DIRECT 3D NUMERICAL SIMULATION OF GRAVITATIONAL TURBULENT MIXING WITH REGARD TO MOLECULAR VISCOSITY

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Original system of equations

$$\frac{d\vec{K}}{dt} + \int_S \rho\vec{u}(\vec{u} - \vec{u}^*)d\vec{S} = \int_V divT dV$$

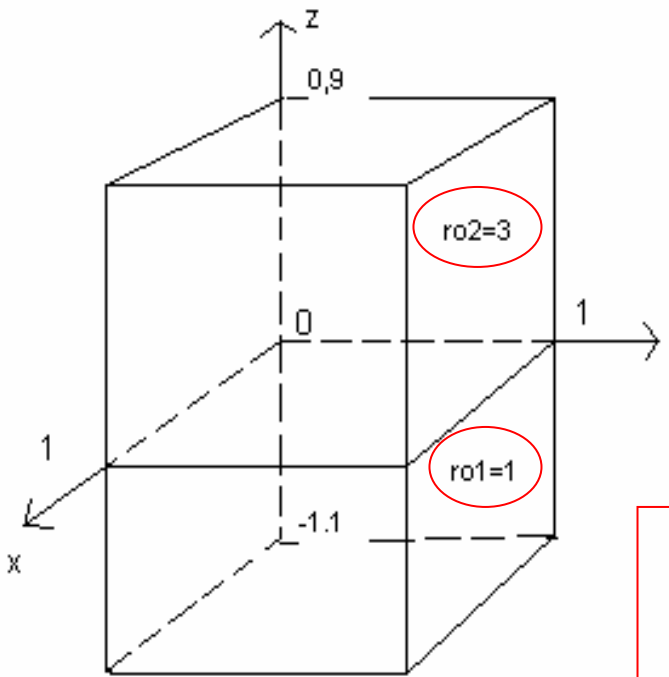
$$\frac{dM_i}{dt} + \int_{S_i} \rho_i(\vec{u}_i - \vec{u}^*)d\vec{S} = 0$$

$$\frac{dE_i}{dt} + \int_{S_i} \rho_i e_i(\vec{u}_i - \vec{u}^*)d\vec{S} = \int_{V_i} Sp(T_i D_i)dV$$

$$\frac{dV_i}{dt} + \int_{S_i} (\vec{u}_i - \vec{u}^*)d\vec{S} = 0$$

$$T^{di}_{\cdot} = \eta\left(\frac{\partial u_j}{\partial x_k} + \frac{\partial u_k}{\partial x_j} - \frac{2}{3}\delta_{jk}\frac{\partial u_j}{\partial x_j}\right)$$

Problem of numerical simulation of gravitational turbulent mixing



$\rho 1=1$, $\rho 2=3$

$\sigma\rho = \pm\rho_1\delta$, where $\delta = 0.1$

Requirement of hydrostatic equilibrium

$$p(z) = p_0 - \int_{z_2}^{z_1} \rho(z) \cdot g \cdot dz$$

$p_0 = 5$, $g_z = -1$

Requirement of incompressibility:

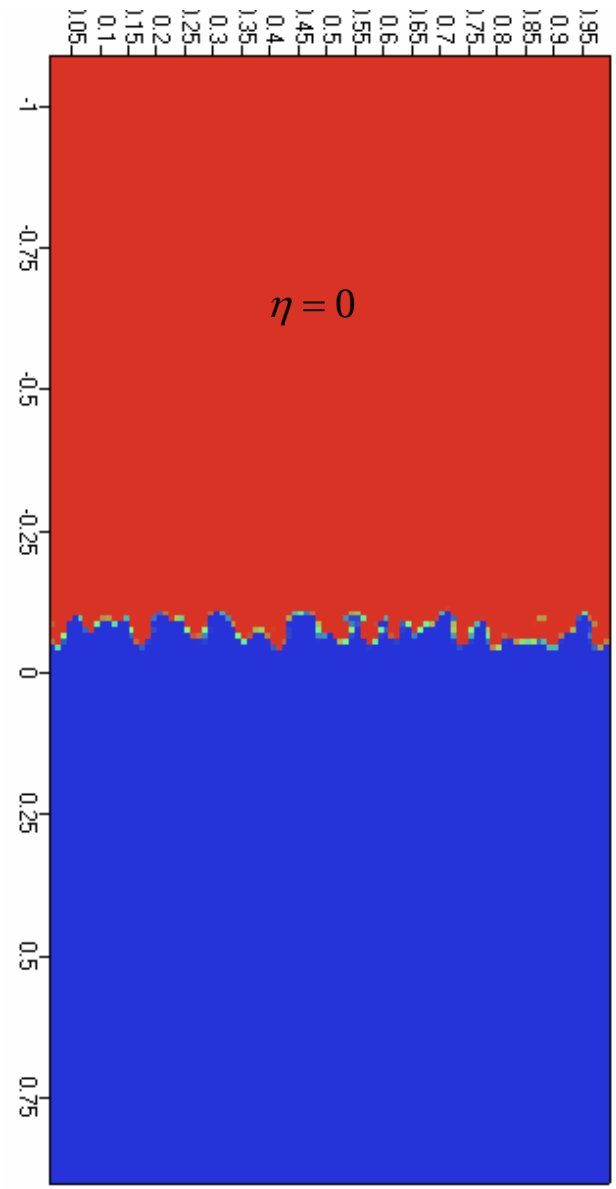
$k=\xi L_t g << \gamma p/\rho$, where $\xi=\text{const}<<1$

$\gamma=1.4$

Grid: $N_x=100$, $N_y=100$, $N_z=200$, $h_x = h_y = h_z = 0.01$

Numbers of computations

№	η
1	-
2	-
3	0
4	$5 \cdot 10^{-3}$
5	$5 \cdot 10^{-4}$
6	$5 \cdot 10^{-5}$
7	$5 \cdot 10^{-6}$


