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Cambridge, UK 19-23 July 2004 IWPCTM9

Growth rate of mixing zone in a direct numerical simulation of Rayleigh-Tailor multimode instability development

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Topics

- Direct numerical simulation of RT&RM instabilities and mixing by codes NUT, MAX, EGAK;
- Comparison with experiments (S.Zaitsev);
- Wavelet analysis;
- Kohonen map for multi-dimensional space visualization;
- Neuro-network forecasting of instabilities and mixing development (Predictor);
- Mixing zone growth rate
- Prospects

Related reports on 9th IWCTM:

•The multiparametric statistical analysis of hydrodynamic instabilities, based on wavelet preprocessing and neuronetwork classification.

Anton Nuzhny et al; Friday, July 23, 9^{20}

•Statistical properties of 2D RT-induced mixing at nonlinear and transient stage for 6-mode ensemble.

Roman Stepanov et al; Tuesday, July 20, Poster 2

•General characteristics of a mixing zone development in a direct simulation of hydrodynamic instabilities with a random phase regular multimode perturbation.

Nikolay Zmitrenko et al; Monday, July 19, Poster 1

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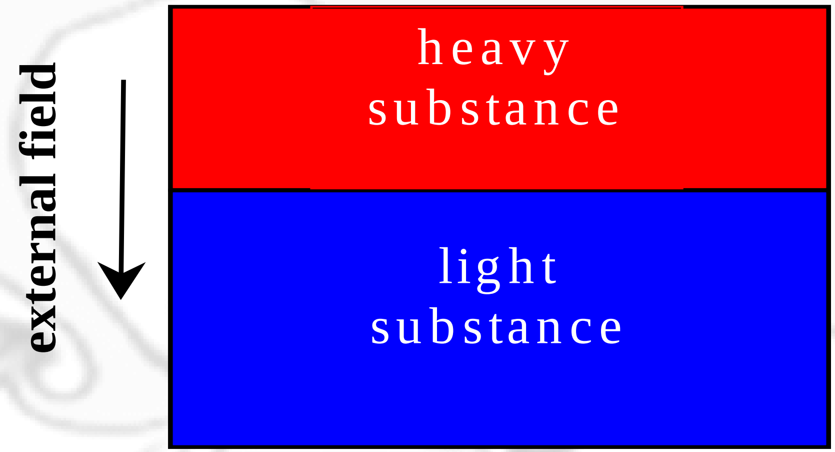
Problem statement (1)

$$p = \frac{R}{\mu}\rho T \qquad \varepsilon = \frac{1}{\gamma - 1}\frac{R}{\mu}T$$



Position of the contact boundary –

$$z(x) = \sum a_i \cos(k_i x + \varphi_i),$$

$$k_i = \frac{2\pi}{\lambda_i} = \frac{2\pi}{L} i, \qquad i = 2,3,5,7,11,13,17,19,23,29,31,37$$

Density field – $\rho(x,z)$

Pressure field – $P(x,z)$

Velocity components fields – $u(x,z), w(x,z)$

Momentum components fields – $p_x(x,z), p_y(x,z)$

Vorticity field – $\Omega(x,z) = \frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}$

Problem statement (2)

Instabilities: RT, RM, limited time of development

Dimension: 2D and 3D

Amplitudes: $a_k{}^0 = a_0(k)$

$$\begin{cases} 1.\ a_k^0 k = \text{const} \Rightarrow a_k^0 = \dfrac{\text{const}}{k} \\ 2.\ a_k^0 = \text{const} \end{cases}$$

Modes: N_{max}=6,8,10,12; i=2,3,5,7,11,13,17,19,23,29,31,37

Phases: random choice on the interval 0 —2π

Attwood number: Ar/Xe A=0.532 ; He/Xe A=0.941

Total number of simulations: 250

Physic values: the kinetic energy: $$E_k = \int_{Z_{min}}^{Z_{max}} \int_{X_{min}}^{X_{max}} \rho \frac{u^2 + w^2}{2} dxdz$$

$$\int_{Z_{min}}^{Z_{max}} \int_{X_{min}}^{X_{max}} \rho w\, dxdz$$: z-component of a momentum

the width of the mixing zone: $L = Z_2 - Z_1$

$$\int_{Z_{min}}^{Z_{max}} \int_{X_{min}}^{X_{max}} C\rho\, dxdz$$: the mass of heavy fluids involved into mixing

the enthropy: $$H = \frac{1}{2} \int_{Z_{min}}^{Z_{max}} \int_{X_{min}}^{X_{max}} \Omega^2(x,z,t)\, dxdz$$

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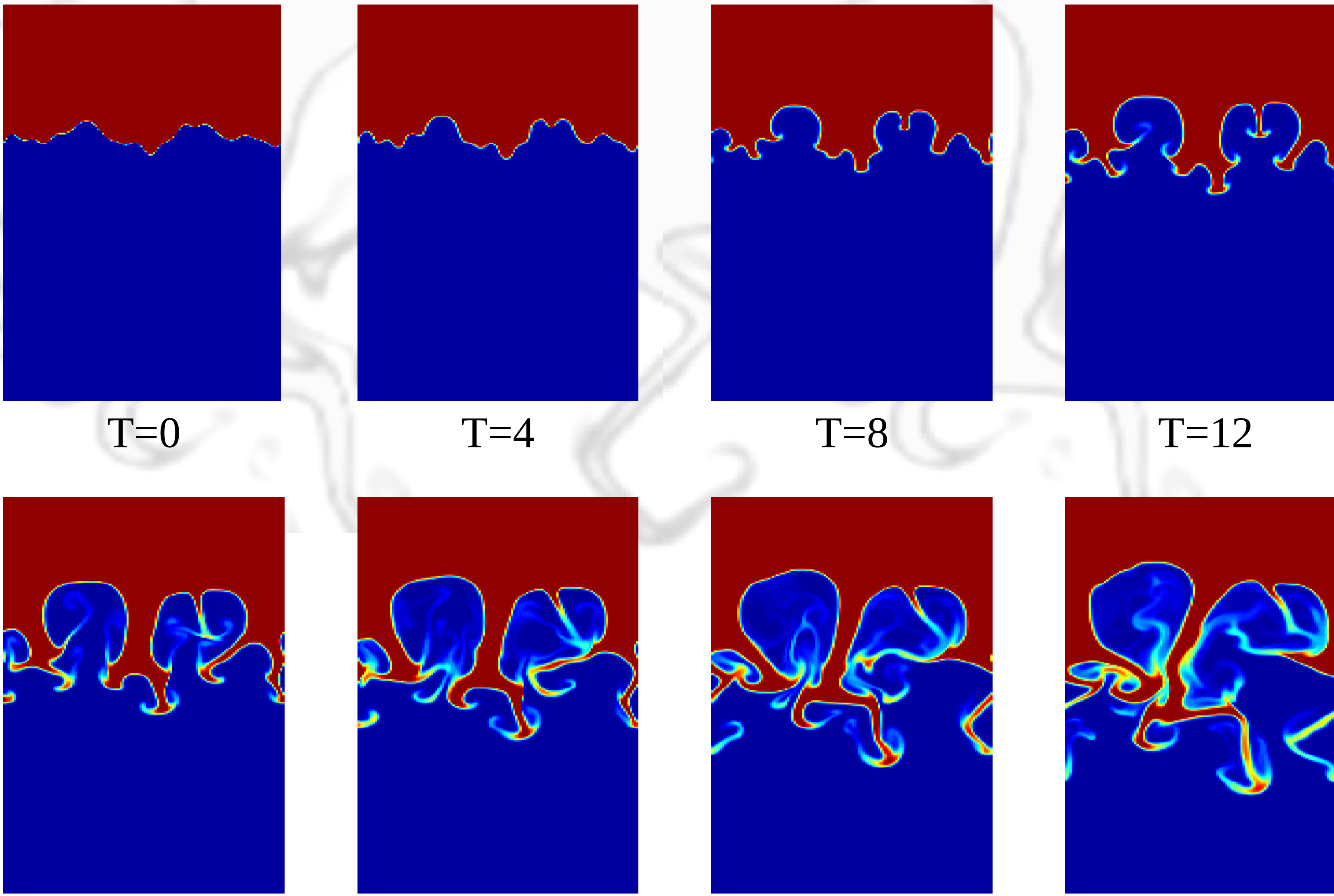
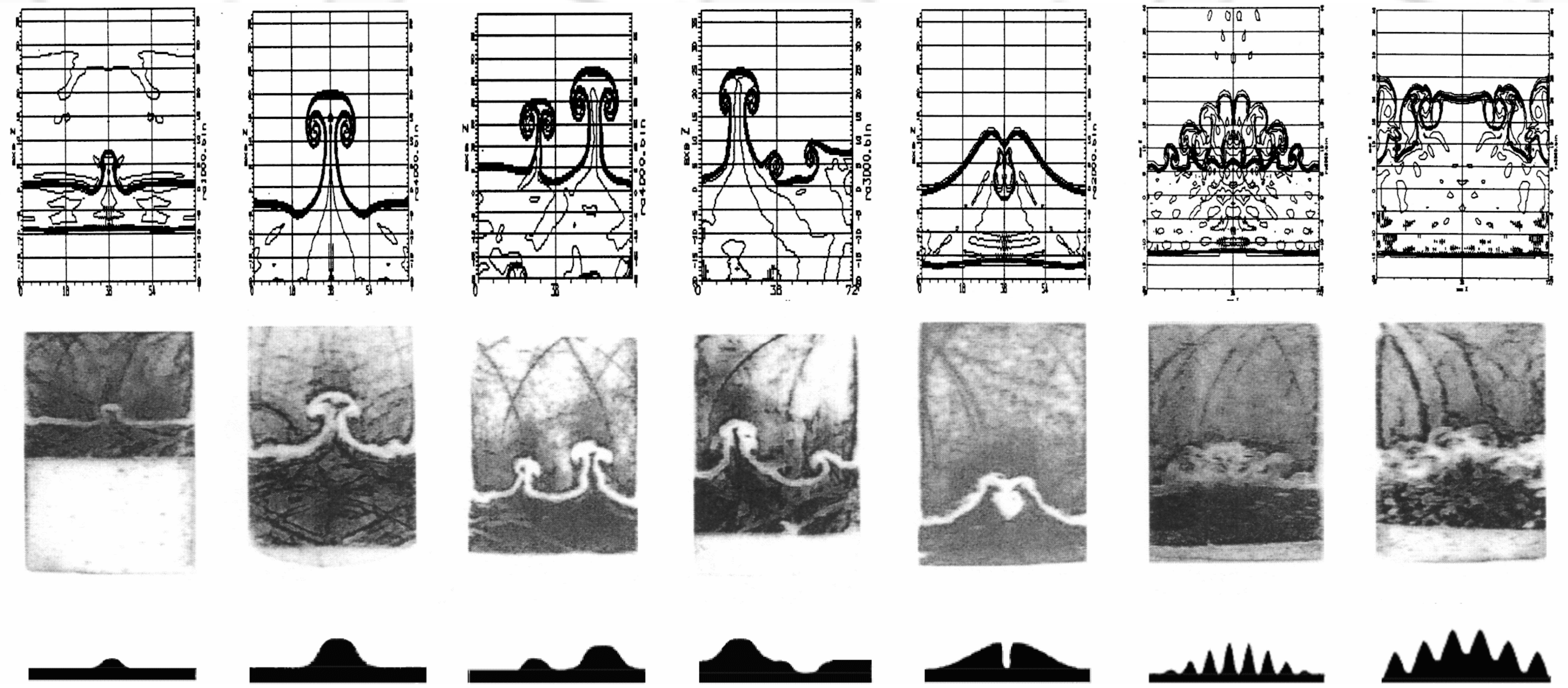
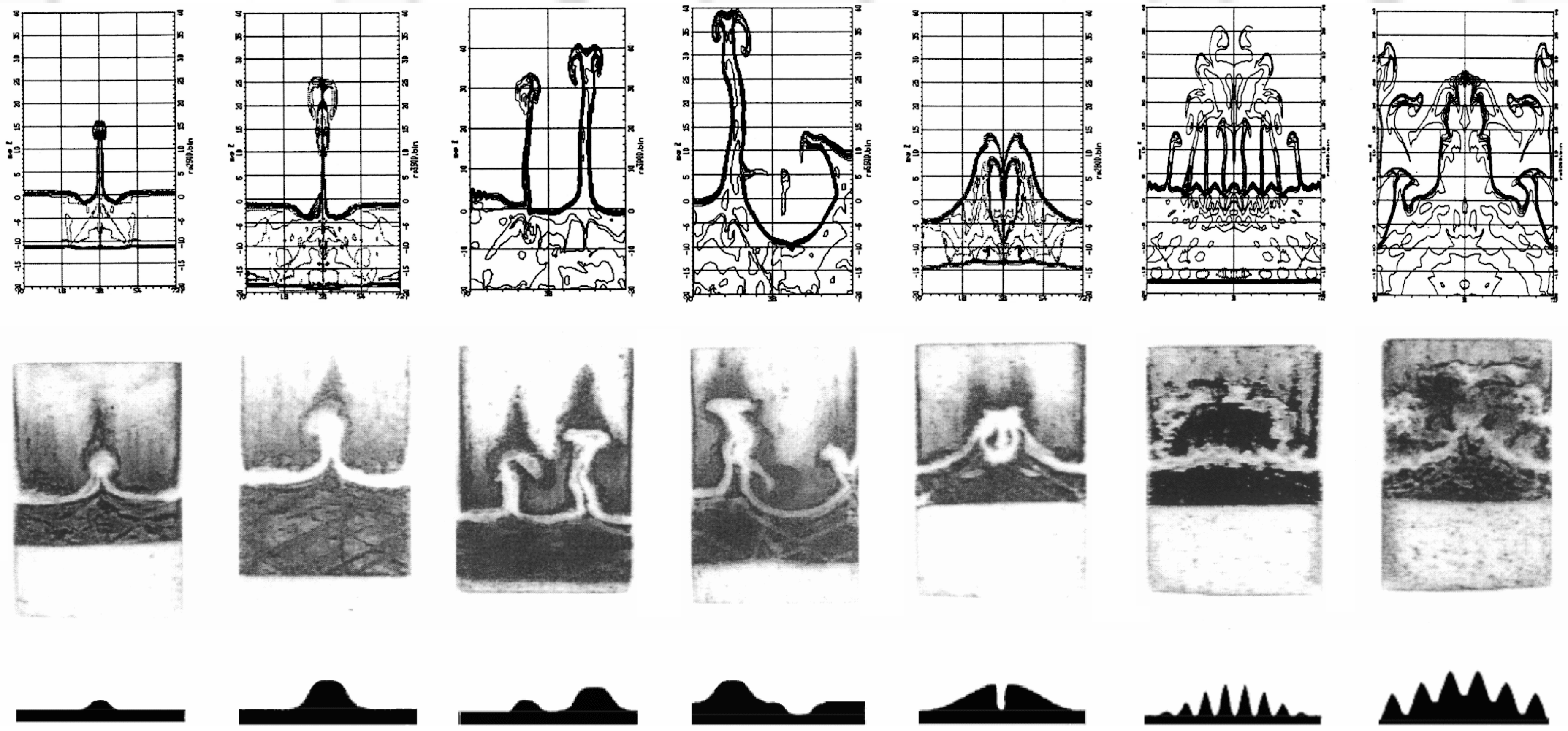
Density field evolution (He/Xe, n=6)



