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Recovery of Rayleigh-Taylor mixing from unstably stratified flow past a cylinder

Wayne N. Kraft & Malcolm J. Andrews

Department of Mechanical Engineering
Texas A&M University
College Station, TX

Tel: (979) 847-8843
E-mail: mandrews@mengr.tamu.edu

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Overview

- Motivation
- Setup of the experiment
- Buoyant wakes and the Rayleigh-Taylor instability
- Visualizations of the wake
- Experiment data and analysis
- Toy model for decaying velocity fluctuations
- Conclusions

Motivation

- To study the interaction of competing equlibria within a turbulent mixing layer. Specifically the exchange of equilibrium between the Rayleigh-Taylor instability and a plane wake will be investigated.

- Investigate the effects of unstable buoyancy on the near wake development.

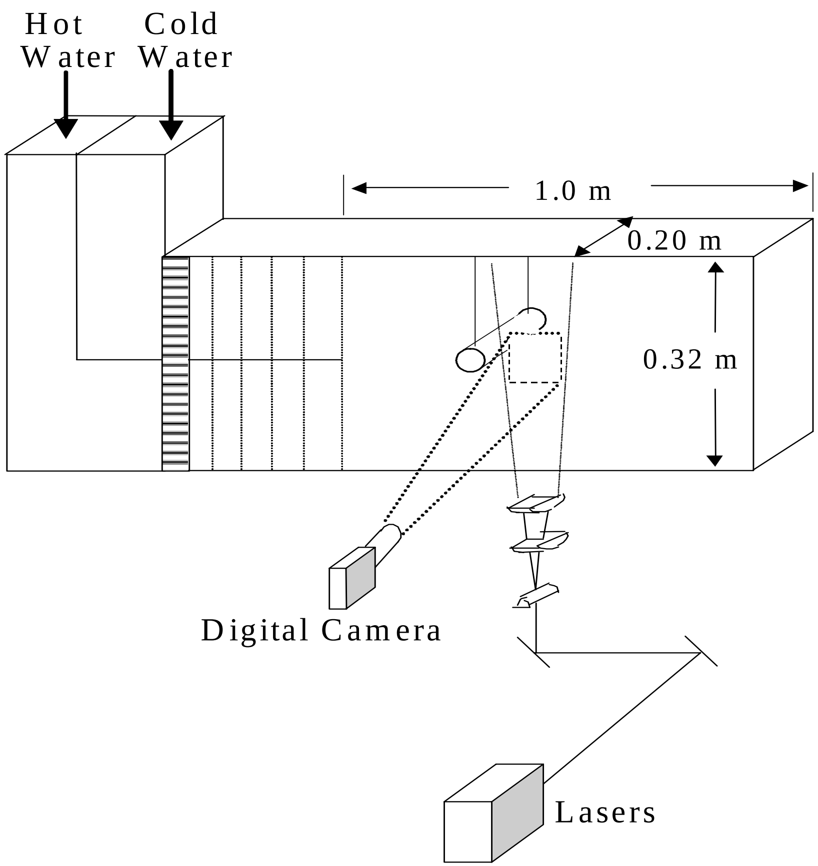
Objectives

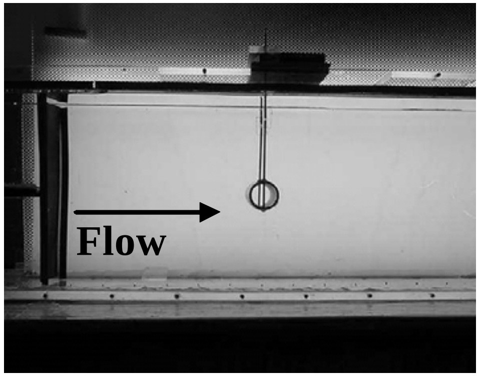
- Disturb a Rayleigh-Taylor mixing layer using a cylindrical obstruction.
- Measure the response of the subsequent wake to unstable stratification.
- Measure the recovery of the mixing layer from a disturbance of its equilibrium.
- Understand the mechanisms for recovery to equilibrium growth of the Rayleigh-Taylor instability.
- Develop a mathematical model to describe the effects of buoyancy on the near wake behavior.

Experimental results to be presented

- Decay of v'_{rms} in the wake of the cylinder
- Change in formation length of shedding vortices with turbulence levels
- Molecular mixing in the wake
- Density spectra in the centerline of the wake
- Recovery of the buoyancy driven turbulence

Setup of the experiment





Diagnostics:
- PIV
- PLIF
- Thermocouple system

Previous research on wakes

- Non-buoyant wake
  - Extensive research on both the near and far wake. A comprehensive review is provided by Williamson (1996)
- Stably stratified wake
  - Recent experiments by Xu et al (1995), Bonnier et al (2002), and Spedding (1997,2002)
- Unstably stratified wake
  - Nothing significant
  - Convection heat transfer studies of heated/cooled cylinders

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Buoyant wake

$\Delta T \cong 5°C$

Mean velocity	U (cm/s)	4.0 - 4.3
Cylinder diameter	D (cm)	1.6
Kinematic viscosity	ν (cm^2/s)	0.01
Reynolds number	Re	640-690
Internal Froude number	Fi^2	-7
Atwood number	A	$5x10^{-4}$

Wake with stable buoyancy:

$$Fi = U/ND$$

$$N^2 = -g/\rho_o (\partial\rho/\partial z)$$

Rayleigh-Taylor instability

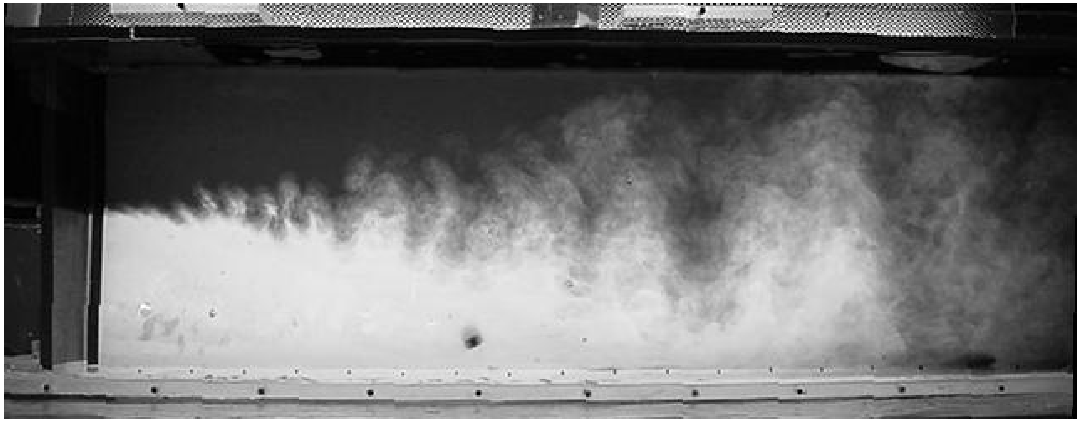
$$\nabla p$$

$$\nabla \rho$$

$$\nabla p \bullet \nabla \rho < 0$$

Governing Parameter:

$$A = (\rho_1 - \rho_2)/(\rho_1 + \rho_2)$$



Self-similar growth
of the mix half-width, h:

$$h = \alpha A g t^2$$

Verification of the experiment

A wake with no buoyancy

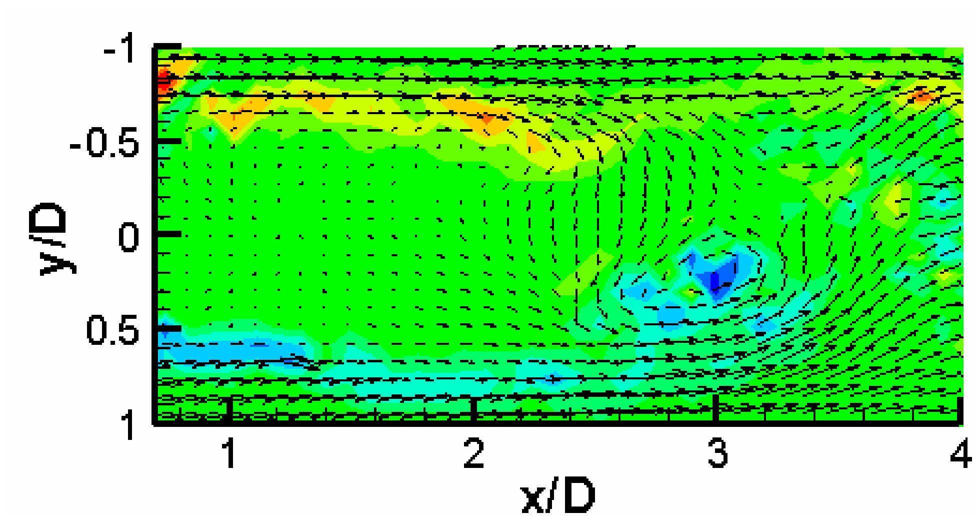
D = 1.6 cm
U = 4.0 cm/s

$f_s \cong 0.49$ Hz

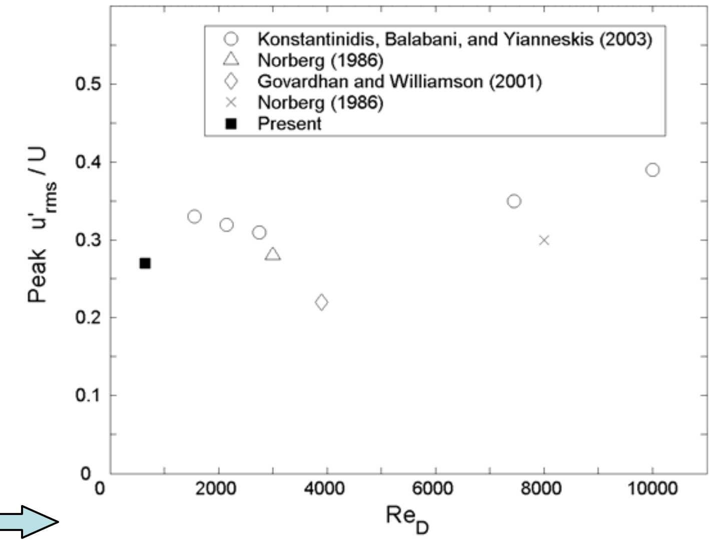
Confirmed through:

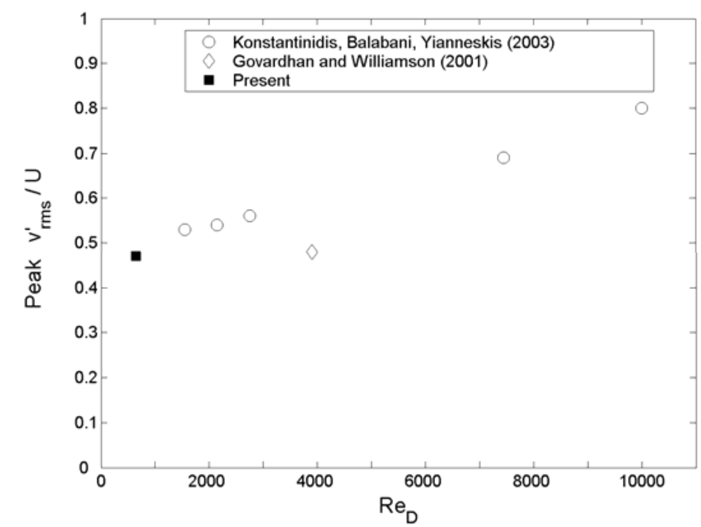
$St = 0.2 = f_s D / U \Rightarrow f_s = 0.5 \text{ Hz}$

Verification of the experiment (cont.)

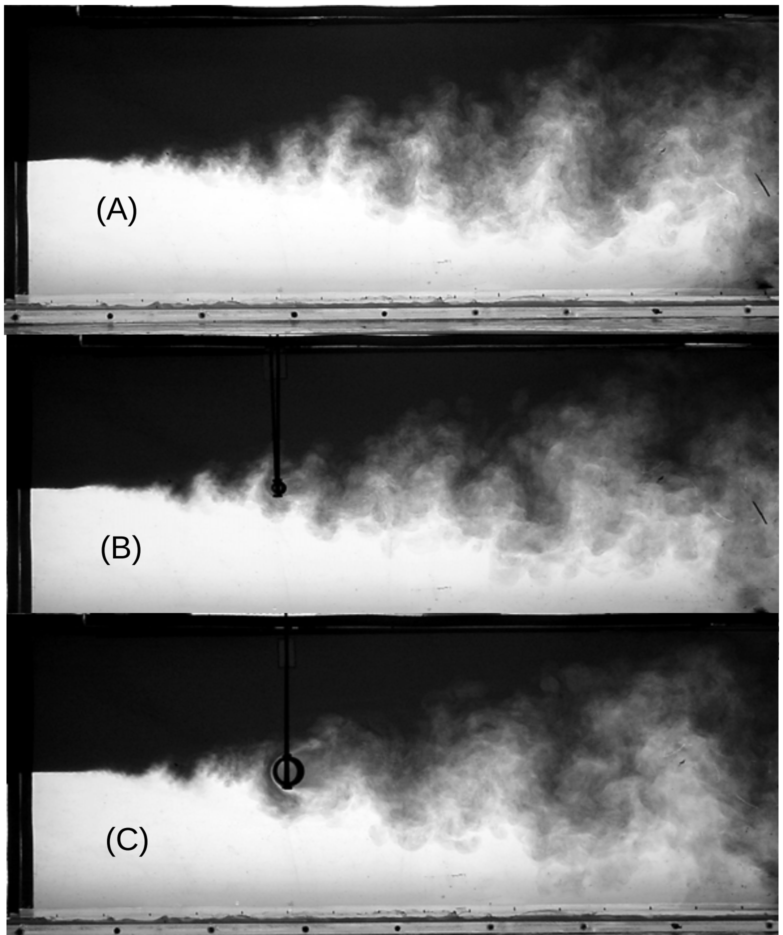


A vector field and vorticity contours for the near field a wake with no buoyancy.





Visualization of the buoyant wake



Visualizations of the buoyancy driven wake using Nigrosene dye.

