

Compressibility effects on Rayleigh-Taylor turbulence

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Introduction

Certain situations, *e.g.*, Inertial Confinement Fusion, involve high accelerations and compressibility effects may become important at laboratory scales. However, analysis of the role of intrinsic compressibility of the turbulent fluctuations in the turbulent stage of the Rayleigh-Taylor problem has not been previously reported. Work has been done on the effect of compressibility on the initial linear stage¹, but our paper is concerned with the *fully turbulent stage*. *A priori*, these compressibility effects could appear if the speed of sound of the fluid is sufficiently reduced (*e.g.*, reducing the reference pressure of the system), and this is the motivation for the present study.

A general overview containing the latest results on compressible turbulence can be found in Chassaing *et al.*² In the case of free shear flows, there is a strong intrinsic compressibility effect: the growth rate of the shear layer thickness and the turbulence levels are significantly reduced as the Mach number increases. The cause has been the subject of much study in the past years³⁻⁸, leading to the following picture: the production term in the turbulent kinetic energy equation reduces as a consequence of the decrease in the pressure-strain correlation, which diminishes the transfer of energy from the streamwise fluctuations to the cross-stream fluctuations. It is reasonable, then, to formulate the same questions for the Rayleigh-Taylor problem, where the input of energy is essentially different.

Henceforth, compressibility will mean *intrinsic compressibility* (density variations due to pressure variations), whose level in the fluctuation fields is determined by the turbulent Mach number,

$$M_t = \frac{q}{\langle c \rangle}, \quad (1)$$

where $q = \sqrt{2K}$, K being the turbulent kinetic energy and $\langle c \rangle$ the average speed of sound, all quantities varying across the mixing layer. The average of any variable ϕ is written as $\langle \phi \rangle$ and it is computed as a plane average at a fixed inhomogeneous location z . They denote Reynolds averages for quantities ϕ per unit volume and Favre averages for quantities ϕ per unit mass, unless otherwise stated.

Large-eddy simulation is used in this work, in particular, a dynamic mixed model is utilized, which

has already demonstrated to provide the temporal evolution of the large-scale three-dimensional fields with reasonable accuracy in different flows⁹. Details about the formulation of the problem, as well as a thorough discussion of the theoretical analysis and results for different possible configurations can be found in Mellado *et al.*¹⁰ Only one of those configurations is presented in this paper for the sake of brevity, although the derived conclusions equally apply to the other configurations.

Compressibility of the turbulence

We consider the hydrostatic equilibrium of a layer of heavy fluid on top of a layer of lighter fluid. Both fluids are treated as ideal gases, the local speed of sound being then given by

$$c = \sqrt{\gamma \frac{R^0 T}{W}} = \sqrt{\gamma \frac{p}{\rho}}, \quad (2)$$

where γ , the ratio of specific heats, lies in the range $1 < \gamma < 5/3$.

With respect to the turbulent kinetic energy, turbulent theory predicts and experiments confirm that, in the incompressible limit, a self-similar state is achieved after a sufficiently long time, in which

$$q_0 = \beta \sqrt{Ag h} \quad (3)$$

is a characteristic scale of the turbulent velocity fluctuations at each time. In this expression, Ag represents the constant external force per unit mass, with $A = (\rho_H - \rho_L)/(\rho_H + \rho_L)$ the Atwood number, and $h(t)$ the thickness of the increasing mixing depth. The heavy fluid (top) is denoted by the subscript H and the light fluid by the subscript L . The coefficient β , of order 1, has to be provided by experimental data.