Comparison with experimental data on Ar-Xe



Comparison with experimental data on He-Xe



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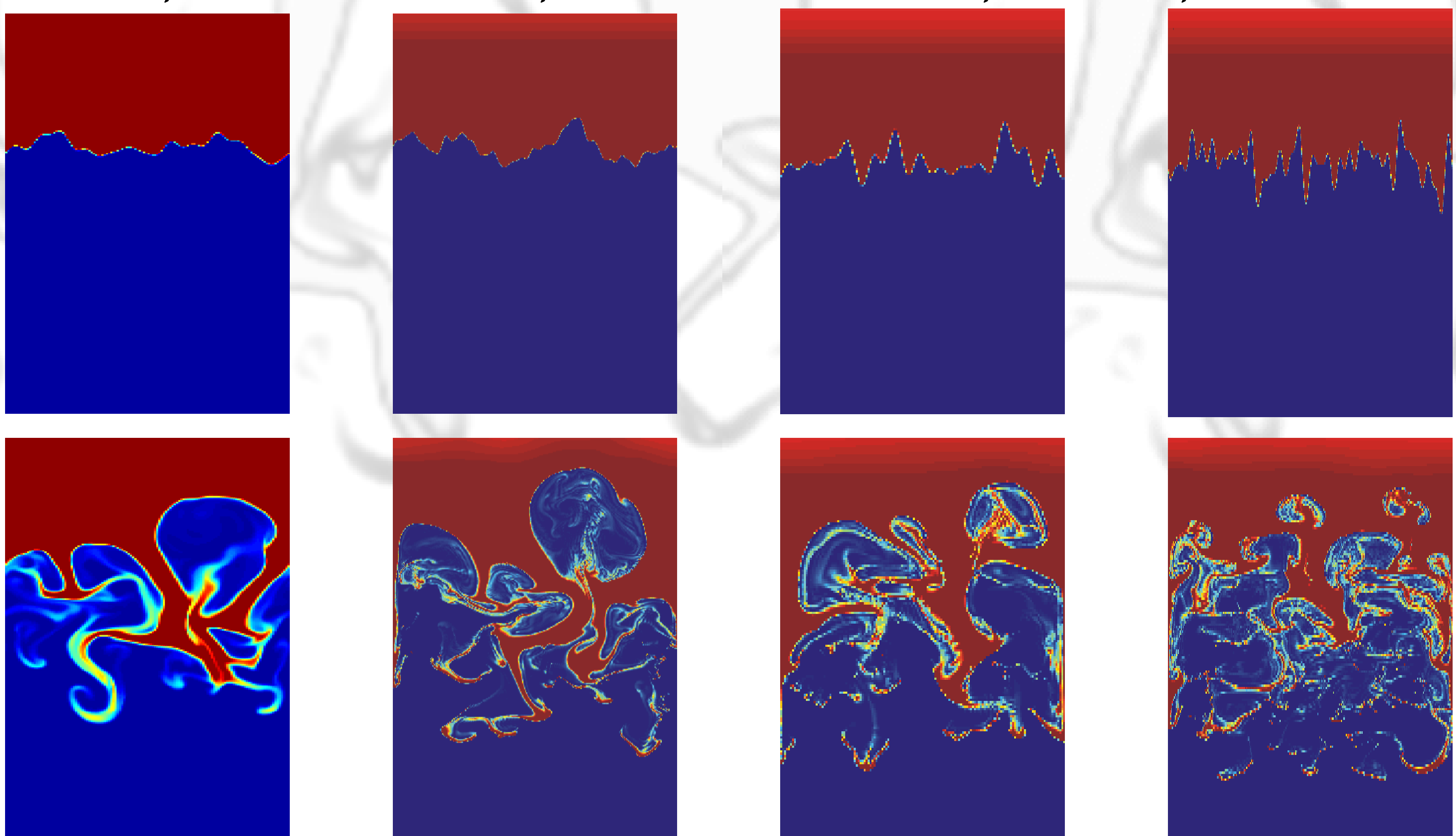
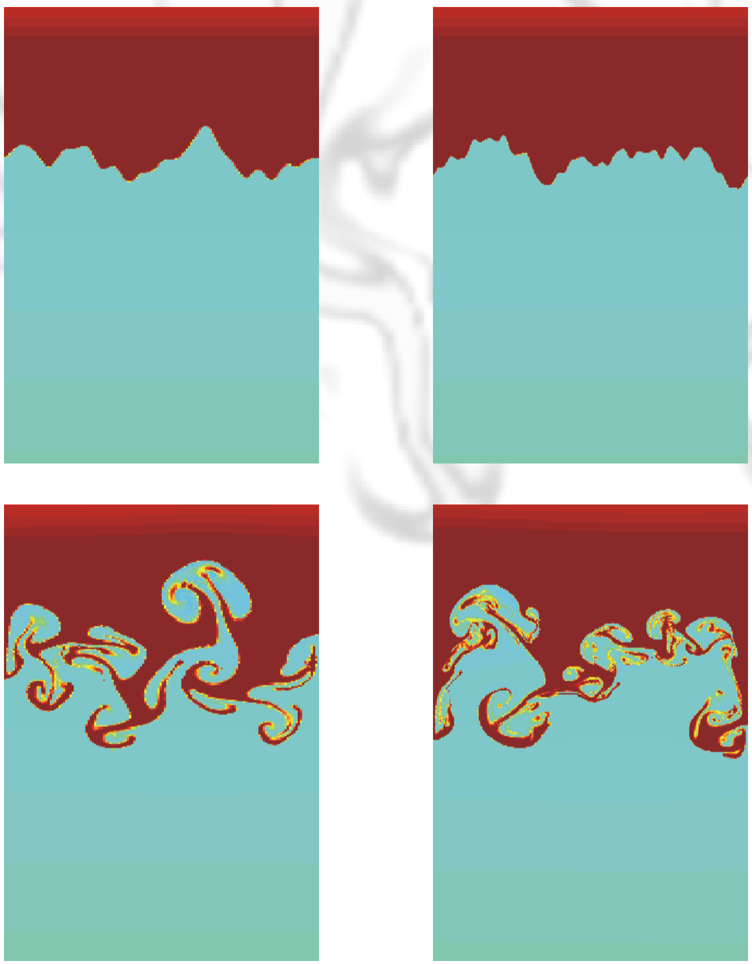
Simulation examples(1) –RT

