Spectral Characteristics of Turbulence Driven by Rayleigh-Taylor Instability

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Outline

- Introduction

Rayleigh-Taylor instability, turbulent spectra, sensitivity to initial conditions, MILES codes

- Simulations

Turmoil code, statistics, initial conditions

- Results

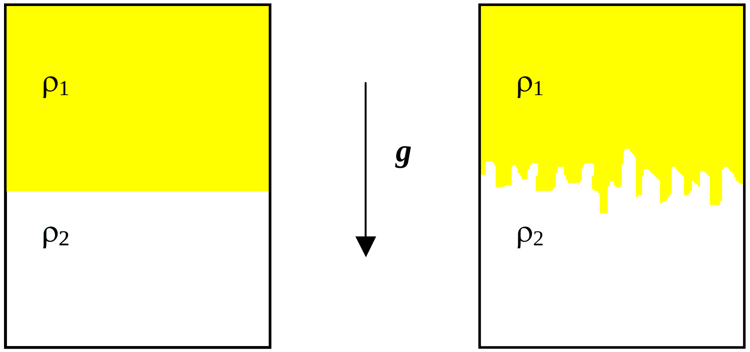
Spectral shape, time evolution, influence of initial conditions, dominant wavenumber, mixing layer width

- Conclusions

Introduction

- R-T instability

$\rho_1 > \rho_2$



Non-dimensional parameter

Atwood number $A = \left(\dfrac{\rho_1 - \rho_2}{\rho_1 + \rho_2}\right) = \dfrac{g'}{2g}$

Linear stability analysis (1D)

For mode of wavenumber k, interface at $h(x,t) = \mathrm{Re}\{h_0 e^{ik.x+\sigma t}\}$, growth rate $\sigma = \sqrt{kAg}$

Small scales are most unstable, unless damped by viscosity
Rayleigh (1883), Taylor (1950)

Nonlinear growth

Dimensional analysis - mean width of mixing region $\bar{h}$ depends on t and Ag, hence $\bar{h} \propto Agt^2$, like t^2. Experimentally and numerically, $\bar{h} = 0.06Agt^2$
Read (1984), Burrows, Smeeton & Youngs (1984)

For an external lengthscale H, timescale $\tau = \sqrt{\dfrac{H}{Ag}}$

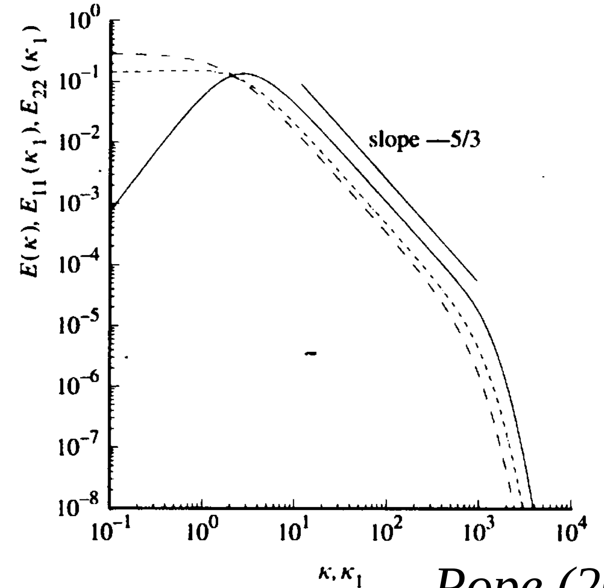
Larger scales observed at later times

Introduction

- Turbulent spectra

Homogeneous isotropic turbulence

At high enough Reynolds number, inertial range $E(k) \propto k^{-5/3}$, between energy input scale and dissipation range. At high k, spectra decay faster than a power law.