In the compressible situation, in general, the ratio of specific heats, γ , the Prandtl number and a characteristic speed of sound, c_0 , appear as additional parameters. There is no externally imposed velocity scale so that the Mach number based on the local velocity fluctuations, M_t , is the relevant one in this problem. Thus, for a Prandtl number of order unity, the situation considered here,

$$q_0 = \beta(\gamma, M_t) \sqrt{Ag h}. \quad (4)$$

If M_t is large, intrinsic compressibility effects could become important, in principle. However, the fact that the motion of the fluids is determined by the initial thermodynamic variables, namely, the zone where the fluid on top has density larger than that below, links the maximum of q_0 over the time to c_0 such that $M_t(t)$ is bounded from above. Hence, M_t , which is small at early times because we start from a steady configuration, might not become large enough for intrinsic compressibility effects to be strong; in particular, the evolution of the turbulent kinetic energy could be well approximated by the incompressible result, Eq. (3).

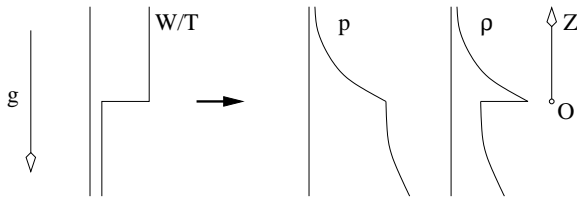


FIG. 1: Profiles of the thermodynamic variables in a two-layer system that is Rayleigh-Taylor unstable.

The two-layer configuration is shown in Fig. 1, the ratio between the molecular weight W and the temperature T of the mixture varying between two well-defined levels, W_L/T_L at the bottom and W_H/T_H (larger) at the top. The pressure, obtained from the equation of state and the hydrostatic equilibrium condition, is

$$p(z) = p_0 \exp\left(-\frac{gW_H}{R^0T_H} \int_0^z \frac{W(\zeta)/W_H}{T(\zeta)/T_H} d\zeta\right), \quad (5)$$

p_0 being the value at the middle plane, $z = 0$. The two characteristic scales of the problem are

$$L_i = \frac{R^0T_i}{gW_i}, \quad i = L, H, \quad (6)$$

the *scale-heights*¹¹ of each layer.

A characteristic speed of sound c_0 is required. The average speed of sound, $\langle c \rangle$, varies along the inhomogeneous direction, being minimum in the denser (top) fluid, according to the profiles of T/W and Eq. (2). However, the maximum turbulent kinetic energy is expected to be close to the center plane (as it is shown later by the simulations), and, as the two fluids mix, a reasonable choice for the characteristic speed of sound is

$$c_0 = \sqrt{\frac{\gamma}{2} \left(\frac{L_H}{L_L} + 1 \right) gL_L}, \quad (7)$$

which has been obtained using the mean value $(T_H/W_H + T_L/W_L)/2$ in Eq. (2) and the definitions of Eq. (6).

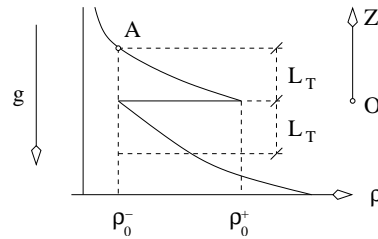


FIG. 2: Definition of the region, $2L_T$, that can participate in the turbulent Rayleigh-Taylor mixing.

In order to estimate q_0 , self-similar analysis cannot be used because h is not the only scale of the problem, and there is no available data in the literature (to the best of our knowledge) in order to estimate the evolution of the turbulent kinetic energy. One possible approach is to calculate the available potential energy of the system, since this constitutes an upper limit to the kinetic energy released to the flow. It is assumed that the turbulent stage develops over a length scale $2L_T$, as shown in Fig. 2. The length L_T includes all fluid heavier than the light fluid at the interface, that is, from $z = 0$ up to the point A in Fig. 2, where $\rho = \rho_0^-$. Mathematically,

$$L_T = L_H \ln \frac{L_L}{L_H} = L_H \ln \frac{\rho_0^+}{\rho_0^-}. \quad (8)$$

This length scale L_T is more relevant than L_H in this two-layer problem because it retains information about the density jump at the interface. Then, the depth-integrated available potential energy can be shown to be¹⁰

$$\frac{E_{p,max}}{mass} = \phi \left(\frac{\rho_0^+}{\rho_0^-} \right) gL_L, \quad (9)$$