NUT He/Xe ak=0.5, n=6 | MAX He/Xe ak=0.8, n=10 | MAX He/Xe n=6, a=const, n=10

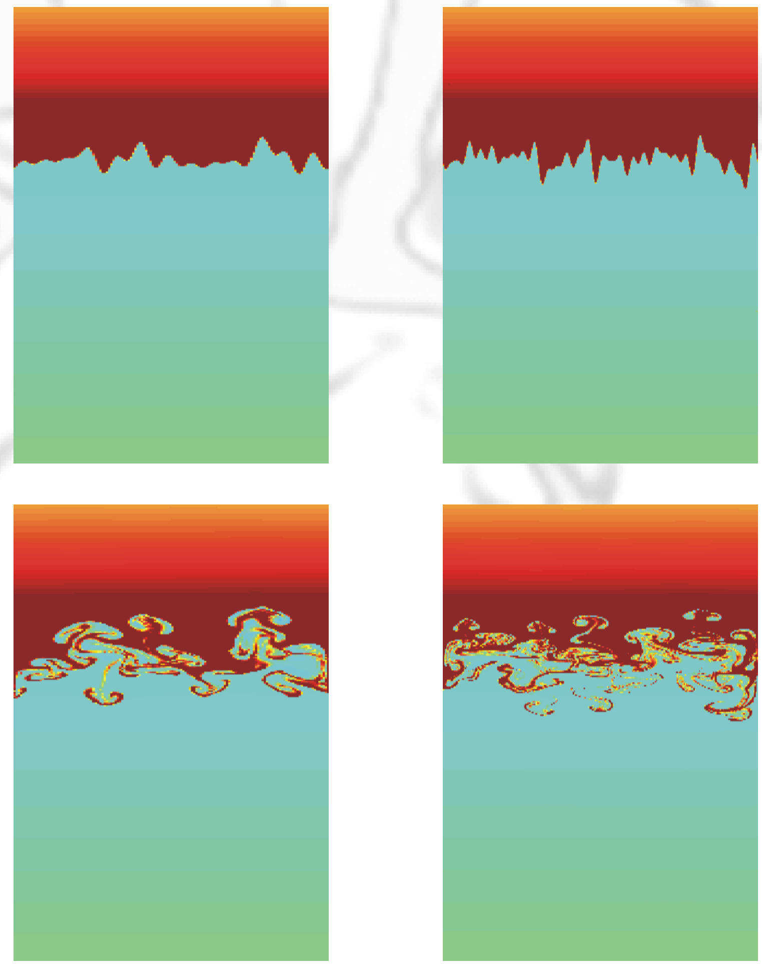


Simulation examples$^{(2)}$ –RT

MAX Ar/Xe
n=6, ak=0.8, n=10



MAX Ar/Xe
n=6, a=const, n=10



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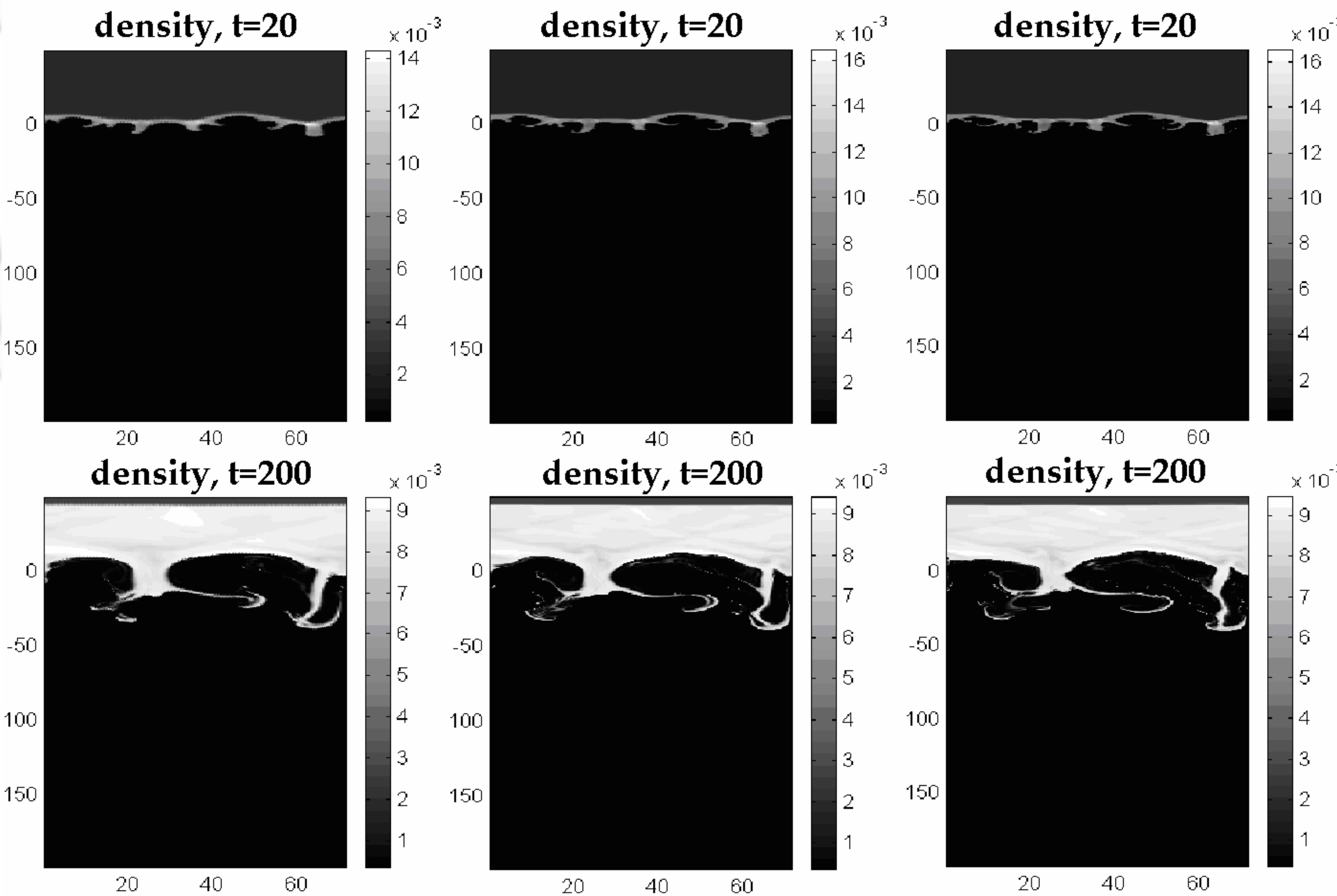
Simulation examples(3) –RM He/Xe



6 harmonics **8 harmonics** **10 harmonics**

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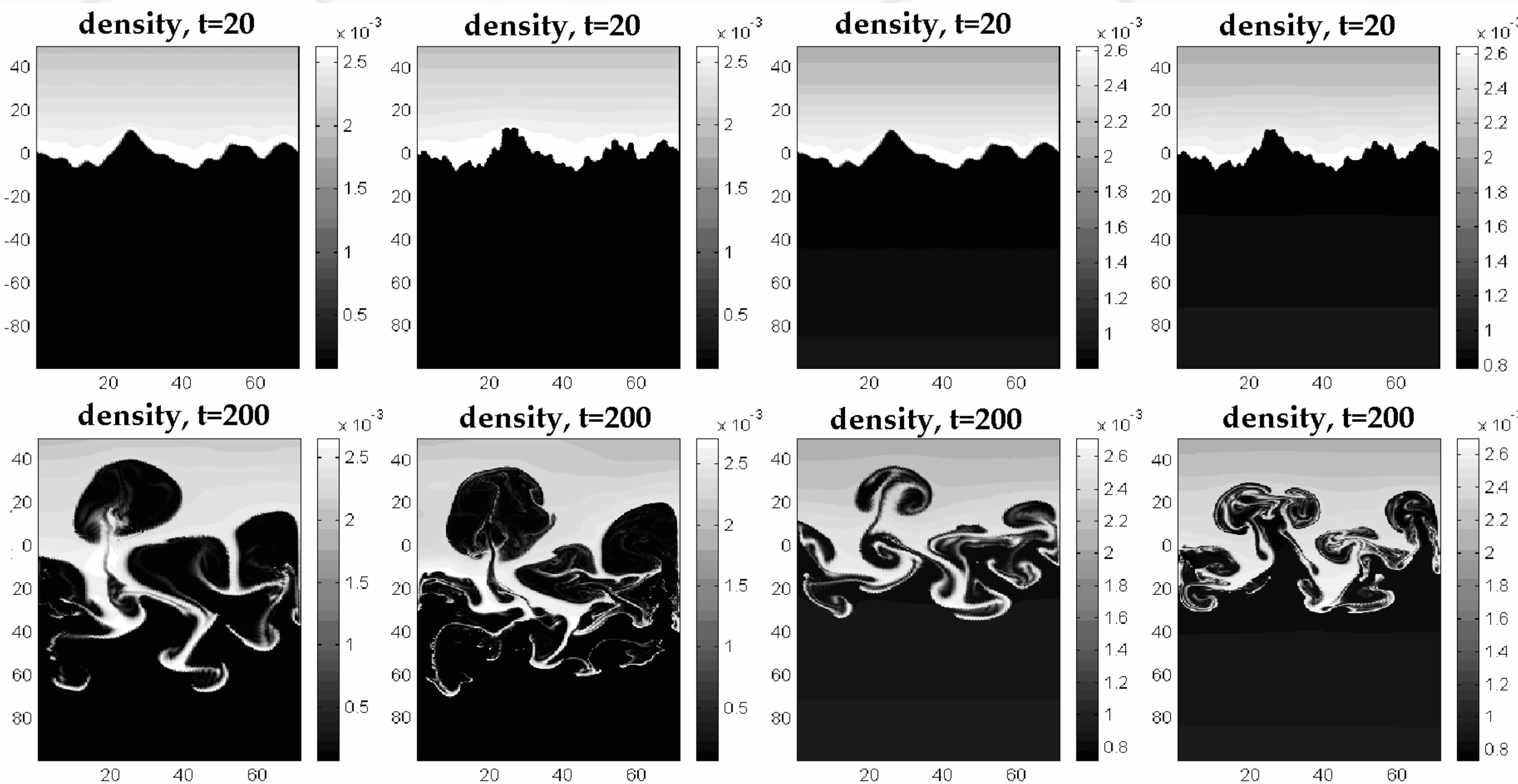
Simulation examples(4) –RT, n=6,12

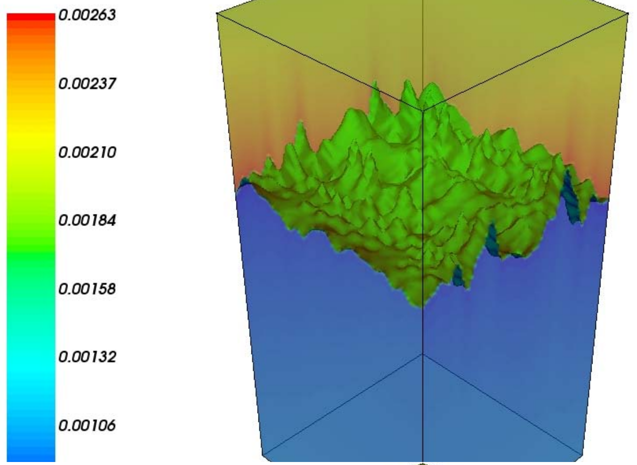


6 harmonics | 12 harmonics | 6 harmonics | 12 harmonics

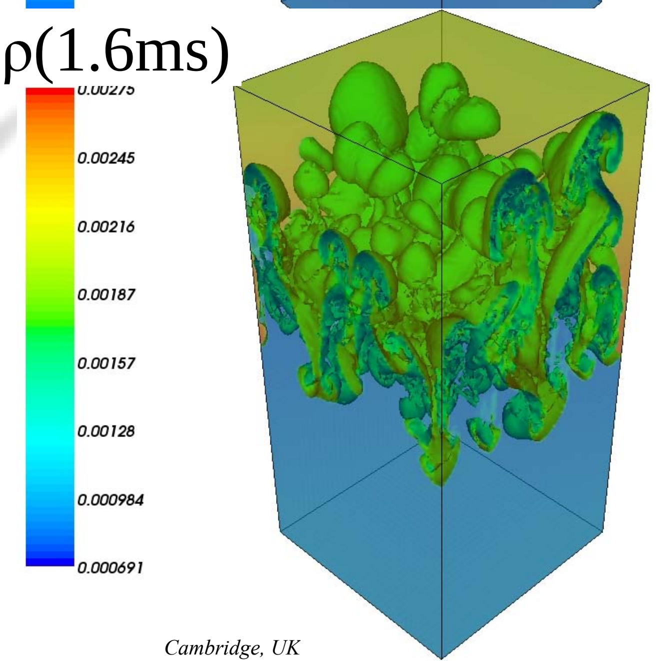
He/Xe | Ar/Xe

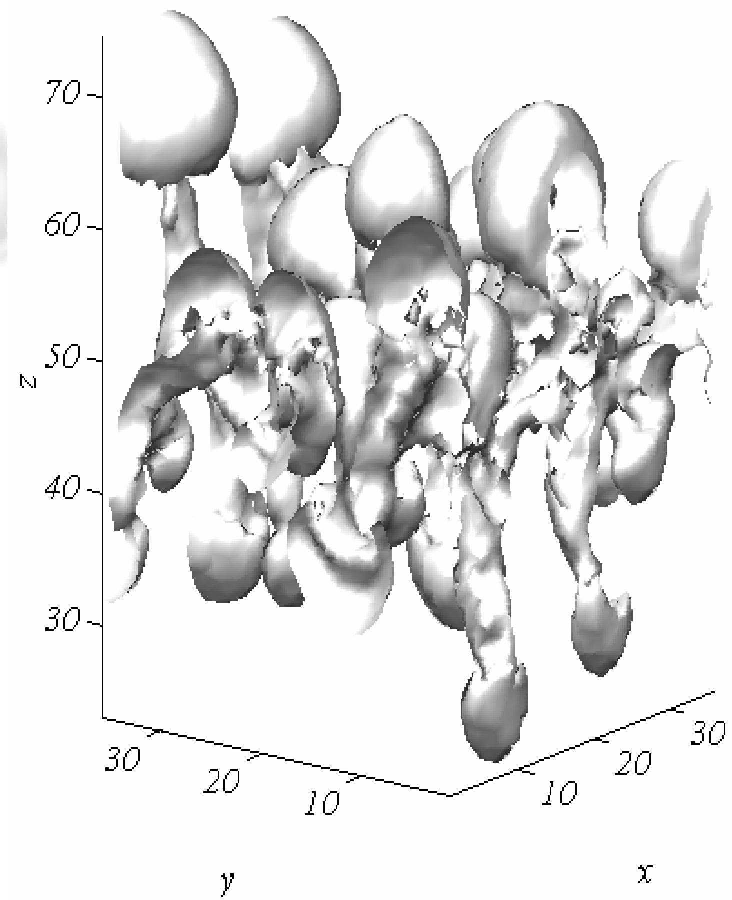
Simulation examples(5) –RT,3D

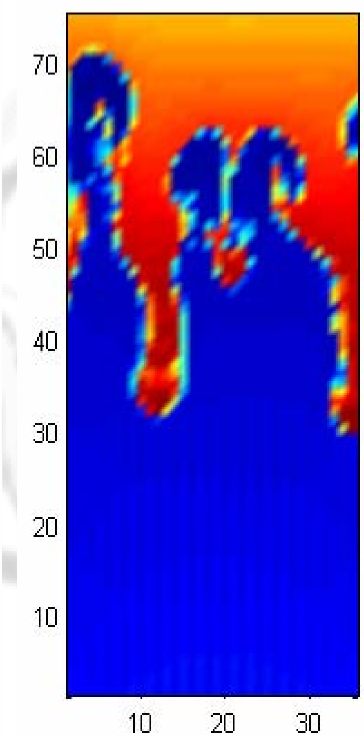
ρ(0μs)