$\eta = 0$
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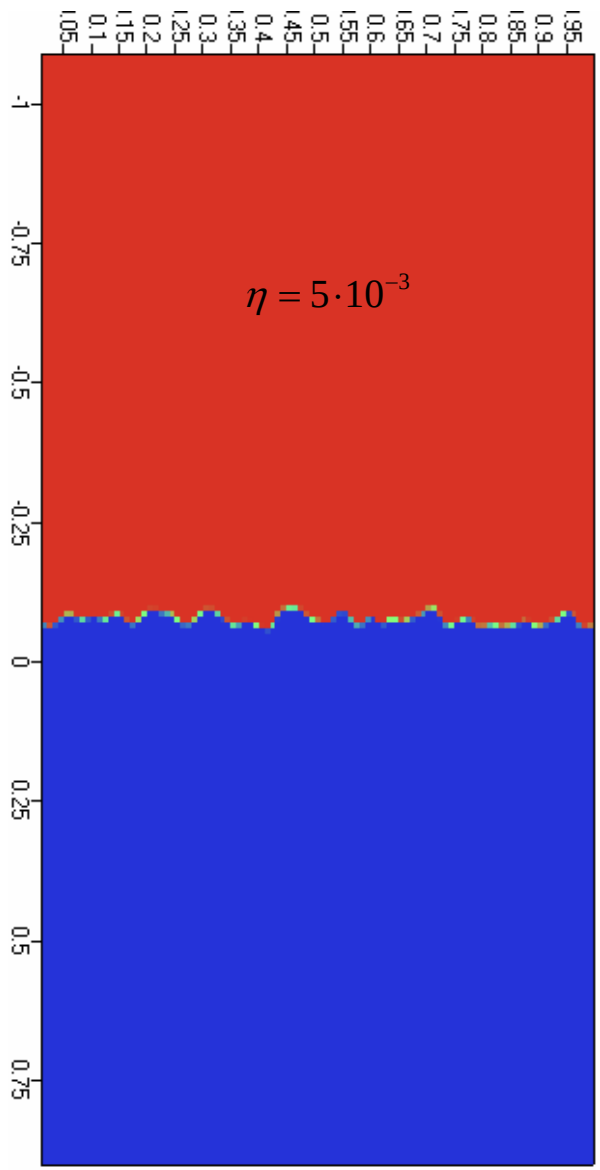
$\eta = 5 \cdot 10^{-3}$
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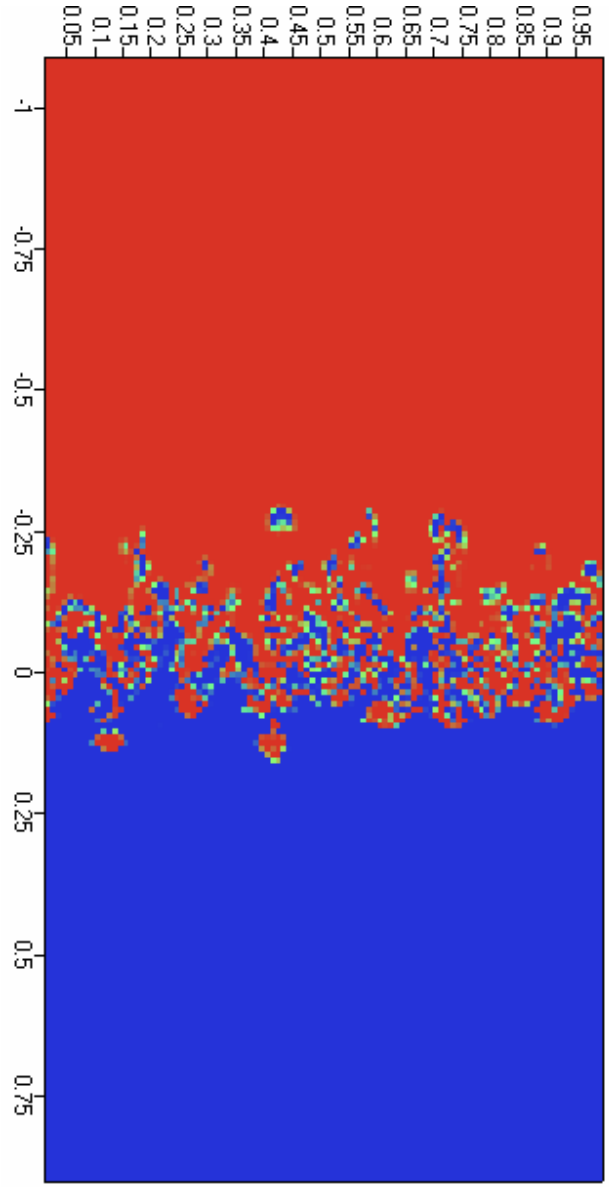
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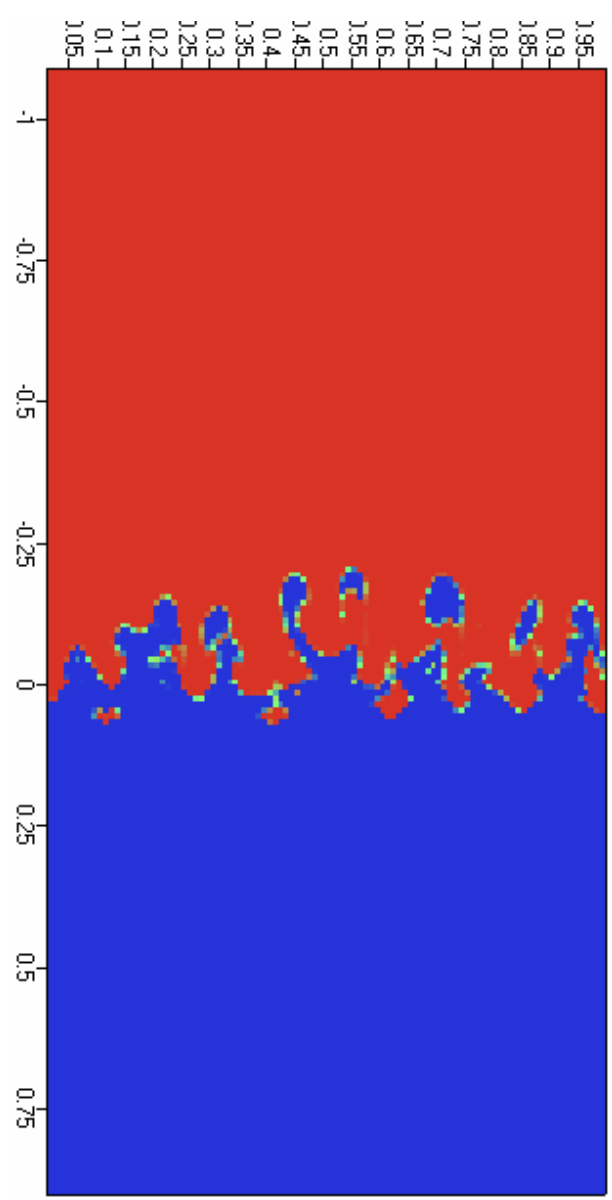
