TURBULENT MIXING BY BUOYANCY DRIVEN RAYLEIGH-TAYLOR INSTABILITY

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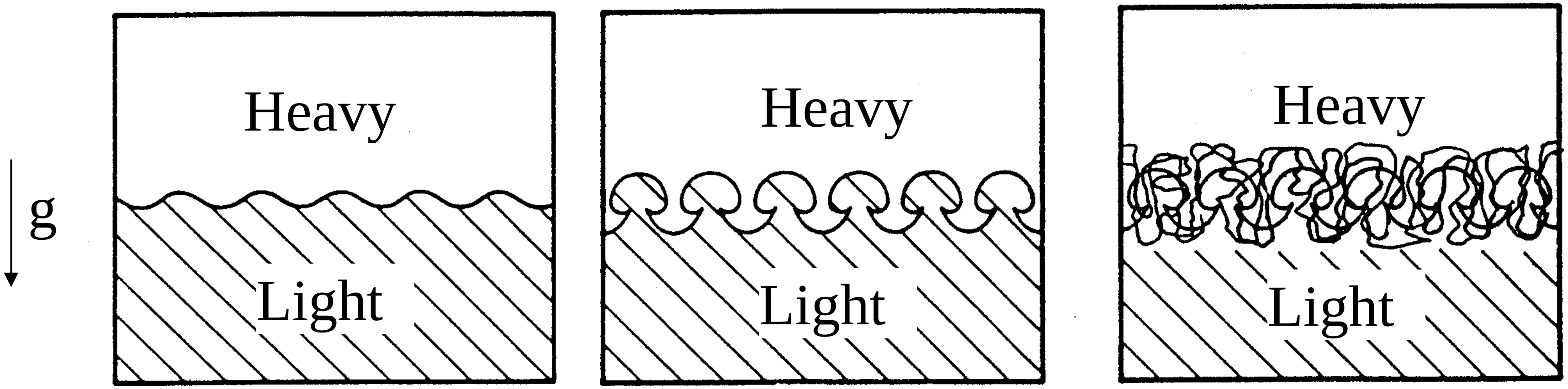
What is Rayleigh-Taylor Mixing?



Linear growth Non-linear growth Turbulent mixing

Main non-dimensional number: Atwood: $At \equiv (\rho_1 - \rho_2)/(\rho_1 + \rho_2)$

Interface is unstable if: $\nabla p \bullet \nabla \rho < 0$

Baroclinic generation of vorticity: $\frac{1}{\rho^2} \nabla p \times \nabla \rho$

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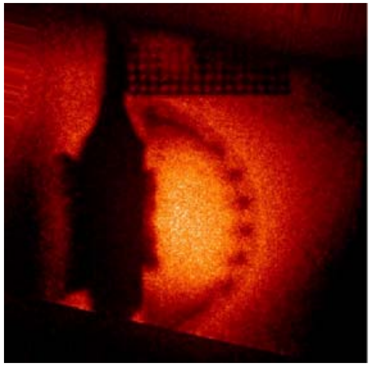
Applications

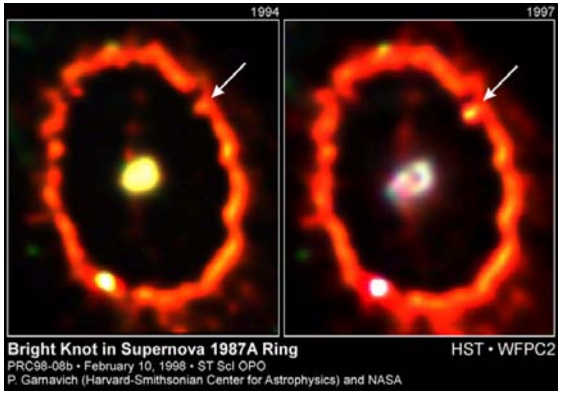


Technology:

- Degradation of ICF capsules (10^{-12}s).
- Formation of oil trapping salt domes (10^{15}s).
- Counter-gradient transport in engine cylinders with swirl.
- Modulation of heat transfer with twisted tapes in tubes.
- Atmospheric temperature inversions (clear air turbulence).
- Multi-phase mixing - drop disintegration.

Space:

- Super-Nova Remnants (SN1987A).
- g-Jitter - Bridgman crystal growth.



Bright Knot in Supernova 1987A Ring
PRC98-08b • February 10, 1998 • ST ScI OPO
P. Garnavich (Harvard-Smithsonian Center for Astrophysics) and NASA

Overview

- Rayleigh-Taylor *mix* experiments are difficult!
- Modern turbulent mix models involve statistical quantities and demand extensive experimental data sets for validation.
- Transient Rayleigh-Taylor experiments do not lend themselves to statistical data collection.
- Over the past 8 years we have developed a statistically steady R-T experiment that facilitates statistical data collection.
- Our Rayleigh-Taylor mix data is used to validate models for the description and understanding of hydrodynamic instabilities that develop during the implosion phase of ICF capsules.

Previous Experiments

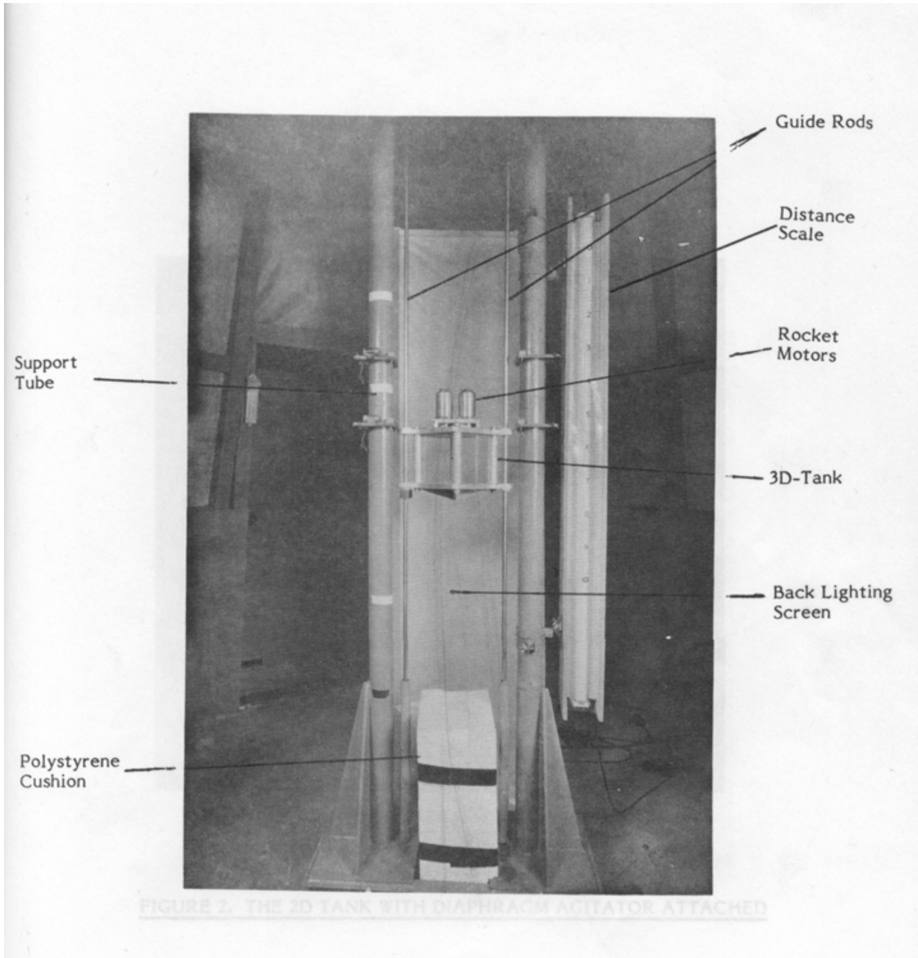
- Read (1980) - Rocket Rig ($0 \leq At \leq 1$).
- Andrews (PhD, 1986) - Overturning tank ($0 \leq At \leq 0.05$).
- Redondo & Linden, Dalziel (1989) - Sliding plate ($0 \leq At \leq 0.05$).
- Dimonte (1992) - LEM ($0 \leq At \leq 1$).
- Kucherenko (1991) - High acceleration ($0 \leq At \leq 1$).

Previous Experiments cont.

Ken Read (1980)

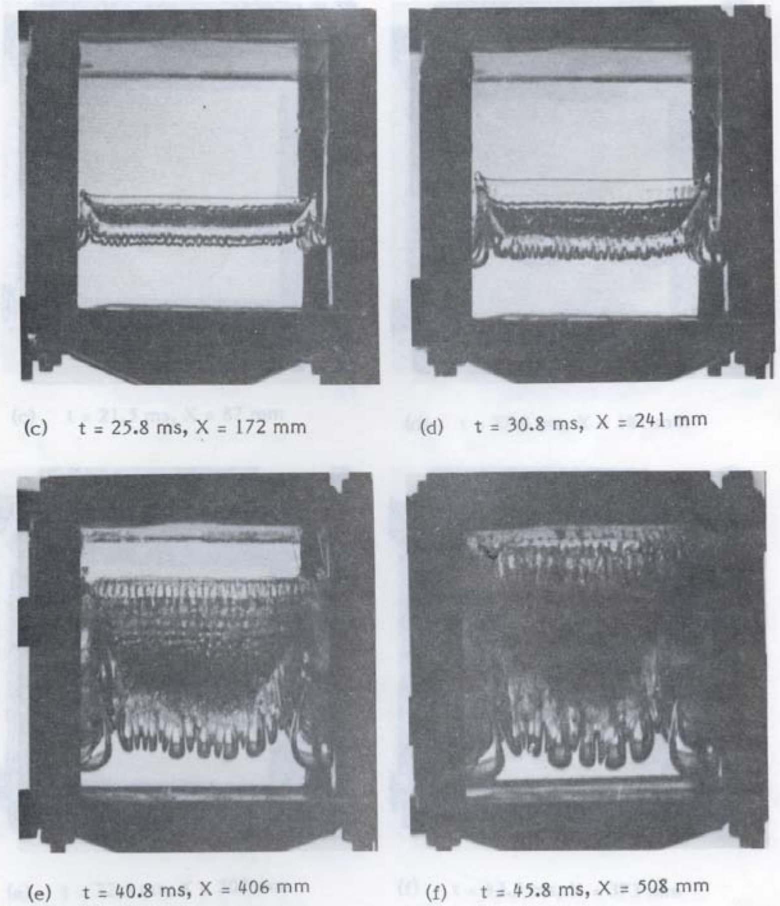
The “Rocket Rig”

Aldermaston, UK.



Previous Experiments cont.

Rocket rig



(c) t = 25.8 ms, X = 172 mm

(d) t = 30.8 ms, X = 241 mm

(e) t = 40.8 ms, X = 406 mm

(f) t = 45.8 ms, X = 508 mm

FIGURE 10. EXPERIMENT 29, ALCOHOL/AIR, 3D TANK, NO IMPOSED PERTURBATIONS

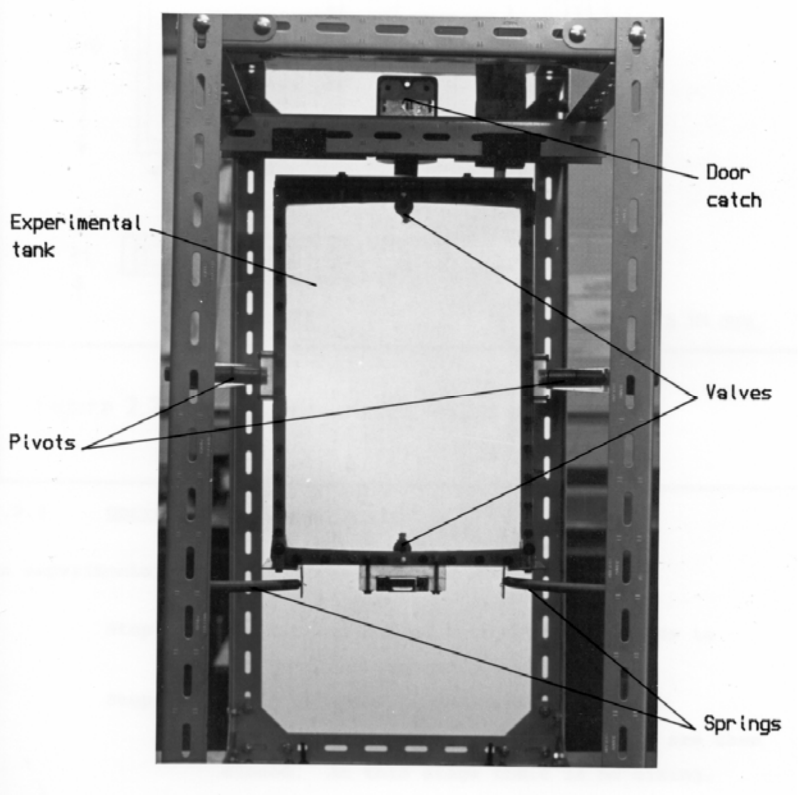
Previous Experiments cont.

Andrews, PhD (1986).

The “2-D Turning Tank”.

Imperial College, UK.