ρ(1.6ms)







isosurface of density at t=1600 μs and density distribution at x=36 mm plane

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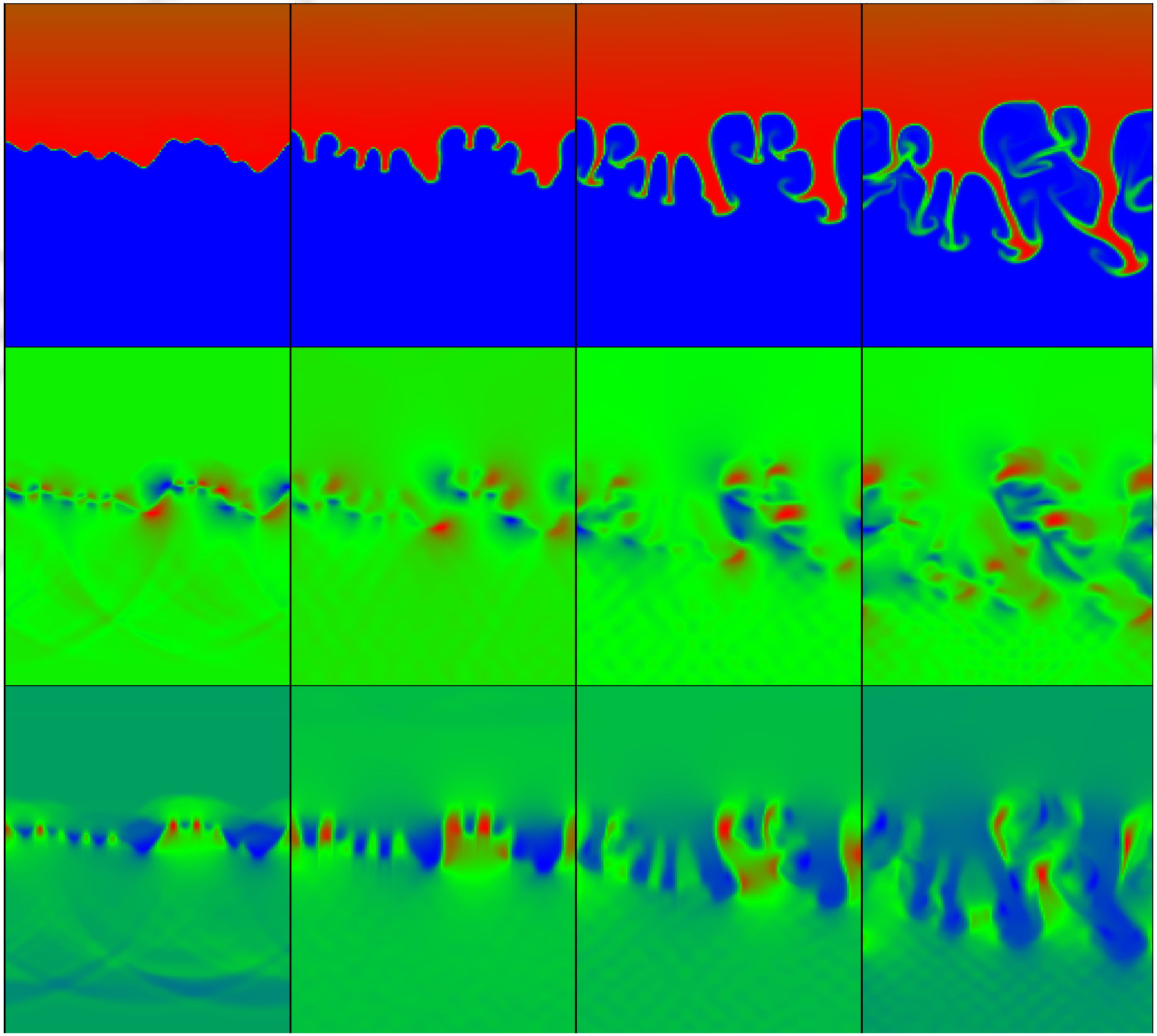
Simulation examples(6) –V(x,z,t), RT



Density

V_x

V_z

He/Xe,

6 modes

t=0,300,600, 900 μs

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Integral values (1)

$$z(x,y) = -\sum_{j=1}^{N}\sum_{i=1}^{N} a_{ij}\cos(k_i x + \varphi_i)\cos(q_j y + \psi_j)$$

$$k_i = \frac{2\pi}{\lambda_{xi}} = \frac{2\pi n_i}{L_x}, \qquad q_j = \frac{2\pi}{\lambda_{yj}} = \frac{2\pi n_j}{L_y}$$

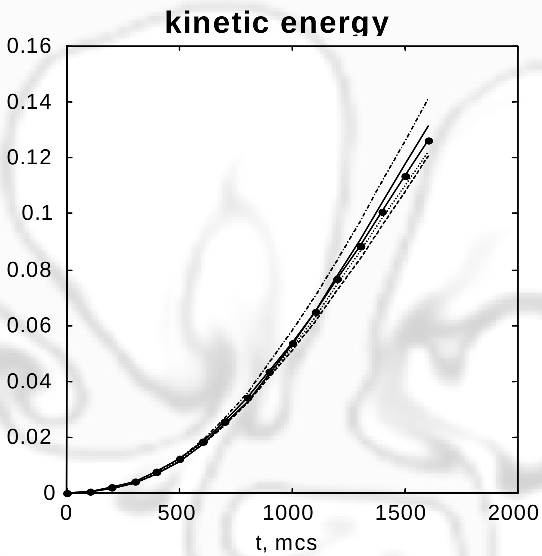
$$(n_i = 2, 3, 5, 7, 11, 13, \ldots)$$

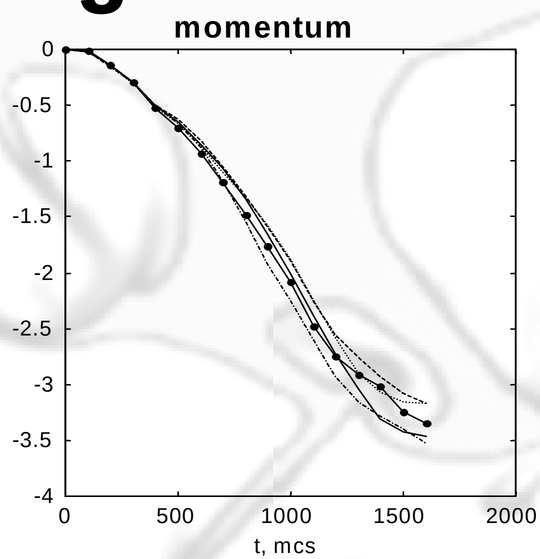
$$a_{ij} = \frac{a_0}{\sqrt{k_i^2 + q_j^2}} = \frac{L}{2\pi}\frac{a_0}{\sqrt{n_i^2 + n_j^2}}$$

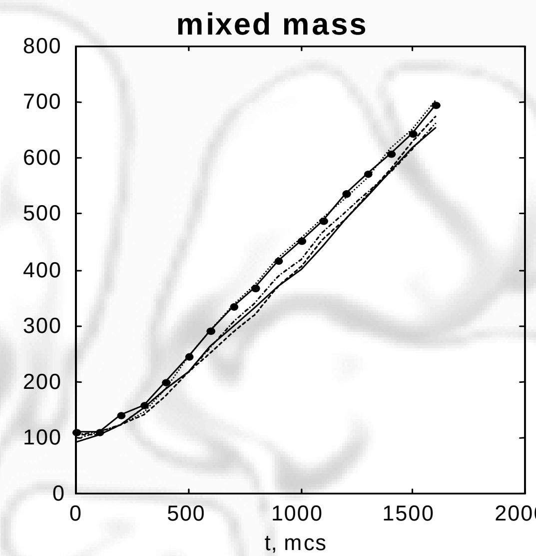
$$a_0 = 0.8, \qquad \varphi_i = \frac{2\pi m_i}{8}, \qquad \psi_j = \frac{2\pi l_j}{8}$$

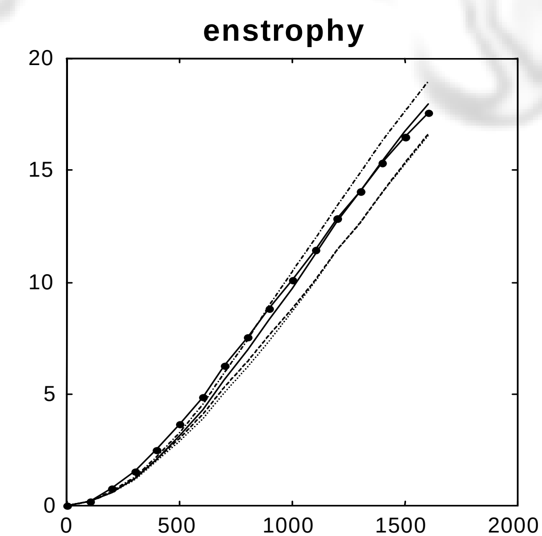
$$m_i = \{4, 1, 5, 2, 7, 3\}, \qquad l_i = \{6, 5, 5, 4, 1, 3\}$$

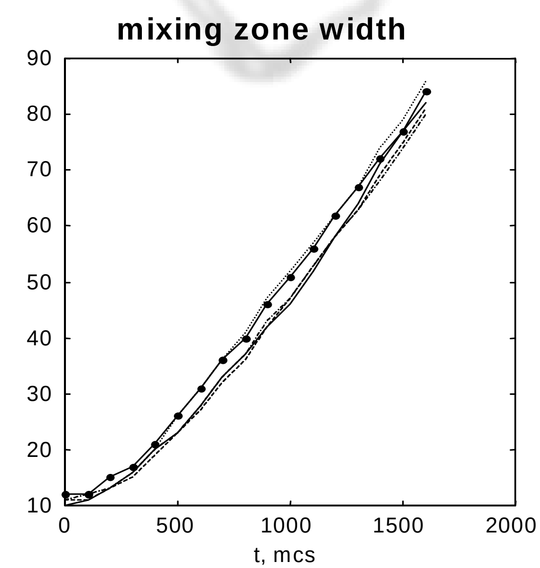
Integral values (2)