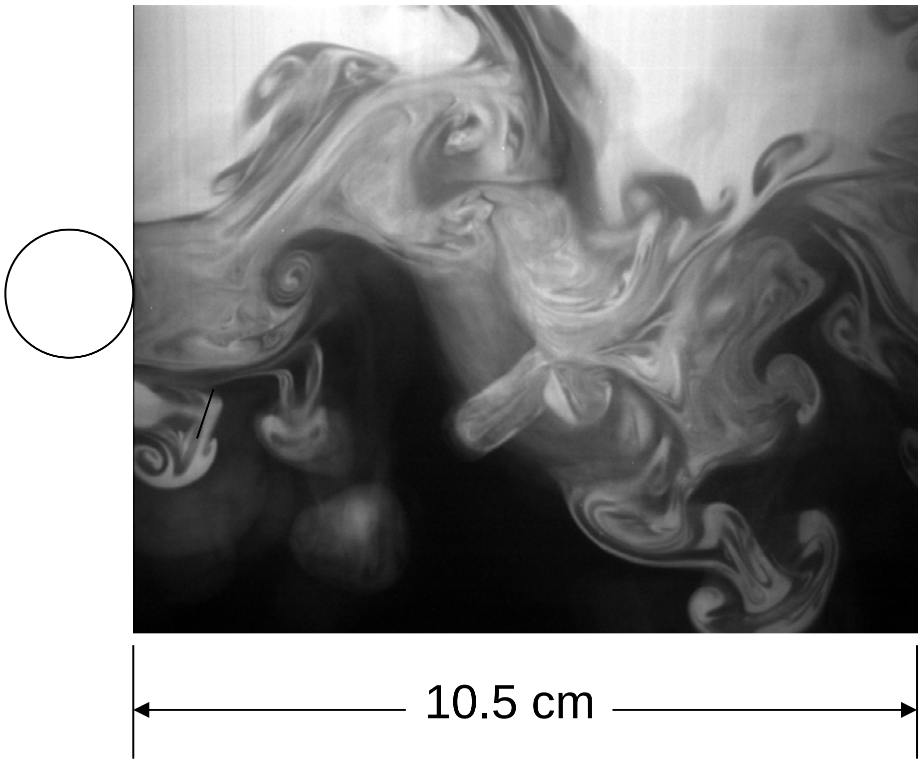
(A) Buoyancy driven-mixing layer

(B) Buoyancy-driven mixing layer and wake with a cylinder of D = 1.6 cm

(C) Buoyancy-driven mixing layer and wake with a cylinder of D = 3.25 cm.

U = 4 cm/s for all three experiments.

PLIF of the buoyant wake



D = 1.6 cm
U = 4.0 cm/s
$A = 5 \times 10^{-4}$

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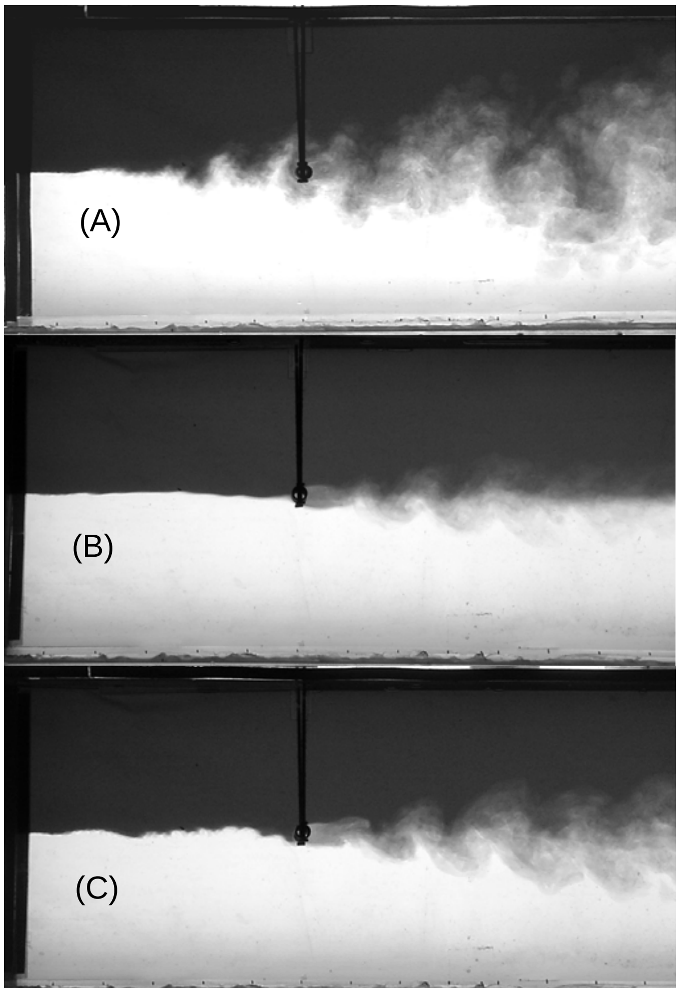
Visualization of buoyancy effects



Visualizations of buoyancy effects on wakes using Nigrosene dye.

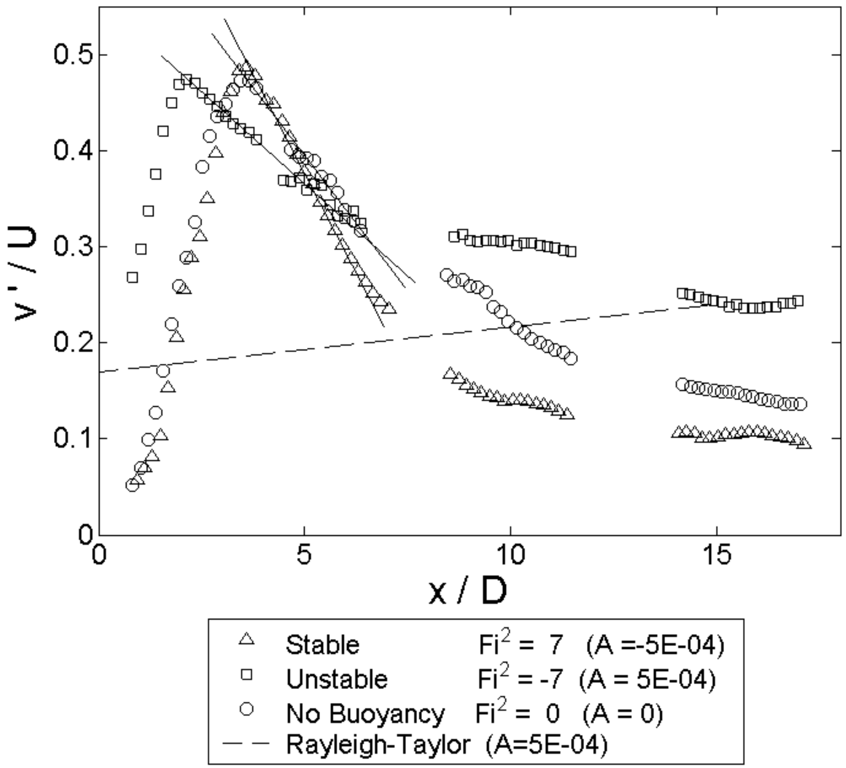
(A) Buoyant wake

(B) Wake with stabilizing buoyancy

(C) Wake without buoyancy effects.

All three wakes are formed from flow around a cylinder of D = 1.6 cm and U = 4 cm/s.

Decay of v'_{rms} in the wake



D = 1.6 cm
U ≅ 4 cm/s

Decay of vertical velocity fluctuations, v'_{rms}, in the wake of a cylinder with stable buoyancy (triangles), unstable buoyancy (squares), no buoyancy (circles), and a typical Rayleigh-Taylor mixing layer for the same experimental conditions (dashed line).

