LES of Rayleigh-Taylor turbulence with compressible miscible fluids

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Introduction

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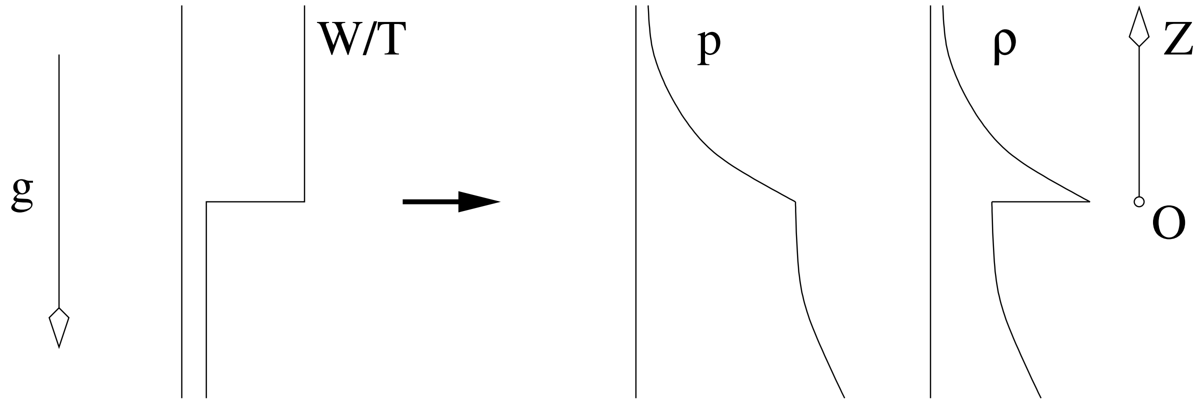
- Certain situations involve high accelerations and compressibility effects become important at laboratory scales.
- Previous approaches have considered the *turbulent stage* in the incompressible limit.
- How does intrinsic compressibility, measured by the turbulent Mach number

$$M_t = \frac{q}{\langle c \rangle} = \frac{\sqrt{2K}}{\langle c \rangle} \, ,$$

modify the evolution of the flow ?

Two-layer configuration

- A layer of large molecular weight (cold) fluid on top of light (hot)



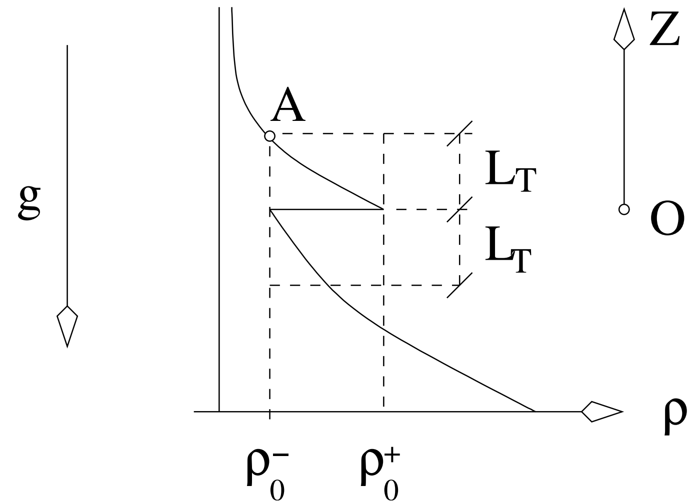
- From equation of state (ideal gas) and hydrostatic equilibrium,

$$p(z) = p_0 \exp\left[-\frac{1}{L_H}\int_0^z \frac{W(\zeta)/W_H}{T(\zeta)/T_H} d\zeta\right] ,$$

where the two large length-scales of the problem are

$$L_i = \frac{R^0 T_i}{g W_i} \qquad , \qquad i = L, H$$

Available potential energy



Assuming the turbulent stage over a length scale L_T, the depth-integrated available potential energy is

$$\frac{E_p}{mass} = \phi(\frac{\rho_0^+}{\rho_0^-})\frac{\rho_0^+}{\rho_0^-} gL_H \ ,$$

where ϕ, function of density jump at the center plane ρ_0^+/ρ_0^-, has a maximum 0.10.