Tank size: 25cm x 36cm x 0.5cm



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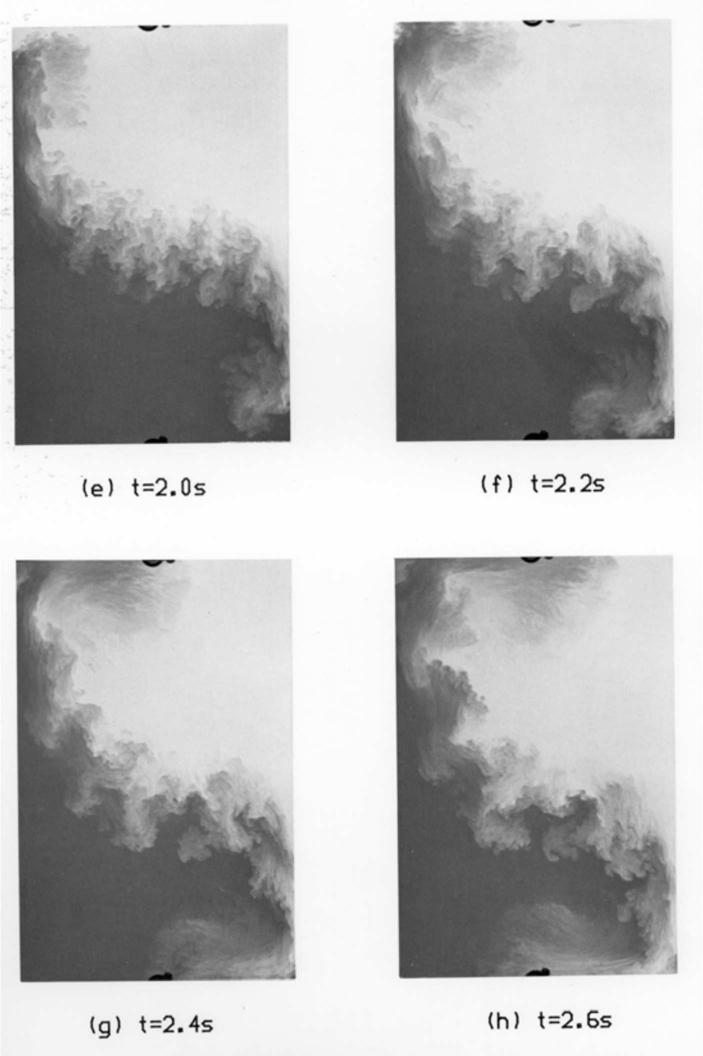
Previous Experiments cont.

2-D Turning Tank - Tilted-rig

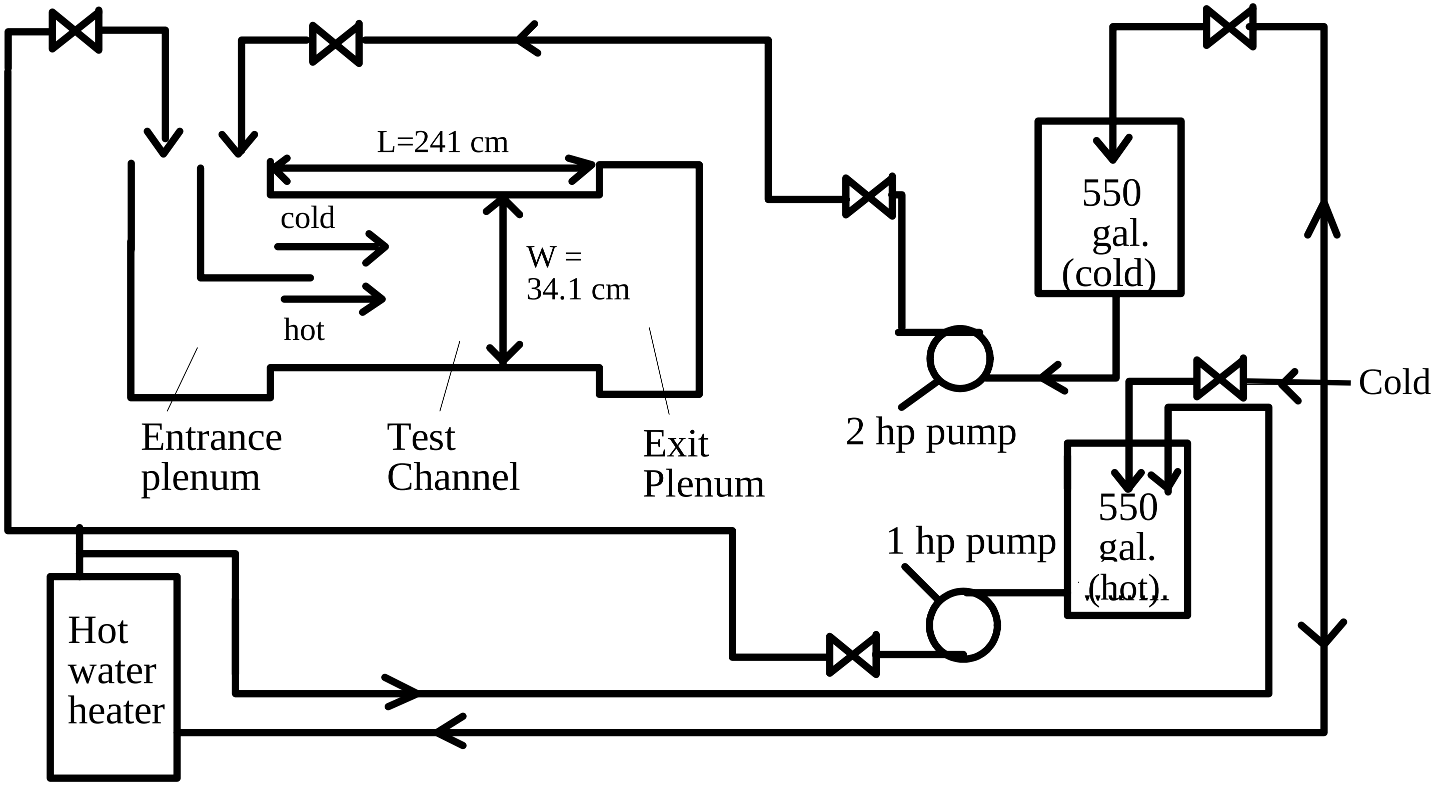
Tilt angle = 55’

ρ_1=1.1 g/cm^3 (brine)

ρ_2=1.0 g/cm^3 (water)



Schematic of TAMU R-T Water Channel



Experimental Details

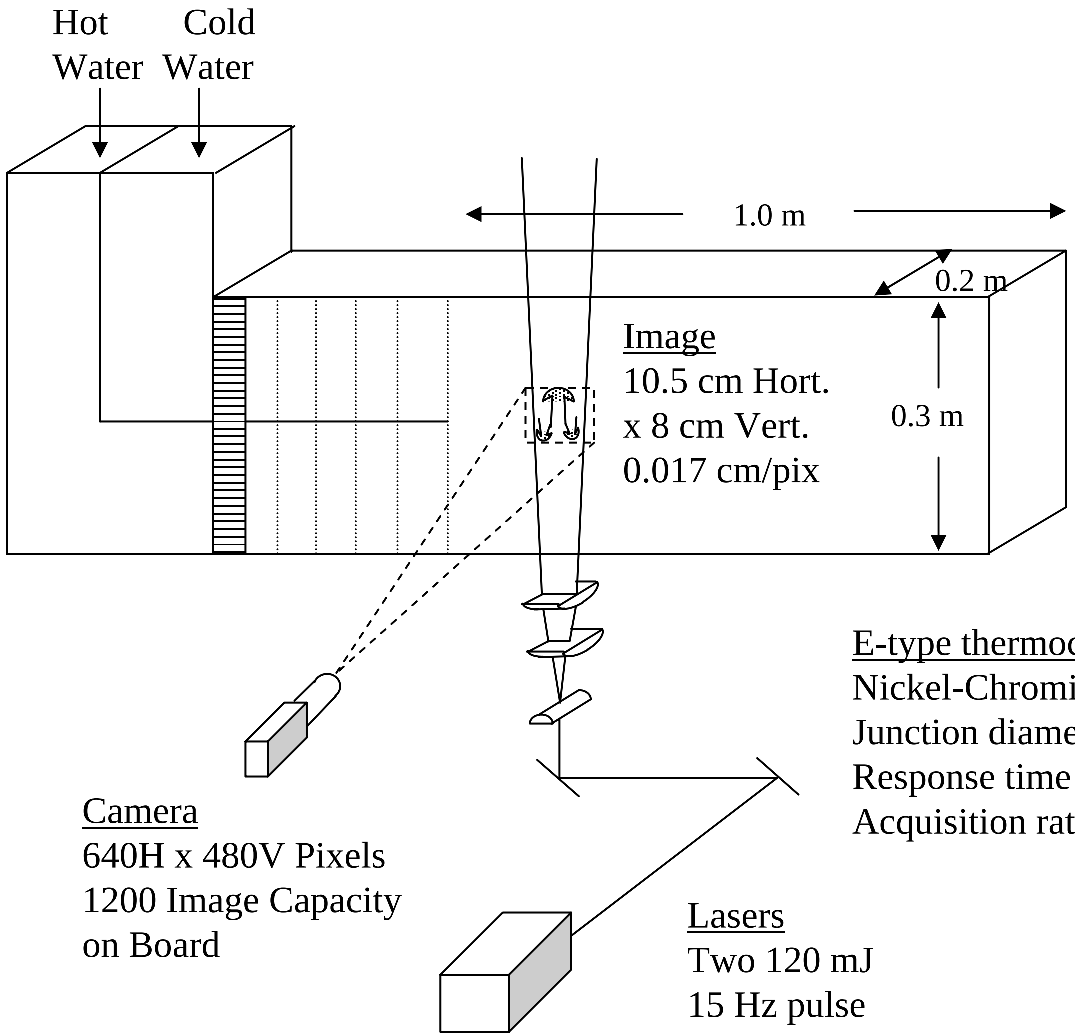


Image
10.5 cm Hort.
x 8 cm Vert.
0.017 cm/pix

PIV particle
concentration:
3mL/2000 L

E-type thermocouples
Nickel-Chromium and Constantan
Junction diameter of 0.01-0.02 cm
Response time 0.001 s/^{o}C
Acquisition rate 8 kHz.

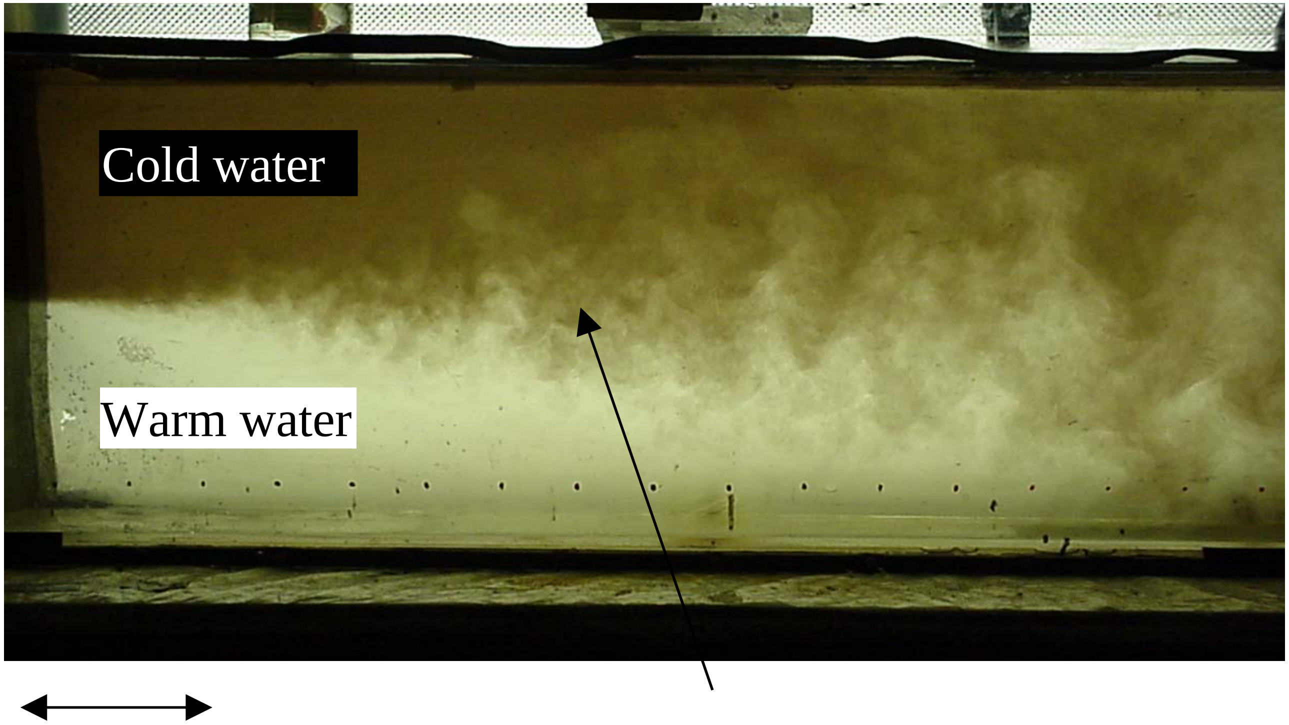
Camera
640H x 480V Pixels
1200 Image Capacity
on Board

Lasers
Two 120 mJ
15 Hz pulse
Sample rate: 30/sec.

R-T Water Channel

- Statistically steady.
- Density difference by hot/cold water thermal expansion.
- Small Atwood numbers (20^{o}C): $0 \le At \le 0.01$.
- Time is x/U_0 (parabolic flow).
- Long collection times (up to 15 minutes).
- Symmetric mix (bubbles and spikes same).
- Good diagnostics available.