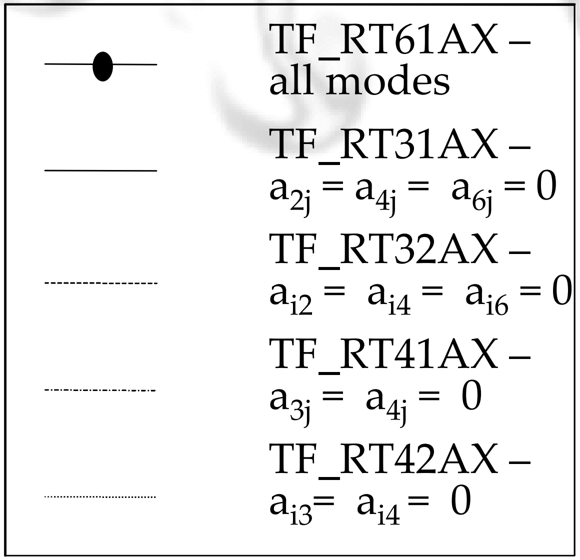












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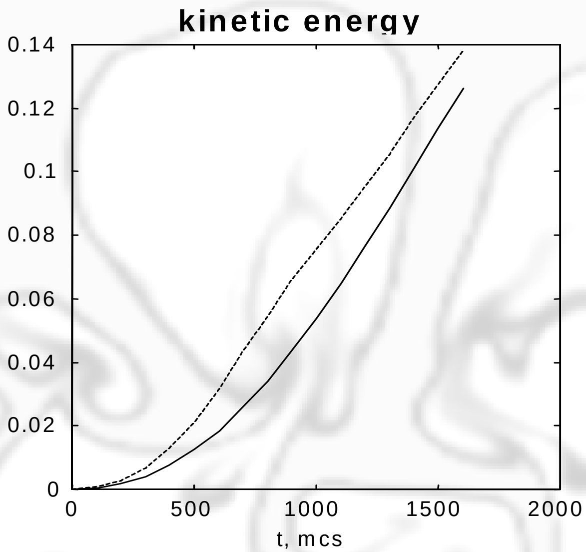
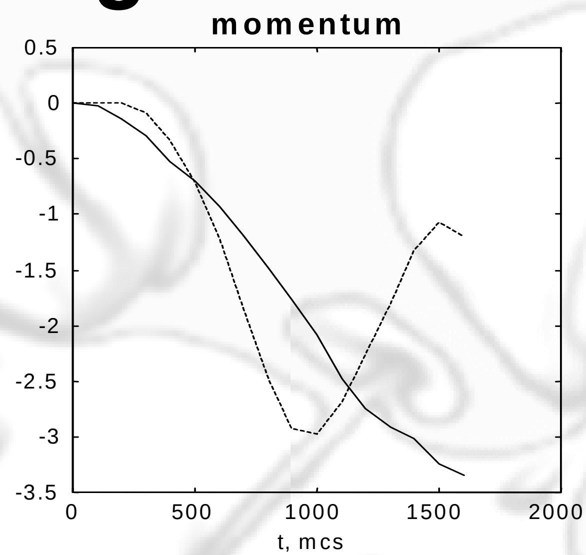
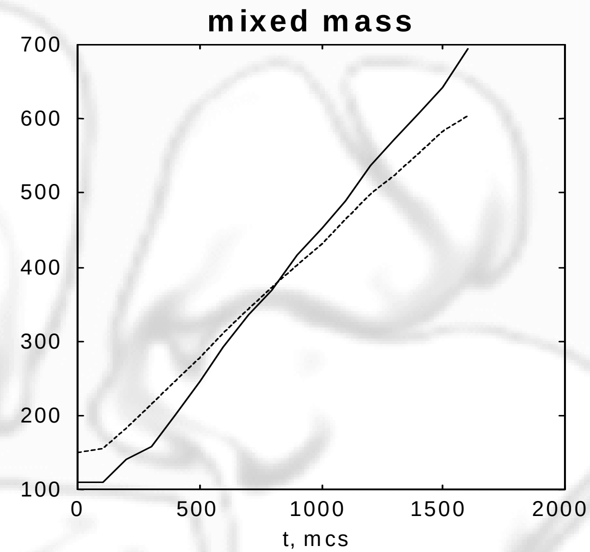
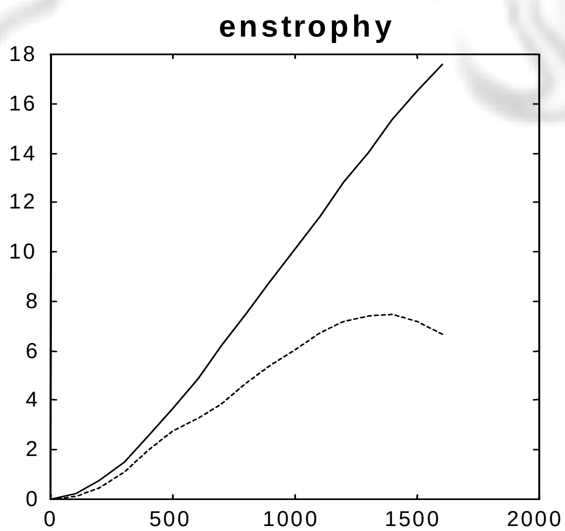
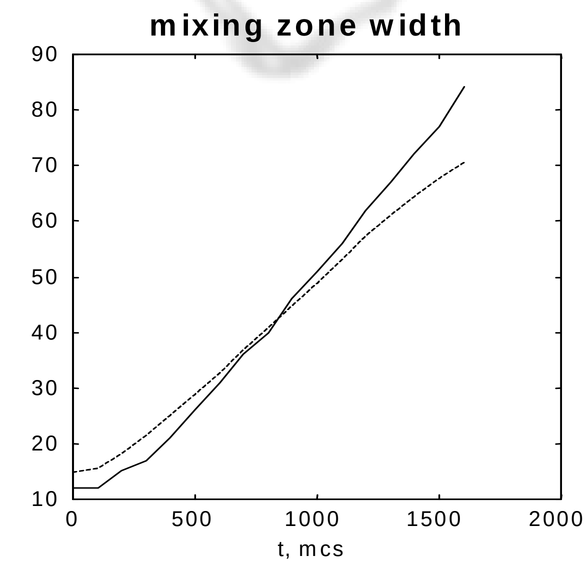
Integral values (3)







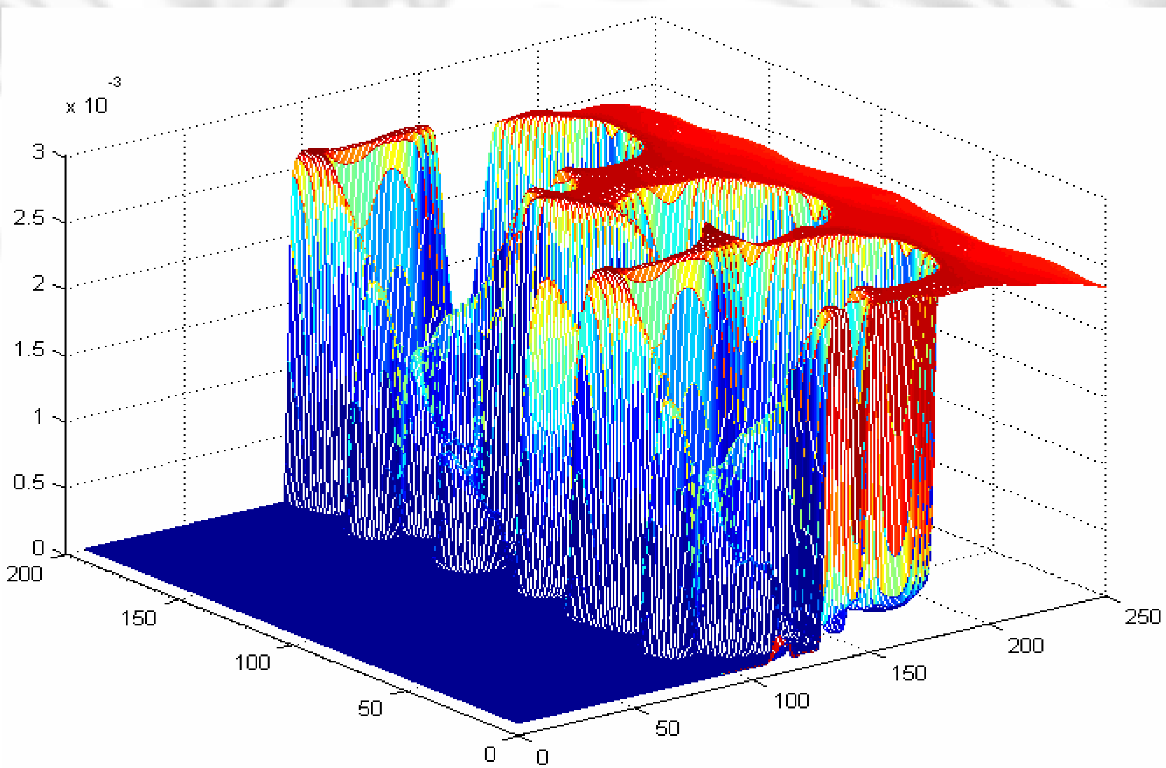




3D (solid) against 2D (dashed) instability.

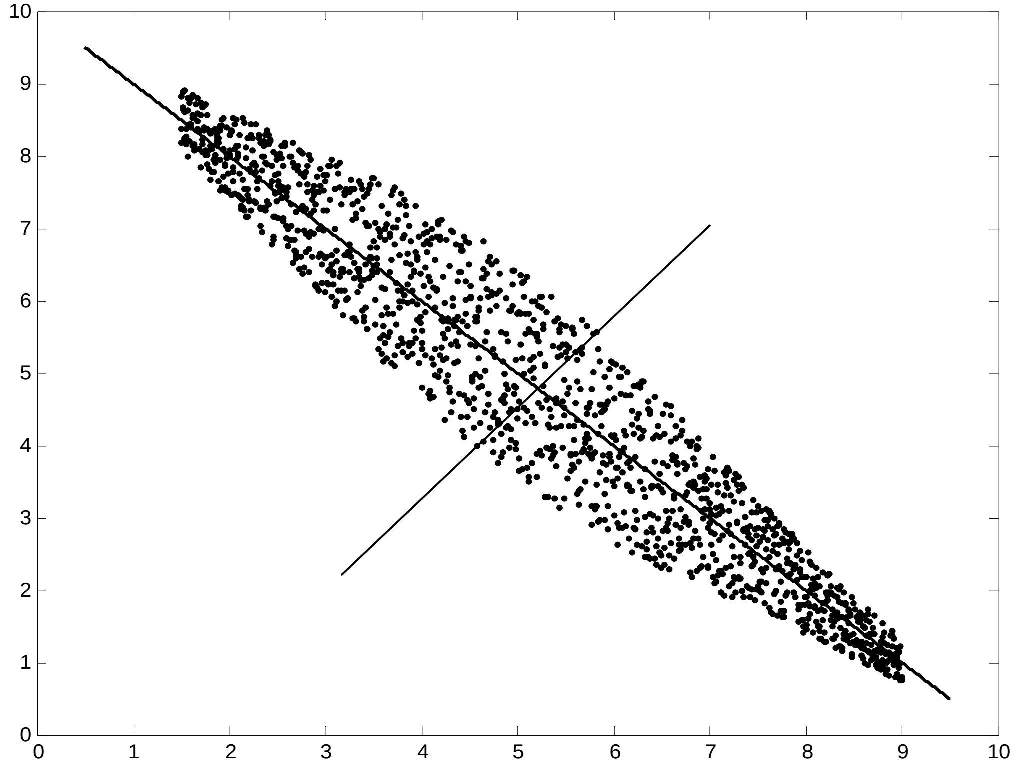
Wavelet-decomposition of density field

$$\rho = \sum c_i \psi_i \qquad c_i = \int \rho \psi_i \, dx dz$$



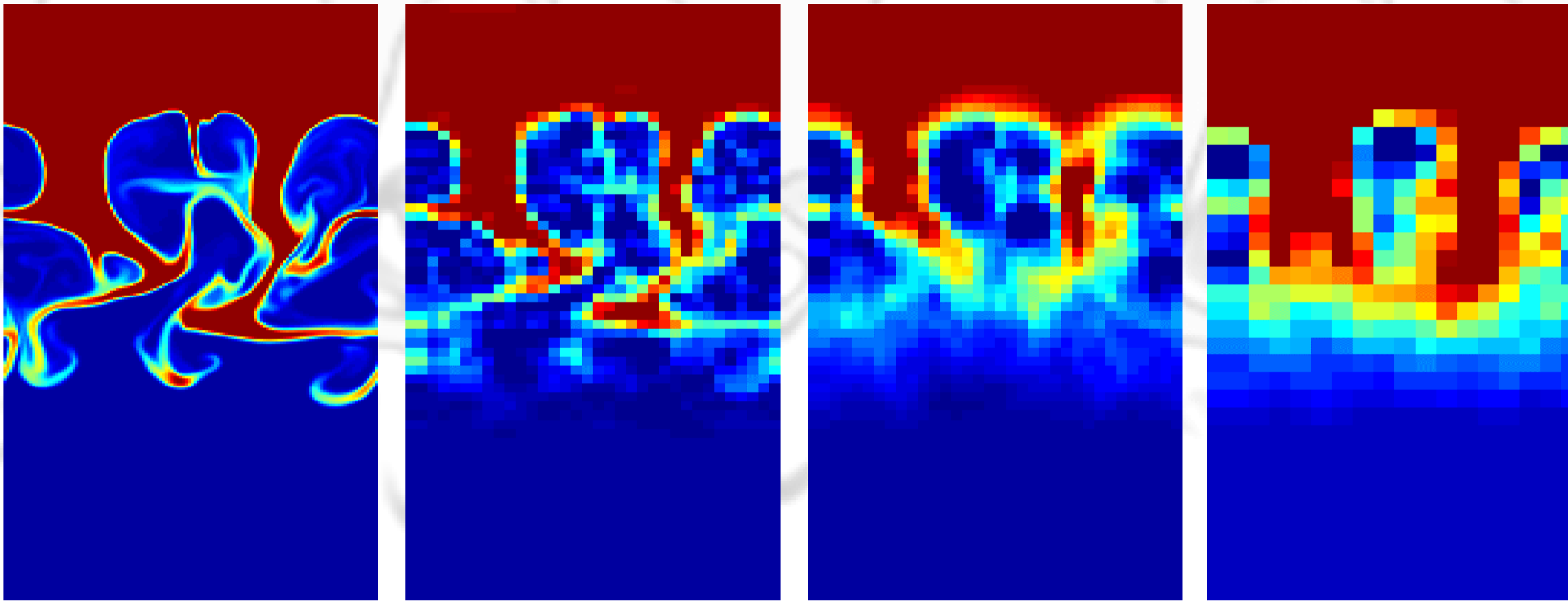
$$\longleftrightarrow \begin{pmatrix} c_1 \\ c_2 \\ \bullet \\ \bullet \\ \bullet \\ c_N \end{pmatrix}$$

Linear compression: Principal Components



$$\{c_1, c_2, \ldots, c_N\} \Rightarrow \{p_1, p_2, \ldots, p_N\} \Rightarrow \{p_1, p_2, \ldots, p_M\}, M << N$$

