Volume fraction profiles of transport structures in Rayleigh-Taylor turbulent mixing zone: evidence of enhanced diffusion processes

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I. Motivations: 2SFK modeling

A Two-Structure Two-Fluid Two-Turbulence (2SFK) model was first presented in 2001 by A. Llor & P. Bailly at the 8th IWPCTM in Pasadena, which:

- had all the basic physics deemed important,
- but was obtained
  by assembling separate independent elements,
- and involved
  an empirical enhanced diffusion for stability.

Today, our new version of 2SFK:

- has the same basic physics,
- follows from a unified derivation
  (least action principle
  and extended thermodynamics of irreversible processes,
  see other presentation by A. Llor, P. Bailly & O. Poujade),
- and still involves enhanced diffusion which is

here shown to be a *general feature*
of *any* Two-Structure Two-Fluid (2SF) model.

Well-known example of 2SF model other than 2SFK:

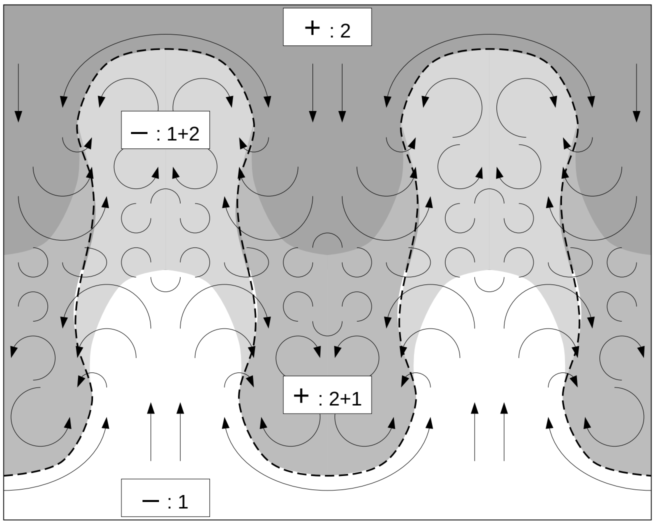
Youngs' model (1995).

II. Basic mass transport equations in two-structure two-fluid models

Most models of Rayleigh–Taylor turbulent mixing involve:

i) light $\boxed{-}$ and heavy $\boxed{+}$ *structures* as basic transport entities,

ii) made of mixed or entrained light $\boxed{1}$ and heavy $\boxed{2}$ *fluids*.



Examples: Youngs', Bubble models, Dimonte's, Dalziel's...
and 2SFK.

Mass conservation equations gives transport (1D):

$$\mathrm{D}_t^{\pm}\alpha^{\pm}\rho^{\pm}c^{m\pm} = -\Phi^{m\pm}_{x,x} \mp \Psi^m,$$

$\mathrm{D}_t^{\pm}a = \partial_t a + (aU_x^{\pm})_{,x}$,
$U_x^{\pm}$ = velocity of structure $\pm$,
$\alpha^{\pm}$ = volume fraction of structure $\pm$,
$\rho^{\pm}$ = density of structure $\pm$,
$c^{m\pm}$ = mass fraction of fluid m in structure $\pm$,
$\Phi_x^{m\pm}$ = flux of fluid m in structure $\pm$ ($\Phi_x^{2\pm} + \Phi_x^{1\pm} = 0$),
Ψ^m = exchange of fluid m between structures $\pm$.