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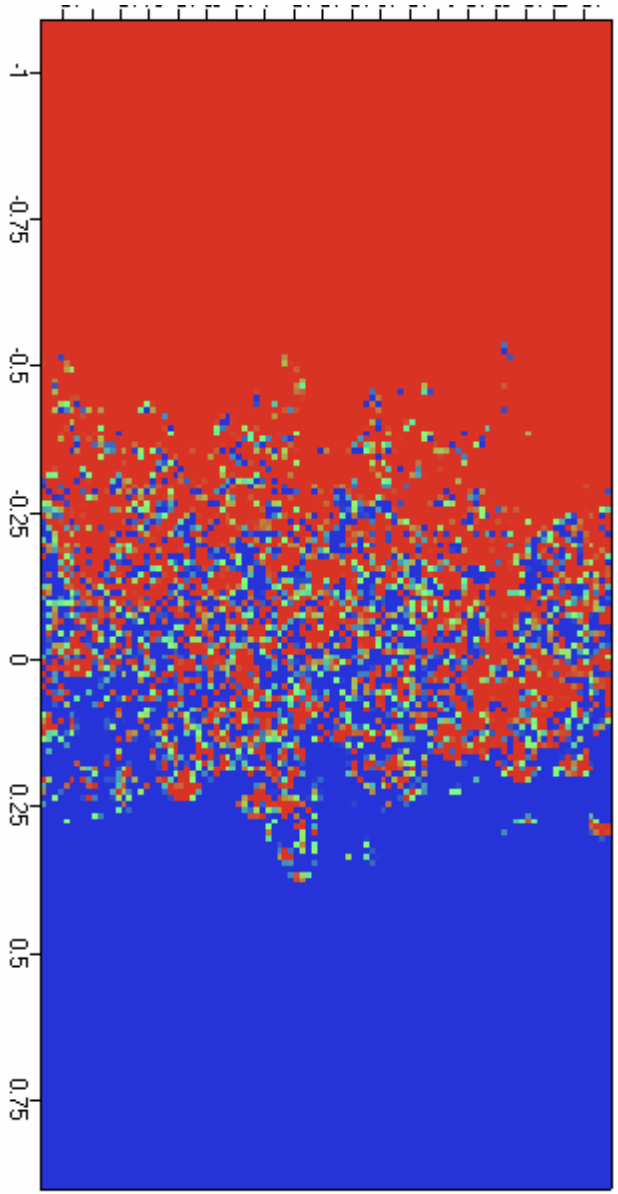
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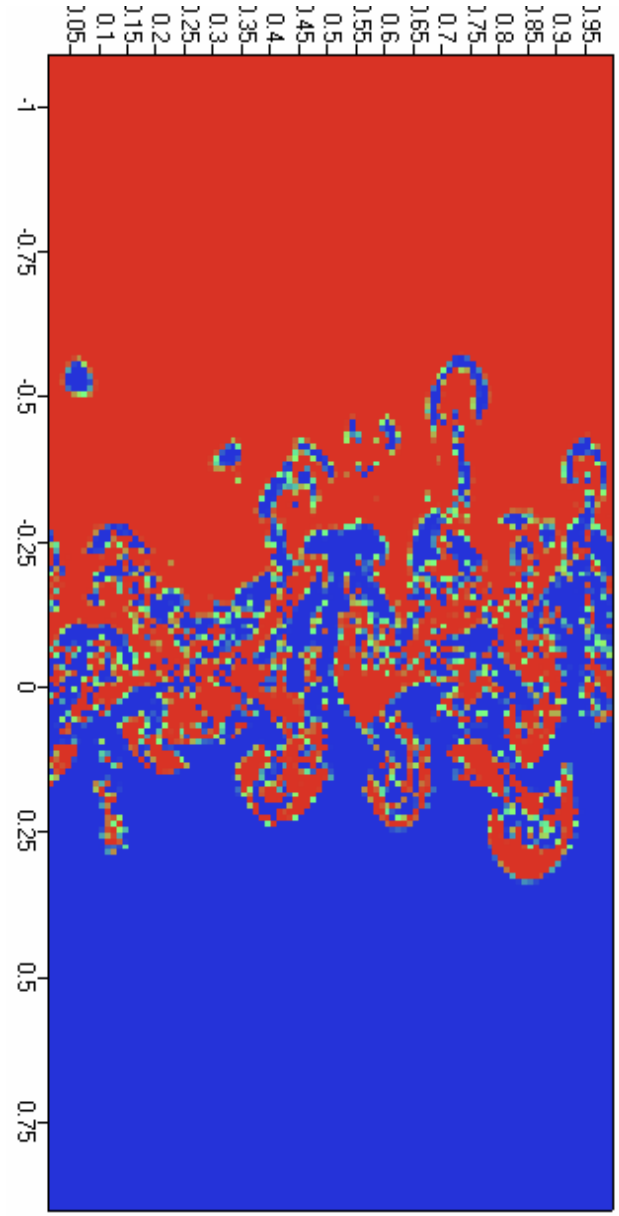
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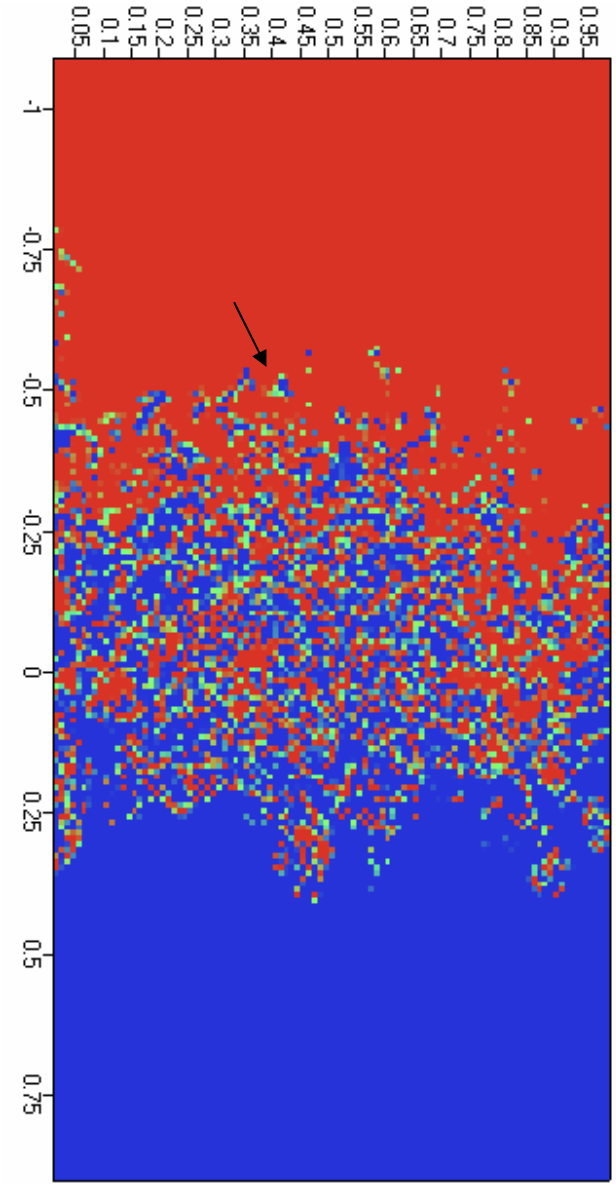