Reconstruction after data compression

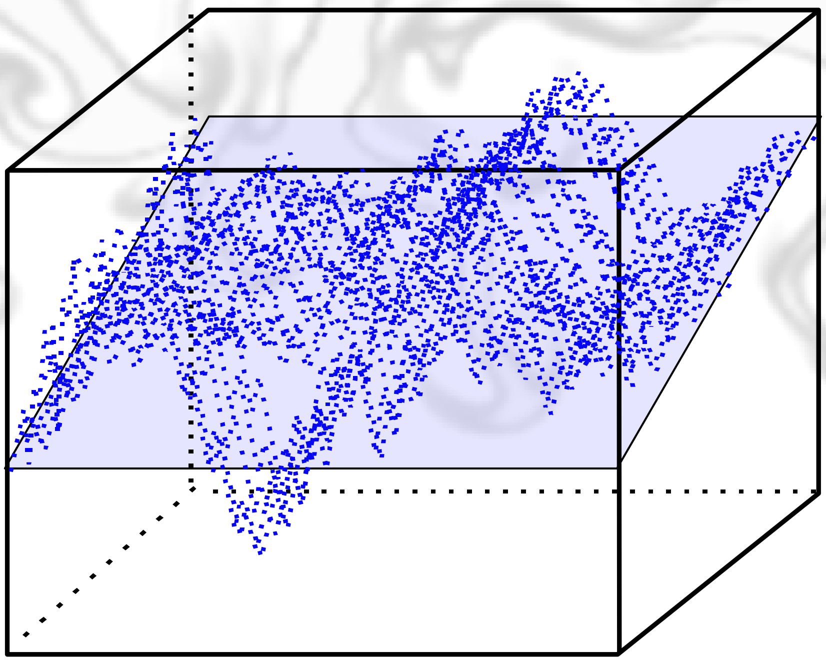


250x130 = 32,500 200 40 17

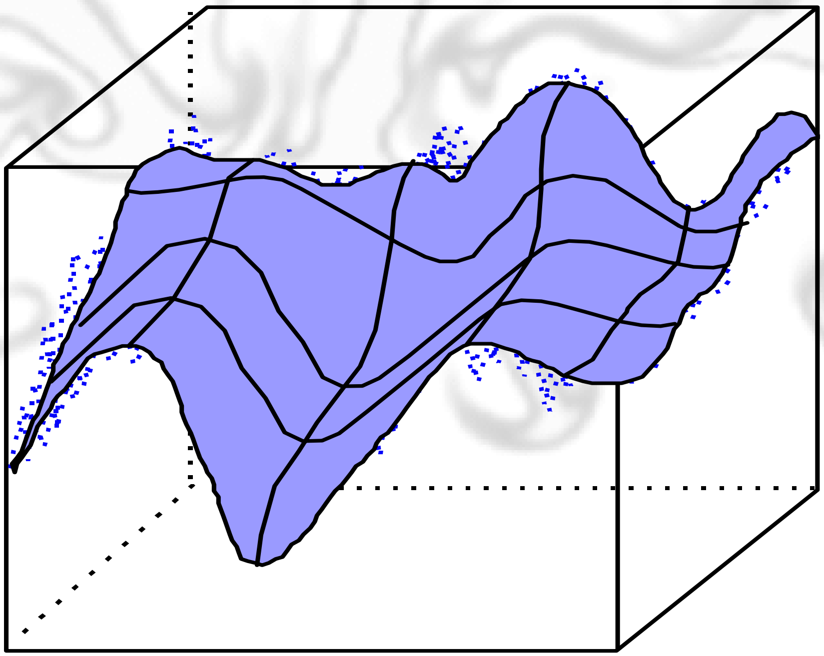
Principal components numbers

Kohonen maps

Linear compression:



Non-linear compression:



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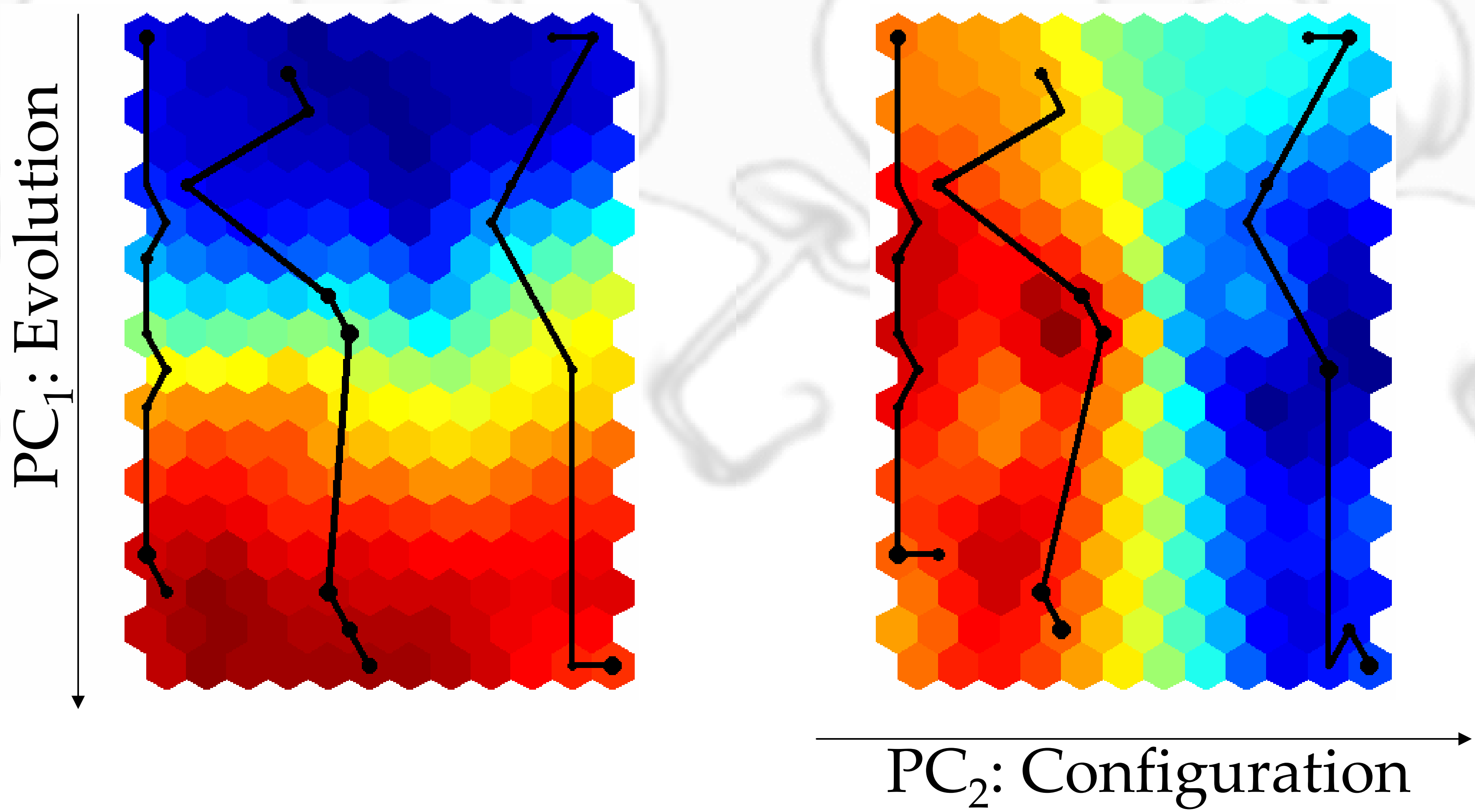
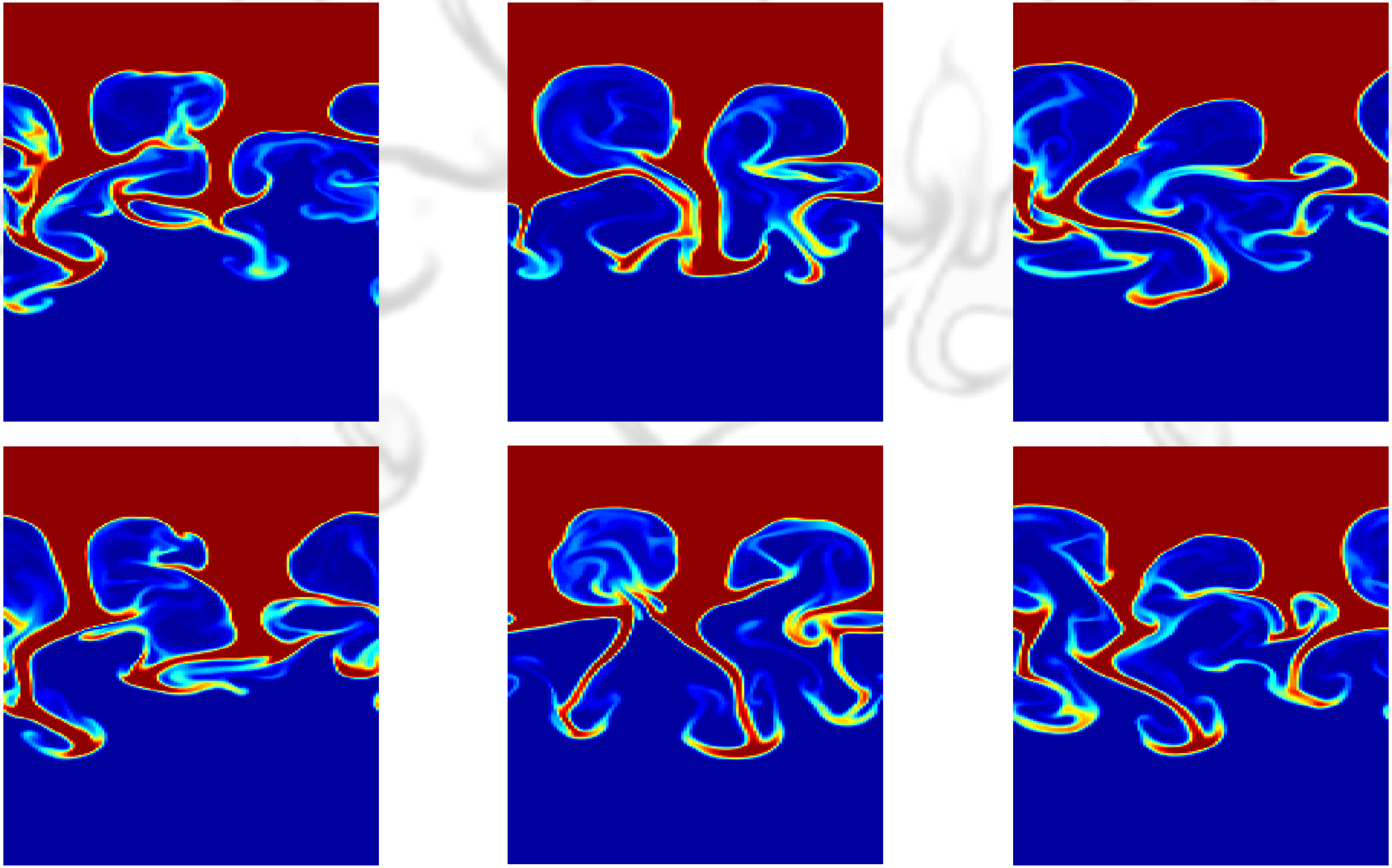
Map interpretation(1)

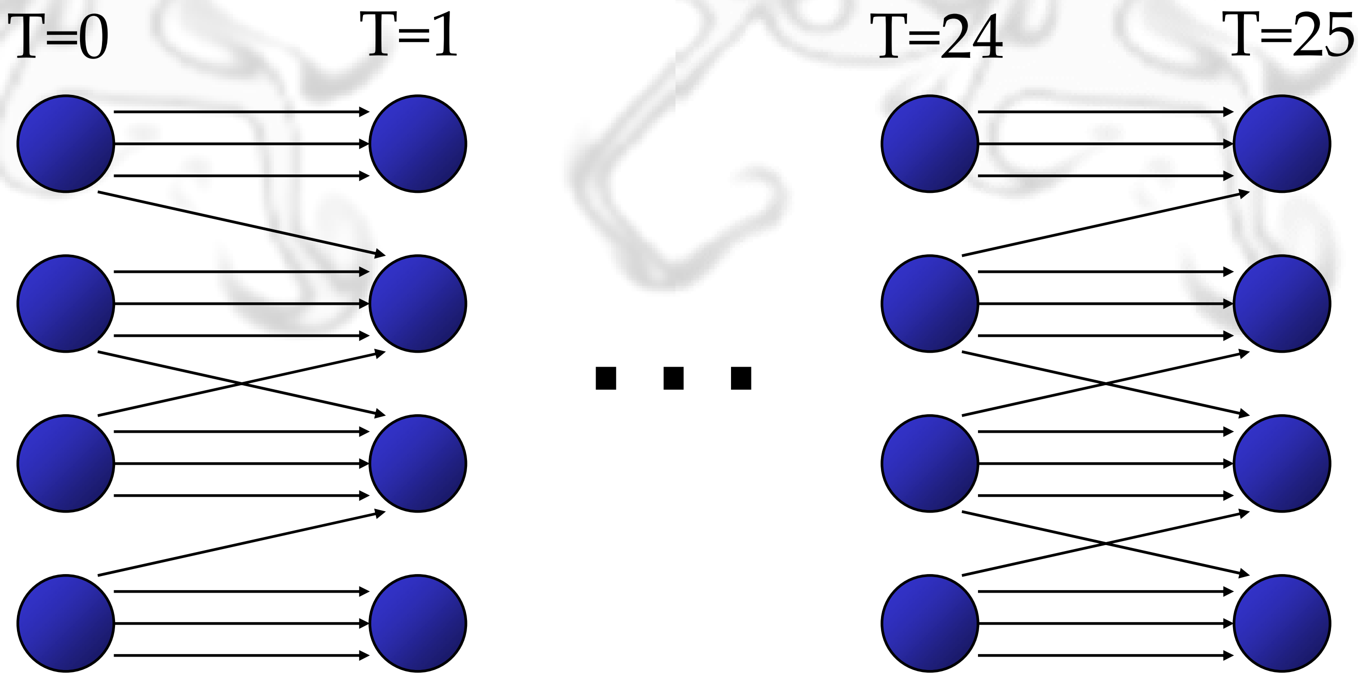


Map interpretation(2)
Processes similarity (He/Xe, n=6)