Bound on M_t

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- Assuming all E_p is transformed into turbulent kinetic energy,

$$q_{0,max} = \sqrt{2K_0} = \sqrt{2\phi_{max}\frac{\rho_0^+}{\rho_0^-}}\sqrt{gL_H}$$

- Assuming that the maximum turbulent Mach number occurs around the center plane. a characteristic speed of sound is

$$c_0 = \sqrt{\gamma R^0 \frac{1}{2}(\frac{T_H}{W_H} + \frac{T_L}{W_L})} = \sqrt{\frac{\gamma}{2}(\frac{\rho_0^+}{\rho_0^-} + 1)}\sqrt{gL_H}$$

- Hence, maximum turbulent Mach number can be estimated by

$$M_{t,max} = \frac{q_{0,max}}{c_0} = \frac{\sqrt{4\phi_{max}}}{\sqrt{\gamma}} = \frac{0.6}{\sqrt{\gamma}}$$

Large-eddy simulations

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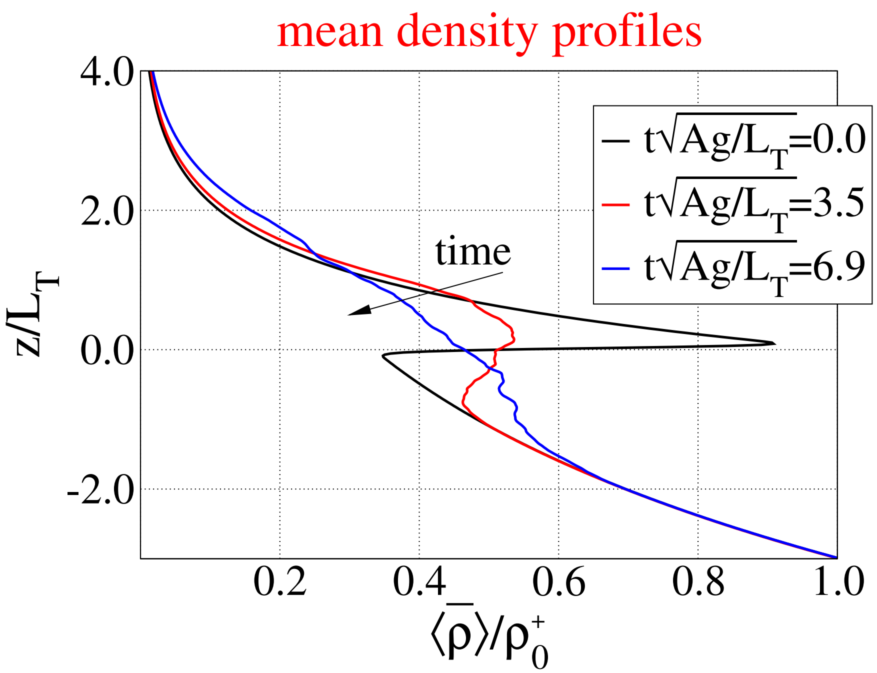
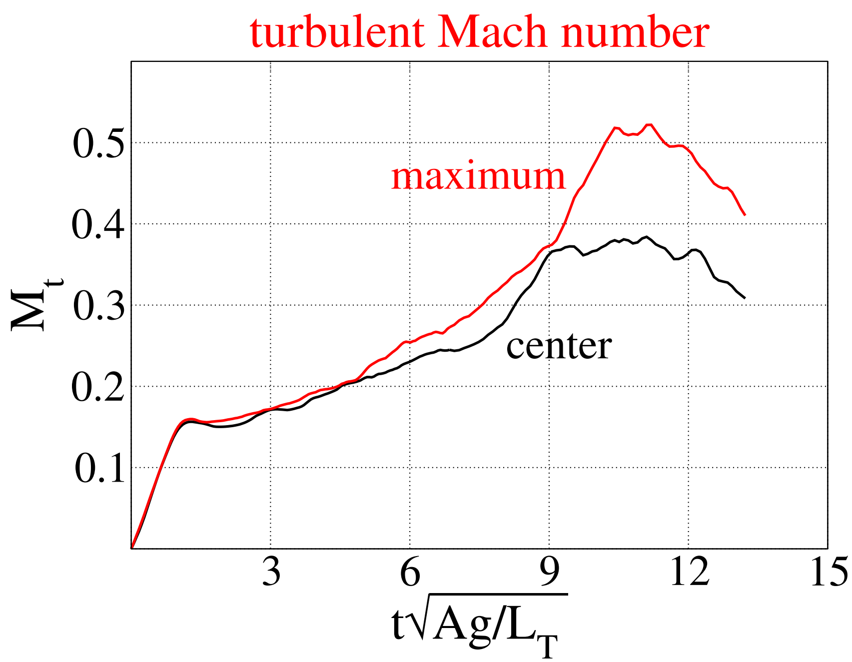
- Dynamic mixed model closure of momentum, energy and species equations. Molecular viscous terms set to zero.
- 6^{th}-order compact scheme.
- 4^{th}-order Runge-Kutta scheme.
- Skew-symmetric formulation of convective terms.
- 4^{th}-order compact dealiasing filter.
- Initial perturbation using a Gaussian spectral density following Cook & Dimotakis (2001).