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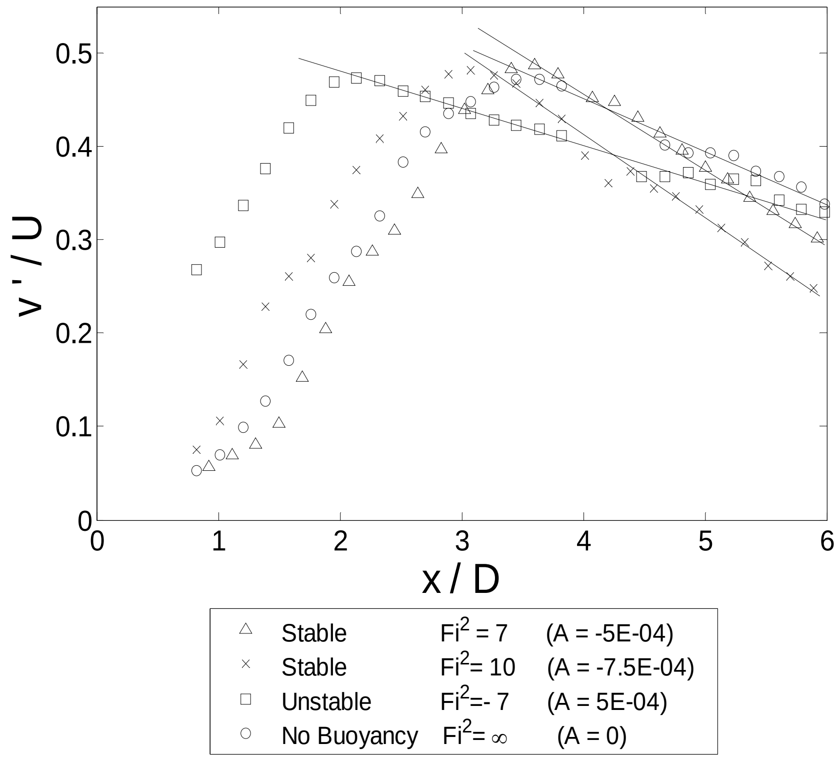
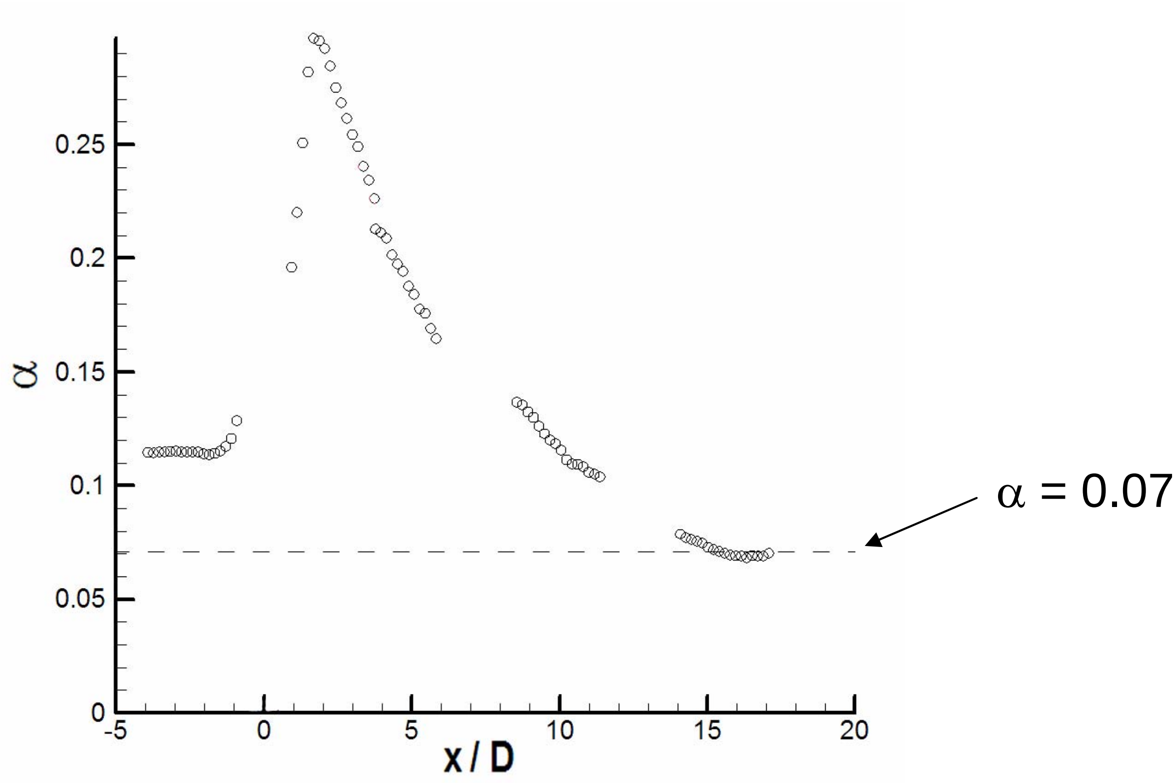
Decay of v'_{rms} in the near field



D = 1.6 cm
U ≅ 4 cm/s

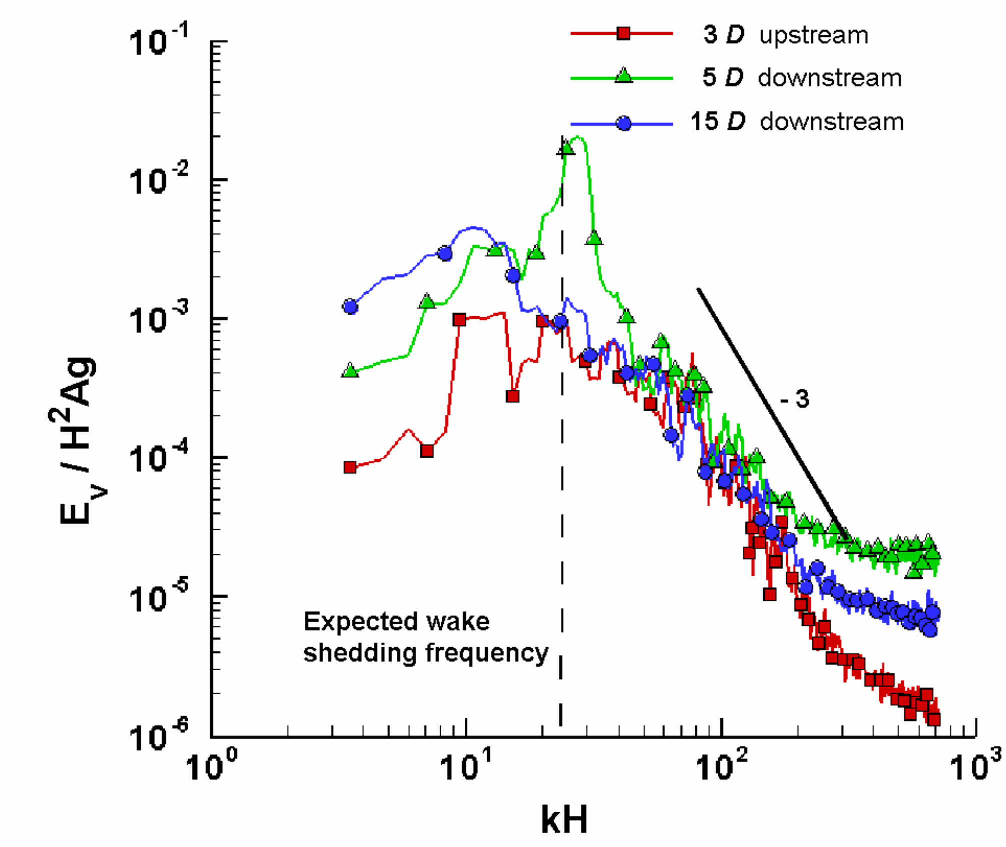
Decay of vertical velocity fluctuations, v'_{rms}, in the very near wake of a cylinder with stable buoyancy, unstable buoyancy, and no buoyancy.