Predictor (1)

$$P_{i,j}(t) \approx \frac{N_{i,j}(t)}{N_i(0)}$$



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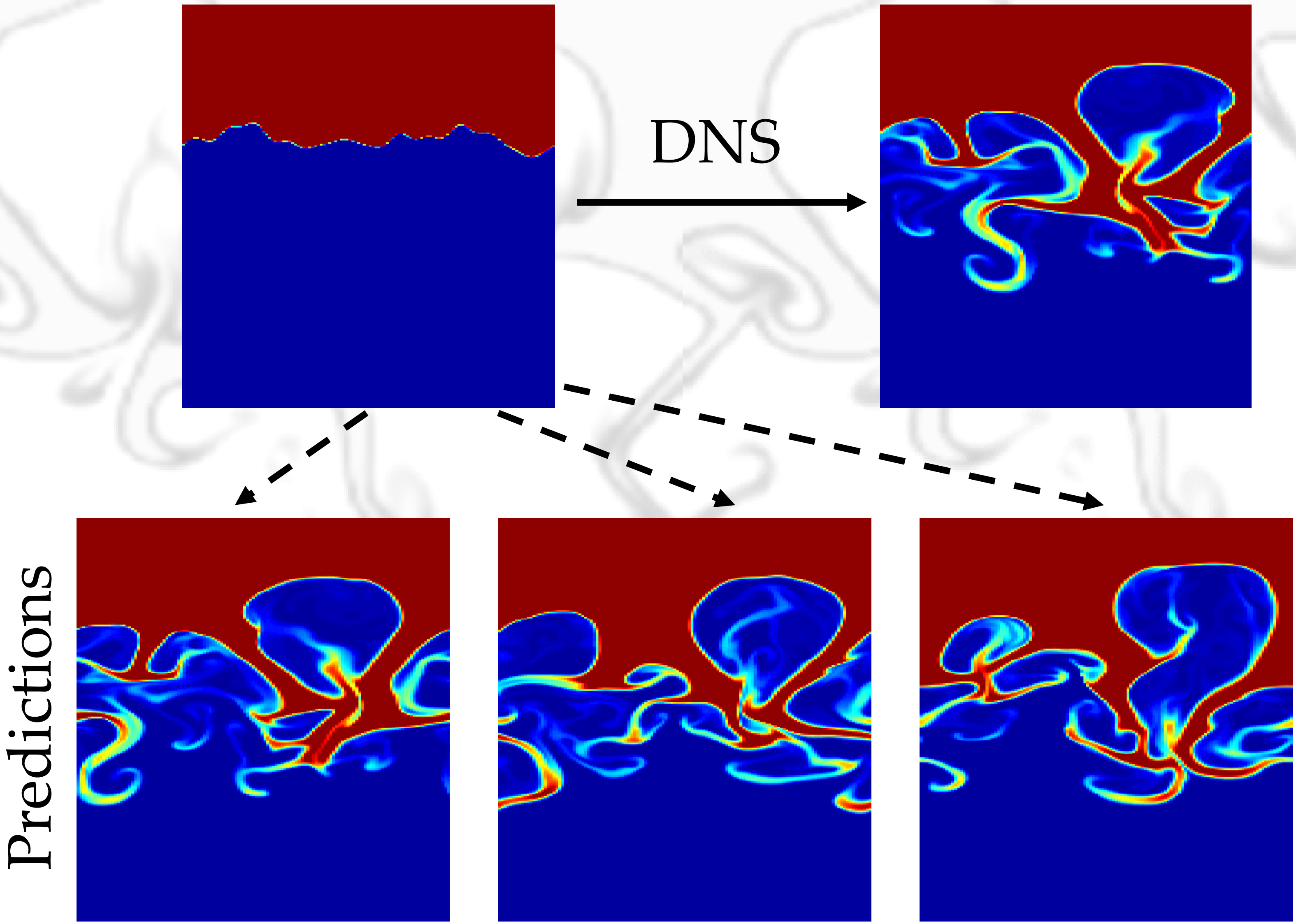
Predictor (2)



Interpretation of principal components[(1)]

$$c_i = \rho\psi_i$$

$$\vec{\bar{c}} \equiv \frac{1}{D}\sum_{\alpha=1}^{P}\vec{c}^{\alpha}, \qquad \Sigma_{ij}^{C} \equiv \frac{1}{D-1}\sum_{\alpha=1}^{D}\left(c_i^{\alpha}-\bar{c}_i\right)\cdot\left(c_j^{\alpha}-\bar{c}_j\right)$$

D – number of density figures

$$\sum_j \Sigma_{ij}^{C} U_{jk} = \lambda U_{jk} \qquad pc_i = \left(c_k - \bar{c}_k\right) U_{ki}$$

$$pc_i = \rho\psi_k U_{ki} - \bar{c}_k U_{ki} \qquad \mathrm{filter}_i = \psi_k U_{ki}$$

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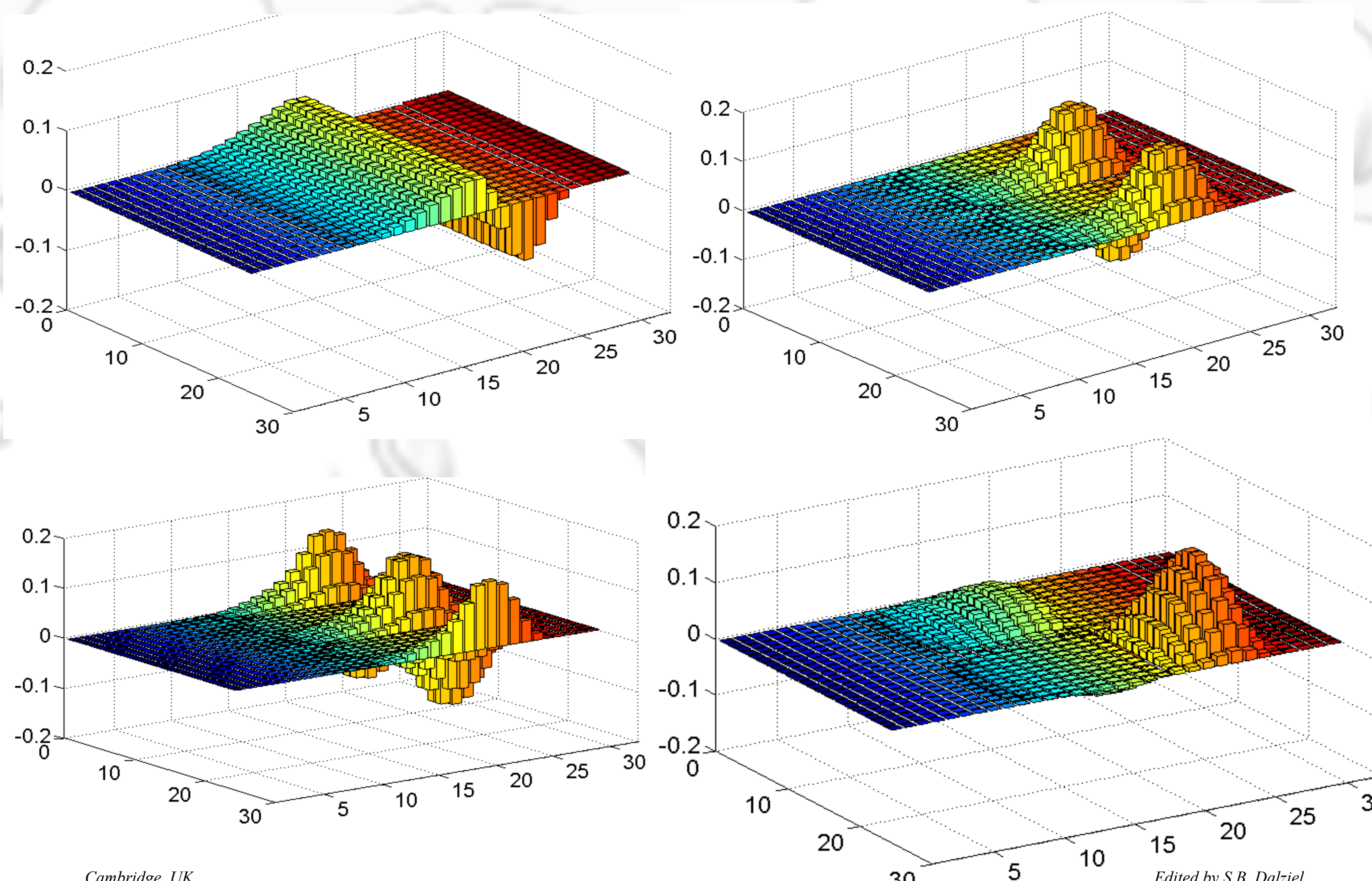
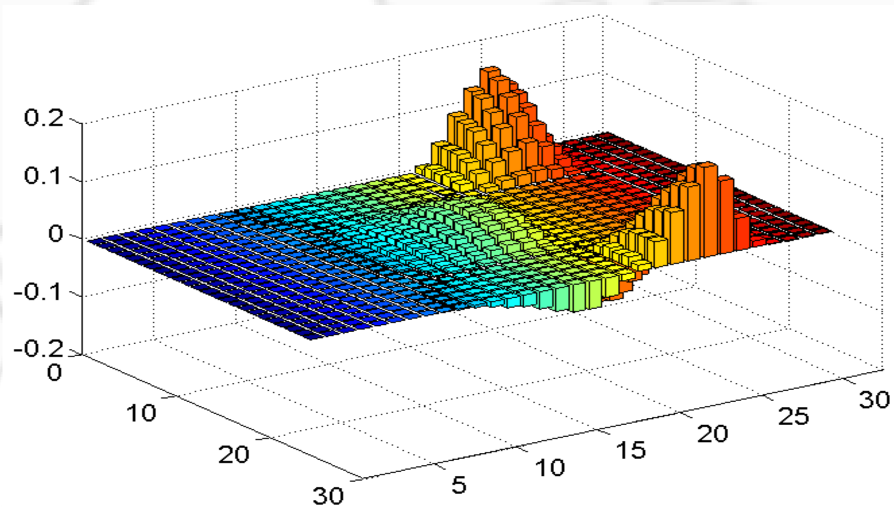
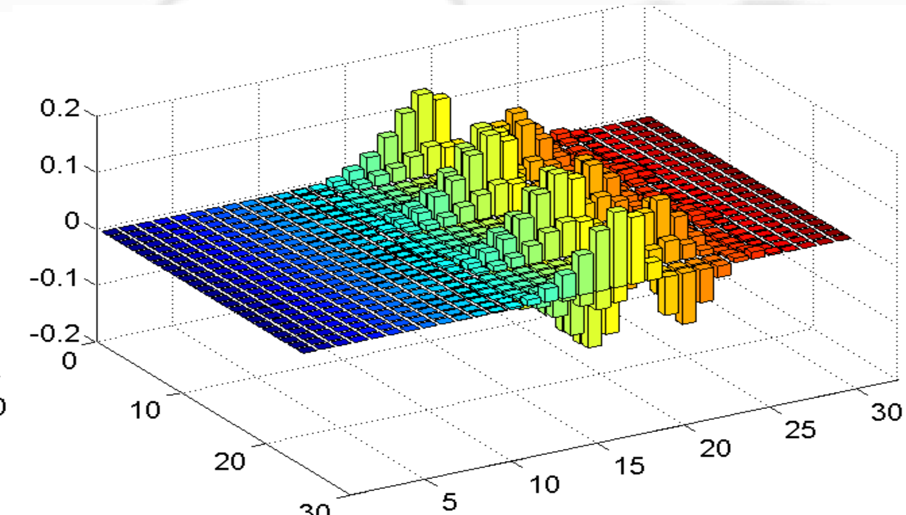
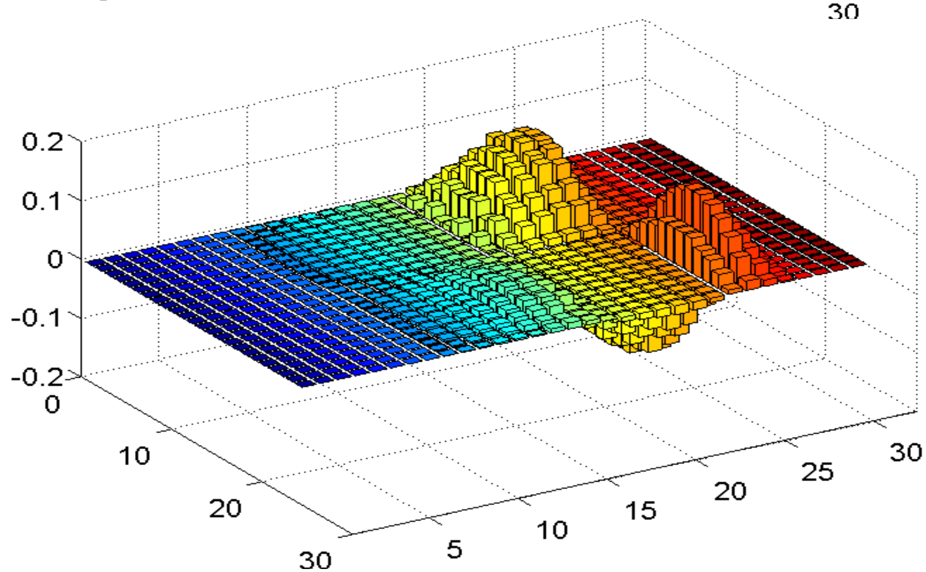
Interpretation of principal components(2)