where ϕ , function of the density jump at the center plane ρ_0^+/ρ_0^- , has a maximum 0.10. The value $q_{0,max}$ obtained from this estimate scales then with $\sqrt{gL_L}$, exactly the same as c_0 , Eq. (7), because $L_L \geq L_H$. Therefore, the ratio between $q_{0,max}$ and c_0 , the maximum turbulent Mach number over the time, is

$$M_{t,max} = \frac{0.6}{\sqrt{\gamma}}. \quad (10)$$

To summarize, the major result of the theoretical analysis presented in this section is that the turbulent Mach number has an upper bound (independent of the density ratio) and may not be large enough for intrinsic compressibility effects to be important in Rayleigh-Taylor turbulence. An assumption underlying the analysis is that the flow is fully turbulent. In this respect, if a large scale perturbation $O(L_T)$ in the two-layer configuration is imposed initially at the interface, then there could be compressibility effects as a blob of pure fluid rises/falls into the opposite pure fluid layer; that is not the case considered

here. Another assumption is that of an ideal gas. The fundamental cause of the limitation of M_t is independent of this latter hypothesis, however, the particular value of $M_{t,max}$ found in the analysis depends on the details of each particular equation of state.

LES results

A two-layer configuration, shown in Fig. 1, is numerically simulated for an isothermal case with a density jump at the interface of 3 : 1. The equations are solved in a rectangular domain $2L_T \times 2L_T \times 11L_T$, where L_T is given by Eq. (8). This size is chosen such that the upper half of the domain spans 6 times this distance L_T . The mesh is $128 \times 128 \times 704$. The initial perturbation of the interface is set following Cook *et al.*¹²

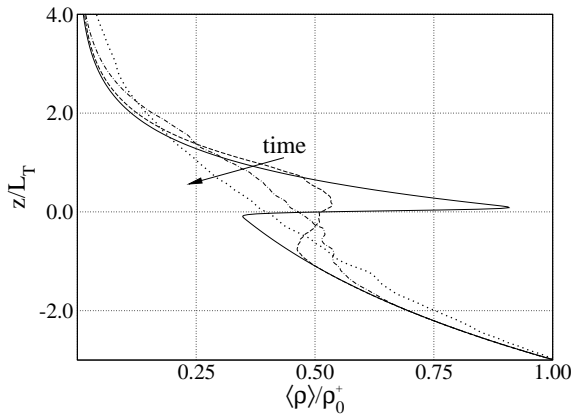


FIG. 3: Normalized mean density profiles at different times $t\sqrt{Ag/L_T}$: — 0, --- 3.5 ($h_Y/L_T = 2$), -·-·- 6.9 ($h_Y/L_T = 4$), ····· 12.0 ($h_Y/L_T = 8$).

Figure 3 presents the vertical profiles of the normalized density at different times. In addition to the initial distribution, the density variation at $t\sqrt{Ag/L_T}$ equal to 3.5, 6.9 and 12.0 is plotted. The mixing zone thickness h_Y , measured by the 1%-points of the mean mass fraction profile, is also indicated. That figure shows that the density jump at $z = 0$ is rapidly reduced due to the mixing process in a time that scales with $\sqrt{L_T/(Ag)}$, the instantaneous Atwood number decreasing with time. By the time that the mixing region grows to $h_Y/L_T = 4$ the regions with static instability of the mean density profile have practically disappeared, clearly observing that the mixing by Rayleigh-Taylor turbulence is restricted to a central zone that scales with the length L_T , as assumed in the previous theoretical analysis.

Figure 4 shows the temporal evolution of the turbulent Mach number. Along with the value at

the center plane, the maximum value, which occurs somewhat off-center toward the upper layer of fluid where the speed of sound is lower, is also plotted.