Recovery of buoyancy-driven turbulence



$$\alpha = v' / 2Agt$$

Power spectra of *v’* in the wake with unstable buoyancy



Power spectra of centerline *v’* in the buoyant wake for a cylinder diameter of 1.6 cm and $A = 5 \times 10^{-4}$.

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Toy model for velocity fluctuations in the buoyant wake

$$\frac{Dk}{Dt} = \nabla \bullet \left(\frac{\upsilon_t}{\sigma_k} \nabla k \right) + P - \varepsilon \qquad \upsilon_t = c_\mu k^{1/2} l_m$$

The assumptions made to simplify the one-equation model:

1) The flow is statistically steady.
2) Mean velocity components in directions other than the x-direction can be neglected.
3) *k* follows a “top-hat” profile across the wake (in the y-direction) so that *k* = *k(x).*
4) The diffusion transport term in the x-direction is negligible compared with convective transport in the x-direction.
5) The velocity fluctuations at the centerline of the wake are locally isotropic.

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Model formulation

$$U\frac{d\langle k\rangle}{dx} = -\frac{\Delta\rho}{\rho} g\langle k\rangle^{1/2} - \frac{C_D\langle k\rangle^{3/2}}{l_m}$$

$$\frac{dv'}{dt} = -\left(\frac{1}{6}\right)^{1/2}\frac{\Delta\rho}{\rho} g - \frac{3^{1/2}}{2^{3/2}}\frac{C_D v'^2}{l_m}$$ where $l_m = D$

No buoyancy: $$\frac{dv'}{dt} = -\frac{3^{1/2}}{2^{3/2}}\frac{C_D v'^2}{D}$$

Stable buoyancy: $$\frac{dv'}{dt} = -\left|-\left(\frac{1}{6}\right)^{1/2}\frac{\Delta\rho}{\rho} g\right| - \frac{3^{1/2}}{2^{3/2}}\frac{C_D v'^2}{D}$$

Unstable buoyancy: $$\frac{dv'}{dt} = +\left|-\left(\frac{1}{6}\right)^{1/2}\frac{\Delta\rho}{\rho} g\right| - \frac{3^{1/2}}{2^{3/2}}\frac{C_D v'^2}{D}$$

Mathematical solutions

No buoyancy:

$$v' = \frac{1}{\left(-a\,t + \frac{1}{v'_{peak}}\right)}$$

Stable buoyancy:

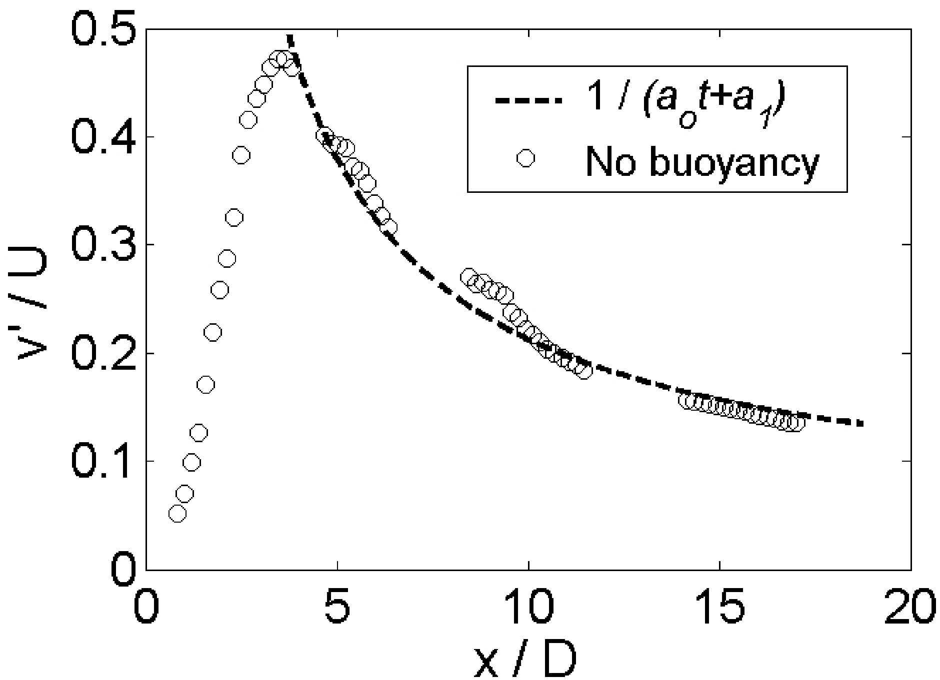
$$v' = \frac{(ac)^{1/2}}{a}\tan\left((ac)^{1/2}t + \arctan\left(\frac{a\,v'_{peak}}{(ac)^{1/2}}\right)\right)$$

Unstable buoyancy:

$$v' = \frac{\left(\frac{-c}{a}\right)^{1/2}}{\tanh\left((-ac)^{1/2}\,t + \text{arctan h}\left(\frac{\left(\frac{-c}{a}\right)^{1/2}}{v'_{peak}}\right)\right)}$$

Where $a = -\frac{3^{1/2}}{2^{3/2}}\frac{C_D}{D}$, $c = -\left(\frac{1}{6}\right)^{1/2}\frac{\Delta\rho}{\rho}g$, and v'$_{\text{peak}}$ is the maximum value of the v'_{rms} at the centerline of the near wake.

