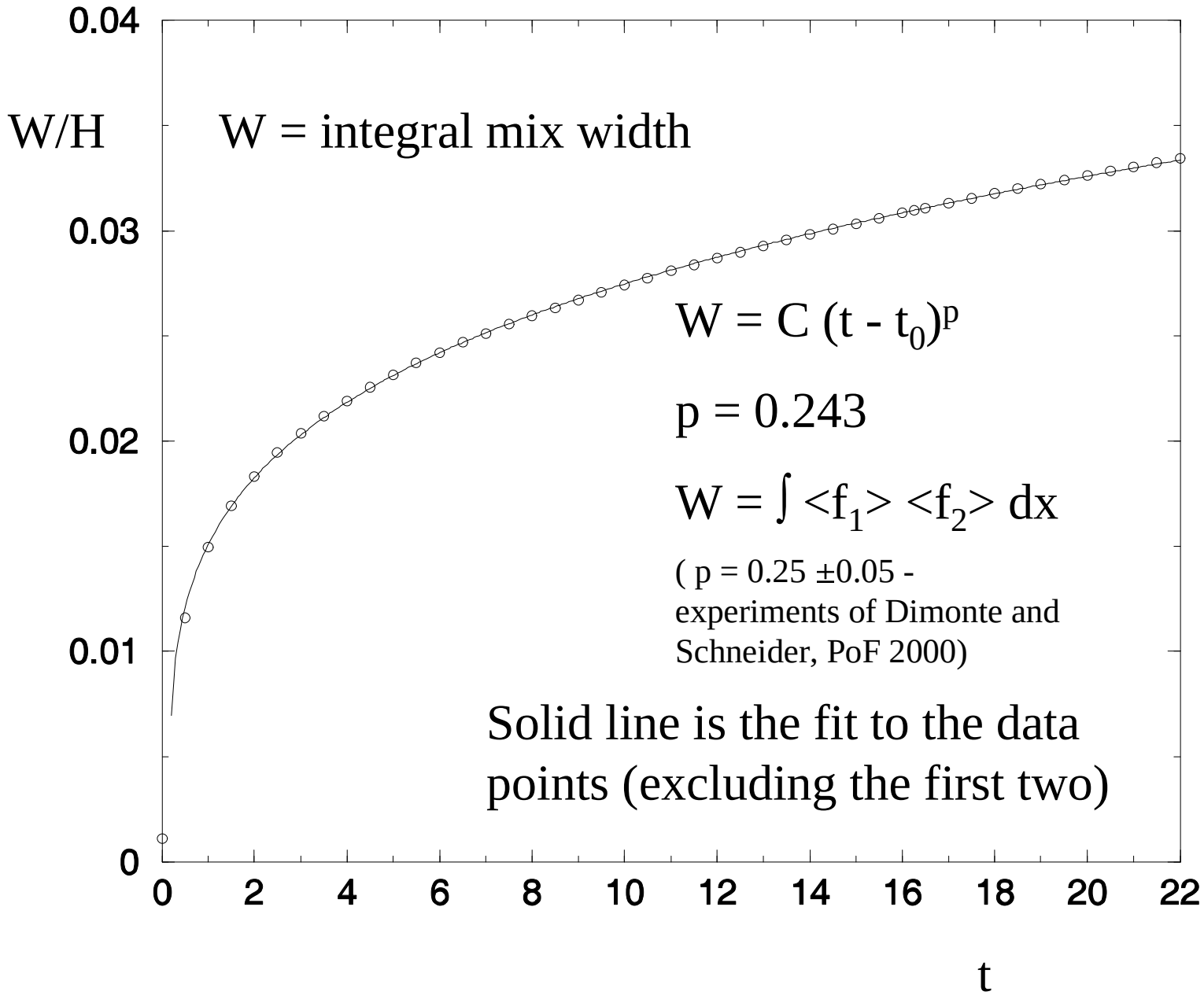
Then $\frac{dW}{dt} = V$

$$\frac{d}{dt}(W V^2) = - cV^3$$

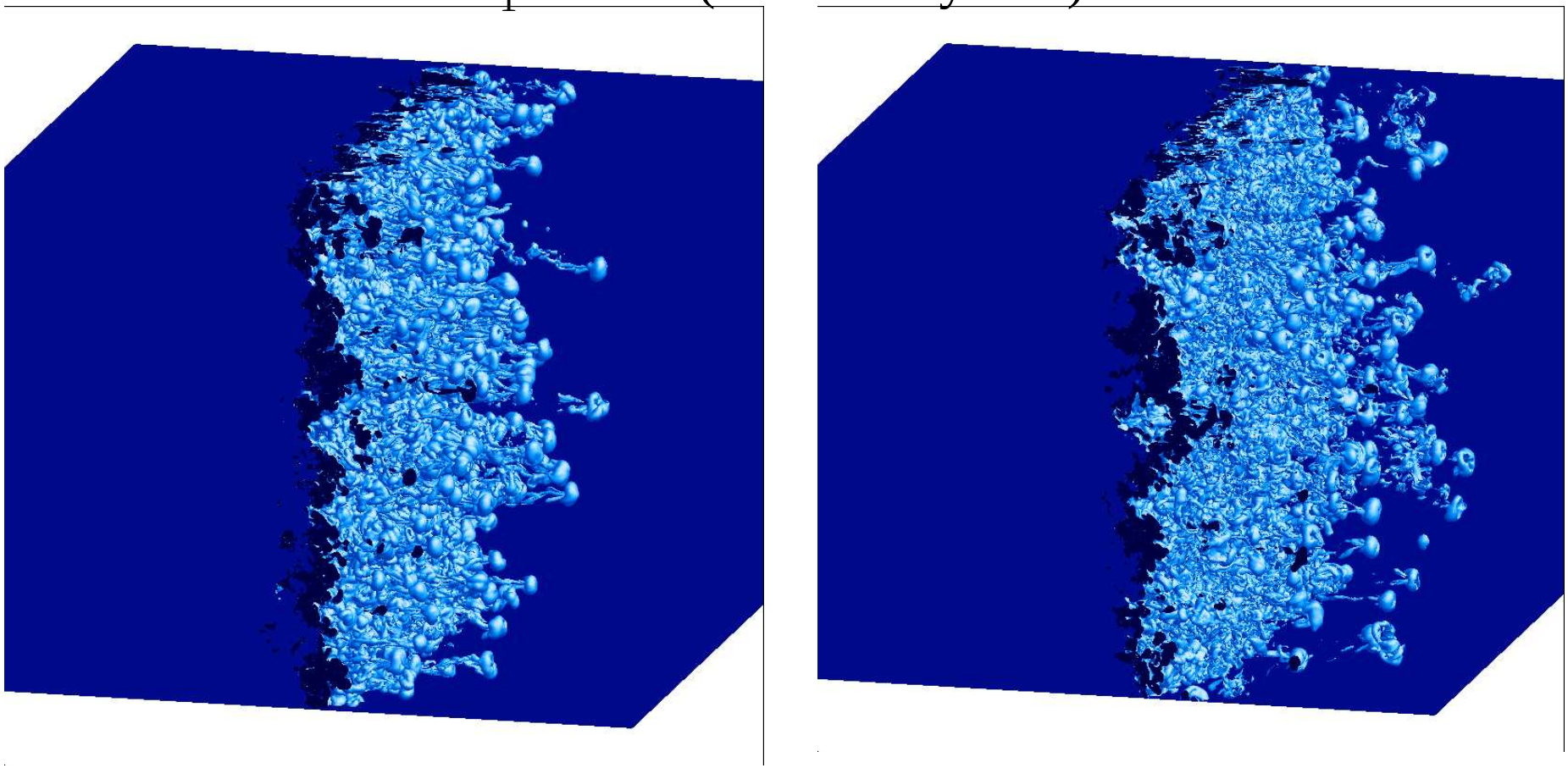
Solution $W = t^p$

where $p = \frac{2}{3+c}$

Note: have assumed that width of the fluid mixing zone, KE zone and KE dissipation zone are all have the same extent.


0.04
W/H
W = integral mix width
0.03
0.02
0.01
0
0
2
4
6
8
10
12
14
16
18
20
22
t
W = C (t - t0)p
p = 0.243
W = ∫ <f1> <f2> dx
(p = 0.25 ±0.05 -
experiments of Dimonte and
Schneider, PoF 2000)
Solid line is the fit to the data
points (excluding the first two)

3D isosurfaces f_1=0.01 (low density side)



t = 6.5 t = 16.5

The mixing zone grows very slowly at late time: small vortex rings escape from the main mixing zone and carry away small amounts of the denser fluid.

RM GROWTH DIRECTLY FROM INITIAL PERTURBATIONS

Initial amplitude power spectrum

$$P(k)=\begin{cases} C\,k^{s} & k_{min} \le k \le k_{max} \\ o & \text{otherwise} \end{cases}$$

For wavelength, $\lambda = \frac{2\pi}{k}$, well inside the (k_{min}, k_{max}) range

Amplitude $a_\lambda = \left\{ \int_{k/2}^{k} P(k')\,dk' \right\}^{\frac{1}{2}} \sim C^{\frac{1}{2}}\,\lambda^{-(s+1)/2}$

Mixing velocity corresponding to wavelength λ

$v_\lambda = \frac{2\pi\, a_\lambda}{\lambda}.A.\Delta$ ΔU = velocity jump Assumes linear theory valid.