Photograph from Experiment



At # = 10^{-3}

$\Delta T = 5^{o}C$

$U_0 = 4$ cm/s

Video of Experiment

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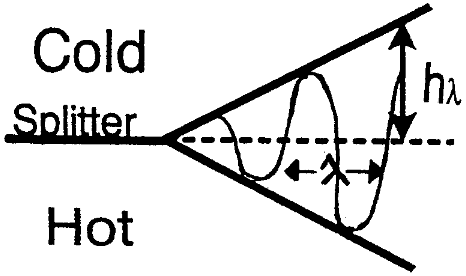
Mechanisms

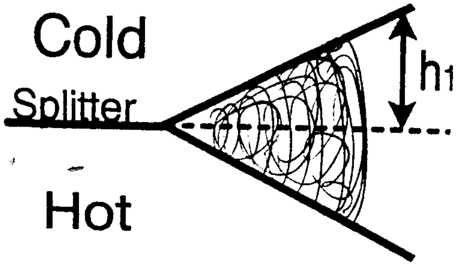
Initial linear saturation, $C_\infty \approx 0.7$

$$h_\lambda = \frac{C_\infty x}{u_0}\sqrt{\frac{A g \lambda}{2}}$$



Quadratic self-similar growth

$$h_1 = \frac{\alpha A g x^2}{u_0^2}$$



$\alpha \equiv$”Universal” growth constant ~ 0.07 (debatable)

Summary of Data Collected

At Atwood numbers of 10^{-3} and $5x10^{-4}$:

- Density profiles across mix; width quadratic growth rate, α
- Ensemble averaged measurements of turbulence R-T mixing correlations:

$$\overline{\rho'^2}, \overline{u'^2}, \overline{v'^2}, \overline{u'v'}, \text{and } \overline{\rho'u'}, \overline{\rho'v'}$$

- Turbulence density fluctuation energy spectra.
- Molecular mix fraction, θ
- Anisotropy tensor, energy dissipation.

Parameter Definitions

$$B_0 = \lim_{T\to\infty}\frac{1}{T}\int_0^T\left(\rho-\overline{\rho}\right)^2 dt/\Delta\rho^2 = \lim_{T\to\infty}\frac{1}{T}\int_0^T\left(\rho'\right)^2 dt/\Delta\rho^2$$

$$B_2 = \overline{\rho^*}\left(1-\overline{\rho^*}\right) = f_1\left(1-f_1\right) \qquad \theta \equiv 1 - B_0/B_2$$

$$\boxed{\rho^* = \frac{\left(\rho-\rho_{\min}\right)}{\left(\rho_{\max}-\rho_{\min}\right)}} \qquad \boxed{\overline{\rho^*} = \frac{\sum_1^n \rho_i^*}{n}} \qquad B_0 = \frac{n\sum_1^n \rho_i^{*2} - \left(\sum_1^n \rho_i^*\right)^2}{n(n-1)}$$

$$B_0(\omega_n) = \frac{2\delta t}{N}\left|\sum_{i=0}^{N-1}(\rho_i *)' e^{2\pi j\omega_n t_i}\right|$$

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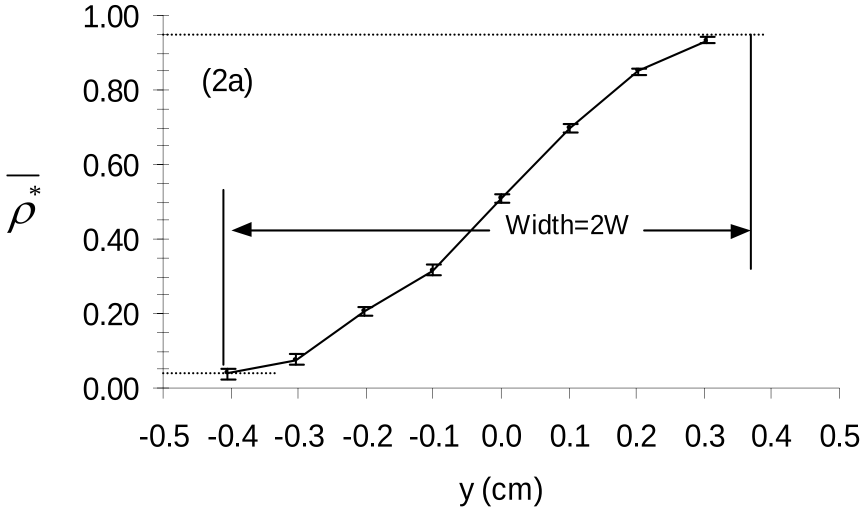
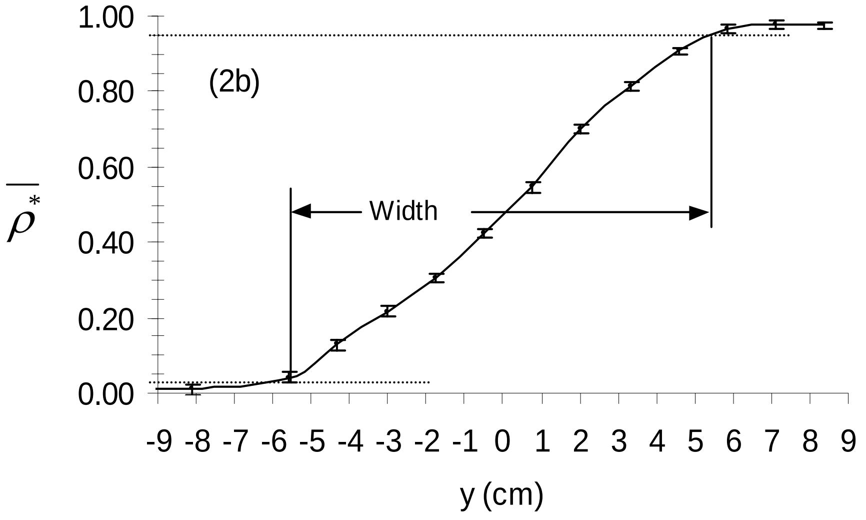
Mean Density Profiles

2.4 cm downstream

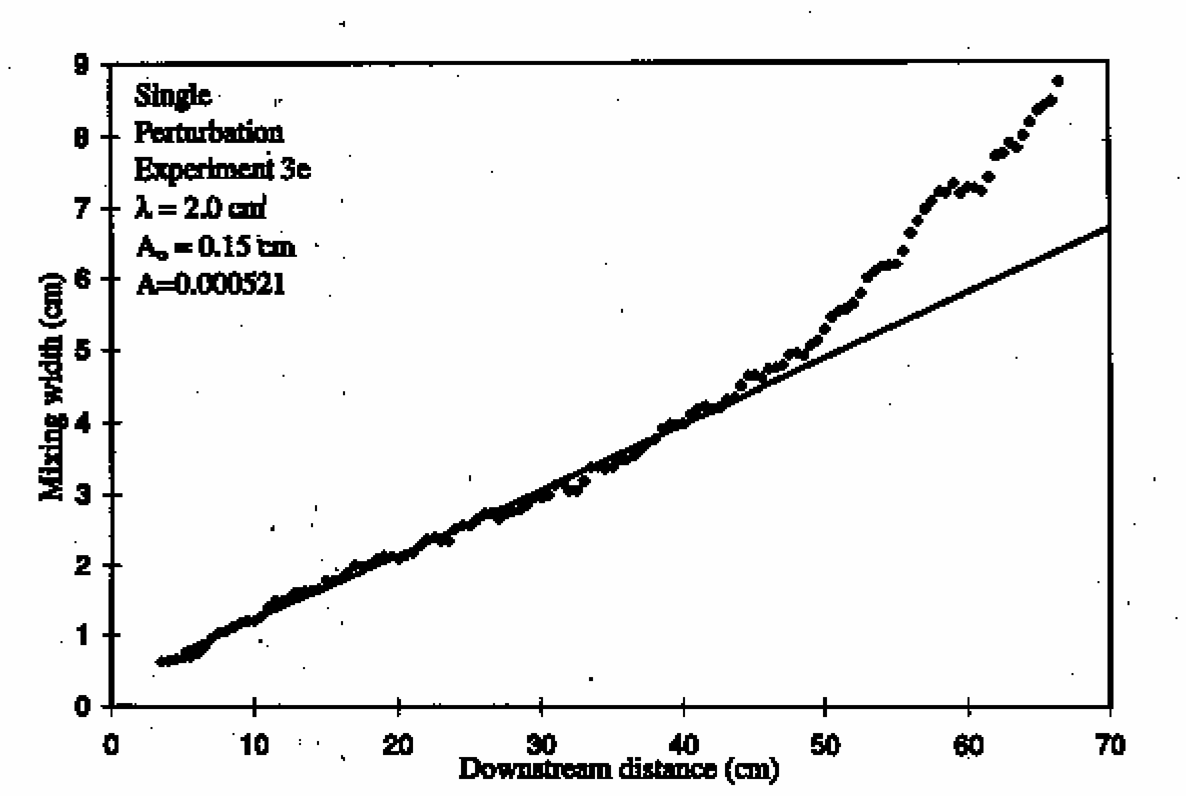


30 cm downstream



Mean density profile taken with thermocouple measurements, and showing error bars.

Mix Width Development



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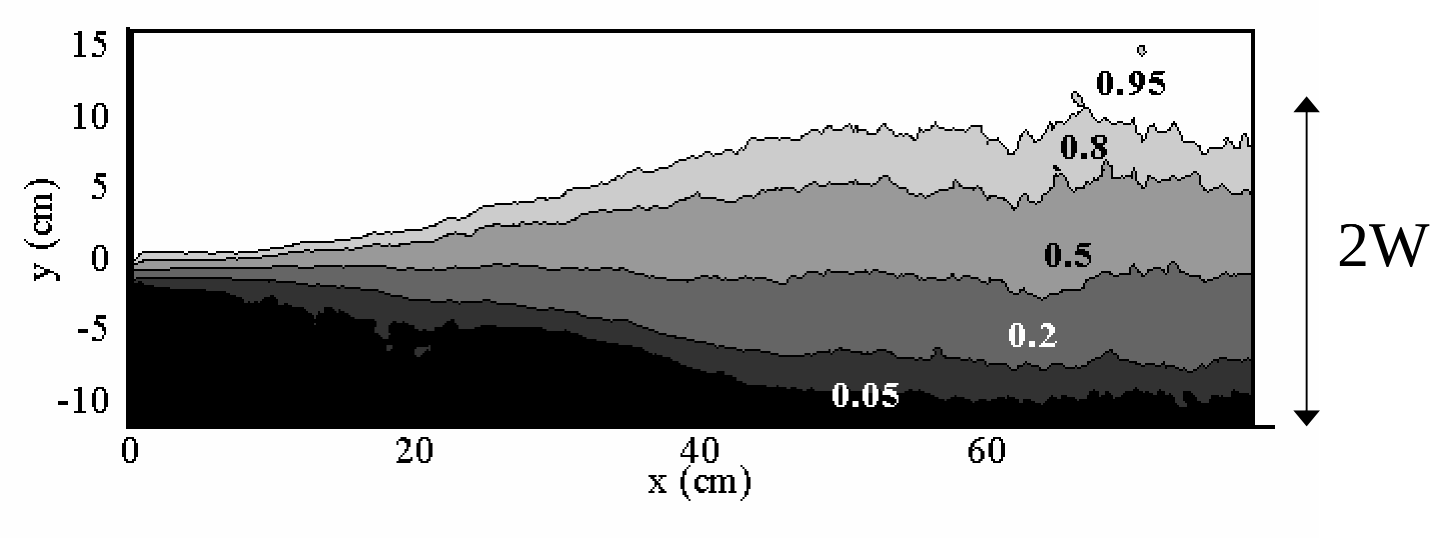
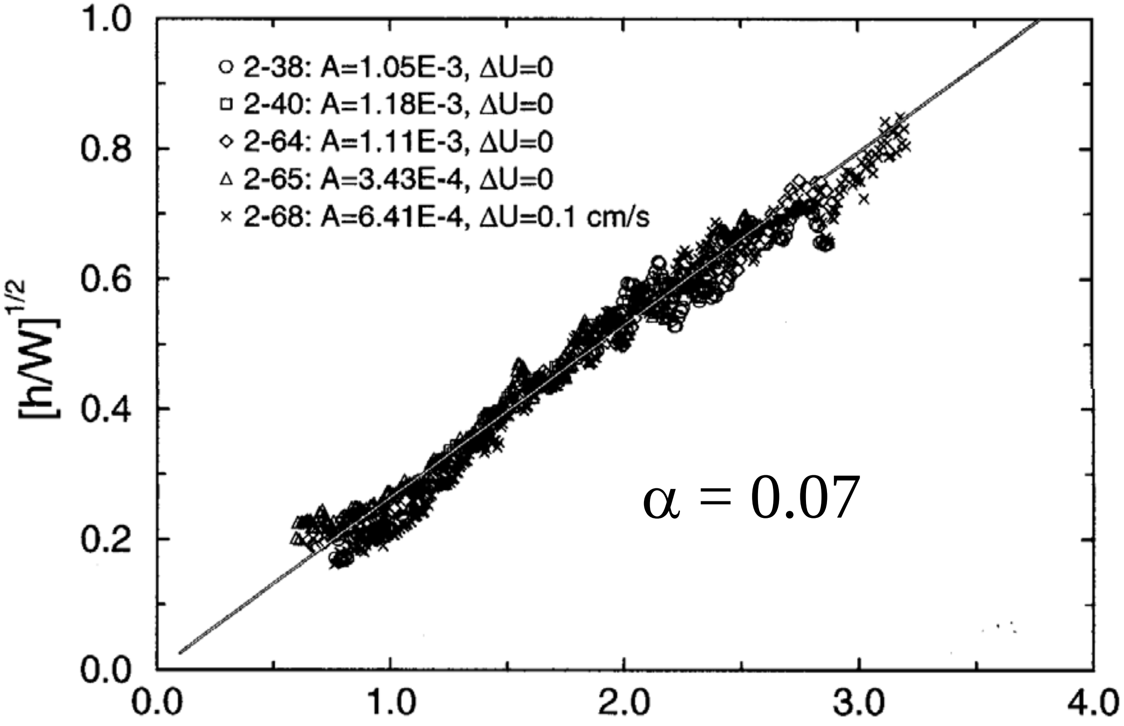
Ensemble Averaged Volume Fractions



140 photos

At=0.00064, ΔT=6^{o}C

Measurement of α



$$\sqrt{(At\,g)/W}\;\frac{x}{U_0}$$

Parameter Definitions

$$B_0 = \lim_{T\to\infty}\frac{1}{T}\int_0^T \left(\rho-\overline{\rho}\right)^2 dt/\Delta\rho^2 = \lim_{T\to\infty}\frac{1}{T}\int_0^T (\rho')^2 dt/\Delta\rho^2$$