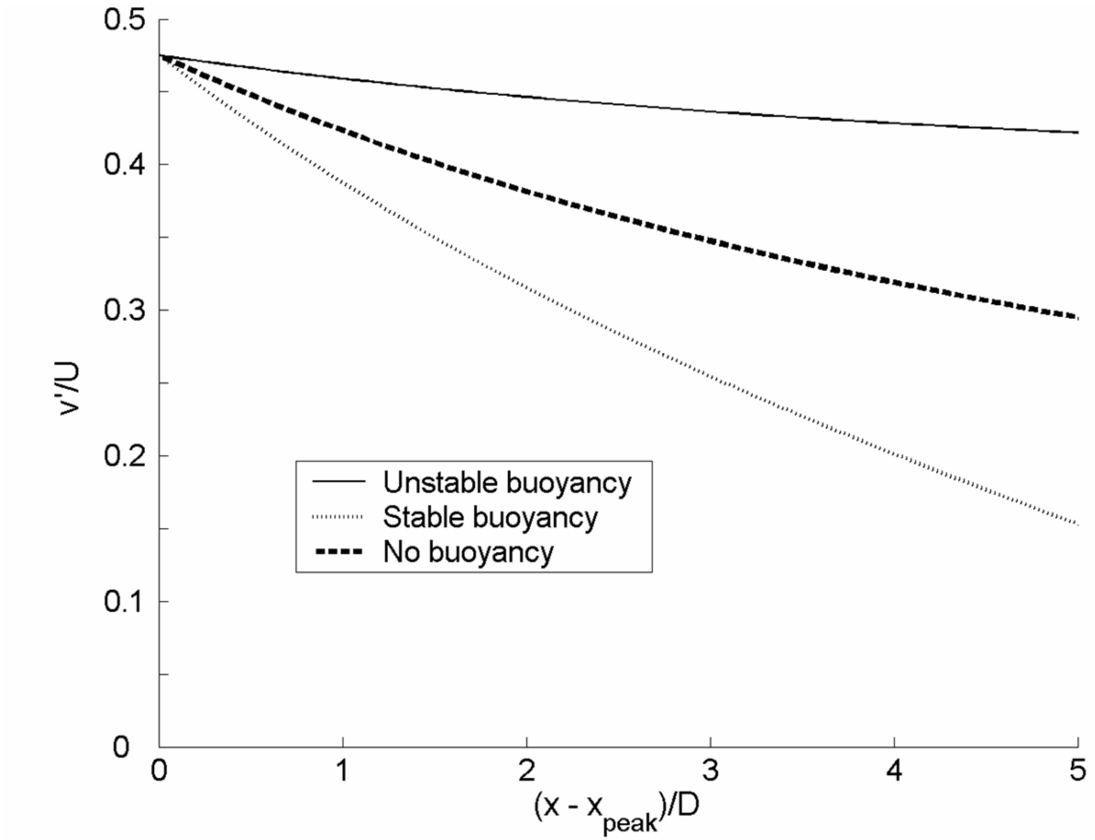
Mathematical solutions



No buoyancy: $$v' = \frac{1}{\left(-a\,t + \frac{1}{v'_{peak}}\right)}$$

Where $a = -\frac{3^{1/2}}{2^{3/2}}\frac{C_D}{D}$, $c = -\left(\frac{1}{6}\right)^{1/2}\frac{\Delta\rho}{\rho}g$, and v'_{peak} is the maximum value of the v'_{rms} at the centerline of the near wake.

Model behavior



Decay of v'_{rms} in the near wake for the cases of no buoyancy, stable buoyancy, and unstable buoyancy as determined from the model solutions using $C_D = 0.42$ and for A= $\pm$ 5x10^{-4}.

Variation of C_D with buoyancy

	Stratification	***A***	***D (cm)***	C_D	**(∂v'/∂x)(D/U)** Experiment	**(∂v'/∂x)(D/U)** *Model*
(1)	Stable	$-7.5\text{x}10^{-4}$	1.6	0.20	-0.085	-0.085
(2)	Stable	$-5.0\text{x}10^{-4}$	1.6	0.29	-0.077	-0.077
(3)	None	0	1.6	0.42	-0.054	-0.054
(4)	Unstable	$5.0\text{x}10^{-4}$	1.6	0.59	-0.038	-0.038
(5)	Unstable	$7.5\text{x}10^{-4}$	1.6	0.77	-0.061	-0.061
(6)	Stable	$-5.0\text{x}10^{-4}$	0.94	0.58	-0.074	-0.074
(7)	None	0	0.94	0.20	-0.024	-0.024
(8)	Unstable	$5.0\text{x}10^{-4}$	0.94	0.59	-0.043	-0.043

$$C_{D\ stable} \approx 0.5 C_{D\ none}$$

$$C_{D\ unstable} \approx 1.5 C_{D\ none}$$

Conclusions

- The transport of energy to the wake from potential energy stored in the unstable density gradients, results in a remarkably rapid return to the characteristic behavior of the Rayleigh-Taylor mixing layer.
- The dynamics of the flow directly behind the cylinder is dominated by the wake, with little influence from the buoyancy.
- The level of buoyancy driven turbulence preceding the cylinder and subsequent wake affects the length of the formation region for shedding vortices.
- The proposed model qualitatively demonstrates the observed decay of centerline vertical velocity fluctuations in the near wake.

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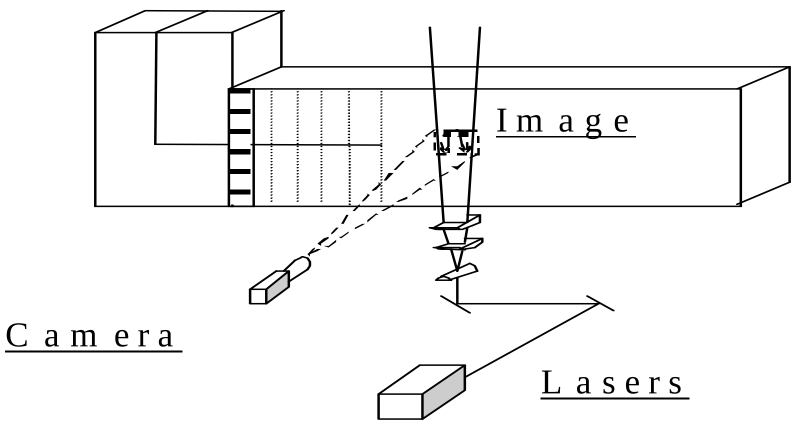
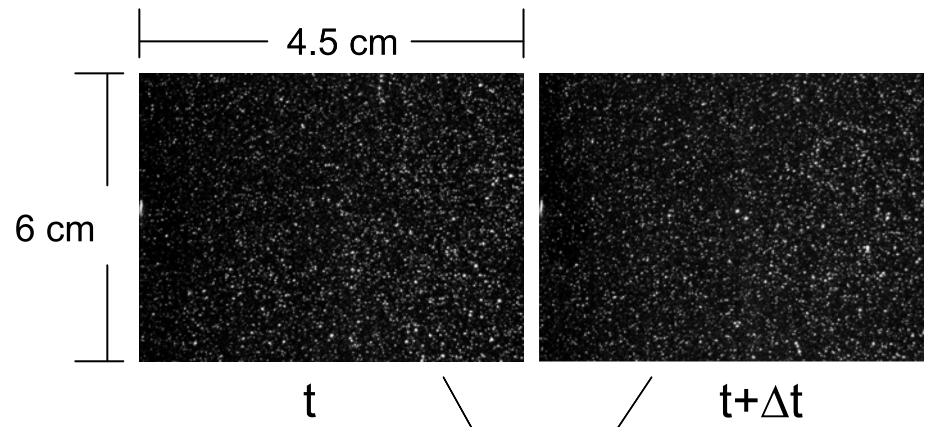
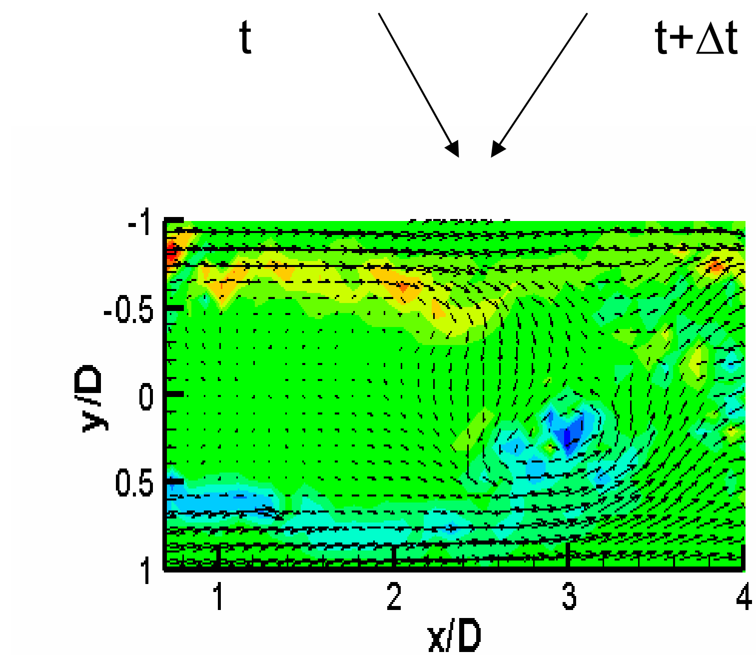
Future work