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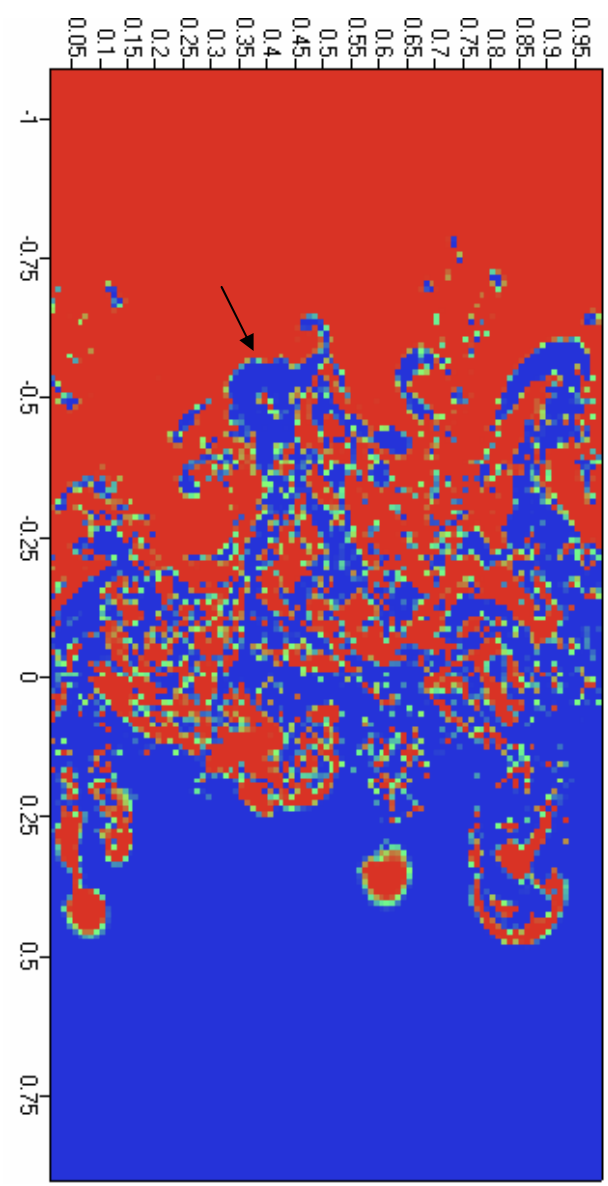
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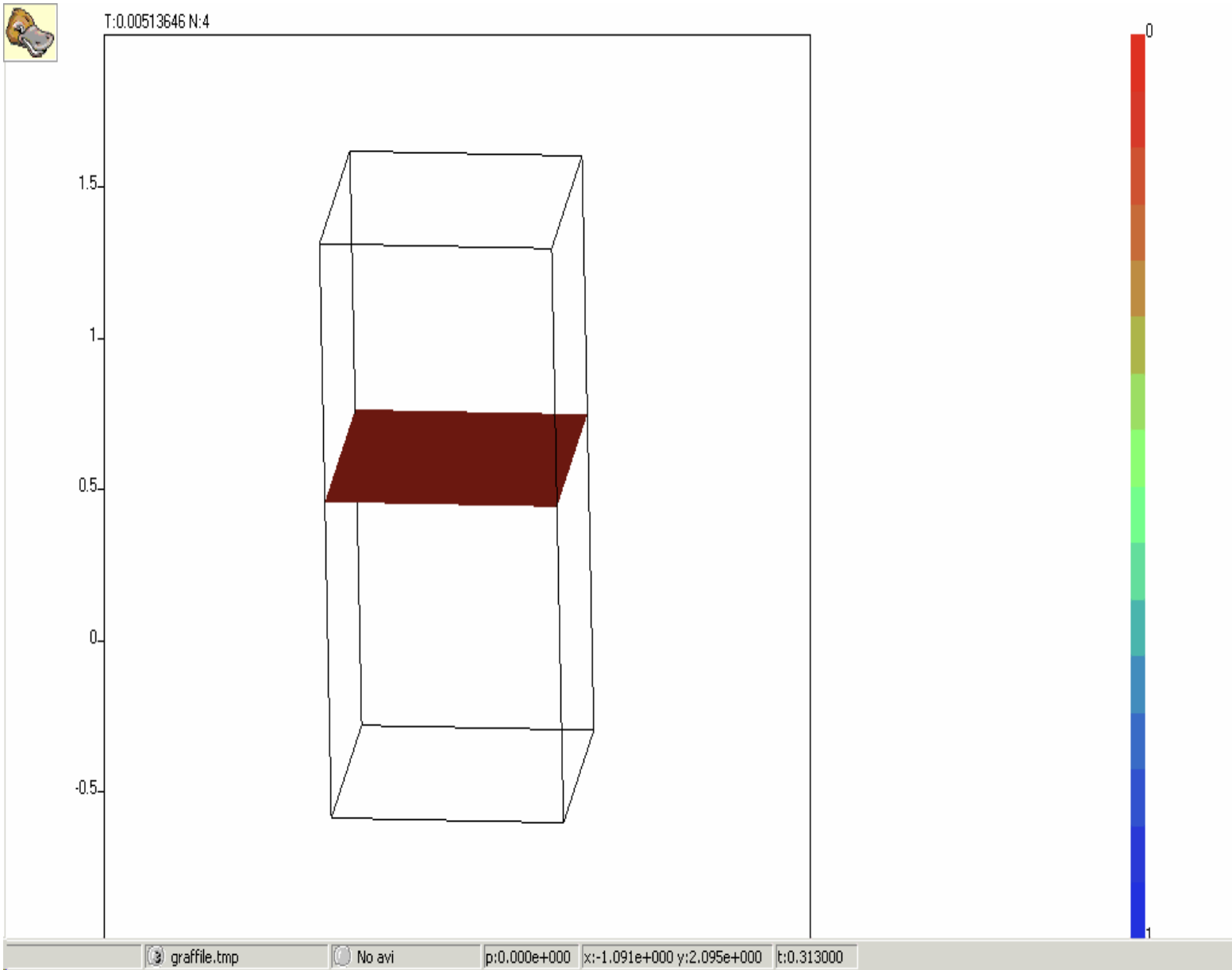
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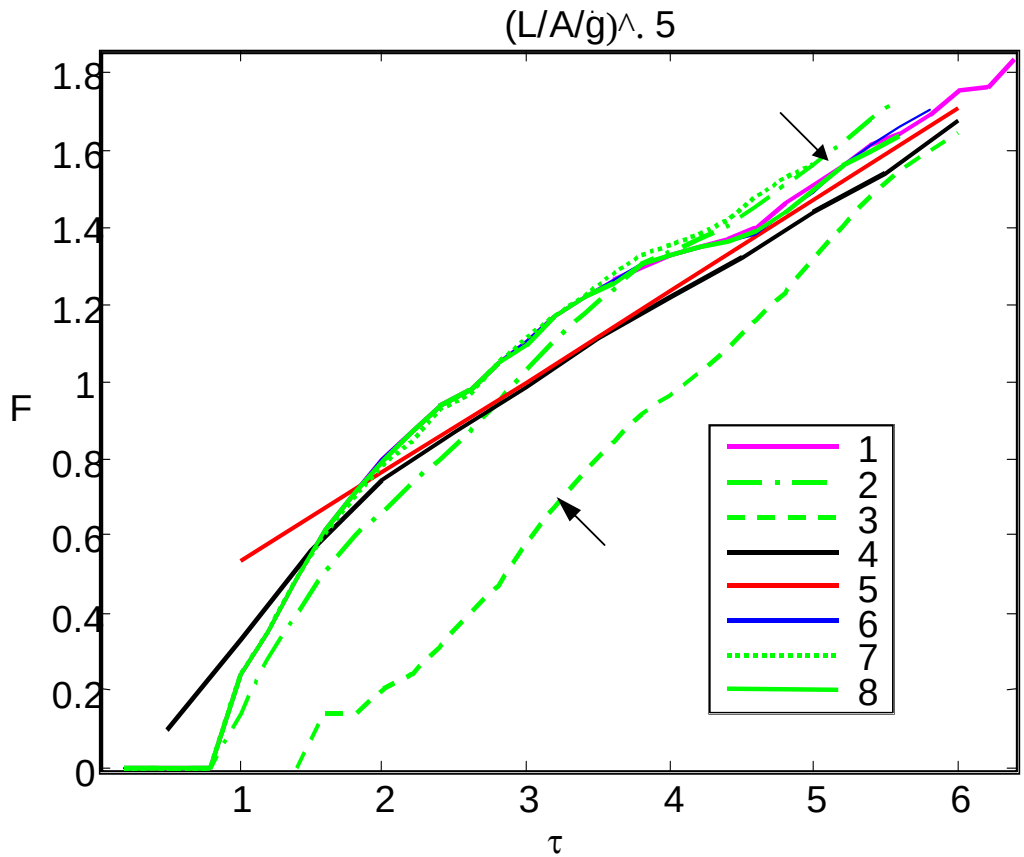
Function F(t) of width TMZ