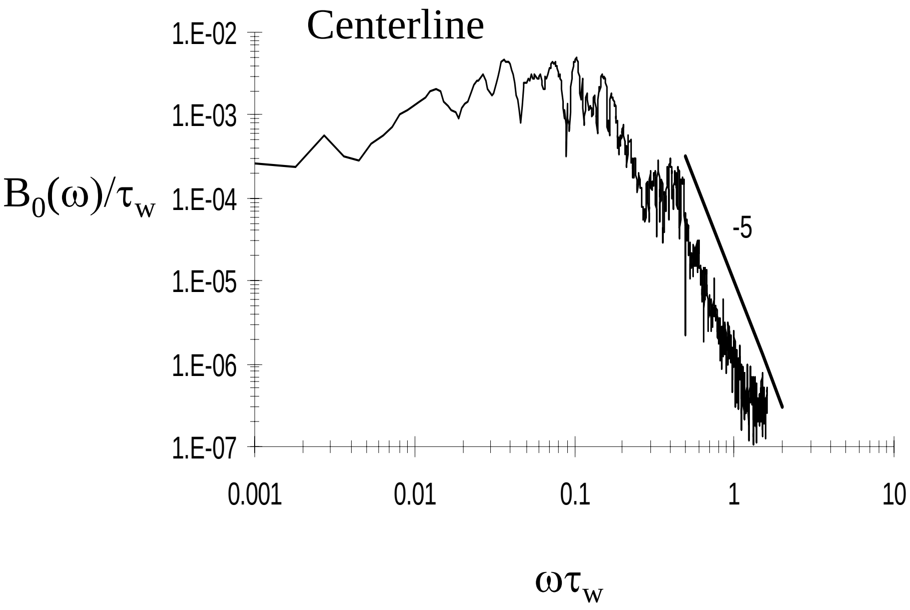
$$B_2 = \overline{\rho^*}\left(1-\overline{\rho^*}\right) = f_1\left(1-f_1\right) \qquad \theta \equiv 1 - B_0/B_2$$

$$\rho^* = \frac{\left(\rho-\rho_{\min}\right)}{\left(\rho_{\max}-\rho_{\min}\right)} \qquad \overline{\rho^*} = \frac{\sum_1^n \rho_i^*}{n} \qquad B_0 = \frac{n\sum_1^n \rho_i^{*2} - \left(\sum_1^n \rho_i^*\right)^2}{n(n-1)}$$

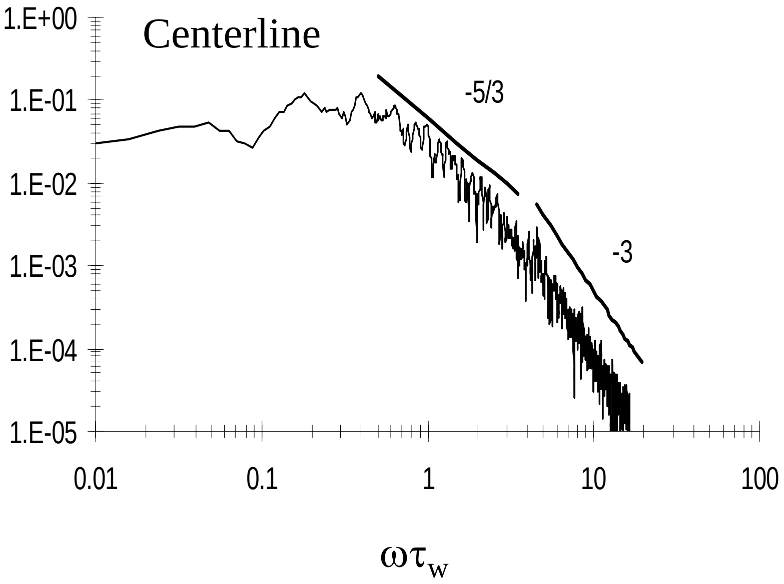
$$\boxed{B_0(\omega_n) = \frac{2\delta t}{N}\left|\sum_{i=0}^{N-1}(\rho_i{}^*)' e^{2\pi j\omega_n t_i}\right|}$$

Density Fluctuation Power Spectra

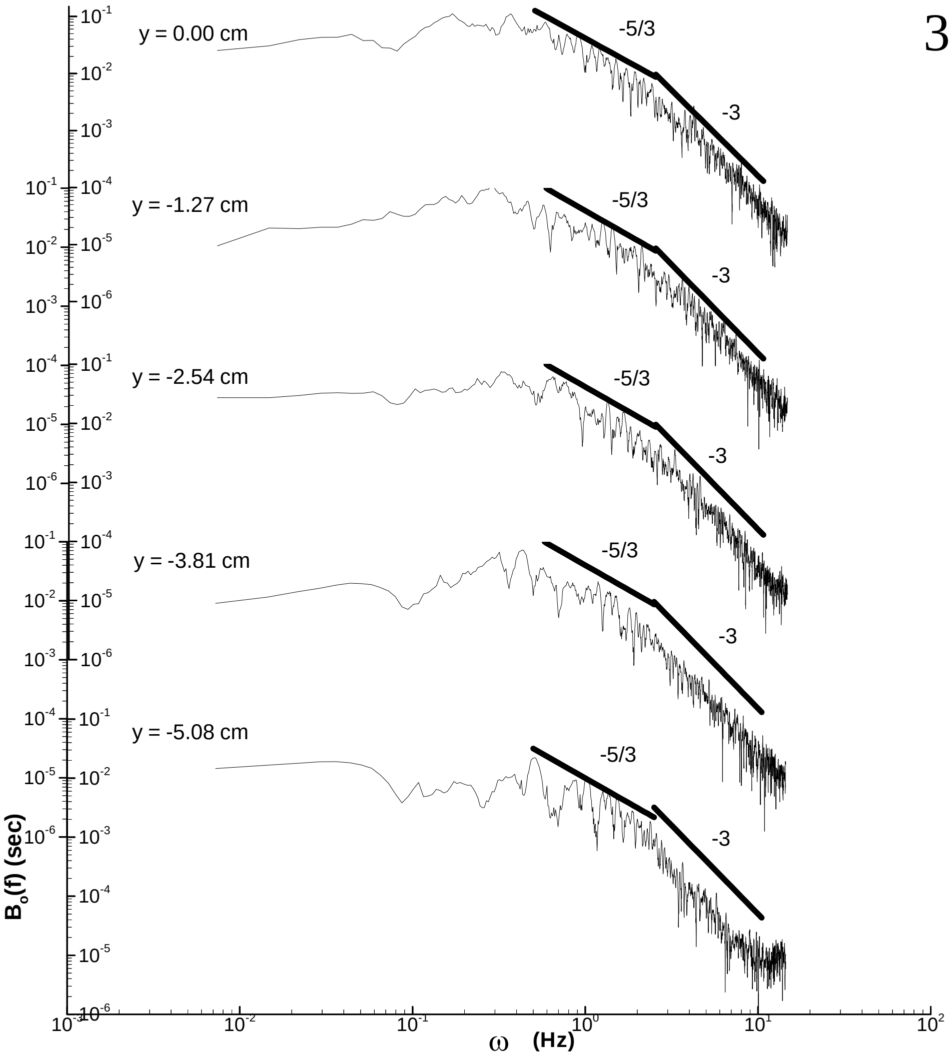
2.4 cm downstream



30 cm downstream



More Power Spectra



30 cm downstream

Centerline

Mix edge

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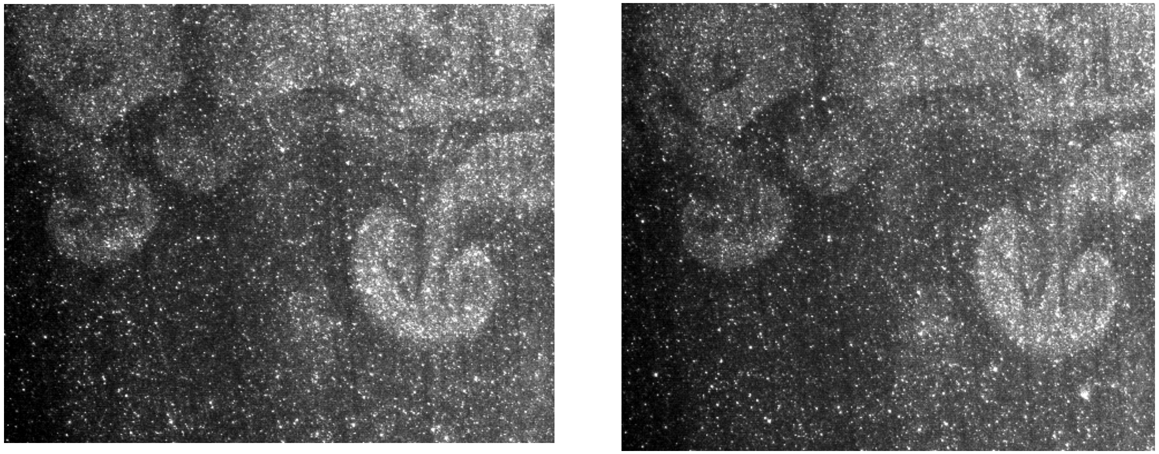
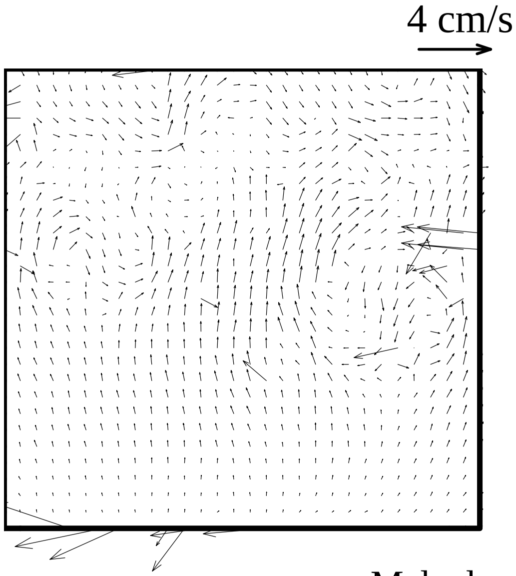
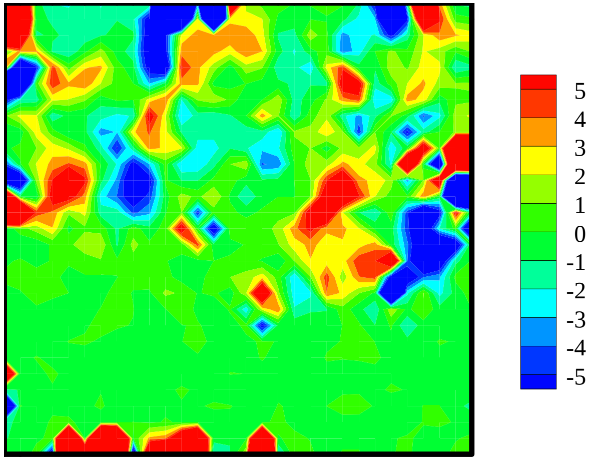
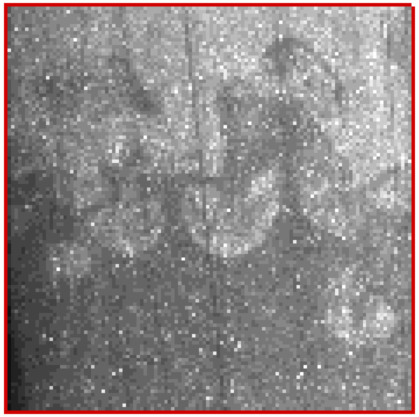
PIV

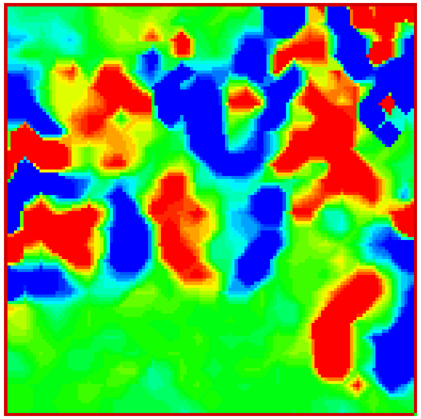




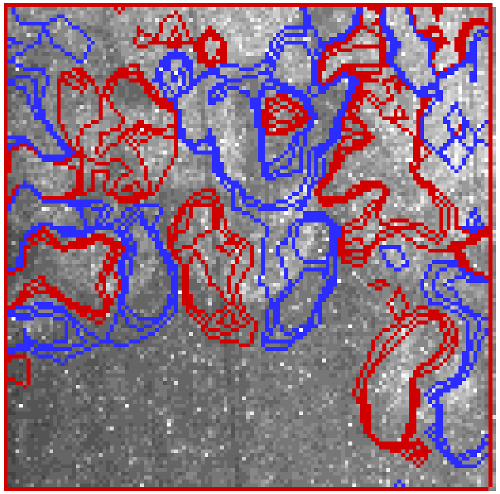




Photographs



Vorticity



Photographs overlaid with vorticity

Parameter Definitions

$$\boxed{B_0 = \lim_{T\to\infty} \frac{1}{T}\int_0^T \left(\rho - \overline{\rho}\right)^2 dt / \Delta\rho^2 = \lim_{T\to\infty} \frac{1}{T}\int_0^T (\rho')^2 dt / \Delta\rho^2}$$

$$\boxed{B_2 = \overline{\rho^*}\left(1 - \overline{\rho^*}\right) = f_1\left(1 - f_1\right)}$$