- Effect of unstable buoyancy on vortex structure
- Change in formation length for vortices due to buoyancy driven turbulence
- Parametric study of the buoyant wake to further characterize the behavior
- Further investigation of the recovery of buoyancy driven turbulence and competing equilibria

Particle image velocimetry (PIV)

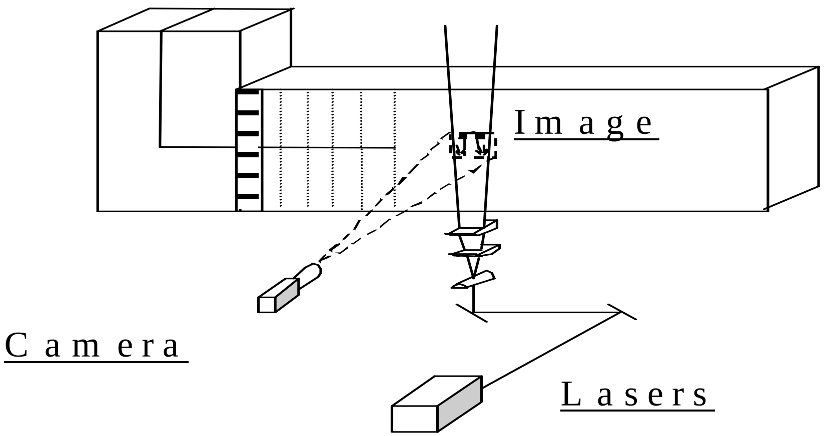


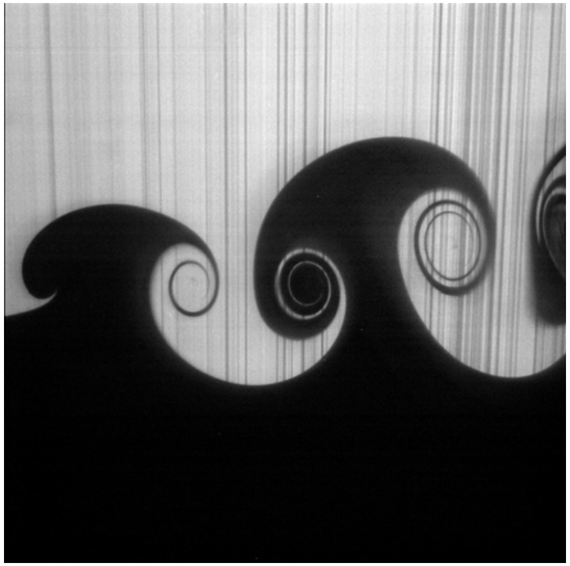




- Laser source – Nd-YAG, 30 Hz
- Laser sheet forming optics
- CCD Camera (640 x 480 pixels)
- Seed particles – (~ 10 μm)

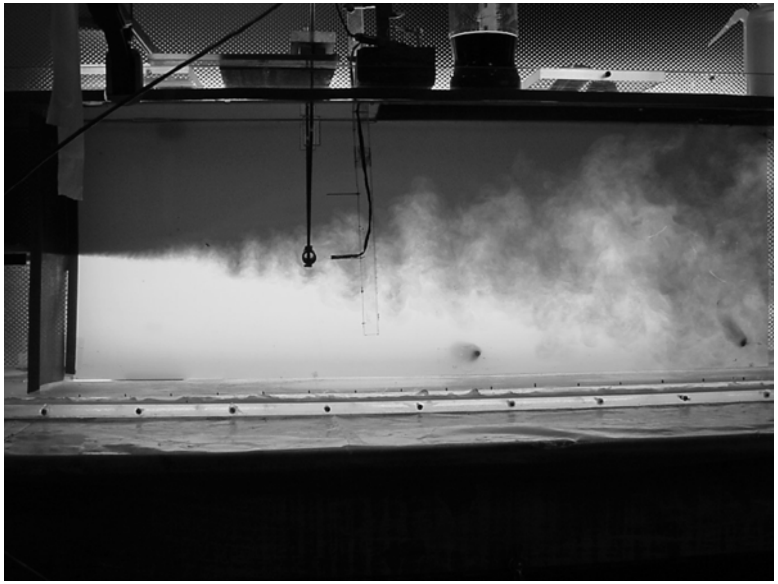
Planar laser induced fluorescence (PLIF)





- Laser source
- Laser sheet forming optics
- Filter
- CCD camera
- Fluorescent dye – Rhodamine 6G

Thermocouple system



E-Type thermocouple

Effective sampling rate of 500 Hz

Sampling time of ~ 120 sec

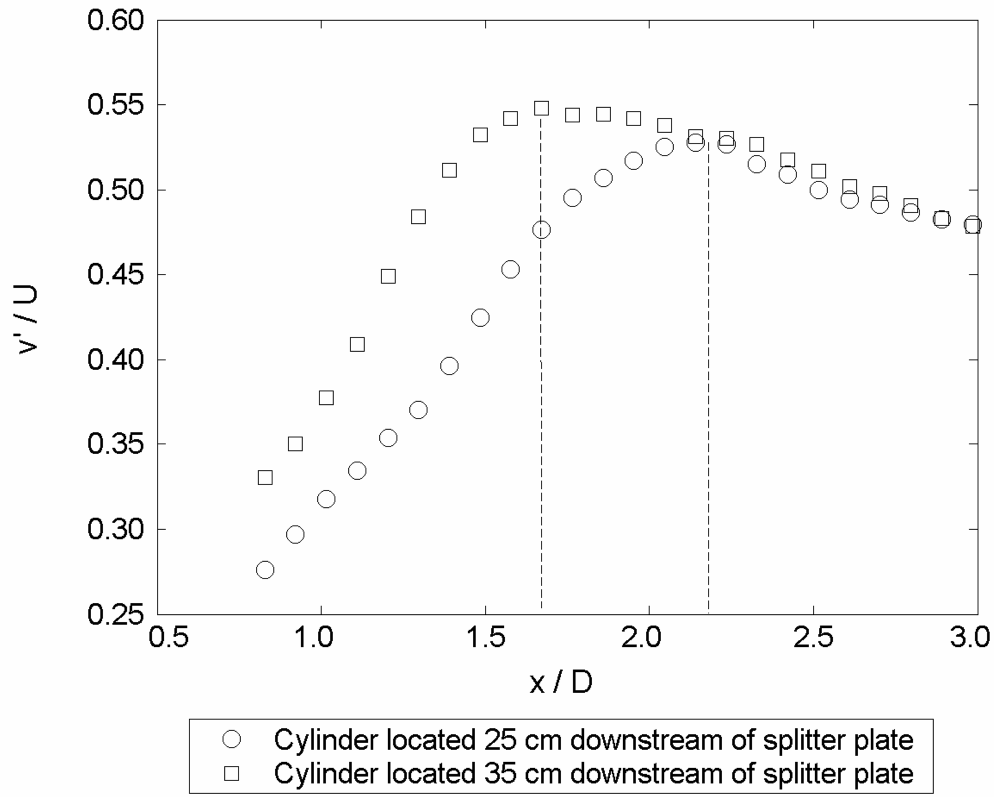
Displacement coefficient:

$$\phi = (T_2 - T_{avg})/(T_2 - T_1)$$

Equation of State (Kukulka 1981):

