Derivation of a minimal 2-structure, 2-fluid and 2-turbulence (2SFK) model for gravitationally induced turbulent mixing layers.

Antoine Llor, Pascal Bailly and Olivier Poujade

email: olivier.poujade@cea.fr
email: antoine.llor@cea.fr

Introduction.

- Aim of modelling.
- Why introduce the idea of turbulent structure ?
- From least action principle.
- Energy chart of 2SFK.
- Conservative skeleton.
- Complete set of equations of 2SFK model.
- Some closures of flux and exchange.
- Growth rate, mixing rate, level of dissipation $(A_t = 0.2)$.
- Volume fraction of light fluid.
- Volume fraction of light structure.
- Conclusion and future prospects.

Aim of modelling.

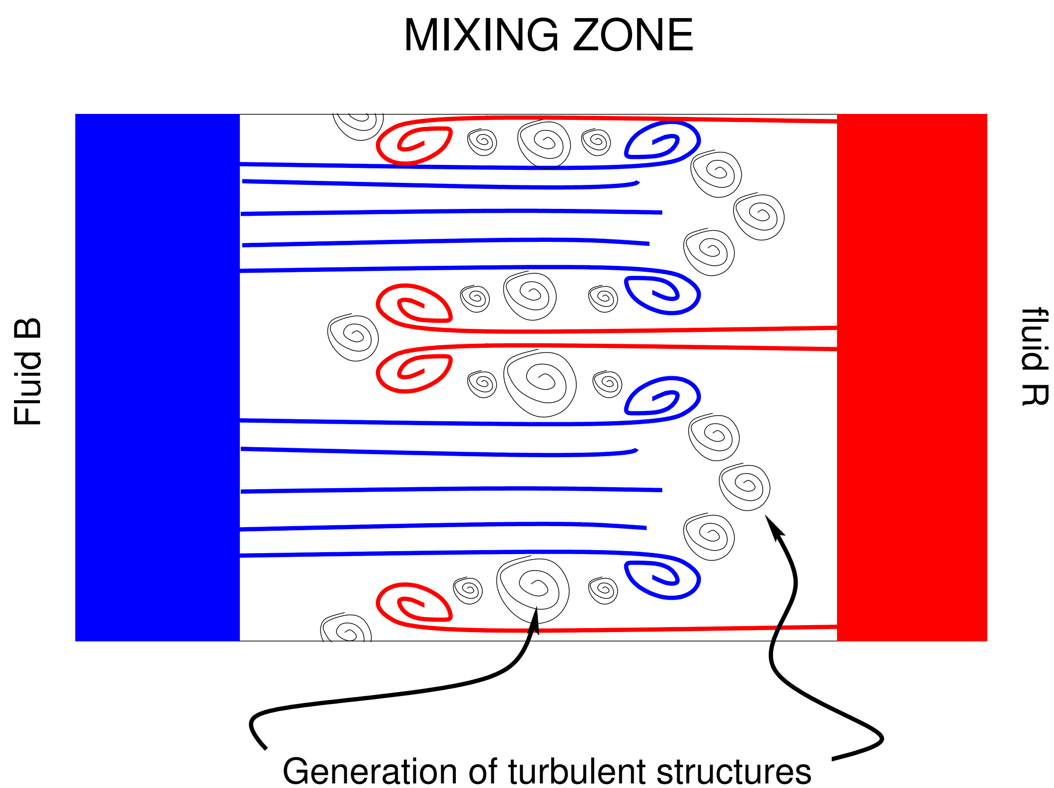
Want to capture some important features of some instabilities :

- Rayleigh-Taylor: when the light fluid pushes the heavy fluid (constant acceleration) $L_s(t) = \alpha_s \mathcal{A} g\, t^2$, $L_b(t) = \alpha_b \mathcal{A} g\, t^2$
- Richtmyer-Meshkov: (impulsive acceleration after a shock) $L_s(t) = \mathcal{L}_s \left(\frac{t}{t_0}\right)^{\theta_s}$, $L_b(t) = \mathcal{L}_b \left(\frac{t}{t_0}\right)^{\theta_b}$