Interpretation of principal components(3)







The width of the mixing zone (1)

$$L_i(t) = L_{0i} + \frac{\lambda_i \cdot f}{\alpha_{eff}} \cdot \left(\sqrt{1 + \frac{(\alpha_{eff} \cdot \gamma_i \cdot t)^2}{2\pi \cdot f}} - 1 \right)$$

$$\gamma_i^2 = \frac{2\pi}{\lambda_i} \cdot gA, \qquad f = \frac{\nu^2(A)}{A}, \qquad \alpha_{eff} = \alpha_0 \cdot \alpha_* / (\alpha_0 + \alpha_*),$$

$$V_\lambda^{asympt} = \nu \cdot \sqrt{g \cdot \lambda}, \qquad k_i = \frac{2\pi}{\lambda}, \qquad \alpha_0 = k_i \cdot a_{0i},$$

$$L(t) = \sum L_i(t) \cdot w_i(t), \qquad L(0) = \sum L_i(0) \cdot w_i(0),$$

$$w_i(0) = \cos(k_i \cdot x_{max} + \varphi_i) - \cos(k_i \cdot x_{min} + \varphi_i),$$

$$w_i(t) = w_i(0) \cdot e^{-\gamma_{KHi} \cdot t} \Rightarrow e^{-\gamma_i \cdot t / n_i} \Rightarrow e^{-(\gamma_i \cdot t / n_i)^2}, \quad \gamma_{KH} \sim \frac{\gamma_i \cdot a_i}{\lambda_i} \Rightarrow \frac{\gamma_i}{n_i}$$

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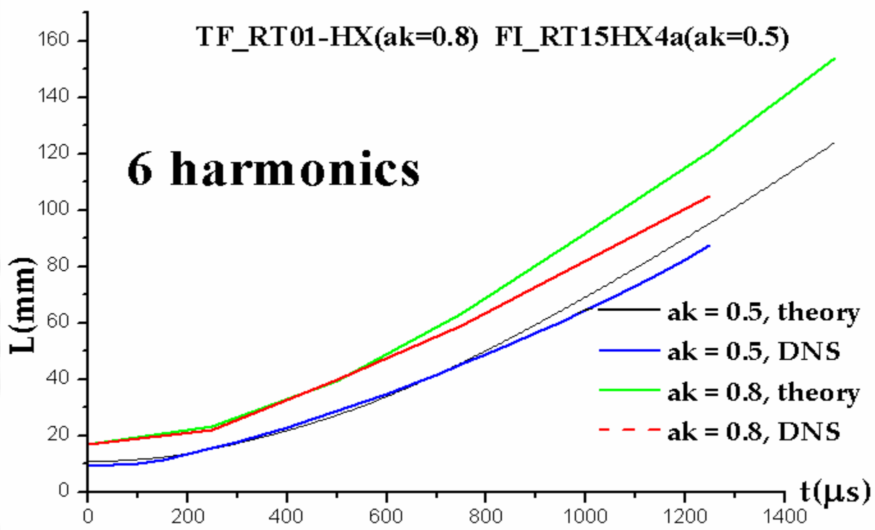
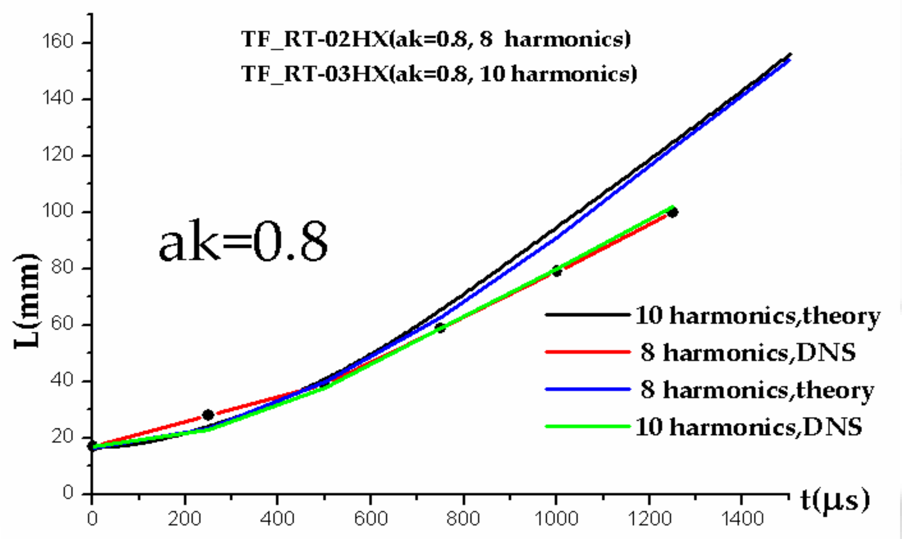
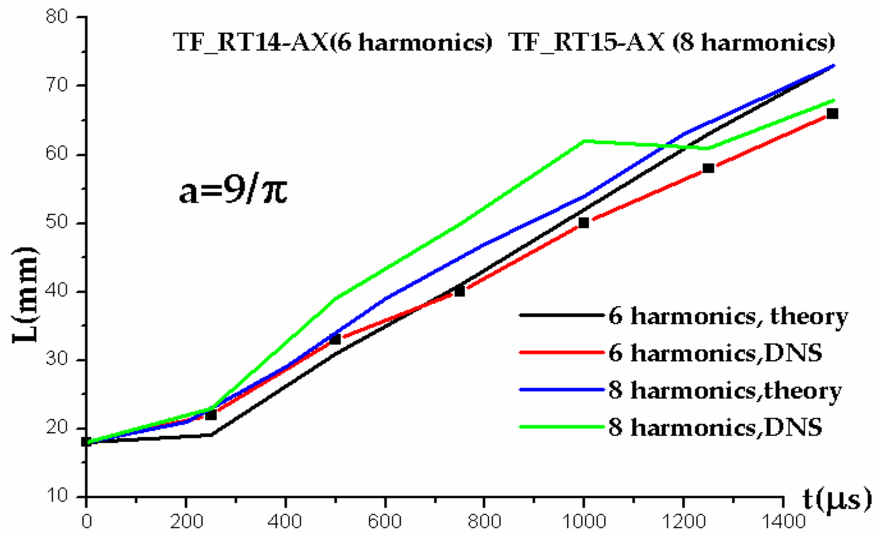
The width of the mixing zone (2)







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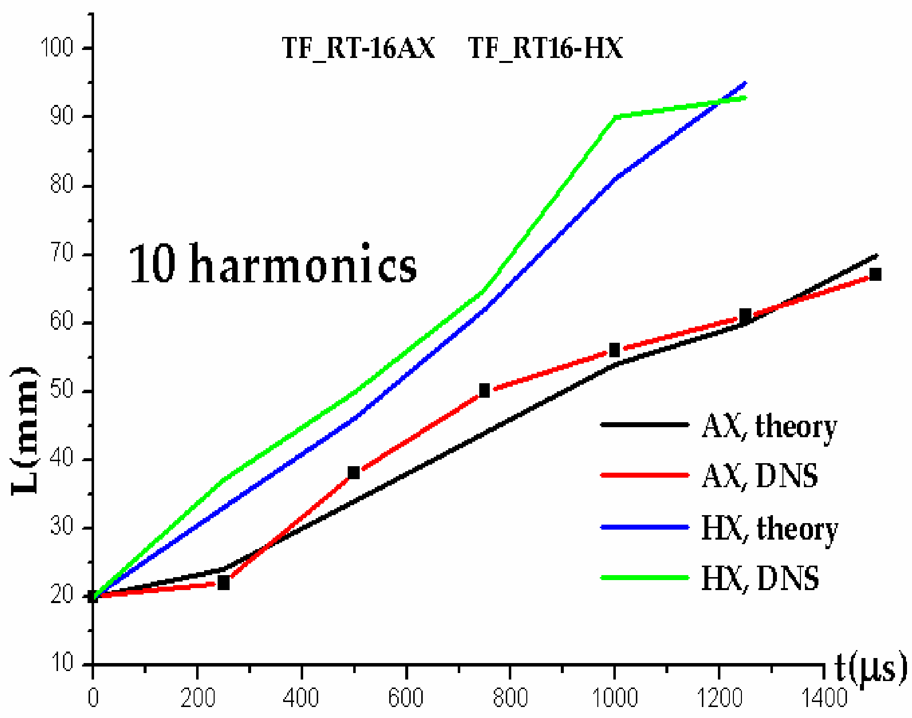
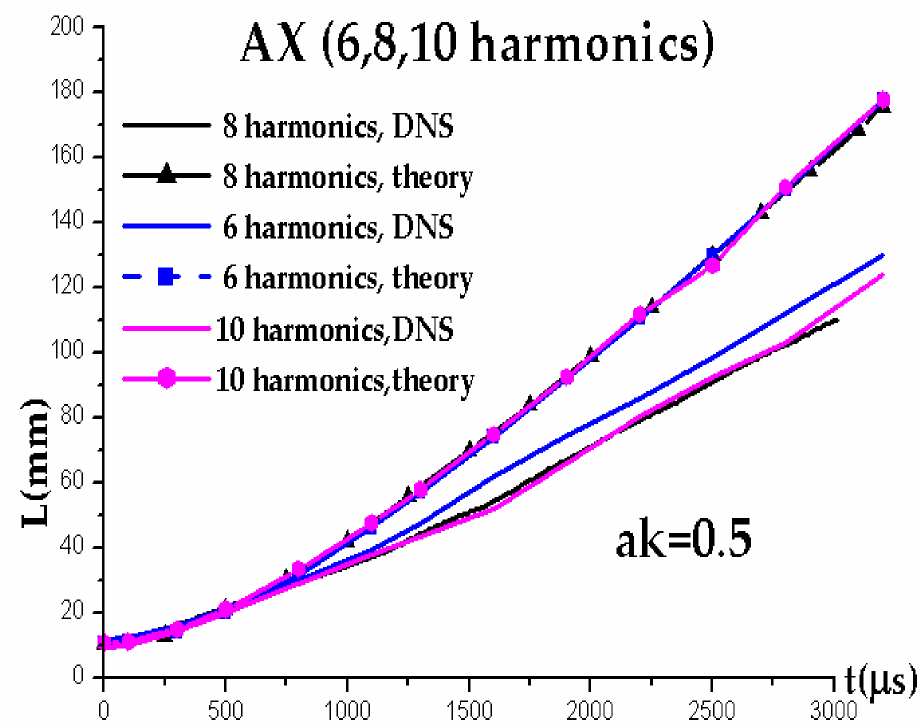
The width of the mixing zone (3)





Conclusion (1)

From DNS

•Database created on the basis of 250 DNS on the development of instability and turbulent mixing of two gases for different regimes:

- RT and RM;
- 2D and 3D;
- different Atwood numbers;
- different mode numbers;
- different amplitudes on the mode number dependencies;

Database contains information on the pressure, velocity, density and enstrophy fields.

•The growth of mixing zone width is close to the linear one with time

•The width of the mixing zone does not depend strongly on the high mode contribution (The width slightly decreases with the inclusion of high modes)

•The width of the mixing zone for 3D and 2D cases for equal condition is approximately equal.

Conclusion (2)

From wavelet analysis

• Wavelet decomposition of density gives adequate representation of the instability and mixing development

• Kohonen maps can distinguish between different examples of the processes and can find similarity of the processes (if it really exists)

• interpretation of principal components of the wavelet space looks a very interesting investigation

• on this basis it is possible to suggest a predictor which can predict final results of instability and mixing development using the information on the initial state only.

Prospects

• 3D instabilities and turbulent mixing development, the wavelet analysis and so on.

• Wavelet analysis of other 2D fields (compare with the density) and their combination.