$$F \equiv \frac{1}{t_L}\sqrt{\frac{L_t}{Ag}}$$

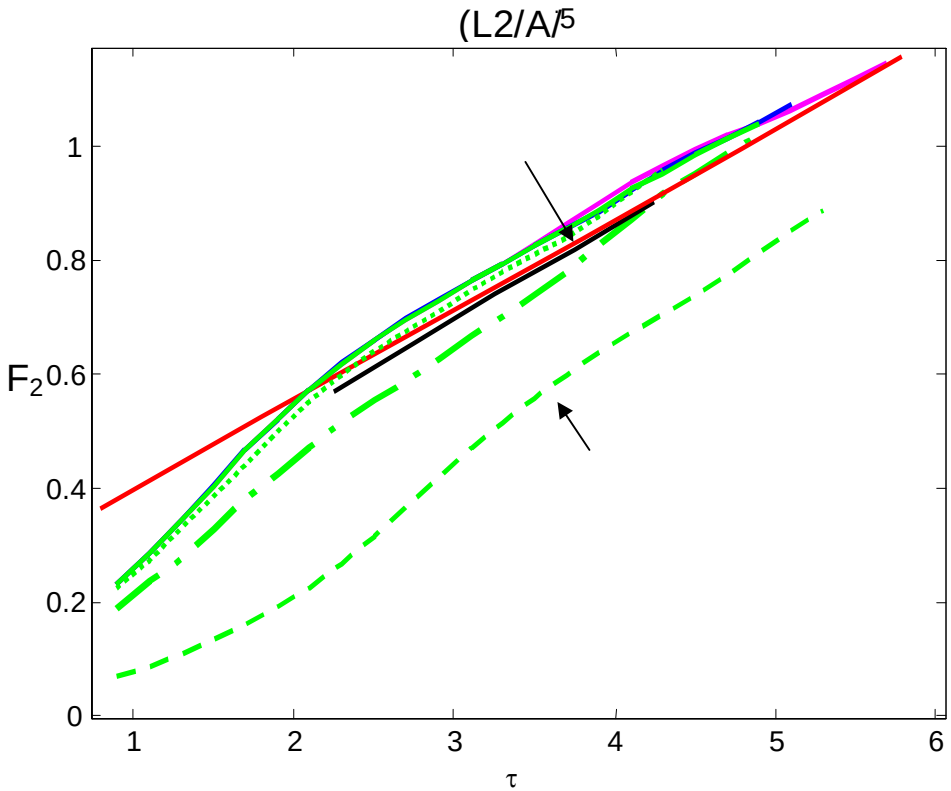
$$t_L \equiv \sqrt{\frac{L_x}{g}}$$

$$A = \frac{\rho_2 - \rho_1}{\rho_2 + \rho_1}$$

$$\alpha = (dF/dt)^2$$

$$L_t = \alpha A g t^2$$

Time dependence of function of coordinate of the light fluid penetration into the heavy fluid