Hence time scale for structure of size λ to appear

$$t = \frac{\lambda}{v_\lambda}$$

$\Rightarrow$ Mixing zone width $W \sim \lambda \sim \left\{ C^{\frac{1}{2}}.\Delta U.t \right\}^{\frac{2}{s+5}}$

(Equivalent result derived by Inogamov, Astrophysics and Space Reviews, Vol 10, part 2,1999)

$P(k) = C\,k^s \qquad \Rightarrow p = 2 / (s+5)$

For growth from initial short wavelength perturbations

$p \sim 0.24$

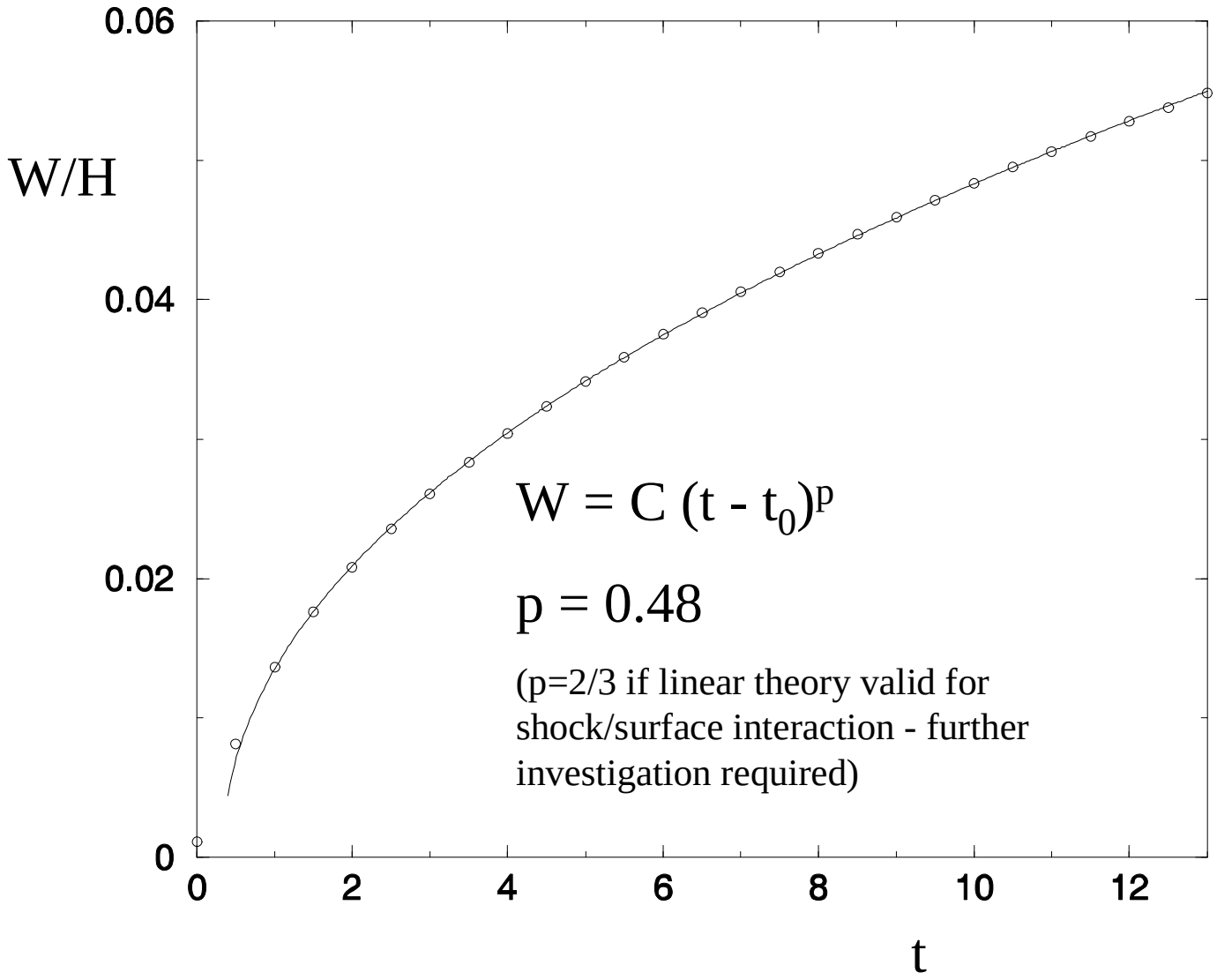
Implies that if $2 / (s+5) > 0.24$ i.e. $s < 3.3$ the exponent p will be determined by the initial conditions.

If s = -3 (as in enhanced self - similar RT simulation) expect $p=1 \Rightarrow$ linear growth (fluid mixing zone smaller than region of velocity fluctuations)

Second simulation performed with

$P(k) = C / k^2$ (s = -2)

$\lambda_{min} = 16\,\Delta x \qquad \lambda_{max} = H / 3 \qquad$ s.d = 0.1 λ_{min} (as before)


0.06
W/H
0.04
0.02
0
0
2
4
6
8
10
12
t
W = C (t - t0)p
p = 0.48
(p=2/3 if linear theory valid for shock/surface interaction - further investigation required)

CONCLUDING REMARKS

• For both RT and KH mixing, 3D simulation of mixing growth by mode coupling gives slower growth than in typical experiments. Suggests that experimental growth rates are enhanced by low levels of long wavelength initial perturbations.

• RM case: exponent p is likely to depend on initial conditions. A low value p ~ 0.24 might be applicable for a shock interacting with a pre-formed turbulent mixing zone (narrow band - large amplitude perturbations). A higher value of p is expected for a shock interacting with an undisturbed rough surface.

• Engineering models: model constants not universal - should be chosen to fit experiments (or 3D simulations) with typical initial perturbations. Expect a weak dependence of model constants on initial conditions (see talk by Guy Dimonte at this workshop)