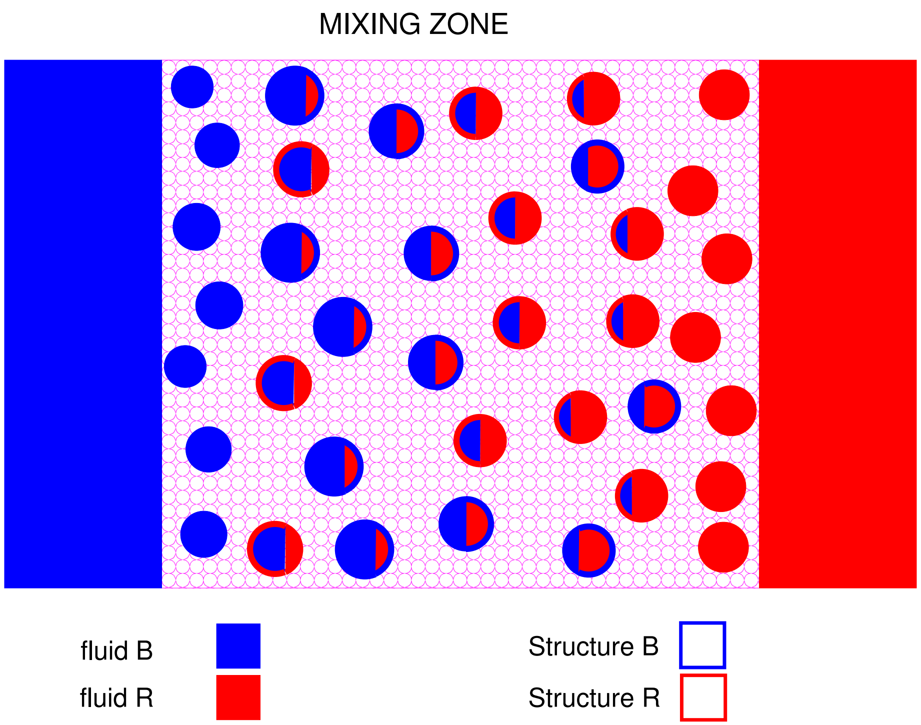
These features are :

- growth rate $\mathcal{Y}_0 = \alpha_s + \alpha_b$ for RT (+ ratio of α_s/α_b), θ_s and θ_s for RM (Youngs 1994; Dimonte et Schneider 1996,1997, 2000; α-group 2004)
- Molecular mixing rate (how fluids have mixed) Θ (Youngs 1991)
- turbulent knudsen number (ratio of turbulent length scale and TMZ width) κ (from Youngs 1994 and α-group 2004).
- Level of turbulence K/K_I (from Youngs 1994 and α-group 2004).
- profiles : volume fractions of fluids must be linear for quasi incompressible fluids at low Atwood ($\leqslant 0.2$).

Why introduce the idea of turbulent structure (1) ?



Why introduce the idea of turbulent structure (2) ?



- For developed turbulent regime.
- Aimed at grasping features of the mixing.
- A fluid is a closed system:
  no exchange possible.
- A structure is an open system:
  exchange possible.
  size of structure $\sim$ turbulent length scale

From least action principle.

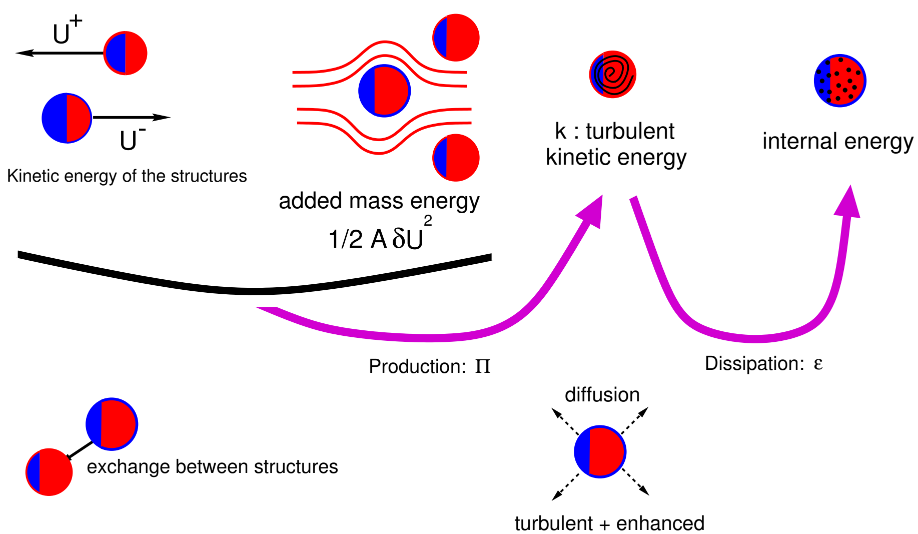
$$S = \int \mathrm{d}^3x\mathrm{d}t \quad \Big\{ \underbrace{\frac{1}{2}\alpha^{\pm}\rho^{\pm}\mathrm{U}^{\pm 2} + \frac{1}{2}\mathcal{A}\delta\mathrm{U}^2}_{kinetic\ energies} - \underbrace{\alpha^{\pm}\rho^{\pm}(k^{\pm} + E^{\pm})}_{potential\ energies}$$