$$F_2(t) = \frac{1}{t_L}\sqrt{\frac{z_l - z_c}{Ag}}$$

$$t_L = \sqrt{\frac{L_x}{g}} \qquad L_x = 1$$

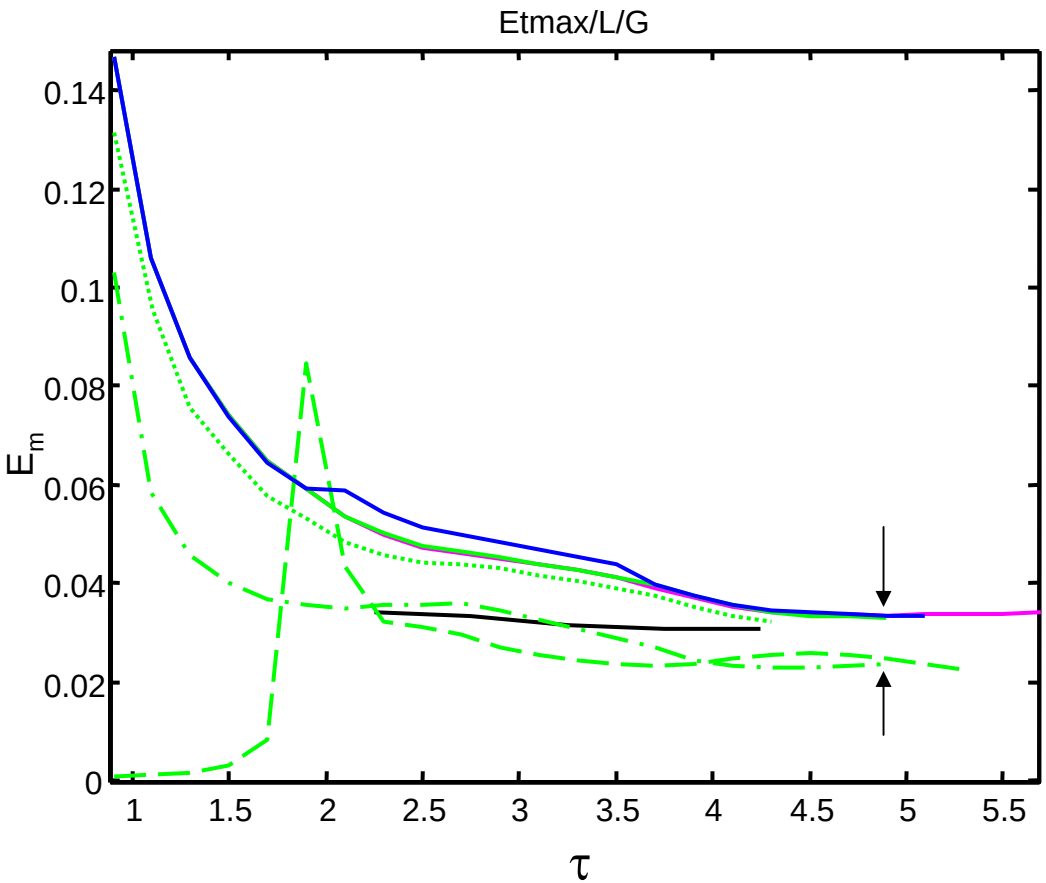
$$Z_2 :< \alpha_2 >_{(X,Y)} < 0.98$$

$$Z_1 :< \alpha_2 >_{(X,Y)} > 0.02$$

$$\alpha = (dF_2 / dt)^2 \qquad L_t \equiv \alpha A g t^2$$

$$\alpha_{(b)Youngs} = 0.025$$

Time dependence of maximum inTMZ of scaled turbulent energy

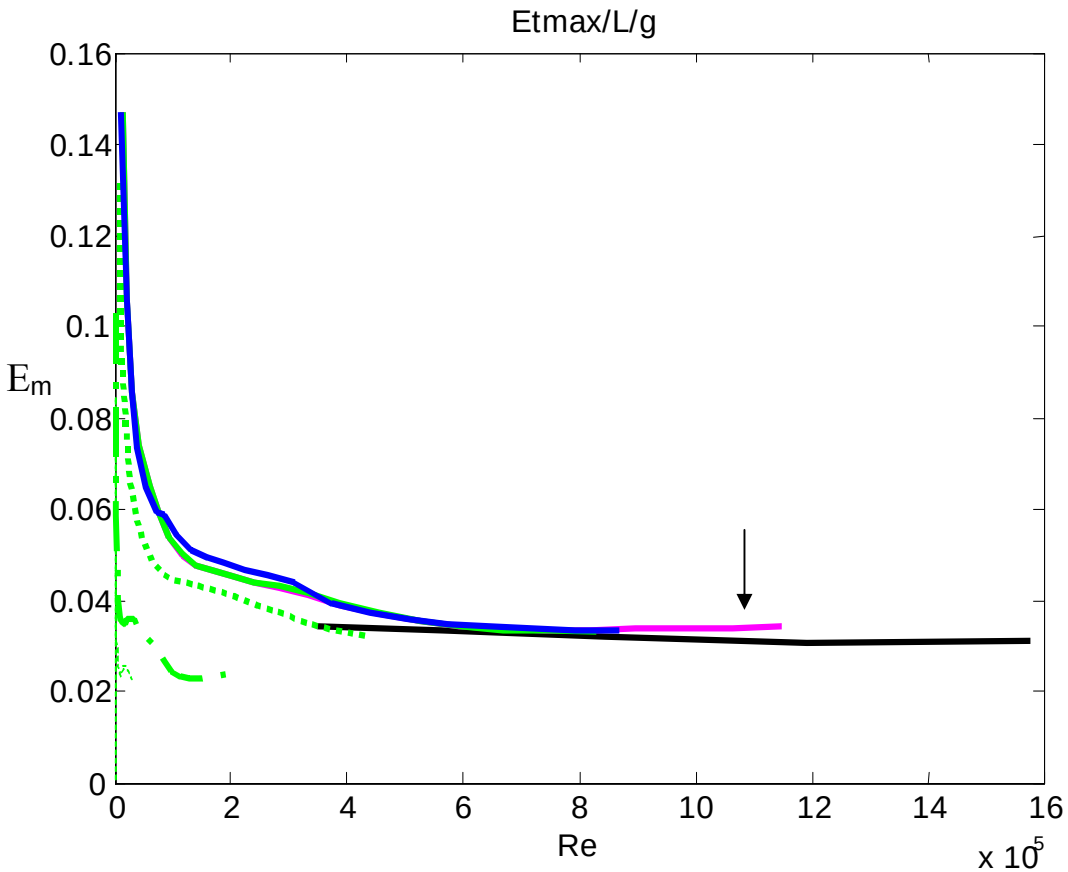


$$E(t) \equiv \frac{k_m}{L_t g}$$

$$r(t) \equiv \max(\langle k \rangle (z,t))$$