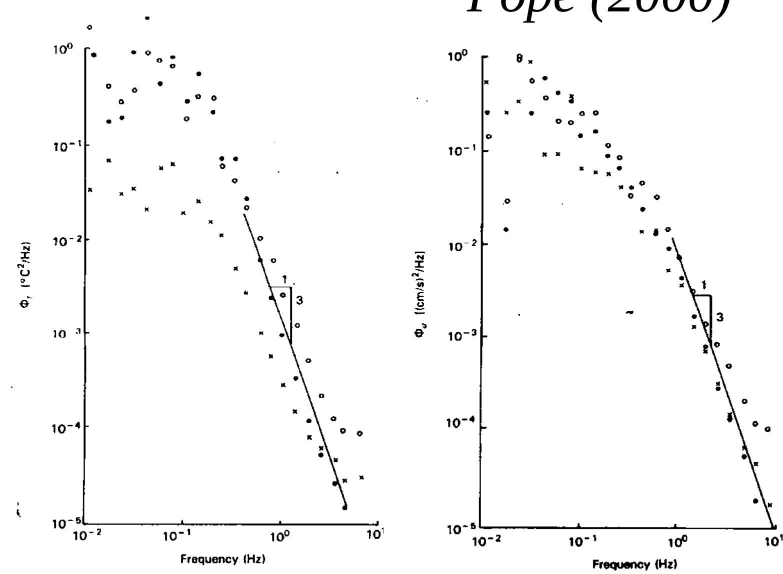


Pope (2000)

Buoyancy-driven turbulence

Clear air turbulence:
$k^{-5/3}$ at high wavenumbers, k^{-3} at lower wavenumbers (provided work done against gravity is small) *Shur (1962), Lumley (1965)*.

Convection adjacent to a heated wall: velocity and temperature fluctuations (right) exhibit k^{-3} spectra



Gebhart et al. (1988)

Introduction

- Sensitivity to initial conditions

Presence of large scales affects mixing layer growth *Dalziel et al (1999)*

Choice of random initial conditions affects DNS simulations *Cook & Dimotakis (1999)*

- MILES codes

Conservation of mass and momentum imposed by the algorithm: loss of resolution at grid scale mimics diffusion of solute and viscous dissipation

In a real fluid, viscosity ν is fixed, and velocity gradients adjust so that dissipation rate ε matches rate of energy supply. Dissipation concentrated at wavenumbers $k > k_\nu = (\varepsilon/\nu^3)^{1/4}$

In code, k_ν is fixed, all energy reaching scales $k > k_\nu$ is dissipated, so viscosity ν varies

Simulations

- Turmoil (David Youngs)

Compressible code, for a mixture of two ideal gases

3D MILES with resolution 200×160×80

Normalisations: choose $H = 1$, $Ag = 1$, $\rho_1 = 1$

Choose parameters to approximate an incompressible fluid. Non-dimensional parameters (ideally small):

density ratio $B = \Delta\rho/\rho = 2/g \approx 0.18$

Mach number $M = \sqrt{(3/5p_0)} \approx 0.08$

incompressibility ratio $I = g^2/10p_0 \approx 0.12$

Compromise $g = 11$, $p_0 = 100$

Simulations

- Statistics in the horizontal mid-plane

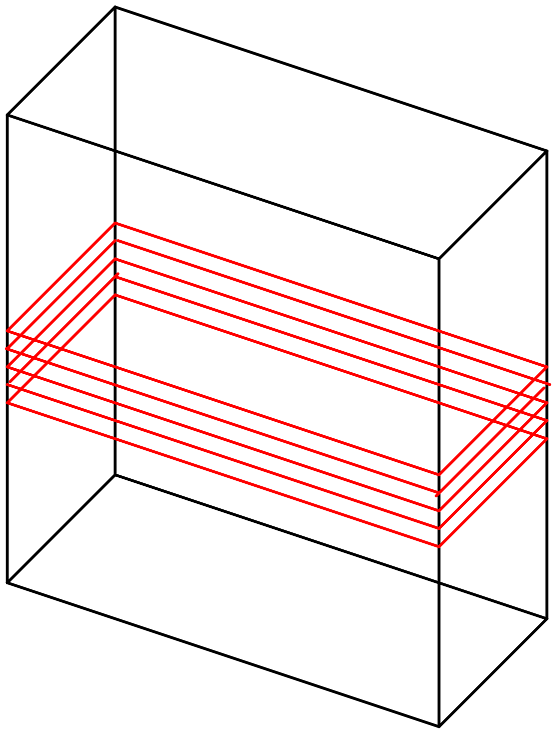
Average over 8 horizontal planes

Data extended using appropriate even/odd symmetry at boundaries to create periodic data