$$\boxed{\theta \equiv 1 - B_0 / B_2}$$

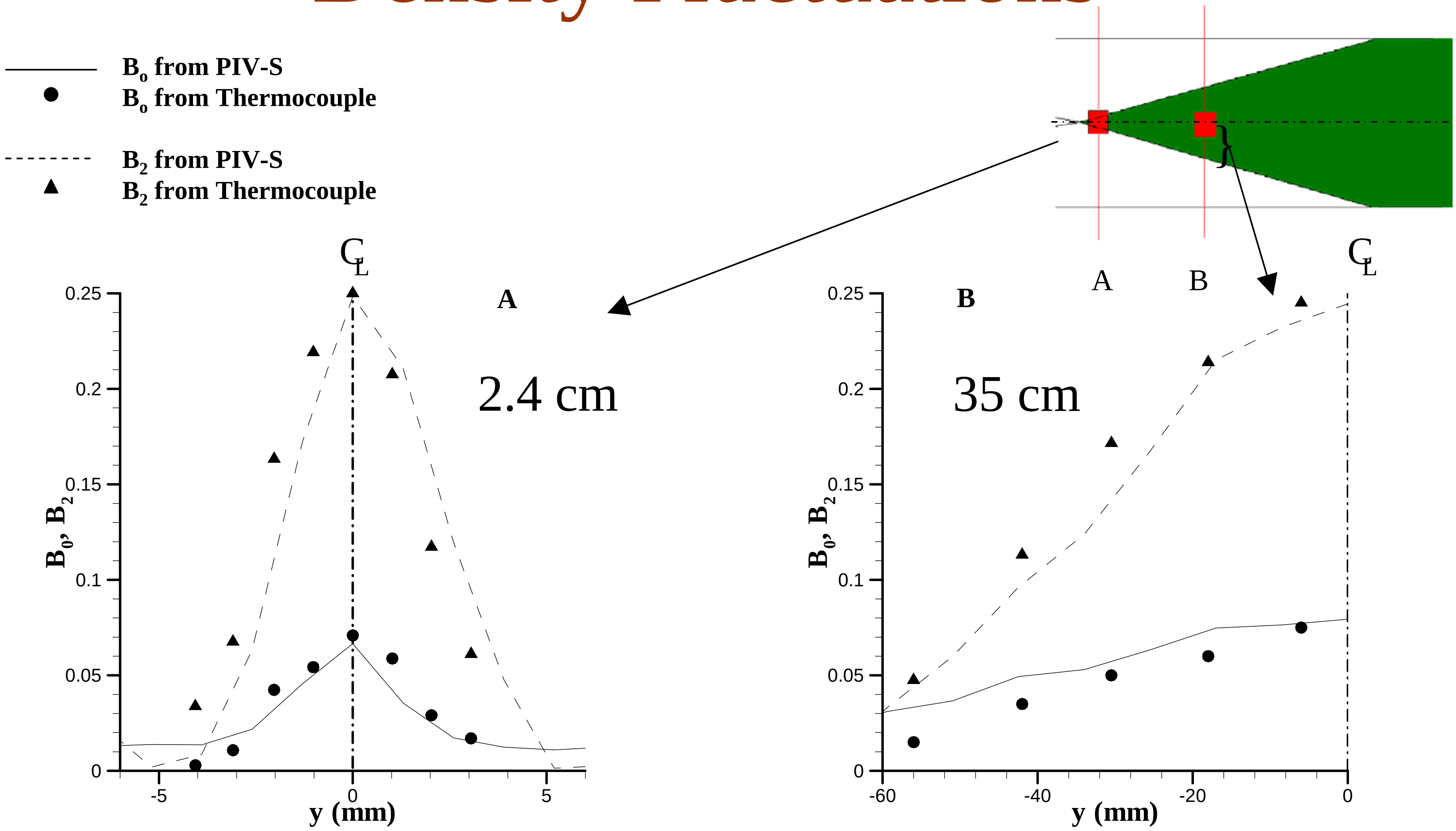
$$\rho^* = \frac{\left(\rho - \rho_{\mathrm{min}}\right)}{\left(\rho_{\mathrm{max}} - \rho_{\mathrm{min}}\right)}$$

$$\overline{\rho^*} = \frac{\sum_{1}^{n} \rho_{\mathrm{i}}^*}{\mathrm{n}}$$

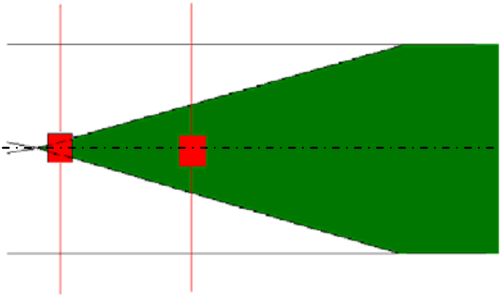
$$B_0 = \frac{\mathrm{n}\sum_{1}^{n} \rho_{\mathrm{i}}^{*2} - \left(\sum_{1}^{n} \rho_{\mathrm{i}}^*\right)^2}{\mathrm{n(n-1)}}$$

$$B_0(\omega_n) = \frac{2\delta t}{N}\left|\sum_{i=0}^{N-1} (\rho_i^*)' e^{2\pi j \omega_n t_i}\right|$$

Density Fluctuations



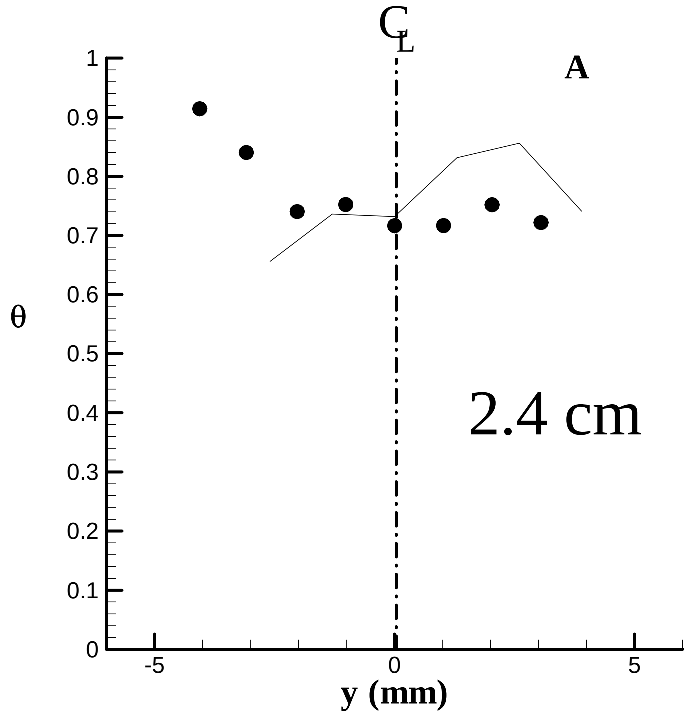
Molecular Mixing

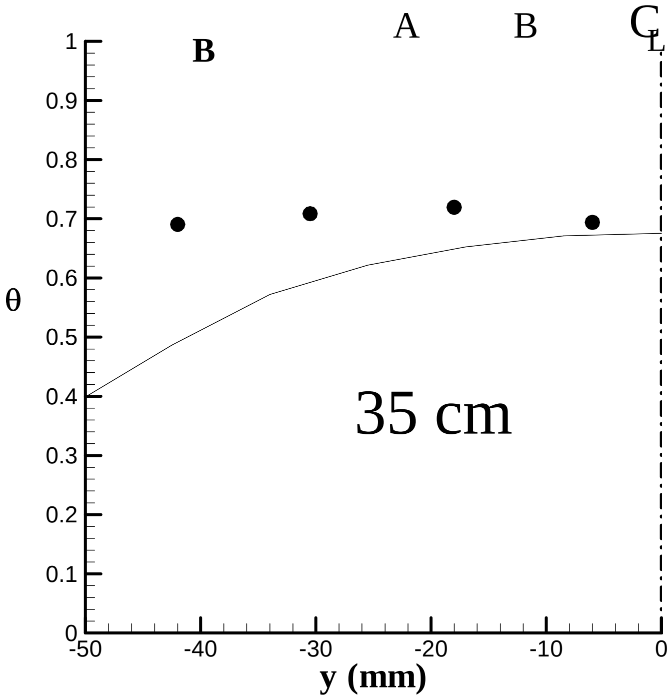


—— **θ from PIV-S**

● **θ from Thermocouple**

$$\alpha_0 = \alpha_2\sqrt{(1-\theta)}$$





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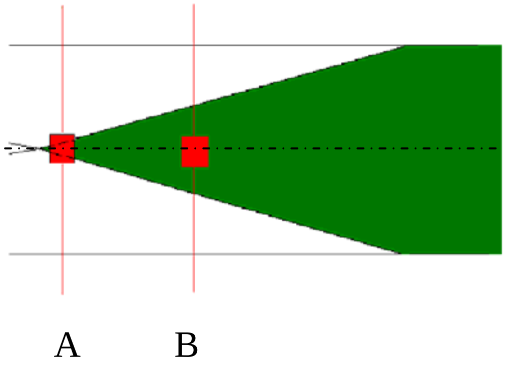
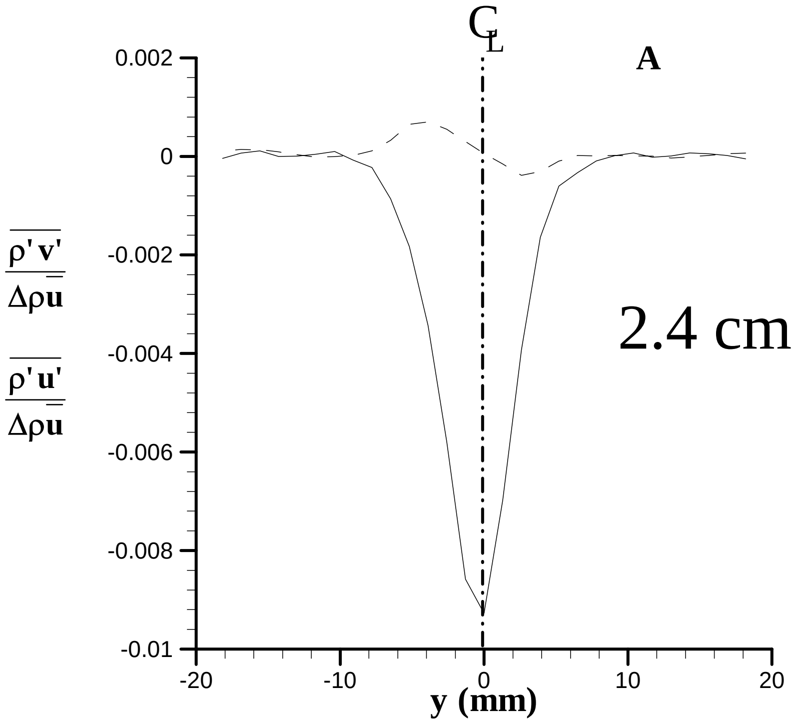
Density/Velocity Correlations