Assuming incompressibility:

$$\mathsf{D}_t^{\pm}\alpha^{m\pm} = -\left(\frac{\Phi_x^{m\pm}}{\rho^m}\right)_{,x} \mp \frac{\Psi^m}{\rho^m}.$$

$\alpha^{m\pm} =$ volume fraction of fluid m of structure $\pm$,
$\rho^m =$ density of fluid m (constant),

$$\alpha^{m\pm}\rho^m = \alpha^{\pm}\rho^{\pm}c^{m\pm}.$$

"Solve" these equations using
experimental data and basic assumptions:

- $At \to 0$, zero Atwood limit,
- $L(t) = 2\alpha \mathrm{At} g t^2$, $2\alpha \approx .08$
  TMZ width self-similar growth, $\xi = x/L$,
- $\alpha^m(\xi) = \alpha^{m+} + \alpha^{m-}$,
  self-similar fluid fraction profiles (linear),
- $\Theta \approx .8 \approx 6\langle \frac{\alpha^{2+}\alpha^{1+}}{\alpha^{2+}+\alpha^{1+}} + \frac{\alpha^{2-}\alpha^{1-}}{\alpha^{2-}+\alpha^{1-}}\rangle$, molecular mixing ratio,
- $\Phi_x^{m\pm} = -D^{\pm}(\xi)(LL') \times \alpha^{\pm}(\rho^1\rho^2/\rho^{\pm})\,(\alpha^{m\pm}/\alpha^{\pm})_{,x}$,
  usual "first gradient" closure, $D^{\pm} \geq 0$,
- $\Psi^m = C(\xi)\,(L'/L) \times\ \alpha^{+}\alpha^{-}\rho^m(\alpha^{m+} - \alpha^{m-})$,
  "first gradient" closure, $C \geq 0$,
- $\alpha^{m\pm}$, assumed profiles,
  simplest consistent with previous constraints.
- $C(\xi) = C$, uniform exchange rate.

4 equations with $\alpha^{m\pm}$ given

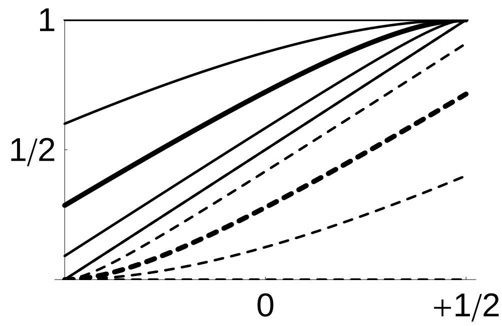
$\downarrow$

4 unknown profiles ($U_x^{\pm}$, $D^{\pm}$) and 1 constant (C)

III. Characteristic volume fraction profiles

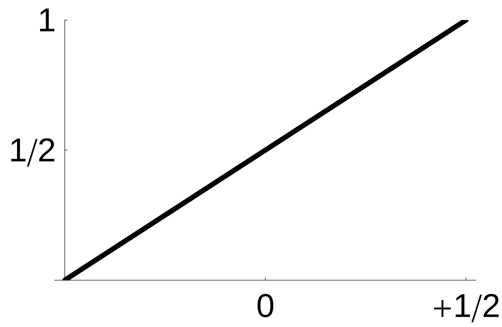
Algebraically-simplest density profiles of structures of zero slope with pure fluids at TMZ edges, and adjusted by pole at ξ_0 for mixing level:

$$\frac{\rho^+ - \rho^2}{\rho^2 - \rho^1} = -\frac{(\tfrac{1}{2} - \xi)^2}{\xi_0 + \tfrac{1}{2} - \xi},$$
$$\frac{\rho^- - \rho^1}{\rho^2 - \rho^1} = \frac{(\tfrac{1}{2} + \xi)^2}{\xi_0 + \tfrac{1}{2} + \xi}.$$

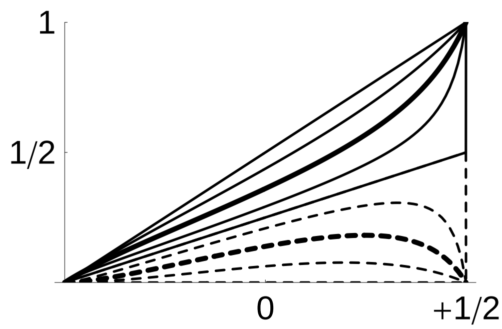


Fluid volume fractions:

$$\alpha^2 = \tfrac{1}{2} + \xi,$$
$$\alpha^1 = \tfrac{1}{2} - \xi.$$

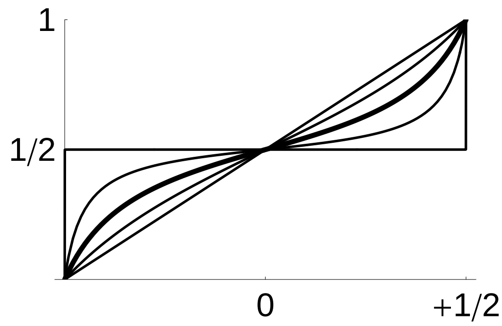


$$\alpha^{2+} = (\tfrac{1}{2} + \xi)\,\frac{\xi_0 + \tfrac{1}{4} - \xi^2}{\xi_0 + \tfrac{1}{2} - 2\xi^2},$$
$$\alpha^{2-} = (\tfrac{1}{2} + \xi)\,\frac{\tfrac{1}{4} - \xi^2}{\xi_0 + \tfrac{1}{2} - 2\xi^2},$$
$$\alpha^{1+} = (\tfrac{1}{2} - \xi)\,\frac{\tfrac{1}{4} - \xi^2}{\xi_0 + \tfrac{1}{2} - 2\xi^2},$$
$$\alpha^{1-} = (\tfrac{1}{2} - \xi)\,\frac{\xi_0 + \tfrac{1}{4} - \xi^2}{\xi_0 + \tfrac{1}{2} - 2\xi^2}.$$



Structure volume fractions:

$$\alpha^{\pm} = (\tfrac{1}{2} \pm \xi)\,\frac{\xi_0 + \tfrac{1}{2} \mp \xi}{\xi_0 + \tfrac{1}{2} - 2\xi^2}.$$



$\Theta = .8$ obtained at $\xi_0 \approx .38$ (thick lines).

IV. Solution of mass transport equations

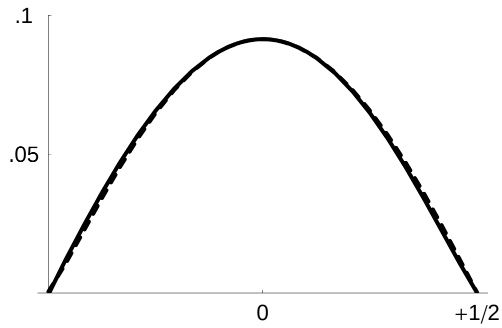
With $\alpha^{m\pm}$ as above, solutions given by simple quadratures of algebraic functions, most conveniently computed numerically and not explicited here.

Limit conditions on $\alpha^{m\pm}$ and $\Phi_x^{m\pm}$ give:

$$C = \frac{\int_{-1/2}^{+1/2} \xi \alpha^{2+}_{,\xi} \mathrm{d}\xi}{\int_{-1/2}^{+1/2} \alpha^+\alpha^-(\alpha^{2+} - \alpha^{2-})\mathrm{d}\xi} \approx 2.$$

Scaled diffusion coefficients D^+ (solid) and D^- (dashed) almost identical.

$D^\pm(0) \approx .0914$.

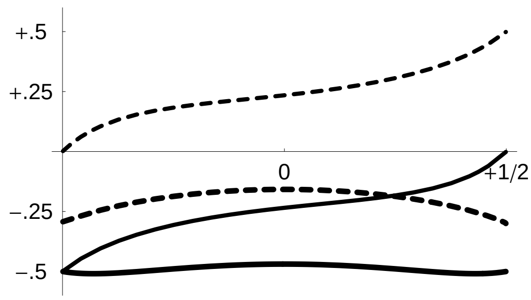


Scaled structure velocities U^+/L' (solid) and U^-/L' (dashed) with nearly constant drift (thick):

$$\delta U_x = U_x^+ - U_x^-,$$

and $\approx 1/4$ diffusive contribution (thick dashed):

$$\delta U_x^\circ = -D^+ \frac{(\alpha^+\rho^+)_{,x}}{\alpha^+\rho^+} + D^- \frac{(\alpha^-\rho^-)_{,x}}{\alpha^-\rho^-}.$$