Calculated every $\tau = 0.25$

Look at energy in concentration variation and velocity components

Integrate over horizontal direction to give 1D spectra



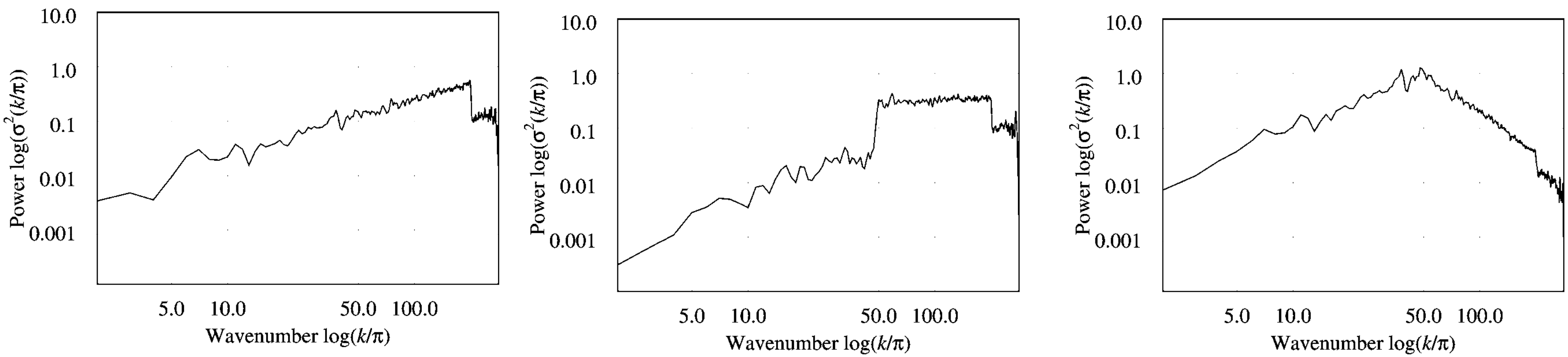
Simulations

- Range of initial conditions

Displace the interface by a few pixels to give random initial perturbation

In some cases, add large scale perturbation in velocity field to mimic experiments in DAMTP

Vary amplitude, smoothness, slope of random noise



Results

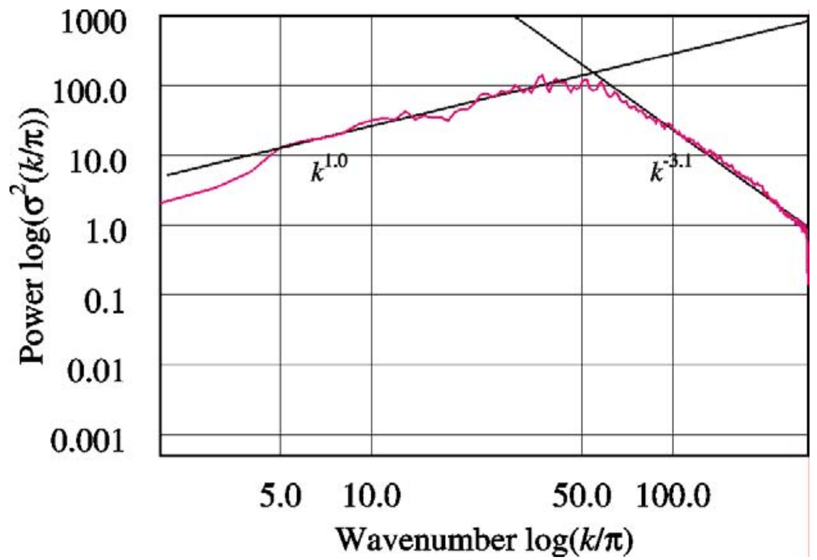
- Spectral shape

For high wavenumber perturbation at $\tau = 1$ (turbulence developing)