$$\underbrace{\nu^{\pm}\,\mathrm{D_t}^{\pm}(\alpha^{\pm}\rho^{\pm}) + \theta^{\pm}\,\mathrm{d_t}^{\pm}(\mathrm{S}^{\pm}) + \tau^{\pm}\,\mathrm{d_t}^{\pm}(\mathrm{k}^{\pm}/\rho^{\pm 2/3})}_{constraints}\Big\}$$

$$\mathrm{D_t}^{\pm}(A) = \partial_t(A) + \partial_i(\mathrm{U}^{\pm}\,A) \qquad \mathrm{d_t}^{\pm}(A) = \partial_t(A) + \mathrm{U}^{\pm}\,\partial_i(A)$$

- Find the conservative part of the equations out of the least action principle.
- Systematic method $\rightarrow$ conservation of momentum and total energy.

Energy chart of 2SFK.



- $KE_{\text{two}} = \frac{1}{2}\alpha^+\rho^+{U^+}^2 + \frac{1}{2}\alpha^-\rho^-{U^-}^2$

$$= \underbrace{\frac{1}{2}\bar{\rho}V^2}_{KE_{\text{single}}} + \underbrace{\frac{\alpha^+\rho^+\alpha^-\rho^-}{2\bar{\rho}}\,\delta\mathrm{U}^2}_{K_D}$$

KE : Kinetic Energy

K : Turbulent Kinetic Energy

K_D : Directed Energy

- $KE_{\text{single}} + K_{\text{single}} = KE_{\text{two}} + K_{\text{two}}$

✔ $\mathrm{K}_{\text{single}} = \mathrm{K}_{\text{two}} + K_D$

- RM: $K_D \ll K_{\text{two}}$; RT: $K_D \sim K_{\text{two}}$

Conservative skeleton.

$$
\begin{aligned}
\mathrm{D_t}^{\pm}(\alpha^{\pm}\rho^{\pm}\mathrm{C}^{\mathrm{m}\pm}) &= 0 \\
\mathrm{D_t}^{\pm}(\alpha^{\pm}\rho^{\pm}) &= 0 \\
\mathrm{D_t}^{\pm}(\alpha^{\pm}\rho^{\pm}\mathrm{U}_i^{\pm} \pm \mathcal{A}\delta\mathrm{U}_i) &= -\alpha^{\pm}\partial_i(\mathrm{P}+\mathrm{P_t}) + \mathcal{S}_i[\mathcal{A}] \\
\mathrm{D_t}^{\pm}(\alpha^{\pm}\rho^{\pm}\mathrm{E}^{\pm}) &= \alpha^{\pm}\mathrm{P}\frac{\mathrm{d_t}^{\pm}(\rho^{\pm})}{\rho^{\pm}} \\
\mathrm{D_t}^{\pm}(\alpha^{\pm}\rho^{\pm}\mathrm{k}^{\pm}) &= \alpha^{\pm}\frac{2}{3}\alpha^{\pm}\rho^{\pm}\mathrm{k}^{\pm}\frac{\mathrm{d_t}^{\pm}(\rho^{\pm})}{\rho^{\pm}}
\end{aligned}
$$

Complete set of equations of 2SFK model.

