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Interacting Thermals

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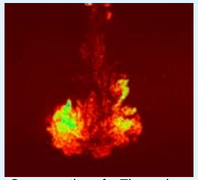
What is a thermal?

- A Thermal is an isolated release of buoyant fluid
- The release is from rest, there is no initial momentum; the motion is driven purely by buoyancy forces.
- At the release point the density difference between the released fluid and the ambient fluid is small so the Boussinesq approximation is applied.
- For small density differences between the thermal and the ambient fluid the motion is the same whether the thermal is ascending or descending.
- As the thermal travels it entrains the surrounding fluid, reducing the difference in density between it and increasing its volume.
- However, the total buoyancy excess remains constant.



Cross-section of a Thermal

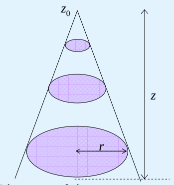
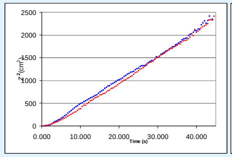
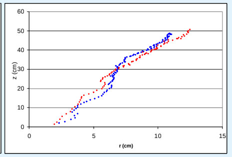


Diagram of the cone swept out as a thermal descends

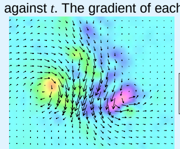
- The thermal is assumed to be self-similar, the general shape of the thermal will remain the same but will increase in size with time.
- As the thermal falls a cone is swept out, the apex of this is known as the virtual origin, z_0, the point at which the thermal would have zero radius.
- The position of the front of the thermal, z, and the maximum radius of the thermal, r, are therefore related by $z = nr$ and the volume can be expressed as $V = mr^3$, where n and m are constants determined by experiment and may vary in each case.
- The first experimental work was done by Scorer 1957, who found n and m to be approximately 4 and 3 respectively.
- From dimensional analysis, the velocity of the thermal is $u^2 = C^2gBr$. Where C is a Froude number and B is the buoyancy ratio.
- Integrating this gives $kz^2 = t$, where $k = \dfrac{m^{\frac{1}{2}}}{2nC(B_0V_0g)^{\frac{1}{2}}}$



Graph of experimental results of z^2 against t. The gradient of each line is k



Graph of z against r. Extrapolating back, the z intercept would be the distance from the release point to the virtual origin.



- The interior structure of the thermal resembles a flattened spherical vortex.

- The first theoretical work was by Morton, Taylor and Turner in 1955. It is assumed that the thermal entrains fluid at a rate proportional to the mean vertical velocity.
- As volume, mass and momentum are conserved, the conservation equations for a thermal can be expressed as:

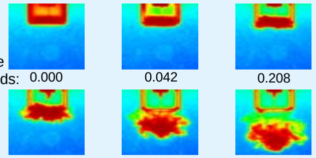
$$\frac{d(\frac{4}{3}\pi r^3)}{dt} = 4\pi r^2 \alpha u,$$

$$\frac{d(\frac{4}{3}\pi r^3 \rho u)}{dt} = \frac{4}{3}\pi r^3 g(\rho_0 - \rho),$$

$$\frac{d(\frac{4}{3}\pi r^3(\rho_1 - \rho))}{dt} = 4\pi r^2 \alpha u(\rho_1 - \rho).$$

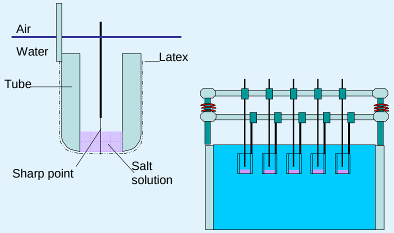
Experimental Setup

The experiments are conducted in a tank of fresh water of dimensions 1.0m×0.8m×0.9m. The results are captured using a digital camera at a rate of 24 frames per second. A sequence of greyscale images at a resolution of 1380×1024 pixels are captured directly into DigiFlow. The images can then be processed to correct for background illumination and to produce a movie where the light intensity relates to the concentration of dye. Using a light sheet and reflective particles it is possible to get a view of the cross section of the thermal and evaluate the velocities within it as well as a view of its interior structure.



Release mechanism

In order to release a thermal a latex membrane is stretched over the end of a tube, between 3.5cm and 5cm interior diameter, and submerged in the tank. Dyed salt water is then run down a pipette to form a layer. The latex is then ruptured to release the thermal. Images from the first 2 seconds after the release are shown above. This technique is repeatable and several tubes can be easily arranged in various formations.



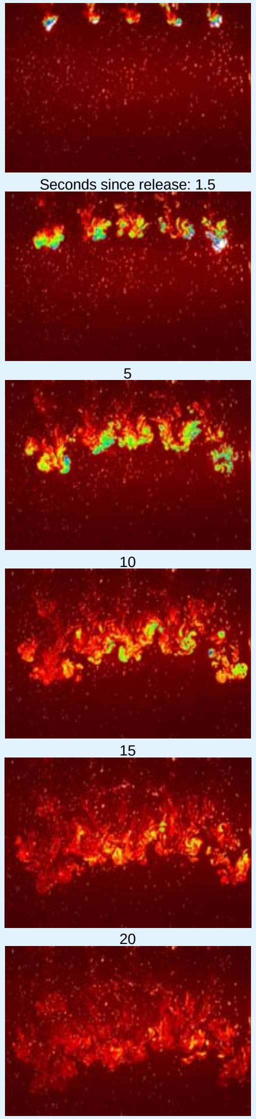
Sketch of the release mechanism, the left hand side shows a close view of the latex covered tube containing salt solution. The right hand side shows several such tubes arranged in the tank.