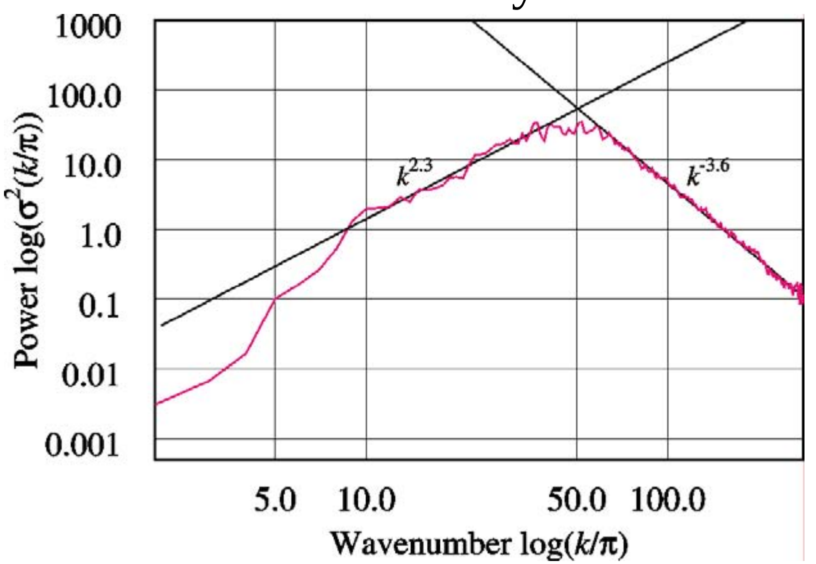
Peak energy where dissipation begins ($\lambda \approx 6\Delta x$ or $k/\pi \approx 67$)