LES results

Domain $2L_T \times 2L_T \times 11L_T$ on a grid $128 \times 128 \times 704$. Density jump $\rho_0^+/\rho_0^- = 3$.





- The turbulent stage takes place over a length L_T.
- M_t is consistent with the analytical bounds.

LES results

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- Decomposition density fluctuations into acoustic and entropic parts (Chassaing *et al.*, 2002)

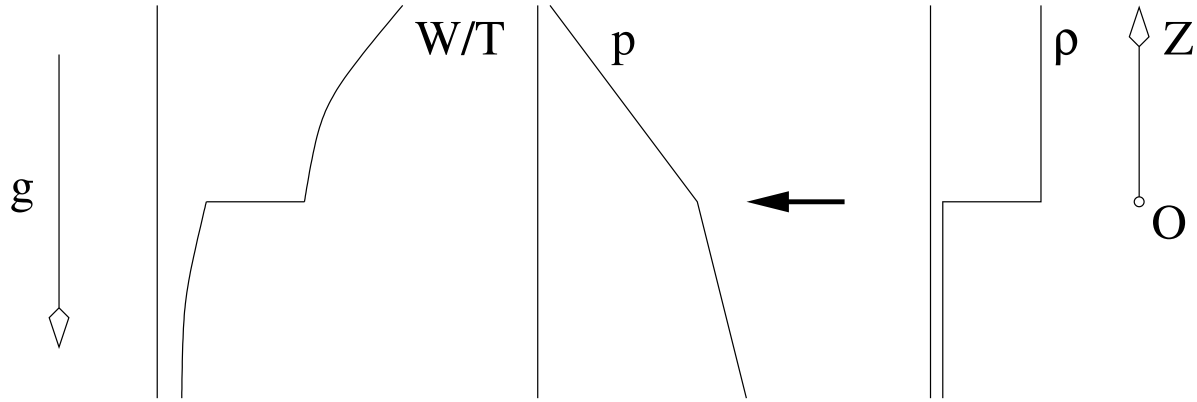
$$\rho'_{ac} = p'/\langle c\rangle^2$$
$$\rho'_{en} = \rho' - \rho'_{ac} \; .$$

  Acoustic part less than 10% of the entropic (composition) fluctuation.

- Pressure-dilatation term less that 10% of the buoyancy-production term in the turbulent kinetic energy transport equation.

Constant-Atwood configuration

- A layer of heavy fluid on top of light



- From hydrostatic equilibrium,

$$p(z) = p_0 \left(1 - \frac{1}{L_H} \int_0^z \frac{\rho(\zeta)}{\rho_H} d\zeta \right) ,$$

where the two large length-scales of the problem are

$$L_i = \frac{p_0}{\rho_i g} \qquad , \qquad i = L, H$$

Bound on M_t

- A characteristic speed of sound is

$$c_0 = \sqrt{\gamma \frac{1}{2}\left(\frac{p_0}{\rho_H} + \frac{p_0}{\rho_L}\right)} = \sqrt{\frac{\gamma}{2} g(L_H + L_L)}$$

- Dimensional analysis yields ($Pr \simeq 1$)

$$q_0 = \beta(\gamma, M)\sqrt{Agh}\ .$$

- Therefore,

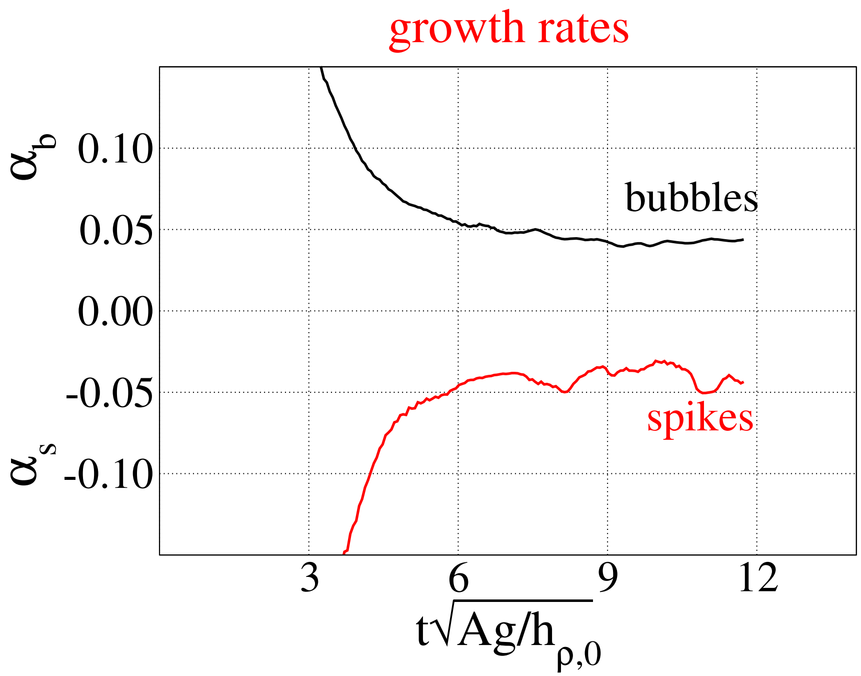
$$M_t = \frac{q_0}{c_0} \simeq \frac{\beta\sqrt{A}}{\sqrt{\gamma}}\sqrt{\frac{2h}{L_H + L_L}} < \frac{\beta\sqrt{A}}{\sqrt{\gamma}}$$