Structure directed energy larger than *fluid* directed energy:

$$\int \alpha^+\alpha^- \frac{(\delta U_x)^2}{2} \mathrm{d}\xi \approx 1.19 \times \int \alpha^2\alpha^1 \frac{(U_x^2 - U_x^1)^2}{2} \mathrm{d}\xi = 1.19 \times \frac{(L')^2}{48}.$$

V. General properties of solutions

Basic properties do not depend on:

- $\alpha^{m\pm}$ profile details (if $\ominus$ preserved),
- $C(\xi)$ non uniformity (up to factor ≈ 2).

More specifically:

- $D^{\pm}(\xi) \neq 0$ at $\xi \neq 0$ if $\ominus \neq 0$
  (otherwise $\exists \xi$ such that $C(\xi) < 0$),
- $D^{\pm}(\pm 1/2) = 0$
  (otherwise $(\alpha^{2+}/\alpha^{+})_{,x} \neq 0$ at $\xi = +1/2$),
- $\boxed{C(\xi) \approx 2}$,
- $D^{\pm}$ bell shaped, $\boxed{D^{\pm}(0) \approx .1}$,
- $\delta U_x(\xi)/L' \approx .5$,
- *Structure* directed energy
  $\approx 20\%$ higher than *fluid* directed energy.

VI. Consistency of $D^{\pm}$ and C with usual k–ϵ estimates

Observationnal data (α-group)
on self-similar Rayleigh-Taylor TMZ:

- TMZ growth coefficient $\alpha \approx .04$ ($L = 2\alpha \mathrm{At} g t^2$),
- reduced potential energy $\dfrac{K_I}{(L')^2} \approx \frac{1}{96\alpha} = .26$,
- *fluid* directed/potential energy ratio $\dfrac{K_D}{K_I} \approx 2\alpha$,
- kinetic/potential energy ratio $\dfrac{K_T}{K_I} \approx .48$,

(consistent selection of α and K_T/K_I values).

Energy balance $\frac{\mathrm{d}}{\mathrm{d}t}(LK_I) = \frac{\mathrm{d}}{\mathrm{d}t}(LK_T) + LE$, yields E.

Then standard k–ϵ closures of rate and diffusion:

$$C_t = \frac{E}{K_T} \times \frac{L}{L'} = 2\left(\frac{K_I}{K_T} - 1\right),$$

$$D_t(0) = \frac{3}{2}\frac{C_\mu}{\sigma_c}\frac{K_T^2}{E} \times \frac{1}{LL'} = \frac{3}{4}\frac{C_\mu}{\sigma_c}\frac{K_T}{(L')^2}\left(\frac{K_I}{K_T} - 1\right)^{-1},$$

and with observed ratios and $C_\mu/\sigma_c \approx .09/.7$:

$$C_t \approx 1.04 \times C,$$
$$D_t(0) \approx .111 \times D(0).$$

Exchange rate correctly captured by k–ϵ approach, diffusion *underestimated by an order of magnitude*!

VII. Energy budget analysis of RT TMZ in adapted k–ϵ approach

Question: are velocity fluctuations responsible for enhanced $D^{\pm}$ associated with new specific turbulent energy reservoir?

Simple model adapted from k–ϵ (similar to 2SFK) where large-scale non-turbulent energies (K_D, added mass...) *are not mirrored in* E:

$$\frac{\mathrm{d}}{\mathrm{d}t}(LK_I) = L\Pi_I,$$

$$\frac{\mathrm{d}}{\mathrm{d}t}(LK_{DA}) = L\Pi_I - L\Pi,$$

$$\frac{\mathrm{d}}{\mathrm{d}t}(LK) = L\Pi - LE,$$

$$\frac{\mathrm{d}}{\mathrm{d}t}(LE) = LC_{\epsilon 1}\frac{E}{K}\Pi - LC_{\epsilon 2}\frac{E^2}{K},$$

(Π_I and Π = gravity and drag works).