Power law in dissipation range

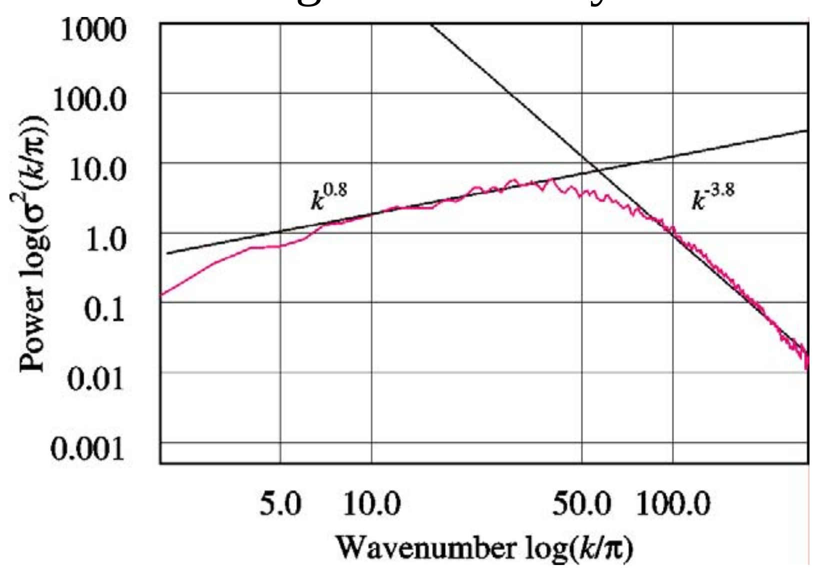
Concentration



Vertical velocity



Along-tank velocity



Results

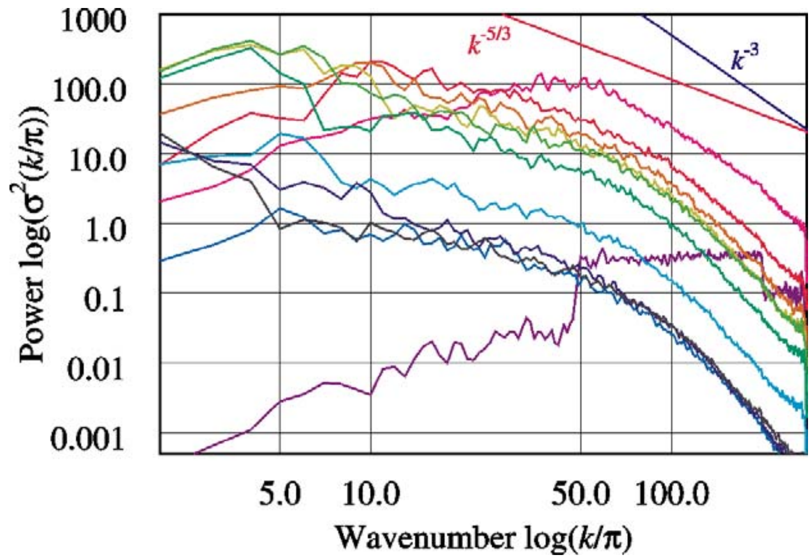
- Time evolution

τ= 0,1,…10 (purple ⇒ blue)

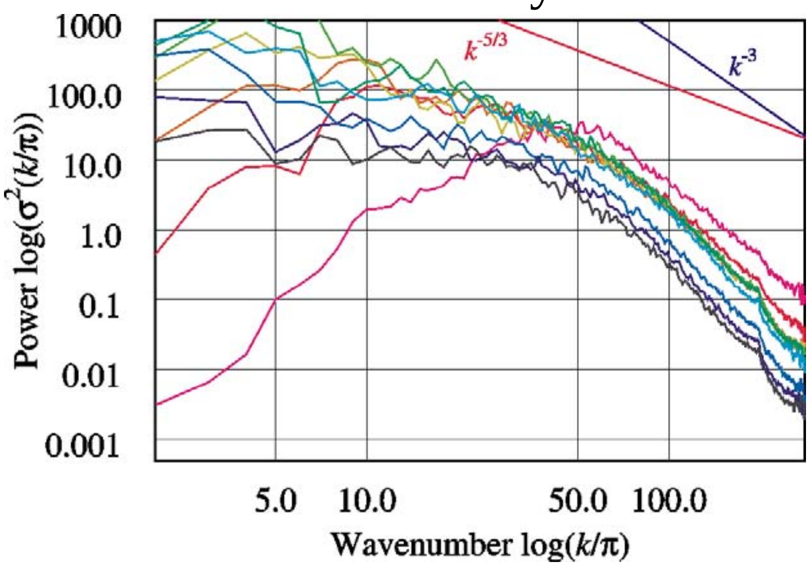
Similarity behaviour in turbulence not constrained by domain size (3<τ<5)

Velocity becomes isotropic as concentration fluctuations decay (τ>8)