- 2SFK stands for 2-Structure, 2-Fluid and 2-K-ϵ
- Conservation per structure of ...

$$
\begin{aligned}
\text{[fluid mass]}\quad \mathrm{D_t}^{\pm}(\alpha^{\pm}\rho^{\pm}\mathrm{C}^{\mathrm{m}\pm}) &= -\Phi^{m\pm}_{j,j}\mp\Psi^{m} \\
\text{[mass]}\quad \mathrm{D_t}^{\pm}(\alpha^{\pm}\rho^{\pm}) &= \mp\Psi \\
\text{[momentum]}\quad \mathrm{D_t}^{\pm}(\alpha^{\pm}\rho^{\pm}\mathrm{U}_i^{\pm} \pm \mathcal{A}\delta\mathrm{U}_i) &= -\alpha^{\pm}\partial_i(\mathrm{P}+\mathrm{P_t}) + \mathcal{S}_i[\mathcal{A}] \\
&\quad -\alpha^{\pm}\mathrm{Q}_{ij,j} - \left(\mathrm{R}_{ij} - \frac{2}{3}\alpha^{\pm}\rho^{\pm}\mathrm{k}^{\pm}\delta_{ij}\right)_{,j} \mp (D_i + \Psi_i^{H}) \\
\text{[internal energy]}\quad \mathrm{D_t}^{\pm}(\alpha^{\pm}\rho^{\pm}\mathrm{E}^{\pm}) &= -\mathrm{P}(\mathrm{D_t}^{\pm}(\alpha^{\pm})\pm\Psi^{\alpha}) + \alpha^{\pm}\rho^{\pm}\Sigma^{\pm} + \alpha^{\pm}\rho^{\pm}\epsilon^{\pm} - \Phi^{E\pm}_{j,j}\mp\Psi^{E} \\
\text{[turbulence : k]}\quad \mathrm{D_t}^{\pm}(\alpha^{\pm}\rho^{\pm}\mathrm{k}^{\pm}) &= \frac{2}{3}\rho^{\pm}\mathrm{k}^{\pm}(\mathrm{D_t}^{\pm}(\alpha^{\pm})\pm\Psi^{\alpha}) + \Pi^{\pm} - \alpha^{\pm}\rho^{\pm}\epsilon^{\pm} - \Phi^{k\pm}_{j,j}\mp\Psi^{k} \\
\text{[dissipation : }\epsilon\text{]}\quad \mathrm{D_t}^{\pm}(\alpha^{\pm}\rho^{\pm}\epsilon^{\pm}) &= -\left(\frac{2}{3}\mathrm{C}_{\epsilon 1} + \mathrm{C}_{\epsilon 3}\right)\rho^{\pm}\epsilon^{\pm}(\mathrm{D_t}^{\pm}(\alpha^{\pm})\pm\Psi^{\alpha}) + \mathrm{C}_{\epsilon 1}\frac{\epsilon^{\pm}}{k^{\pm}}\Pi^{\pm} - \mathrm{C}_{\epsilon 2}\alpha^{\pm}\rho^{\pm}\frac{\epsilon^{\pm 2}}{k^{\pm}} \\
&\quad -\Phi^{\epsilon\pm}_{j,j}\mp\Psi^{\epsilon}
\end{aligned}
$$

Some closures of flux and exchange.

$$\lambda^{\pm} = \frac{k^{\pm 3/2}}{\epsilon^{\pm}} \quad \text{Turbulent length scale}$$

$$\nu^{\pm} = \mathrm{C}_{\mu} \frac{k^{\pm 2}}{\epsilon^{\pm}} \quad \text{Turbulent viscosity} \quad \mathrm{C}_{\mu} = 0.09$$

$$\lambda = \frac{\lambda^{+4} + \lambda^{-4}}{\lambda^{+3} + \lambda^{-3}} \sim \max(\lambda^{+}, \lambda^{-}) \quad \text{Exchange length scale}$$

$$\omega_{\Psi} = \frac{1}{\mathrm{C}_{\mu}} \frac{\nu^{+} + \nu^{-}}{\lambda^{2}} \quad \text{Exchange time scale}$$

$$D_s^{\pm} = 4\mathrm{C_s}\, \alpha^{+}\alpha^{-}\lambda \, || \, \delta \mathbf{U} \, || \quad \text{Sifting diffusion}$$

$$D_i + \Psi_i^H = \frac{\alpha^{-}\rho^{-}H_i^{+} + \alpha^{+}\rho^{+}H_i^{-}}{\alpha^{+}\rho^{+} + \alpha^{-}\rho^{-}} \Psi + \mathrm{C_d} \alpha^{+}\alpha^{-}\omega_{\Psi} \frac{(\alpha^{+}\rho^{+})^2 + (\alpha^{-}\rho^{-})^2}{\alpha^{+}\rho^{+} + \alpha^{-}\rho^{-}} (\delta H_i - H_i^o) \quad \text{Drag}$$