Under self similar RT growth, yields:

$$\frac{K_{DA}}{K_I} = C_\epsilon \frac{K_T}{K_I} - C_\epsilon + 1, \quad \text{where} \quad C_\epsilon = \frac{4\,C_{\epsilon 2} - 3}{4\,C_{\epsilon 1} - 3},$$

$\approx .118$ for K_T/K_I as above, $C_{\epsilon 2} \approx 1.92$, $C_{\epsilon 1} \approx 1.44$.

Observed: $K_{DA}/K_I = 1.2 \times 1.5 \times (K_D/K_I) \approx .144$

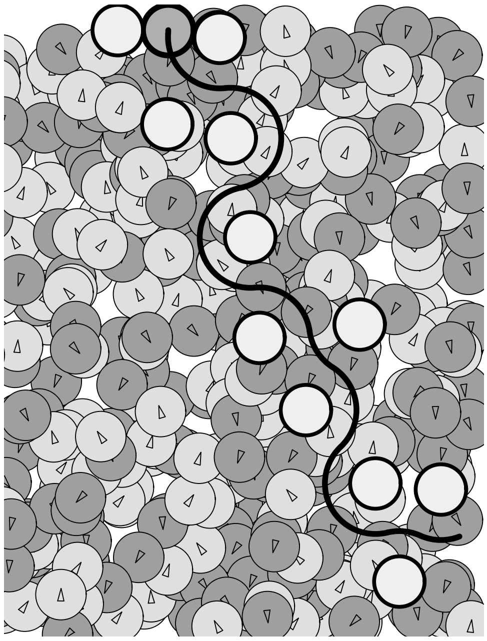
with corrections for structure/fluid and added mass.

No significant energy reservoir other than directed and added mass is expected to be associated with enhanced diffusion.

VIII. Modeling and experiment suggestions

Possible consistent diffusion process:

sifting diffusion or "Pachinko" effect between opposite flowing structures.

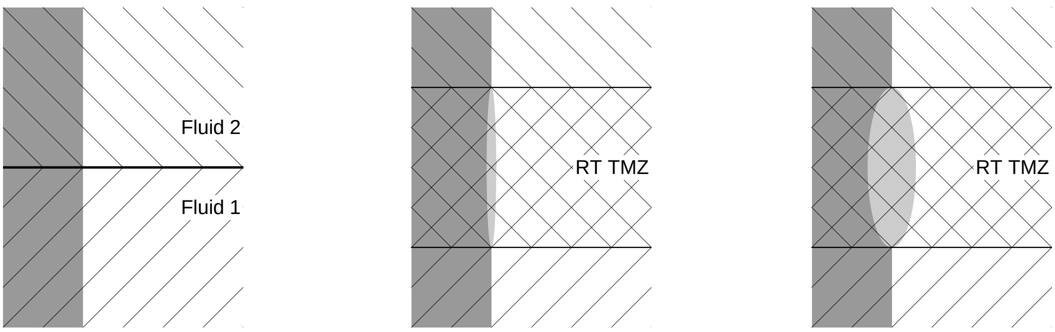


Consistent closure:

$$D^{\pm} = D = 4C_s\alpha^{+}\alpha^{-}\lambda\|\delta\vec{U}\|,$$

where λ is characteristic structure size and $C_s \approx 1$.
$\lambda \approx L/5$ in RT (α-group).
May stabilize models by making them parabolic.

Observationnal confirmation of enhanced diffusion could be achieved despite lack of structure determination methods:



Initial state | Standard k–ϵ diffusion | Consistent enhanced diffusion

Ultimately, structure analysis techniques will provide global insight (not only on diffusion).