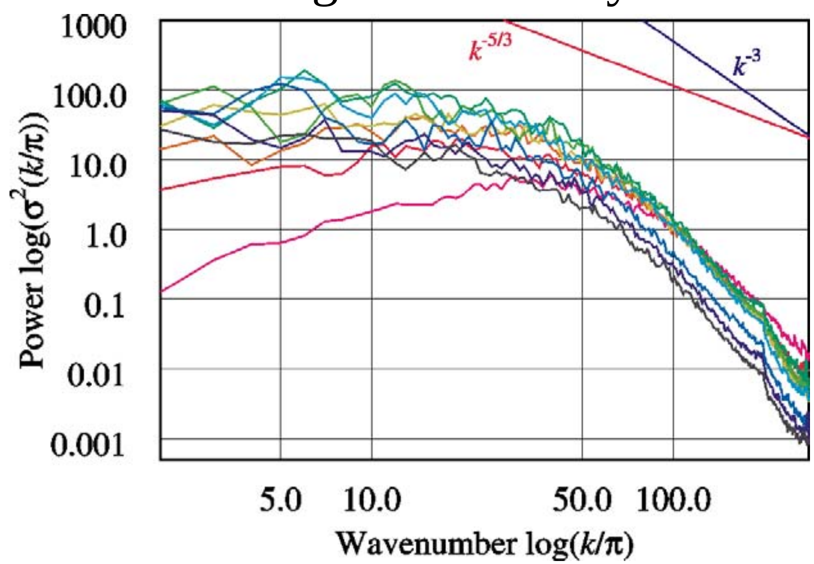
Concentration



Vertical velocity



Along-tank velocity



Results

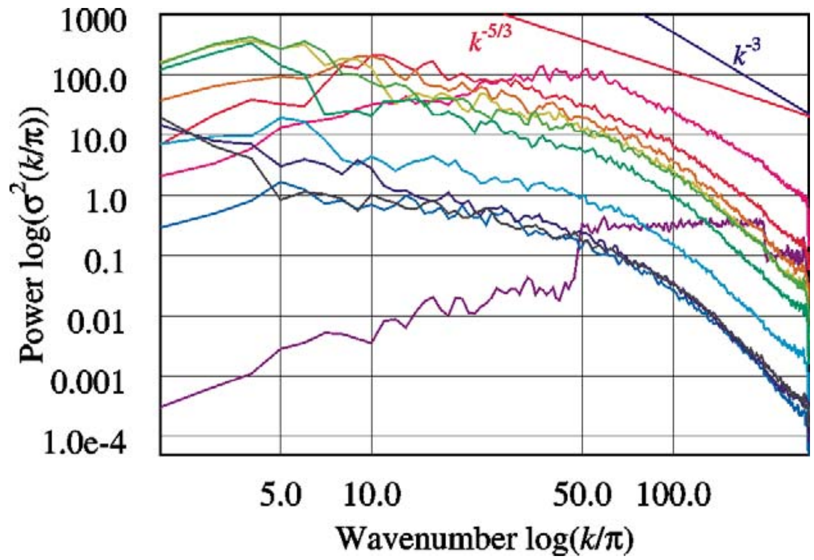
- Varying initial conditions

Concentration spectra

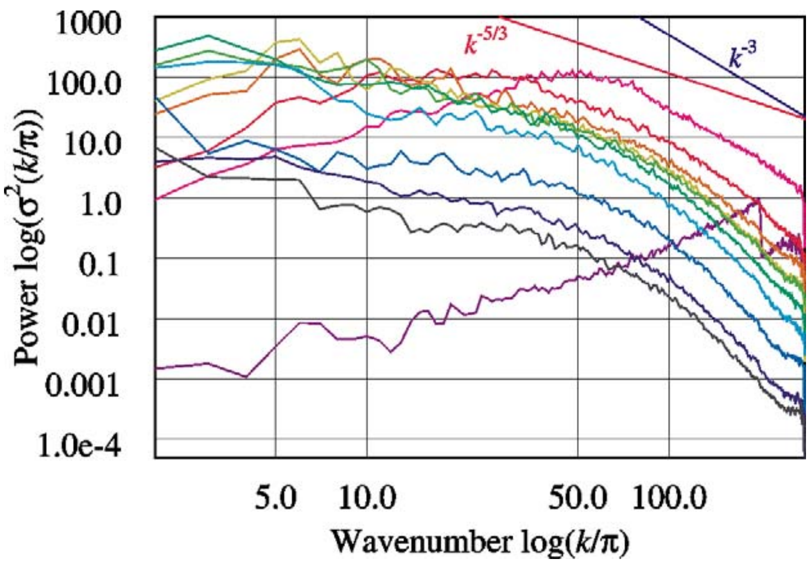
Little difference between extreme initial conditions