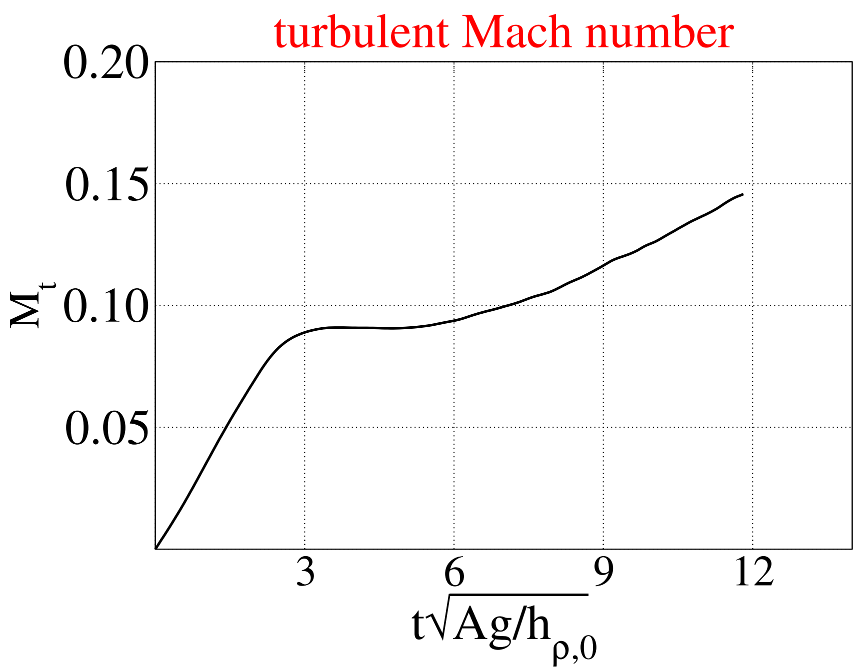
- Estimating $\beta \simeq 0.3 - 0.6$ from incompressible results, $A \leq 1$, it is

$$M_{t,max} = \frac{0.6}{\sqrt{\gamma}}$$

LES results

Domain $L_H \times L_H \times 2L_H$ on a grid $256 \times 256 \times 512$ with a density ratio $\rho_H/\rho_L = 3$.





- LES gives $\beta = 0.36$ and $M_{t,max}$ occurs around the center plane.
- Incompressible scaling and results are obtained: $\alpha_b \simeq 0.04$, $\Xi, \Theta \simeq 0.75 - 0.80$, $KE_{xy}/KE_z \simeq 0.5$.

Conclusion

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- The initial thermodynamic state of the system determines the amount of potential energy per unit mass involved in the turbulent mixing stage, and thus the level of turbulent fluctuations achievable is linked to the characteristic speed of sound such that the turbulent Mach number is limited.
- For the particular case of an ideal gas, $M_{t,max}$ varies between 0.25 and 0.5 for $\gamma = 1.4$, small enough values for compressibility effects to be relatively small.
- LES show that the hypothesis in the analysis hold.
- Results from LES shown that compressibility effects are small and the numerical results agree with incompressible data and incompressible scaling.

References

CHASSAING, P., ANTONIA, R. A., ANSELMET, F., JOLY, L. & SARKAR, S. 2002 *Variable density fluid turbulence*. Kluwer Academic Publishers.

COOK, A. W. & DIMOTAKIS, P. E. 2001 Transition stages of Rayleigh-Taylor instability between miscible fluids. *J. Fluid Mech.* **443**, 69–99.