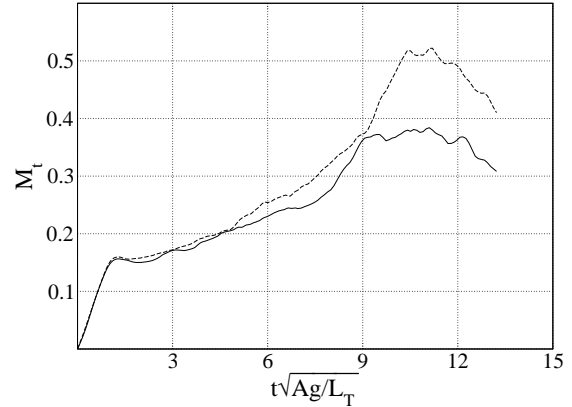


FIG. 4: Temporal evolution of the turbulent Mach number, $M_t = q/\langle c \rangle$: — at the center plane, --- maximum value.

The fact that the mean density evolves in a scale of order L_T as observed in Fig. 3 confirms the overall available potential energy calculated in the theoretical analysis. The speed of sound was estimated by the center plane value and it is observed that M_t at the center plane in the LES is less than the upper bound found in the previous section, approximately 0.5, confirming the analysis. There is a displacement of the maximum value of M_t toward the initially heavy fluid side (upper layer), but the difference observed in Fig. 4 is small enough for the theoretical bounds to hold in general.

In order to further analyze the compressibility of the flow, apart from looking at the turbulent Mach number, it is also customary to split the density fluctuation into an acoustic part and an entropic part, one possible definition being²

$$\begin{aligned} \rho'_{ac} &= p' / \langle c \rangle^2 \\ \rho'_{en} &= \rho' - \rho'_{ac}. \end{aligned} \quad (11)$$

Here, since the flow is practically isothermal, the entropic part originates from the composition fluctuations. Results show that the major part of the fluctuation corresponds to the entropic mode, the acoustic contribution being only 6% of the entropic one at the center plane at $t\sqrt{Ag/L_T} = 3.5$, which is characteristic of a situation with low intrinsic compressibility. At later times, this ratio slightly increases, being 10% at $t\sqrt{Ag/L_T} = 6.9$, which is still a small value. Consistently, the pressure-dilatation term in the transport equation for the turbulent kinetic energy is less than 10% of the buoyancy-production term.

Conclusions

Compressibility effects in Rayleigh-Taylor turbulence with miscible fluids have been studied in a two-layer system formed by a step-like distribution of the ratio between molecular weight and temperature. The density decreases exponentially with increasing height in each layer. It has been shown analytically that the turbulent Mach number is bounded from above, independently of the density jump at the interface. The reason is that the initial thermodynamic state of the system determines the amount of potential energy per unit mass involved in the turbulent mixing stage, and thus the level of turbulent fluctuations that is achievable is linked to the characteristic speed of sound such that the turbulent Mach number is limited.

In the particular case considered here of an ideal gas, this bound on the turbulent Mach number is $M_{t,max} \simeq 0.5$. This value is small enough so that compressibility effects *may* be relatively small. The large-eddy simulations (LES), performed with a density jump at the interface of 3:1, indeed *confirm* that the flow is not significantly affected by intrinsic compressibility, showing that M_t does not exceed the analytical bounds. We note that the Richtmyer-Meshkov configuration, not studied here, can potentially develop a higher turbulent Mach number, since the initial velocity field is set independently of the thermodynamic state.

The compressibility effects have been studied in

the LES decomposing the density fluctuation into the entropic part, due to variation of composition, and the acoustic part, due to intrinsic compressibility. This latter is found to be less than 10% of the total density fluctuation, indicating that intrinsic compressibility effects are indeed small.

A constant-Atwood number configuration defined by a step-like profile of the density itself, the pressure decreasing linearly with height in each layer, has also been considered. The reader is referred to Mellado *et al.*¹⁰ for a thorough presentation. The conclusions are the same as those obtained in the two-layer system presented here, namely, small compressibility effects, and key features such as the quadratic time evolution of the mixing depth, the anisotropy of the Reynolds stresses and the value of the mixing parameters compare well with those observed in the incompressible cases reported in the literature.

Acknowledgments

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