$$k(z) = \frac{\langle (u_j)^2 \rangle - \langle u_j \rangle^2}{2}$$

Maximum in TMZ of scaled turbulent energy versus Re



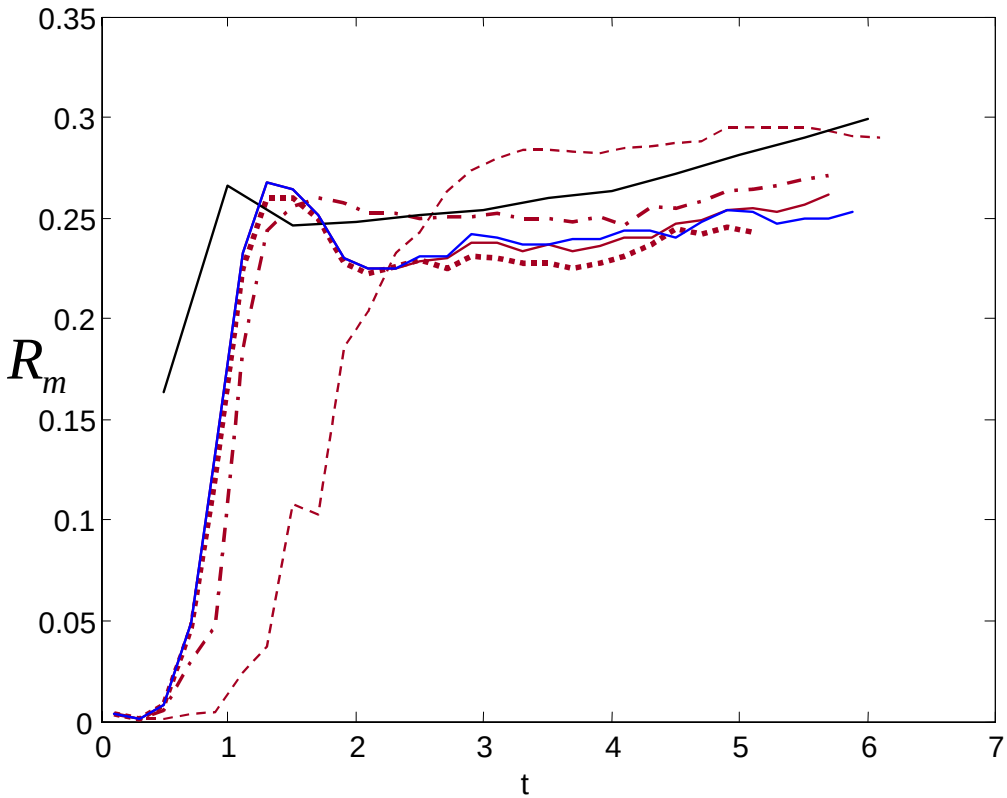
$$E(t) \equiv \frac{k_m}{L_t g}$$

$$r(t) \equiv \max(\langle k \rangle (z,t))$$

$$k(z) = \frac{\langle (u_j)^2 \rangle - \langle u_j \rangle^2}{2}$$

$$\mathrm{Re} = \frac{g^2 t^3}{\tilde{\nu}}$$

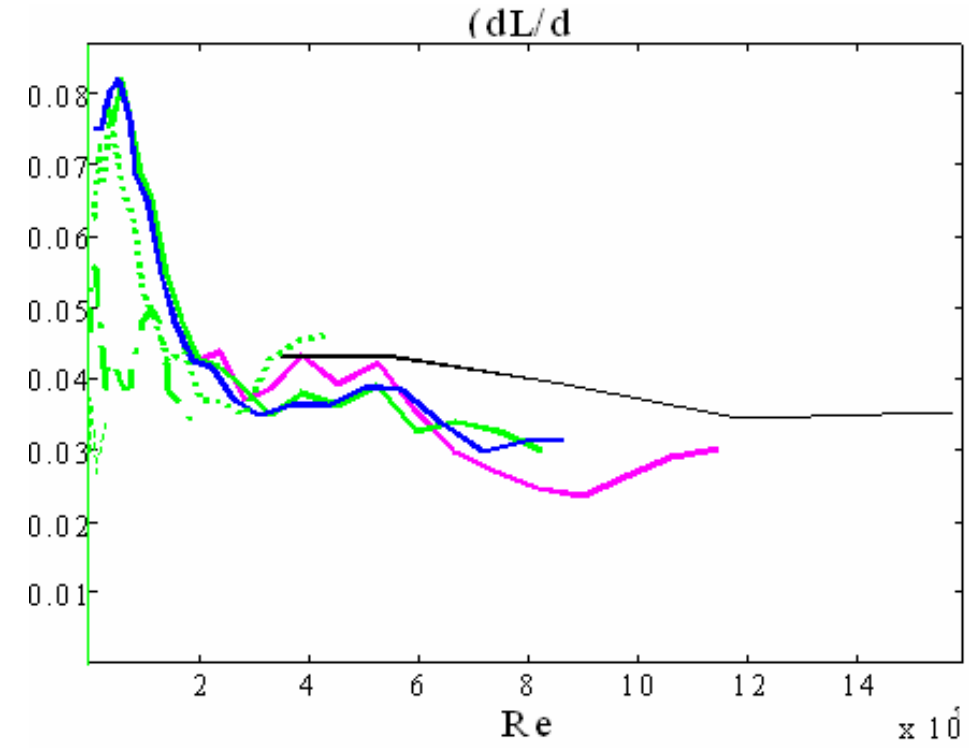
Time dependence of the maximum inTMZ value of the squared density fluctuation ratio