$$H_i^o = -\left(1 + \frac{(\alpha^{+}\rho^{+} + \alpha^{-}\rho^{-})\mathcal{A}}{\alpha^{+}\rho^{+}\alpha^{-}\rho^{-}}\right) \left[D_s^{+} \frac{(\alpha^{+}\rho^{+})_{,i}}{\alpha^{+}\rho^{+}} - D_s^{-} \frac{(\alpha^{-}\rho^{-})_{,i}}{\alpha^{-}\rho^{-}} \right] \quad \text{Diffusive drift}$$

$$\Phi_i^{A\pm} = -\alpha^{\pm}\rho^{\pm} \left(\frac{\nu^{\pm}}{\sigma_A} + D^{\pm} \right) \left[A_{,i}^{\pm} - P_A \left(\frac{\rho_{,i}^{\pm}}{\rho^{\pm 2}} + \sum_m \frac{C_{,i}^{m\pm}}{\rho^{m\pm}} \right) \right] \quad \text{Flux of A}$$

$$\Psi^A = \frac{1}{2} \mathrm{C}_{\epsilon c}\, \alpha^{+}\alpha^{-}\omega_{\Psi} (\alpha^{+}\rho^{+}A^{+} - \alpha^{-}\rho^{-}A^{-}) \quad \text{Exchange of A}$$

Growth rate, mixing rate, level of dissipation ($A_t = 0.2$).

Rayleigh Taylor

Atwood 0.2

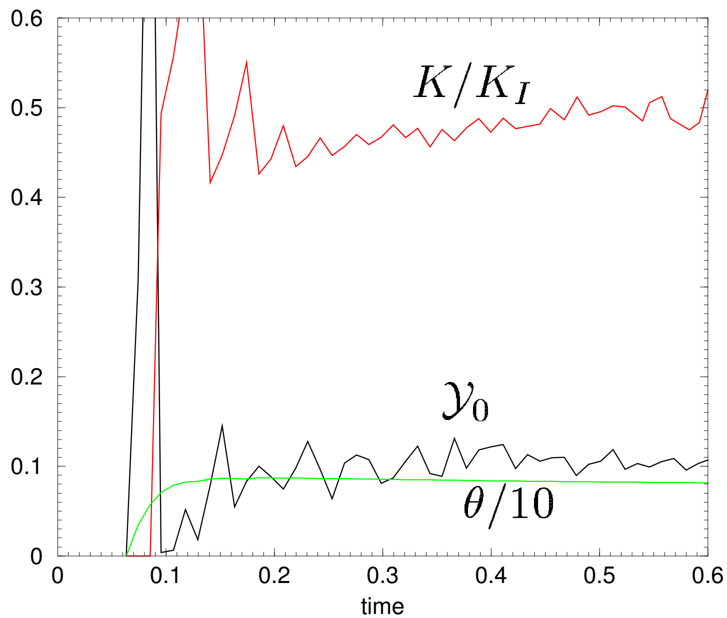
reference:

(α-group results)