Amount of molecular mixing is also very similar

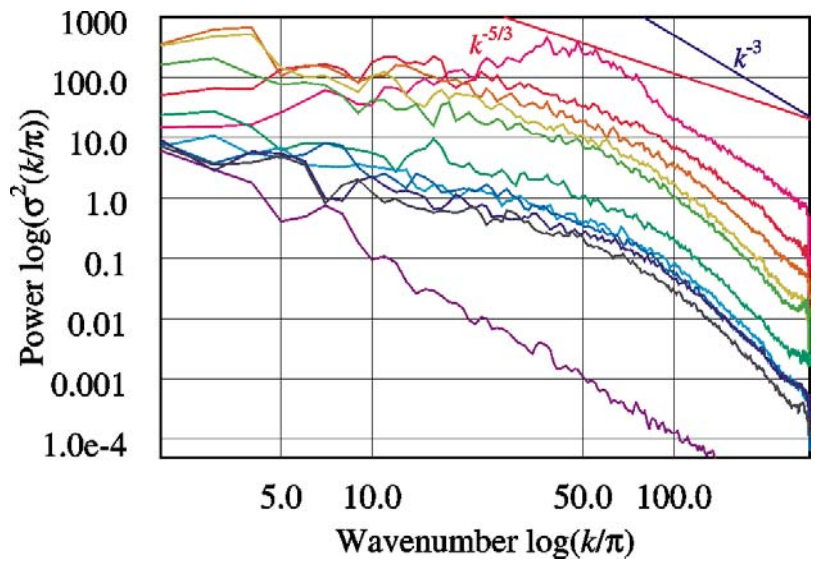
High k



k^3



k^{-3}

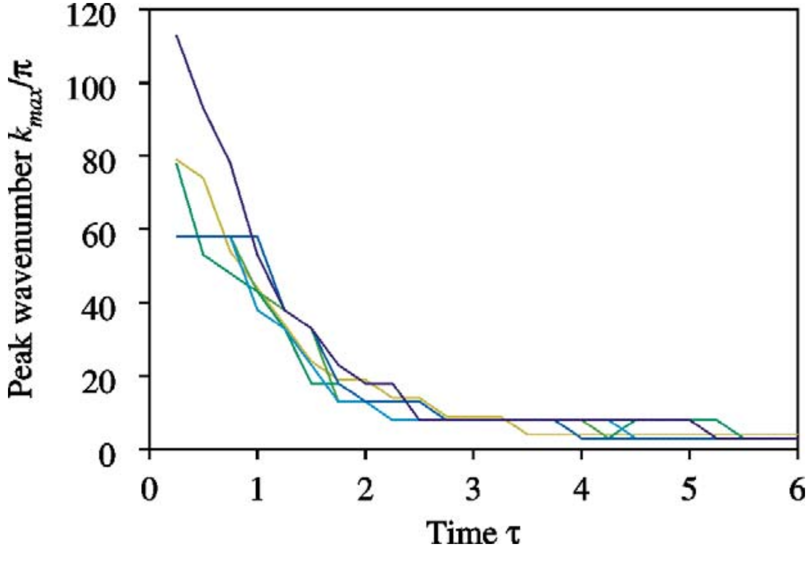


Results

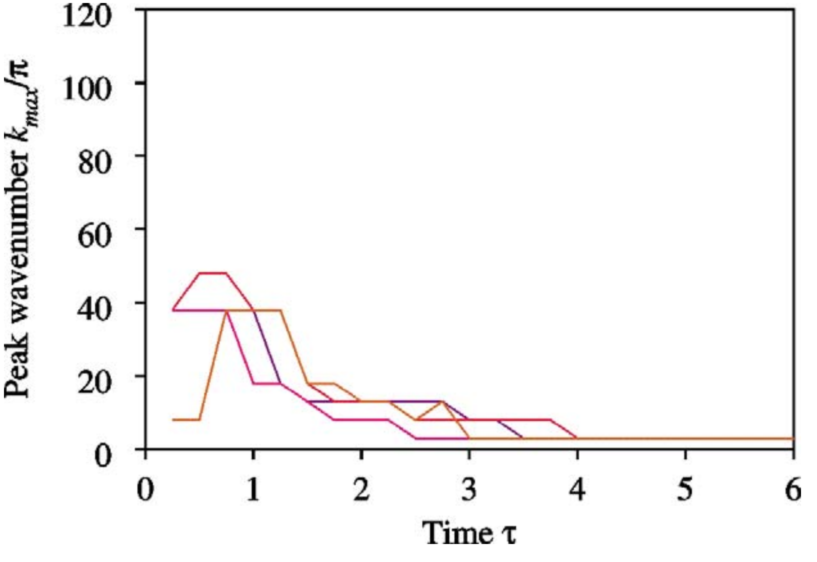
- Dominant wavenumber

Wavenumber of peak vertical velocity disturbance depends on initial spectrum for $\tau < 2$

high wavenumber dominant (k^{1} or higher)