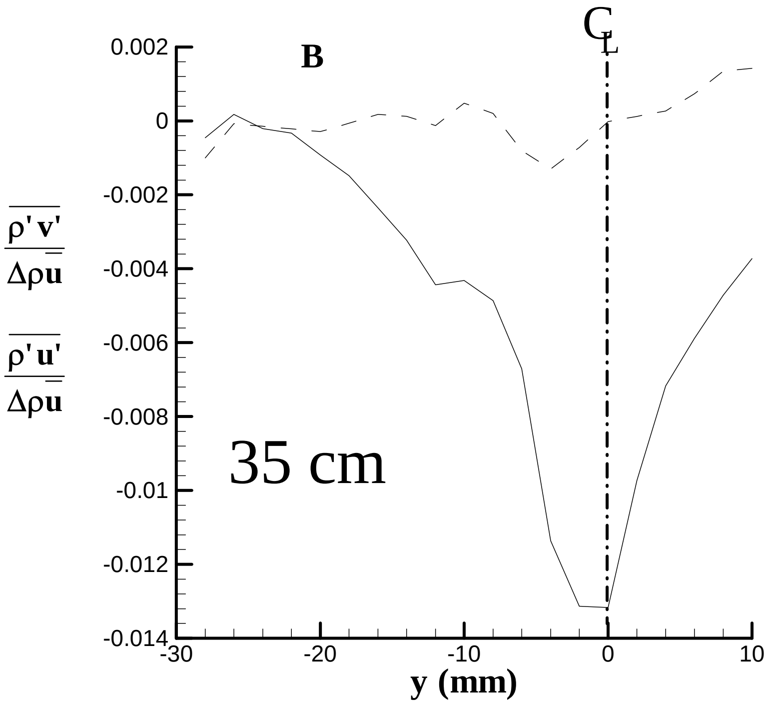


—— $\frac{\overline{\rho' \mathbf{v}'}}{\Delta\rho\bar{\mathbf{u}}}$

- - - - - $\frac{\overline{\rho' \mathbf{u}'}}{\Delta\rho\bar{\mathbf{u}}}$



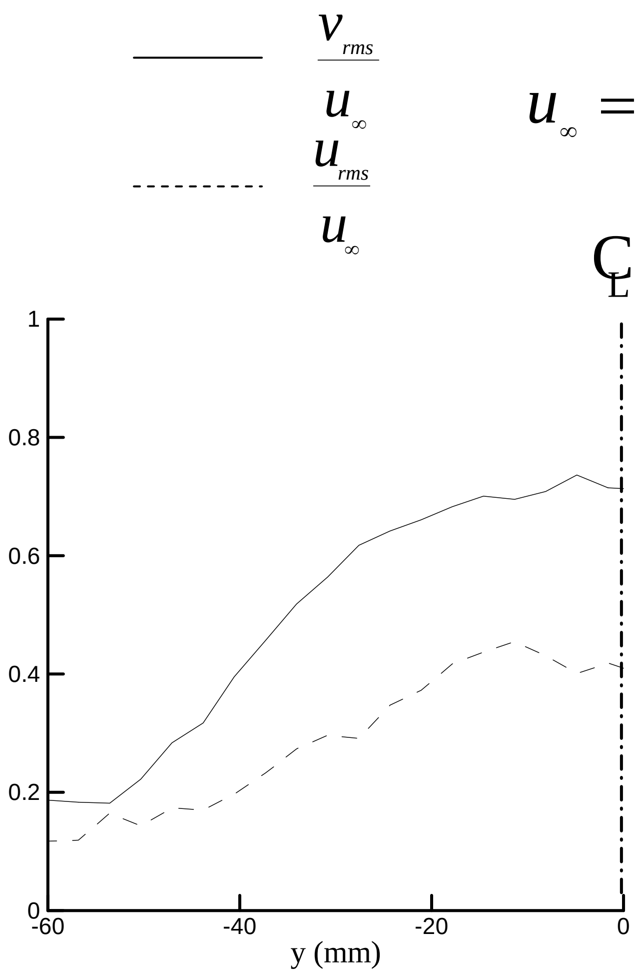
2.4 cm

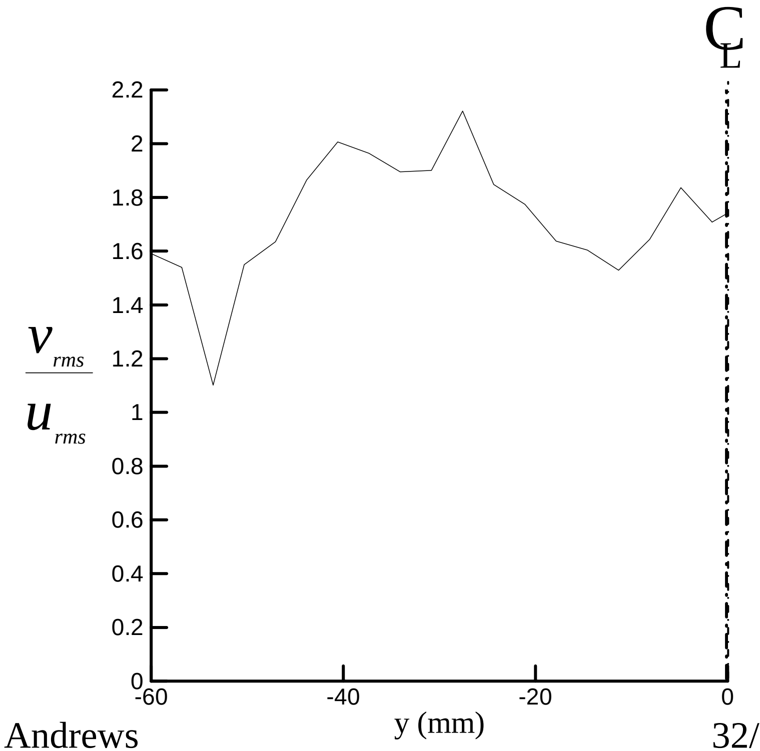


35 cm

Velocity Fluctuations (35 cm)

$$u_\infty = 0.7\sqrt{A_t gH/4}$$





Anisotropy Tensor

$$b_{ij} = \frac{< u'_i u'_j >}{< u'_k u'_k >} - \frac{1}{3}\delta_{ij}$$

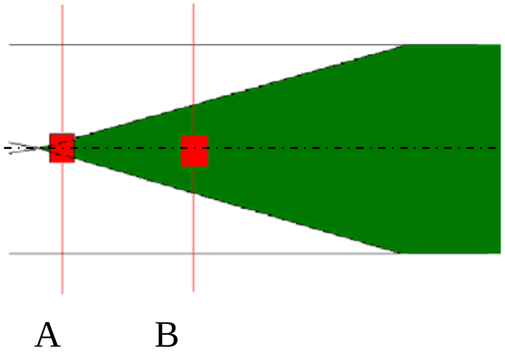
$$< u'_i \, u'_j > = 0 \quad \text{if i} \neq \text{j}$$

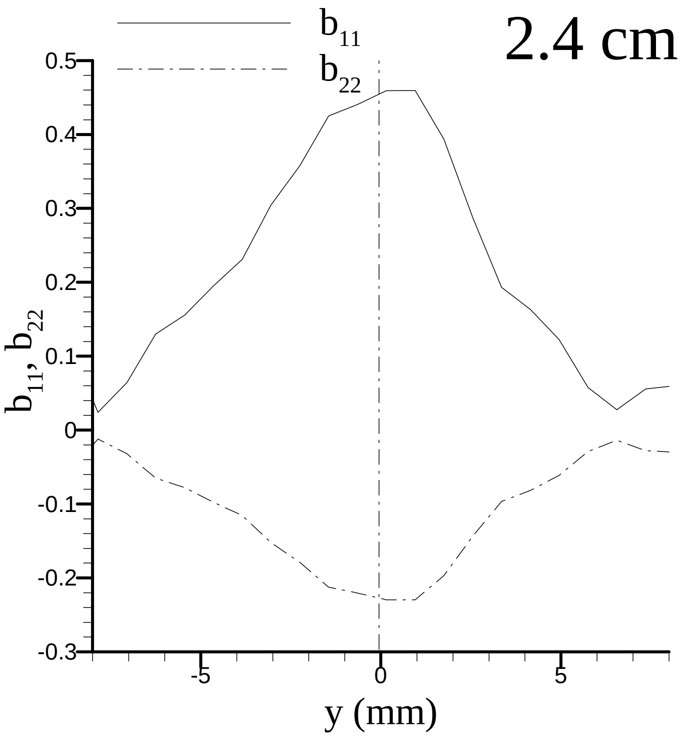
where $$< u'_k \, u'_k > = \overline{u'^2} + \overline{v'^2} + \overline{w'^2}$$

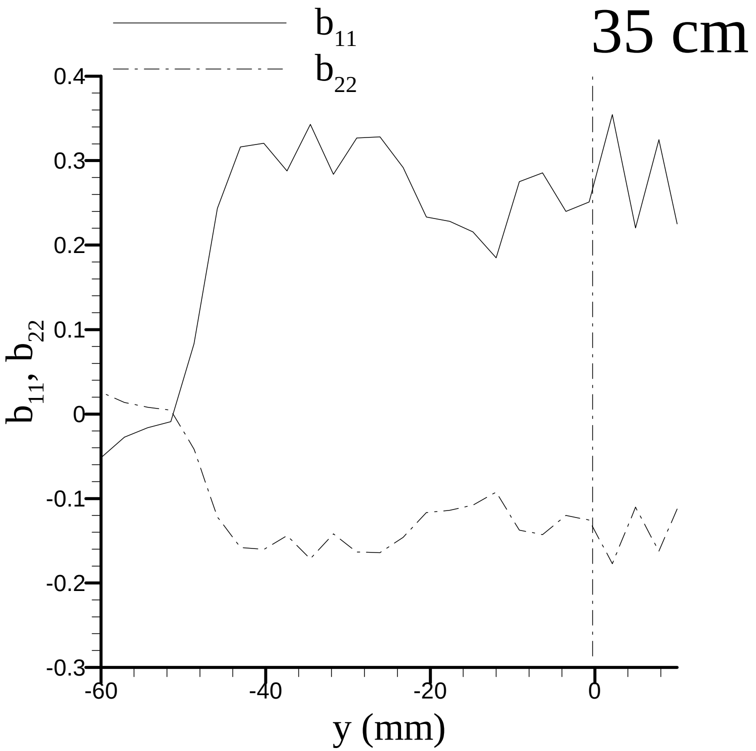
$$\approx 2\overline{u'^2} + \overline{v'^2}$$

Isotropy $\Rightarrow b_{ii} = 0$

Anisotropy Tensor







Energy Dissipation

$$PE_i = \int_0^w \rho_{step}\, z\, dz \quad \Rightarrow \quad \int_0^{\frac{w}{2}} \rho_1 g z\, dz + \int_{\frac{w}{2}}^{w} \rho_2 g z\, dz$$

$$PE_f = \int_0^w \rho_{measured}\, z\, dz \quad \Rightarrow \quad \sum_{i=0}^{n} \rho_i g z_i \Delta z$$

$$PE_{released} = PE_i - PE_f$$

where, $\rho_{measured}$ *is the measured density, and* ρ_{step} *is the step-profile of density at the interface corresponding to the initial condition*

$$KE_i = 0 \qquad KE_{generated} = \tfrac{1}{2}\int_0^w \rho\, v'^2 dz$$

where, $W =$ mix width, $v' =$ rms velocity

Dissipation, $D = PE_{released} - KE_{generated}$

$$\left|\frac{D}{PE_{released}}\right| = 0.49$$

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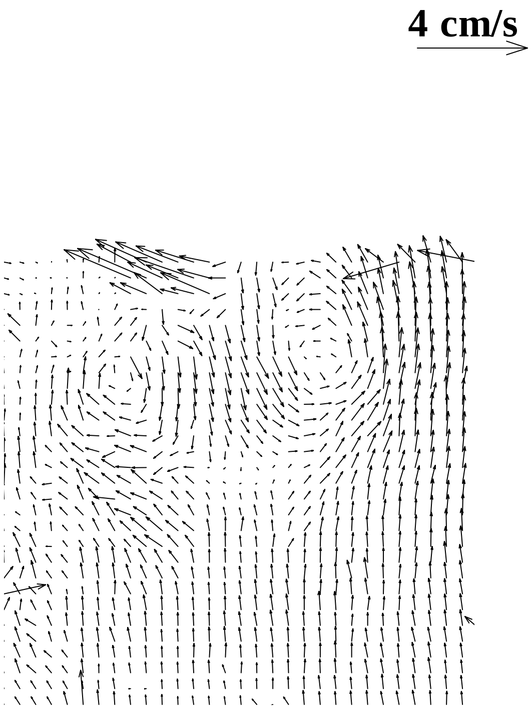
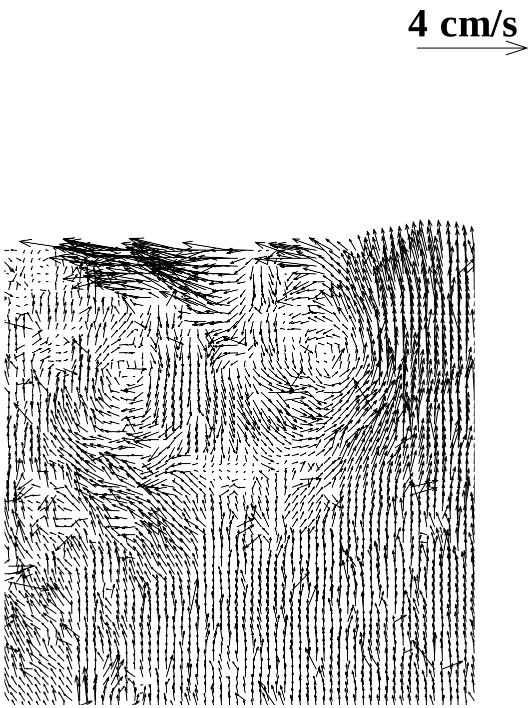
Velocity vectors from PIV

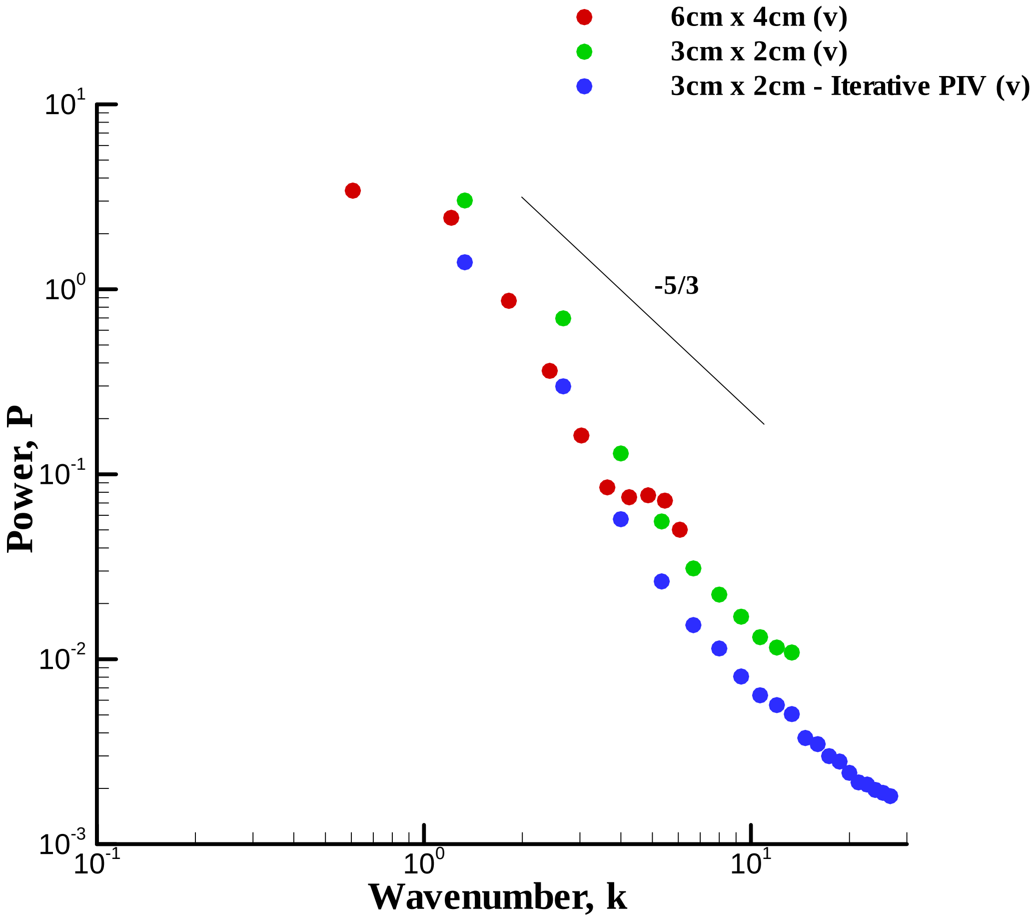


Grid size: 16 x16 pix.



Grid size: 8 x 8 pix.