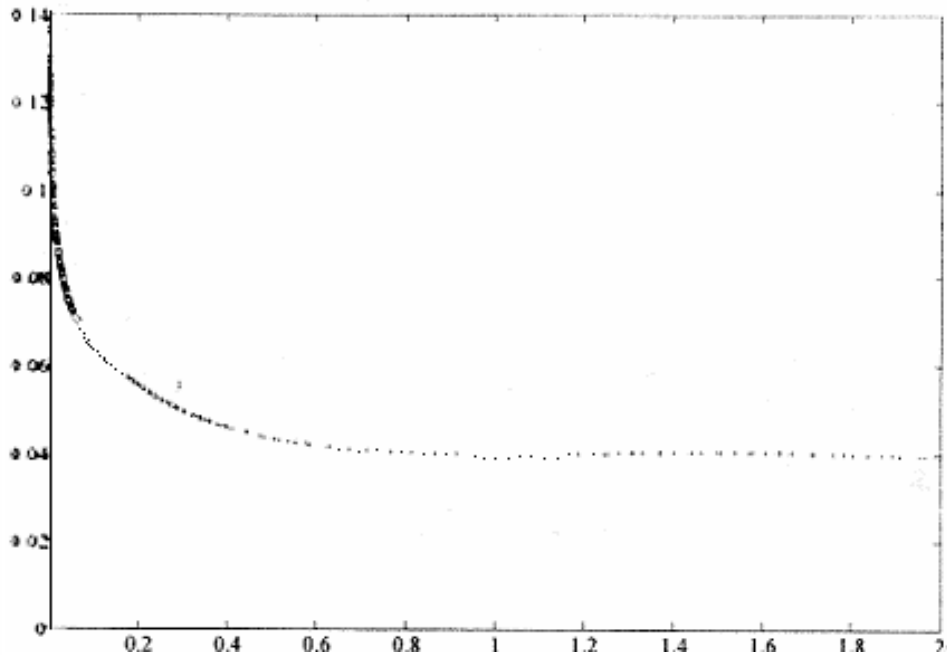


$$R_m \equiv \max(R)$$

$$R \equiv \frac{<\rho>^2 - <\rho^2>}{<\rho>^2}$$

Scaled coordinate of the light fluid penetration into the heavy fluid versus Re





$$\alpha_b \equiv \frac{1}{A}\frac{\partial(z_2 - z_c)}{\partial S}$$

$$S \equiv gt^2$$

$$\mathrm{Re} = \frac{g^2 t^3}{\tilde{\nu}}$$

Velocity fluctuation spectrum

$$E^{n}_{ii}(x,y) = < u_i^2 >_l + < u_i >_l^2,$$

where $l = rh$, $r = 2, n_x$

$n = 1, n_z$

$i = x,y,z$

h – size of cell

$$E_{ii}(z) = < E^{n}_{ii}(x,y) >_N,$$

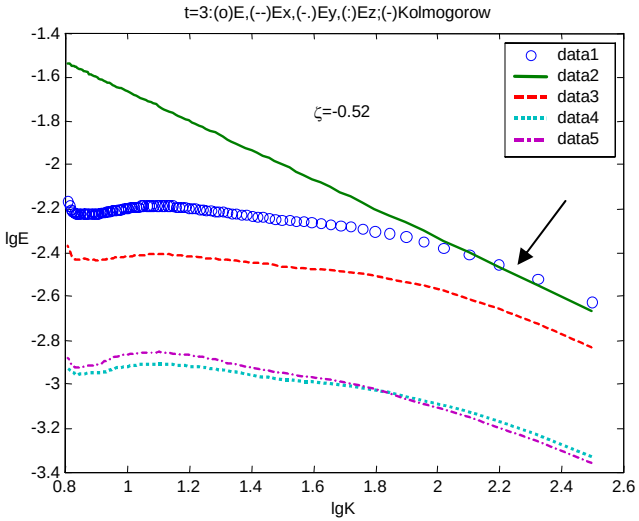
where $N = n_x n_y$

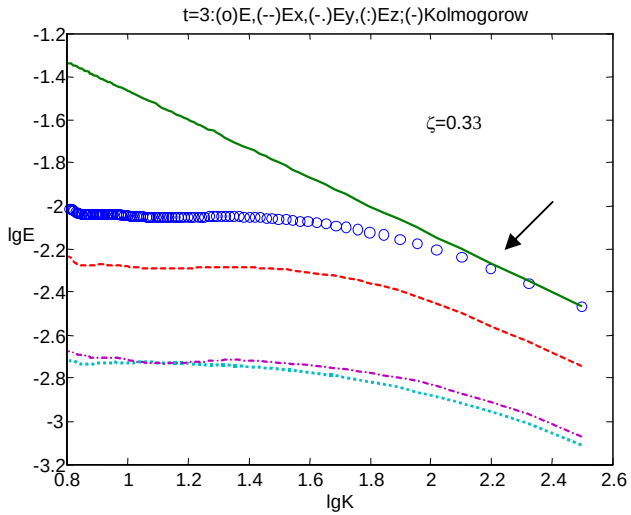
$$E = \sum_{i=1}^{i=3} E_{ii}$$

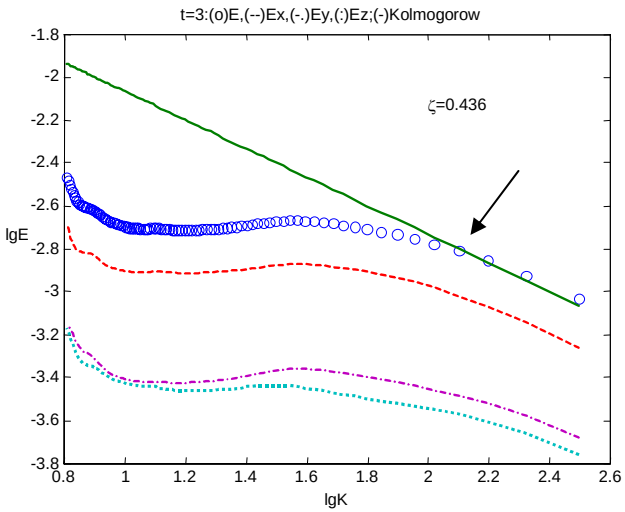
$K = 2\pi/rh$ – wave number,

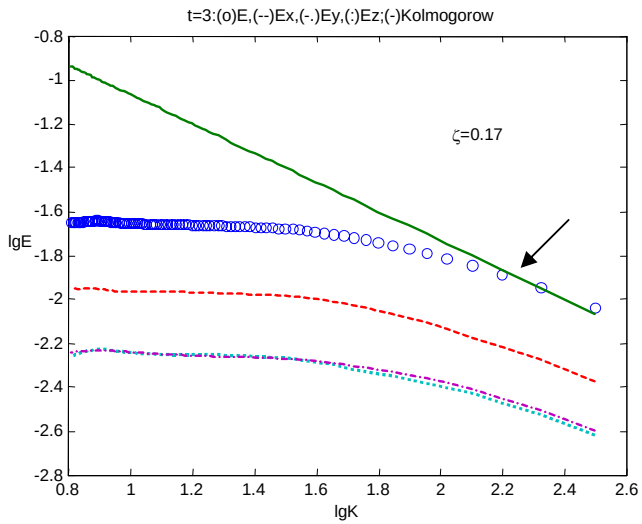
$r = 2,N$

$\lg E = -2K/3 + \text{const}$ – Kolmogorov spectrum









$$\zeta \equiv \frac{z - z_c}{L_t}$$

Efficiency of parallelization

number of processors	Coefficient of efficiency
9	-
18	95
24	95
30	95
40	95
54	90
60	86

$$K_{ef} = \frac{t_1}{t_p * p} * 100\%$$

$$K_{ef}^{(9)} = \frac{t^{(9)} * 9}{t_p * p} * 100\%$$

Cunclusion

1. With large enough viscosity values computation results significantly differ from the results of computations without viscosity: turbulence is suppressed and self-similar phase of mixing is not achieved.

2. With small enough viscosity values computation results are close to results of computations without viscosity. This means the value of viscosity $\nu = 10^{-5}$ is comparable to the scheme viscosity value.

3. The function of scaled coordinate of the light fluid penetration into the heavy fluid versus Re determined by the summarized value of the molecular and scheme viscosities has been constructed. The dependence is qualitatively the same as that obtained by Anisimov V.I., Kozlovskikh A.S. and Baban' S.A. : α value decreases, as Re increases, and achieves the limiting self-similar value.