$K/KI \sim 0.46$

$\mathcal{Y}_0 \sim 0.12$

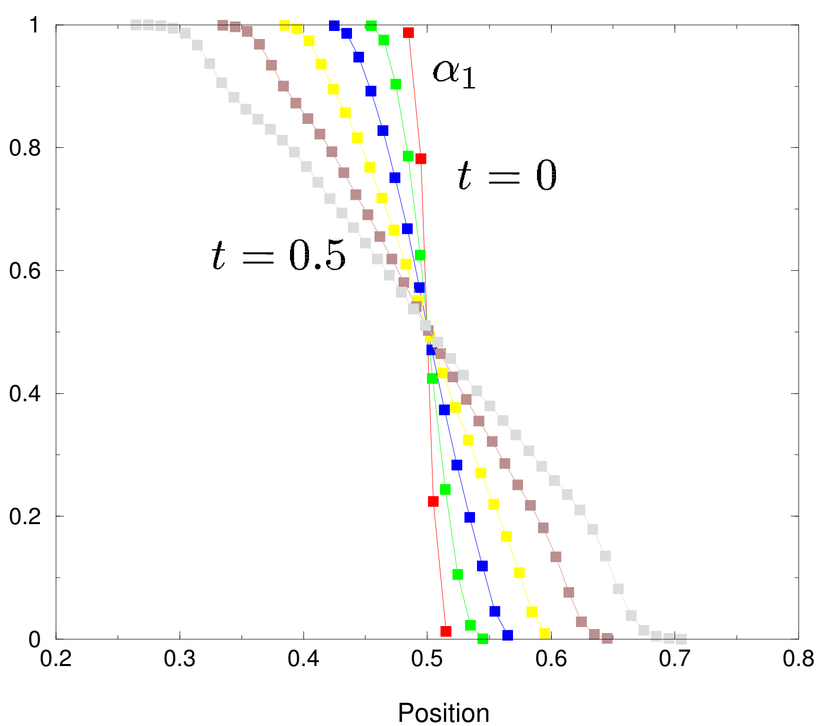
$\theta \sim 0.8$



- ✔ K is the single-fluid turbulent kinetic energy.
- ✔ K_I is the gravitational energy input.
- ✔ $\mathcal{Y}_0$ is the growth rate.
- ☛ $L(t) = \mathcal{Y}_0 A_t g t^2$
- ✔ θ is the molecular mixing rate.

Volume fraction of light fluid.

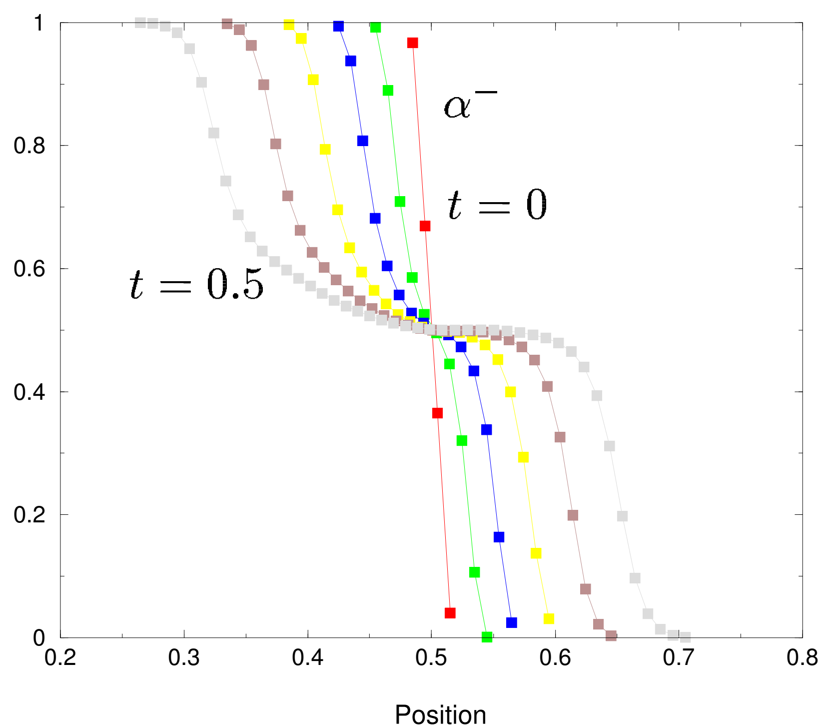
elf similar evolution
developed turbulence
in the TMZ)



☛ Volume fractions of fluids are linear over the TMZ for low Atwood 0.2. In agreement with EXP and DNS

Volume fraction of light structure.

elf similar evolution
developed turbulence
in the TMZ)



- There is a significant part of the TMZ where $\alpha^+ \approx \alpha^-$
- DNS simulation would allow to verify these profiles

Conclusion and future prospects.

- ✔ From the least action principle the set of equations conserves momentum and total energy.
- ✔ There are 3 adjustable constants $\mathrm{C}_{\epsilon\mathrm{c}}$ (exchange), C_{s} (sifting), C_{d} (drag). One is fixed for a physical reason $\mathrm{C}_{\epsilon\mathrm{c}} = 1.01$. $\mathrm{C}_{\mathrm{s}} = 1.0$ and $\mathrm{C}_{\mathrm{d}} = 0.9$ are adjusted to fit profiles and to match growth rate, molecular mixing rate, turbulent Knudsen number and Level of turbulence.
- ✔ 1D code called BBB (fully explicit, bi-lagrange and remap, second order in space and time).
- ☛ How do we model shocks in mixing layers ?