V-velocity Wavenumber spectra
x = 35cm, A=0.00075



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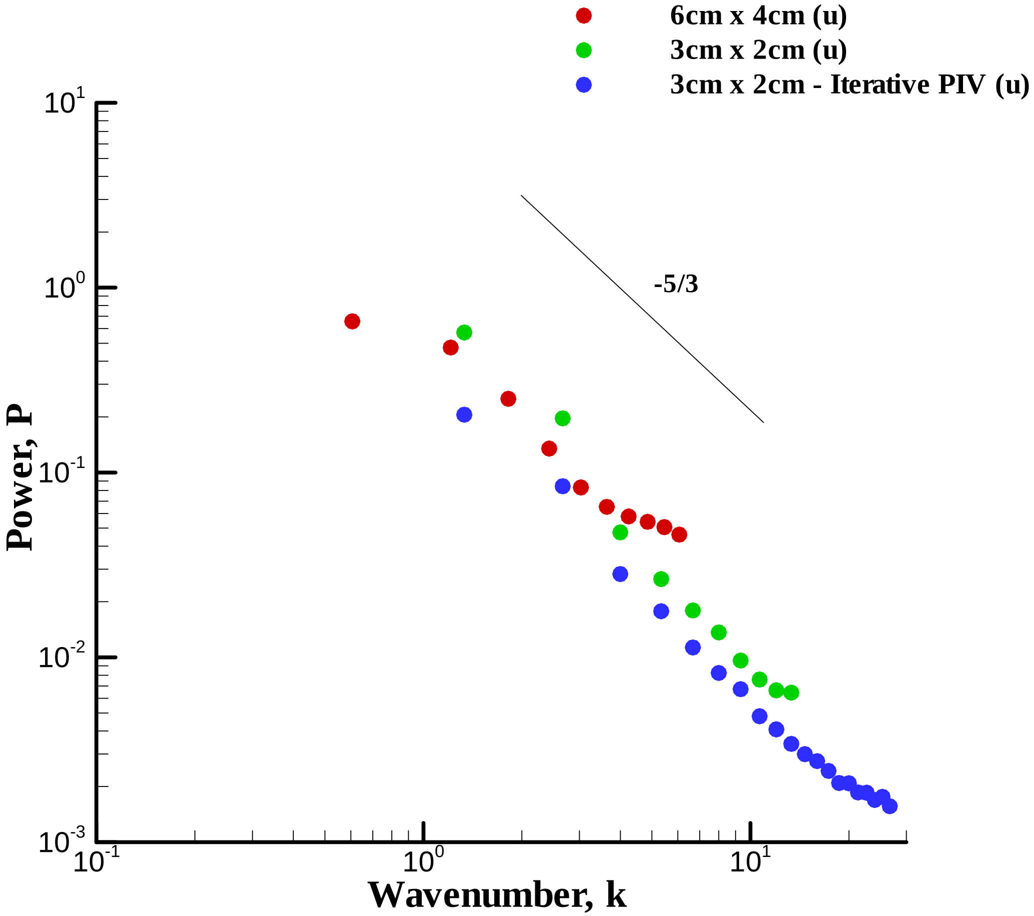
U-velocity Wavenumber Spectra
x = 35cm, A = 0.00075



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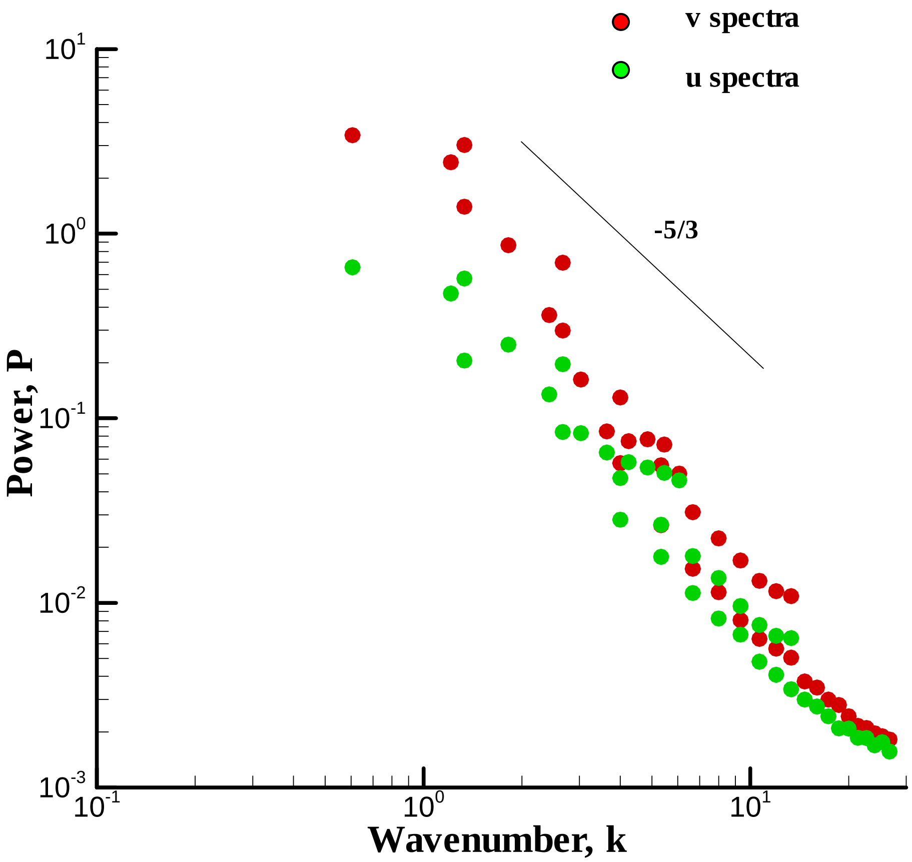
U- and V- velocity Wavenumber Spectra
x = 35cm, A = 0.00075



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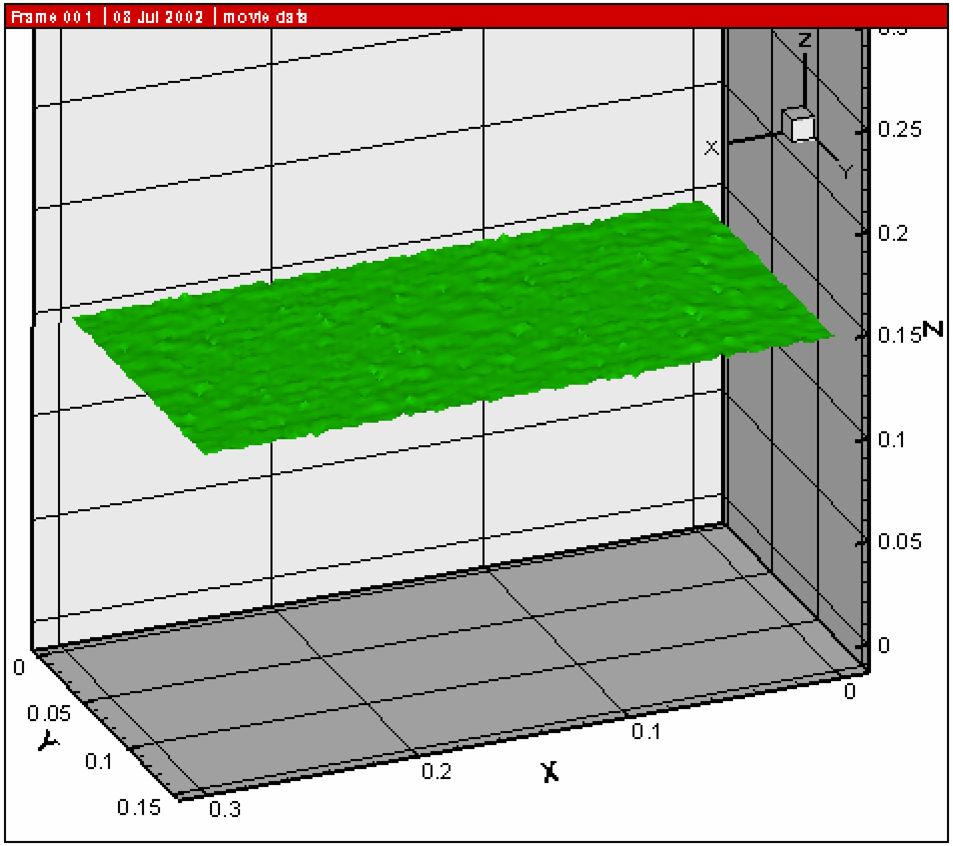
3-D MILES Simulations

- CFD code called 3D-RTI from Andrews.
- Transient 3-D VOF method.
- Euler, incompressible (MILES).
- 3rd order Van-Leer limiters used to prevent non-physical oscillations (momentum and volume fractions).
- Initial velocity field set by velocity potential.
- nx*ny*nz: 96:48:96
- X*Y*Z: 30cm:15cm:30cm

3-D MILES Simulations



A New High At Experiment

(under construction with funding from the DOE)

- $0 \le At \le 0.75$
- Air/Helium (gases at room temperature).
- Statistically steady.
- Lewis # ~ 1 (ratio of thermal & mass diffusion).
- Heat air & use temperature as fluid marker.

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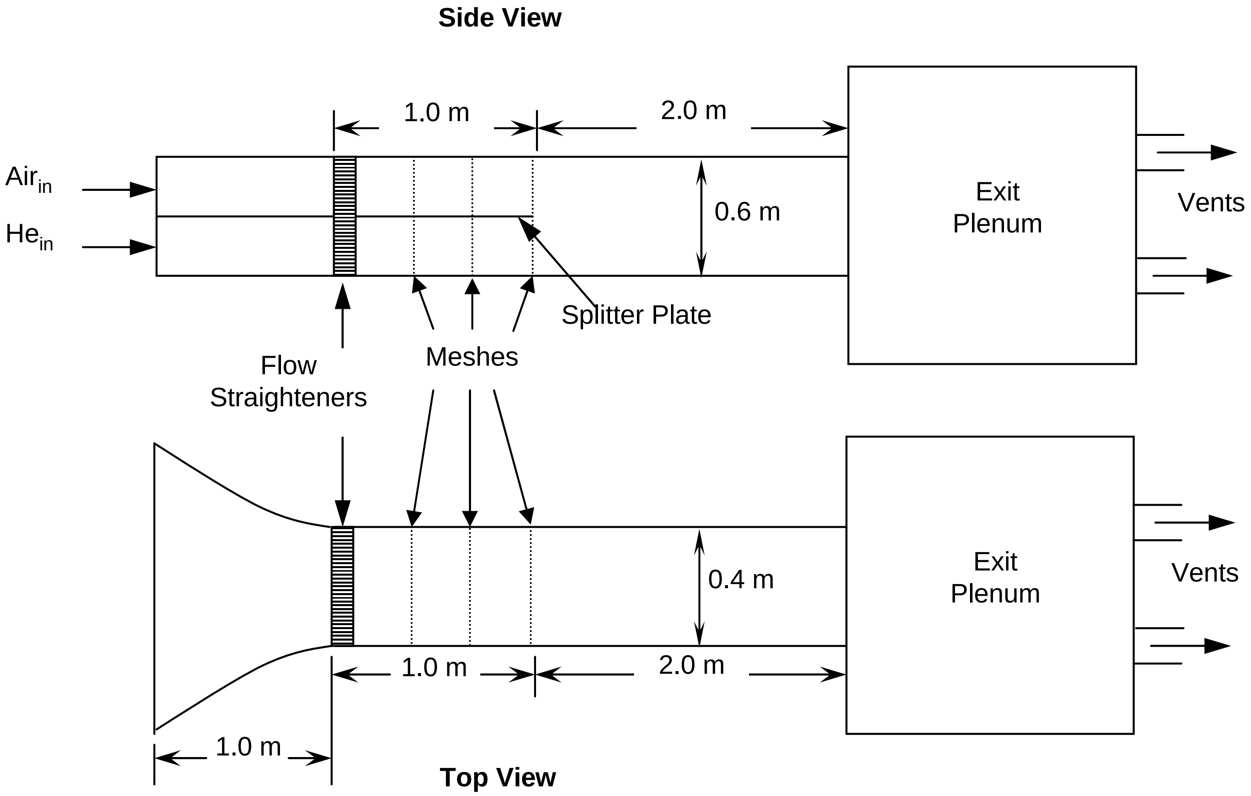
Schematic of New TAMU High At Experiment



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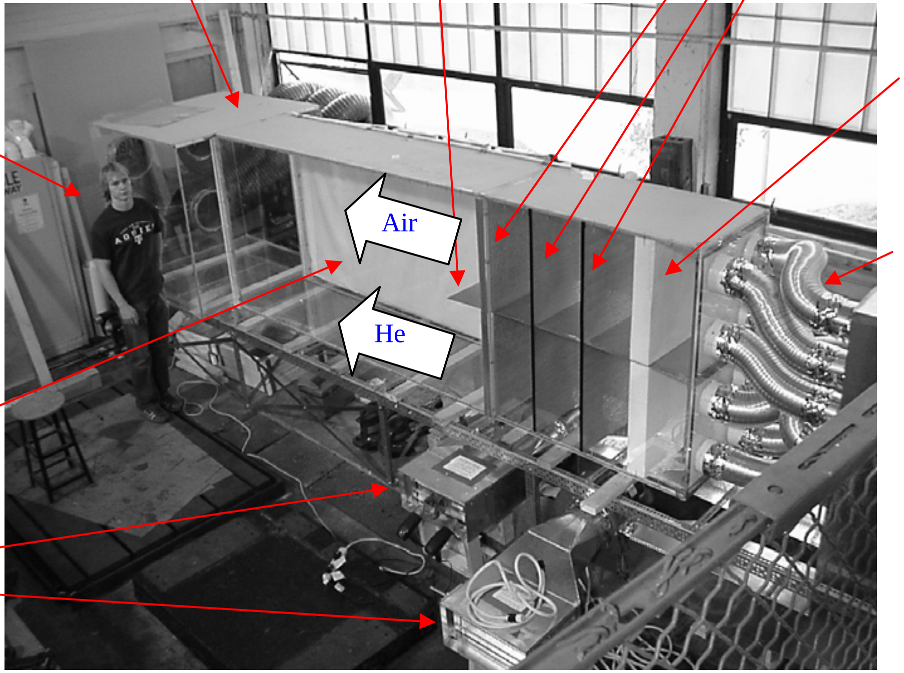
Photograph of facility