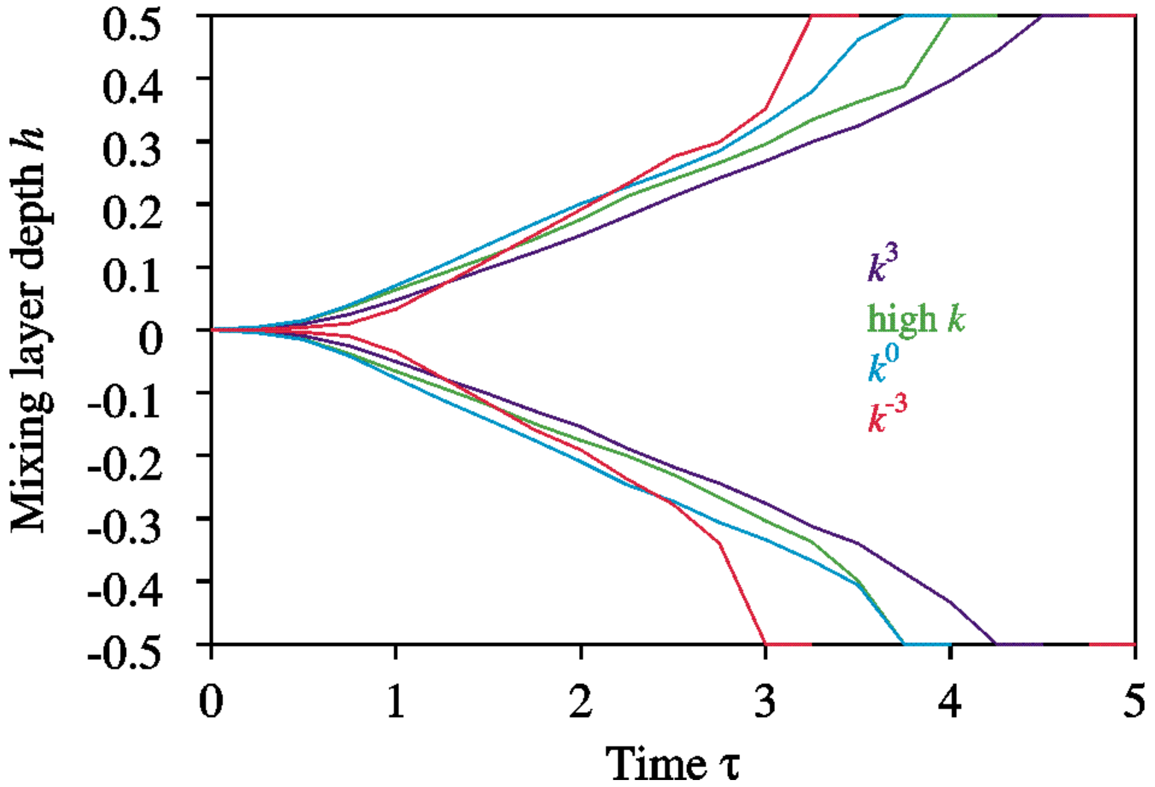
low wavenumber dominant (k^{-1} or lower)



Evidence of period-doubling

Results

- Mixing layer width



Bias towards energy at low *k* in simulation with initial k^{-3} spectrum gives

slower initial growth

faster late time growth

Conclusions

- Spectra evolve rapidly ($\tau<1$) to similar shapes
- Similarity phase: spectra approximately constant for $3 < \tau < 5$
- High k spectra and amount of molecular mixing are not sensitively affected by the initial conditions. Power law behaviour which steepens with time ($k^{-3} \rightarrow k^{-5}$)
- Low k spectra, early dominant k_{max} and time origin are sensitively affected by the initial conditions. In particular k^{-3} spectrum gives particularly rapid growth