References

Scorer, R. S. 1957
Experiments on convection of isolated masses of buoyant fluid, J. Fluid Mech. **2** 583-594

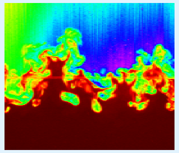
Morton, B.R., Taylor, G. & Turner, J.S. 1956
Turbulent gravitational convection from maintained and instantaneous sources. P. Roy. Soc. Lond. A. Mat. **234**, 1-23.

DigiFlow website: http://www.damtp.cam.ac.uk/lab/digiflow/



Motivation

A thermal entrains the ambient fluid rapidly. When falling in isolation, the thermal is free to expand and entrain the fluid surrounding it in all directions. When thermals are in proximity to each other, the fluid between them becomes limited and they compete for it. In an array this becomes more complicated as each thermal competes with its neighbours. When the initial density or size of the thermals are not equal one may be deflected or completely entrained by the other. Arrays may include a number of thermals with different densities in a variety of formations. A compact array of thermals can be considered as a model for the Rayleigh-Taylor instability



Courtesy of JeanPaul Jeffrai

Rayleigh-Taylor Instability

- The Rayleigh-Taylor instability develops from a layer of fluid lying above a layer of less dense fluid.
- This becomes unstable when perturbed and mixing occurs.
- One aim of this project is to model the Rayleigh-Taylor instability as two arrays of interacting thermals, one ascending and one descending.

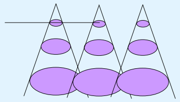
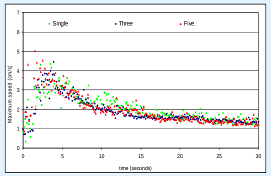


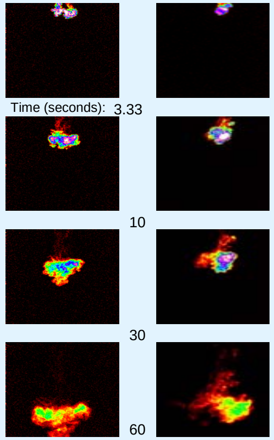
Diagram of how thermals might be expected to collide and merge if they behaved as in the isolated case, due to entrainment this happens much sooner.

Interaction between Thermals

The simplest model might expect thermals to fall as if in isolation, sweeping out the same cone and starting to merge at the point at which their cones intercept. However, because of the rapidity of the entrainment, thermals will collide well before this point. The amount of interaction depends on the initial separation between the thermals, if there is sufficient spacing between the thermals they will start falling as if isolated but will begin to pull towards each other. Fluid will be entrained from one to the other whilst they will remain as separate structures. If the thermals are close enough they will eventually completely merge.



The Maximum speed in a release of one, three and five thermals with the same initial conditions. The time sequence to the left is of 5 equal thermals falling in line.

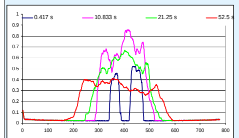


Two Thermals

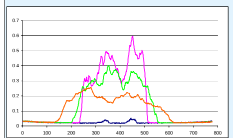
The sequence to the left is of two experiments, in each one two thermals of equal density were released side by side. The initial conditions were the same except that in the experiment on the right only the right hand thermal contained dye. This shows how fluid from one thermal is entrained into the other and they eventually merge into a single structure. The merged thermals behave in a similar way to a single thermal, however it is not known if the internal structure evolves to become axisymmetric.

Merging

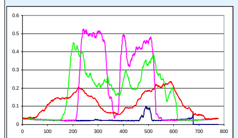
The vertical axis of each chart opposite is the pixel position along the length of a frame, the horizontal axis is the averaged intensity along the y-axis of a frame. The intensity at each point in the frame is related to the density of the thermal at that point. The top chart is for two thermals which are released two initial radii apart. At the earliest time two distinct peaks are visible, corresponding to the two separate thermals. By 10 seconds after the release there are fluctuations in the density but no separate peaks, by 52 seconds the density is almost homogeneous and so the thermals have completely merged. The middle chart shows the same thing with the thermals released 3 initial radii apart. The final chart has an initial separation of 5 radii and, although there is some overlap, at 53 seconds there are still two distinct thermals.



Two thermals, 2 initial radii apart



Separation of 3 initial radii



Separation of 5 initial radii

Acknowledgements

This PhD is funded by an EPSRC mathematics CASE studentship with AWE

At AWE, Aldermaston: David Youngs

At Cambridge: The researchers and technical staff of the GK Batchelor Lab.

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