Exit plenum

Splitter plate

Meshes

UG Research Asst. (6ft tall)

Flow straightener

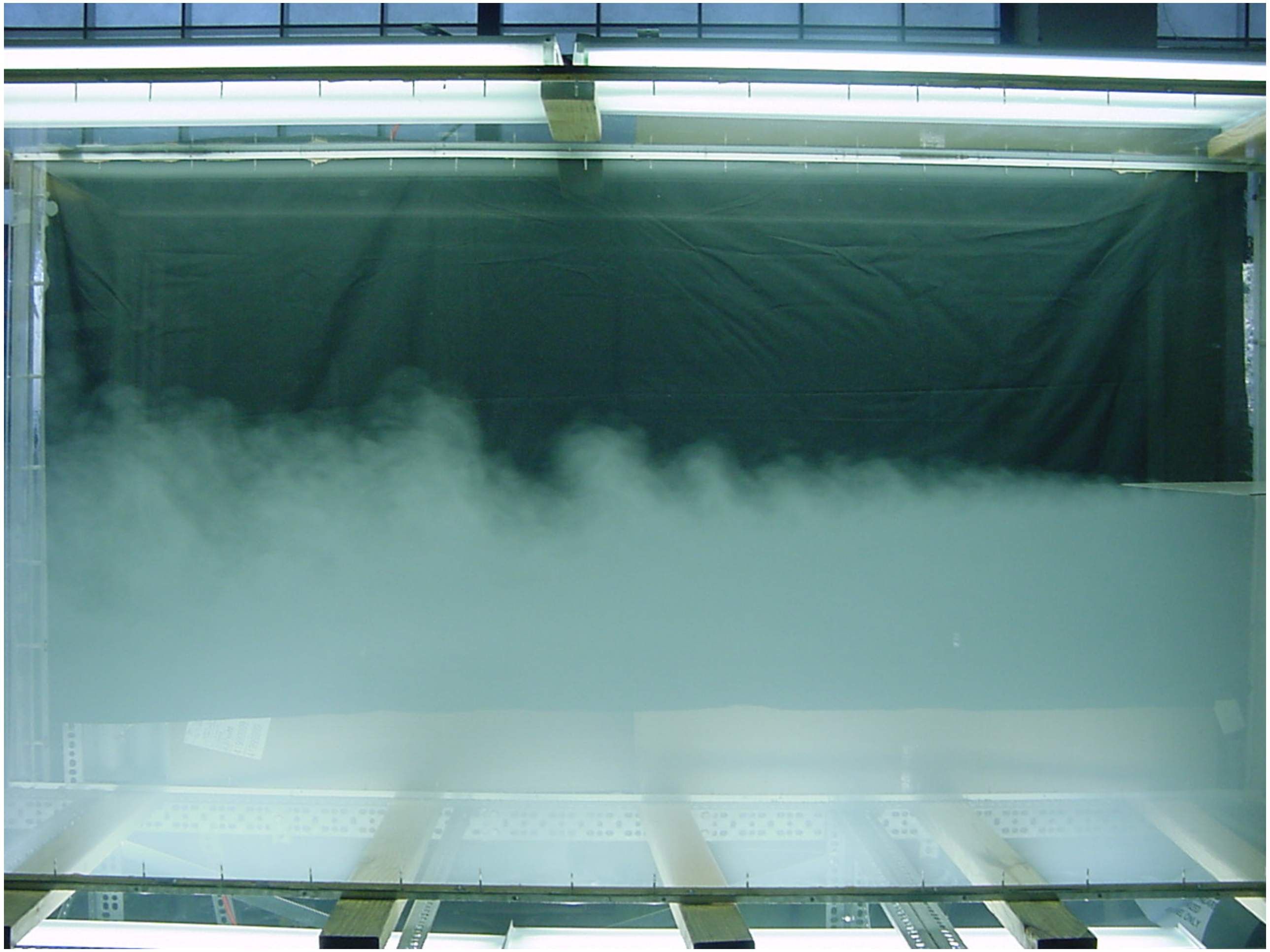
Inlet ducting

Flow channel

Fans

Figure 2. Air/Helium Facility as of December 8, 2003.

Photograph from facility



Current & Future Work

- Effect of initial conditions and coupling to CFD (LLNL).
- Buoyancy driven wakes (DOE).
- Gas channel (DOE).
- CFD Simulation & modeling - RM (Jacobs @ Arizona).
- 3-D MILES of RT mixing (LANL).
- Environmental and Naval applications.

Recent Publications

Wilson, P.N., Andrews, M.J., and Harlow, F.H., “Spectral Non-Equilibrium in a Turbulent Mixing Layer,” *Physics of Fluids A*, Vol. 11, No. 8, pp. 2425-2433, August, 1999.

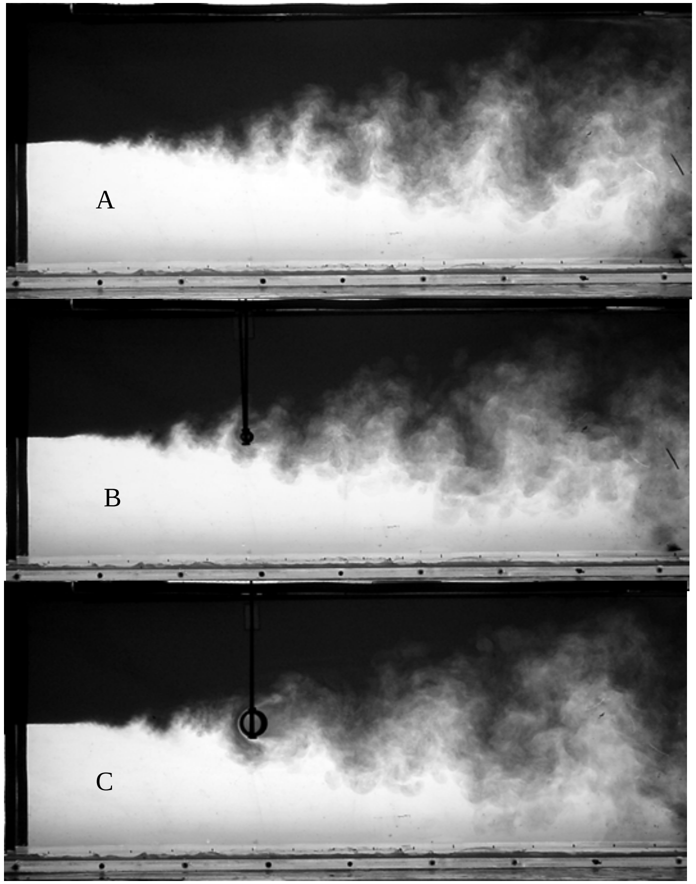
Wilson, P.N., and Andrews, M.J., “Spectral Measurements of Rayleigh-Taylor Mixing at Low Atwood Number,” *Physics of Fluids A*, Vol. 14, No. 3, pp. 938-945, March, 2002.

Ramaprabhu, P., and Andrews, M.J., “Simultaneous Measurements of Velocity and Density in Buoyancy Driven Mixing,” *Experiments in Fluids*, Vol. 34, pp. 98-106, 2003.

Dimonte, G., Youngs, D.L., Dimits, A., Weber, S., Marinak, M., Calder, A.C., Fryxell, B., Biello, J., Dursi, L., MacNeice, P., Olson, K., Ricker, P., Rosner, R., Timmes, F., Tufo, H., Youns, Y.-N., Zingale, M., Wunsch, S., Garasi, C., Robinson, A., Ramaprabhu, P., and Andrews, M.J., “A Comparative Study of the Turbulent Rayleigh-Taylor (RT) Instability Using High-Resolution 3D Numerical Simulations: The Alpha Group Collaboration,” to appear in *Physics of Fluids*. Accepted, December, 2003.

Ramaprabhu, P., and Andrews, M.J., "Experimental Investigation of Rayleigh-Taylor Mixing at Small Atwood Numbers," *Journal of Fluid Mechanics*, Vol. 502, pp. 233-271, March, 2004.

Buoyant Wakes



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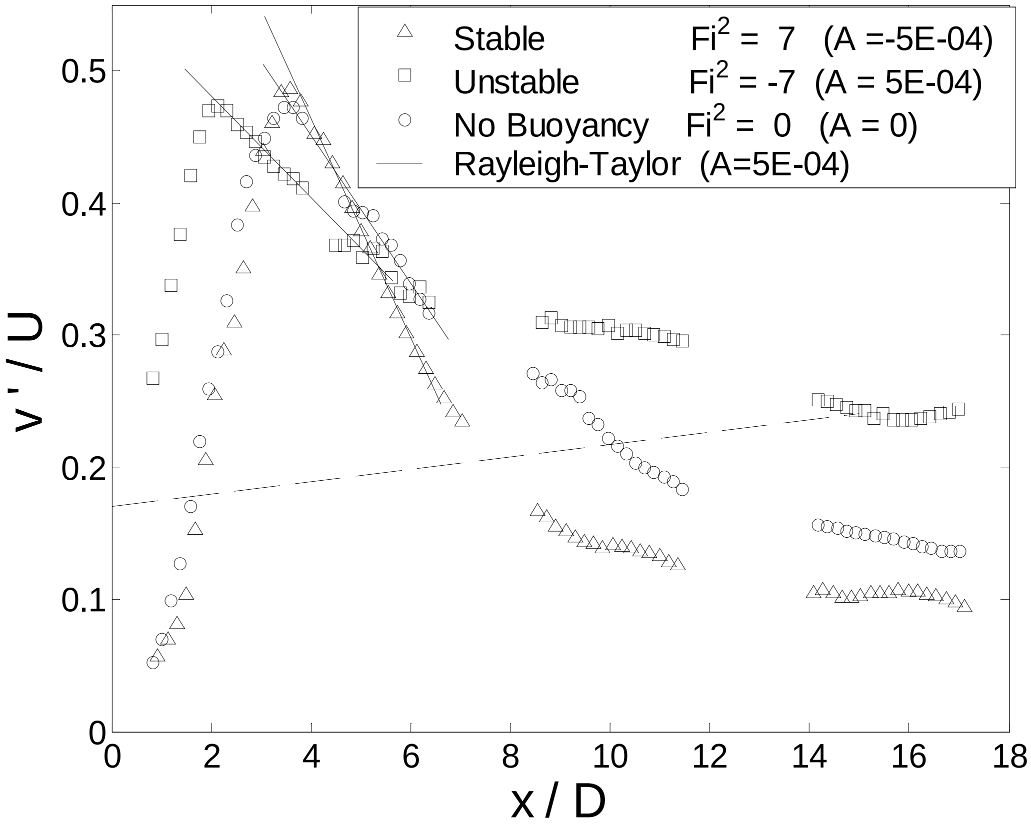
Buoyant Wakes



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Properties

Properties at 20^{o}C	**Water**	**Air**	**Helium**
Density (kg/m^3)	998.0	1.19	0.166
Viscosity (N s/m^2)	1.003 E-3	1.80 E-5	1.97 E-5
Kinematic viscosity (m^2/s)	1.005 E-6	1.51 E-5	1.19 E-4
Prandtl #	7.0	0.7	0.7
Schmidt #	600 (H_2O/NaCl)	0.22 to 1.73 (varies across mix)	

Key Design Considerations

- Keeping the flow parabolic so that the Taylor hypothesis may be used to relate time and space as $t = x/U$, where U is the channel flow speed and x the distance downstream.

- The maximum Reynolds number for the mix.

- The cost of helium, which is related to the x-sectional area and the flow speed U.

- How to measure the instantaneous density.

Parabolic Flow

$$h_b = 0.07\,At\,g\,t^2 \text{ or } h_b = 0.07\,At\,g\left(x/U\right)^2$$

Parabolic if: $\dot{h}_b / U < \tan(15^o) \approx 0.25$

Setting At=0.75, $h_{b,max}$=0.3 m at x_{max}=1 m, gives U ~ 1.0 m/s

Max Re

$$\mathrm{Re} = \frac{h_b \dot{h}_b}{\nu_{centerline}}$$

Setting At=0.75, $h_{b,max}$=0.3 m at x_{max}=1 m, gives:

Cold/Hot water: $\mathrm{Re}_{max}=700$

Air/Helium: $\mathrm{Re}_{max}=2400$

Cost of He

Cost of He ~ \$4/m^3

Setting U= 1m/s, X-section area=0.3*0.4=0.12 m^2

Gives the max volume flow rate of 0.12 m^3/s

Allow 20 flow lengths (1 m) gives 2.4 m^3/expt.

Filling channel ~ 2.6 m^3

Total volume/expt. = 5m^3. **Cost/expt. = \$20**

Measuring Density

- Density measurement is the problem with two-component gases.
- But in gases the thermal and mass diffusivities are close.
- So by heating the air say 10^{o}C above the He, the temperature of the air becomes a marker (like dye in the water channel).
- The air temperature marker is better because it matches the mass diffusivity and so can provide measurements of:
  - Instantaneous density
  - Mean density
  - Molecular mix

Design Summary

Parameter	**Small At # (hot/cold)**	**Large At # (air/helium)**
At # $=(\rho_1-\rho_2)/(\rho_1+\rho_2)$	1.0 E-3	0.755
Length	1.0 m	2.0 m
Height	0.3 m	0.6 m
Depth	0.2 m	0.4 m
Re_{max}	~700	~2400
U	0.05 m/s	1.0 m/s
Cost/run	~$0	$20
Diagnostics	Thermocouple Dye PIV	Thermocouple Smoke Hot wire PIV?

Diagnostics

Flow visualization: Smoke

Thermocouple measurements: Density

PIV: Velocities (see next)

Hot-wire anemometry: Velocities