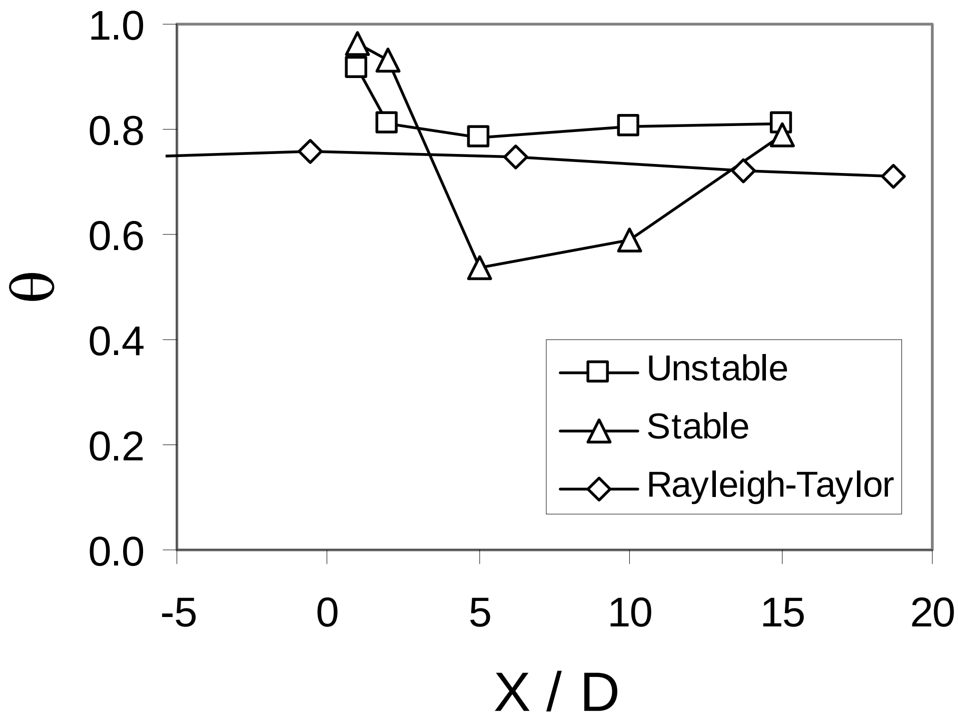
$$\rho = \frac{999.8396 + 18.2249T - 0.007922T^2 - 55.448\times10^{-6}T^3 + 149.756\times10^{-9}T^4 - 393.295\times10^{-12}T^5}{1 + 18.159\times10^{-3}T}.$$

Variation of the location of v'_{peak}



Variation of the location of peak v'_{rms} with mixing layer turbulence level. Flow conditions for both cases were with a *U* = 4.4 cm/s, *A*= 5x10-4 and a cylinder diameter of 1.6 cm.

Molecular mixing in the wake



$$f_1 = \lim_{T\to\infty} \frac{1}{T}\int_0^T \frac{\rho-\rho_2}{\rho_1-\rho_2}dt$$

$$f_2 = 1 - f_1$$

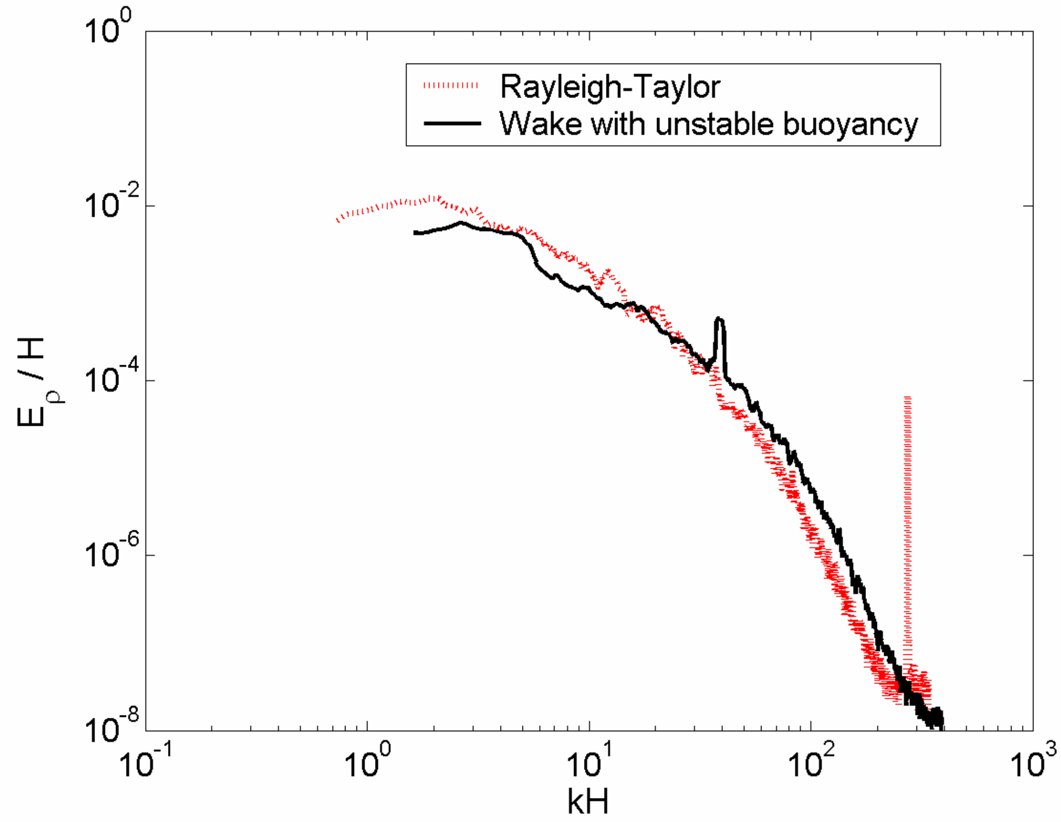
$$B_o = \lim_{T\to\infty} \frac{1}{T}\int_0^T \left(\rho-\overline{\rho}\right)^2 dt / \Delta\rho^2$$

$$B_2 = f_1 f_2$$

$$\theta = 1 - \frac{B_o}{B_2}$$

For two completely molecular mixed fluids θ is equal to 1 and for two immisicble fluids θ is 0.

Power spectra of centerline density fluctuations in the wake



A comparison of power spectra of centerline density fluctuations in the buoyant wake (solid black line) and a mixing layer driven by the Rayeigh-Taylor instability (dotted red line) obtained from Ramaprabhu (2003).