

# Numerical simulation of pulsar wind and Supernova shell interaction and mixing.

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# Model of SN (Type II) Remnants (Jun, 1998)

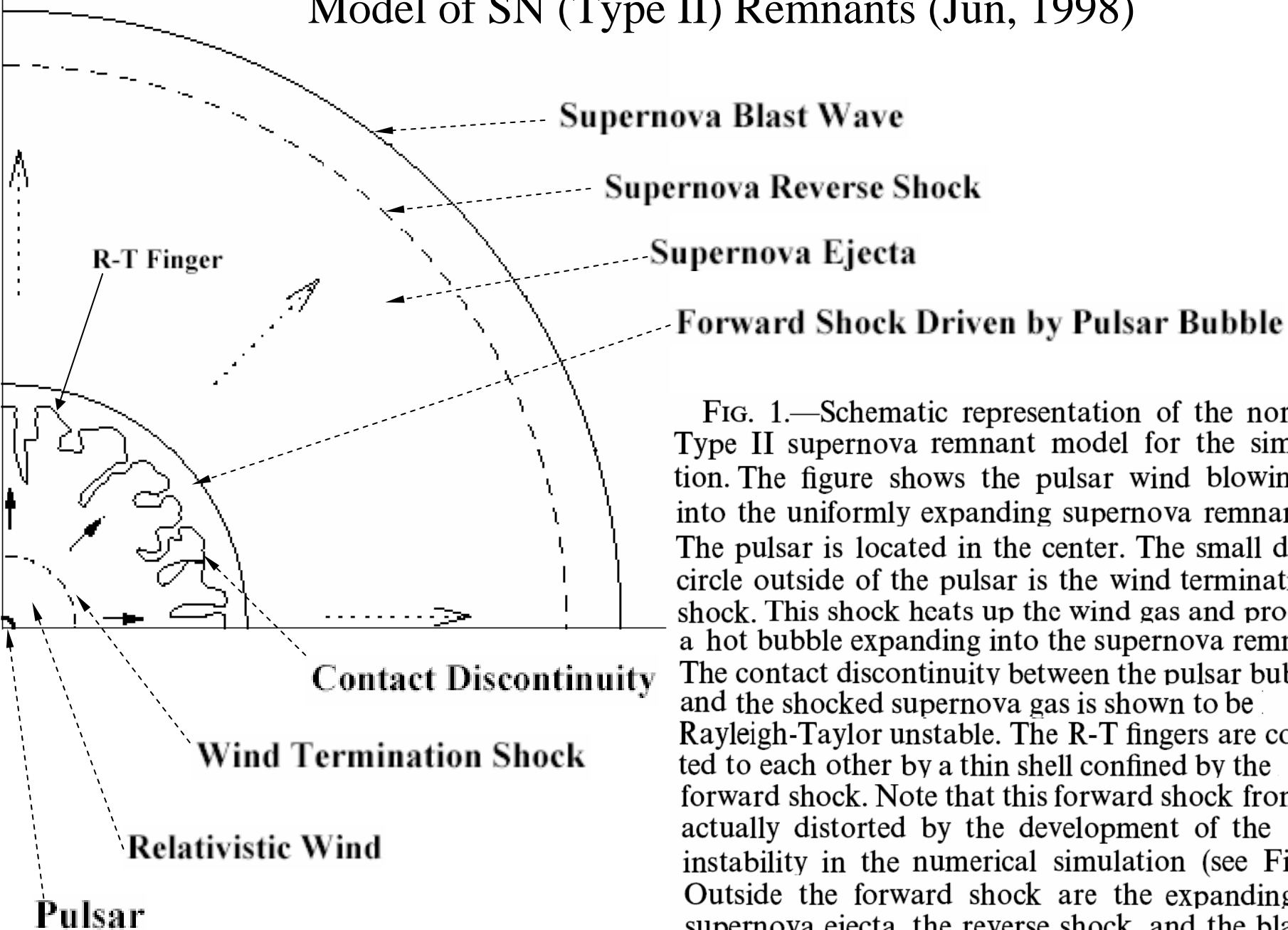
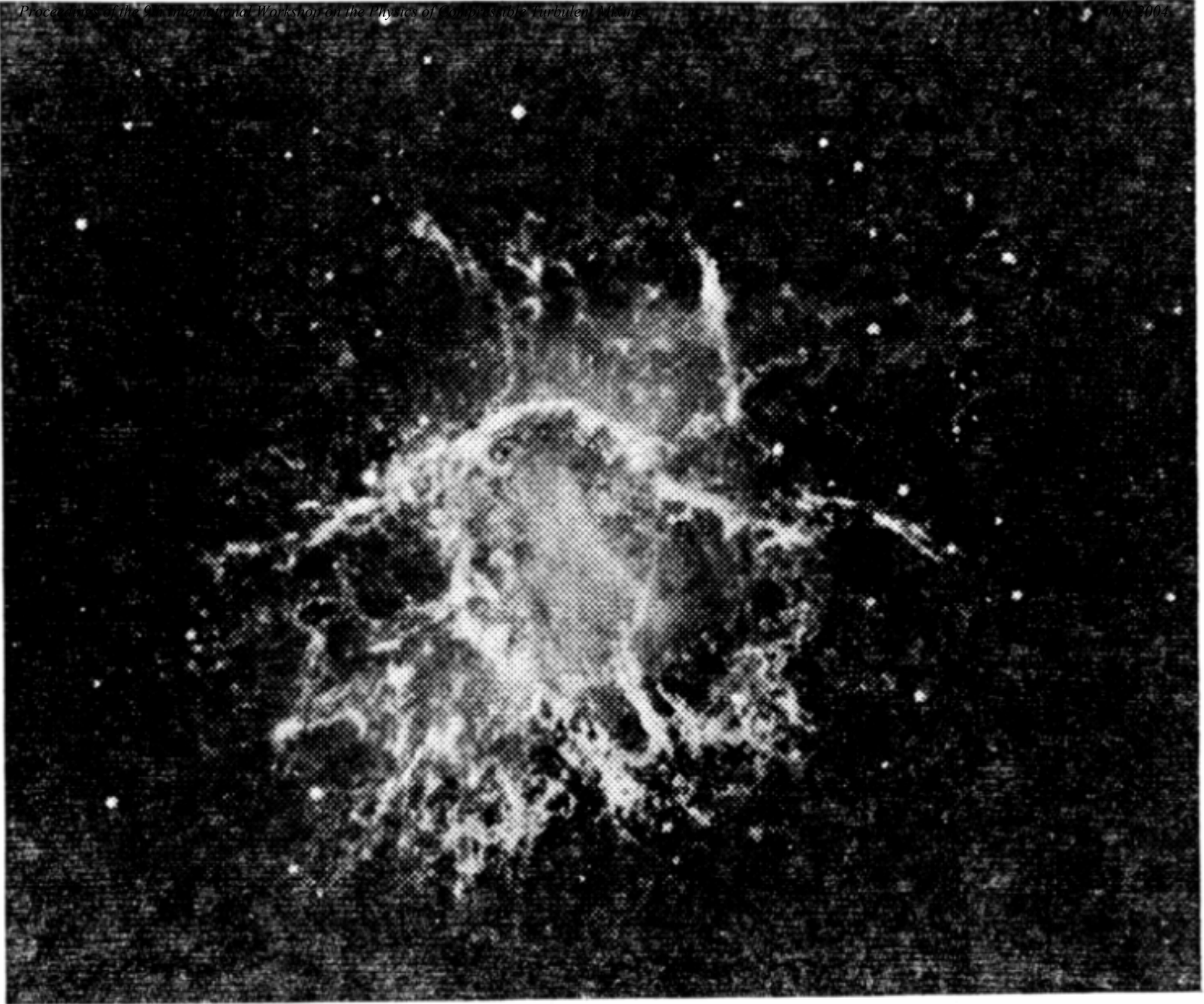


FIG. 1.—Schematic representation of the normal Type II supernova remnant model for the simulation. The figure shows the pulsar wind blowing into the uniformly expanding supernova remnant. The pulsar is located in the center. The small dotted circle outside of the pulsar is the wind termination shock. This shock heats up the wind gas and produces a hot bubble expanding into the supernova remnant. The contact discontinuity between the pulsar bubble and the shocked supernova gas is shown to be Rayleigh-Taylor unstable. The R-T fingers are connected to each other by a thin shell confined by the forward shock. Note that this forward shock front is actually distorted by the development of the instability in the numerical simulation (see Fig. 6). Outside the forward shock are the expanding supernova ejecta, the reverse shock, and the blast wave of the supernova.



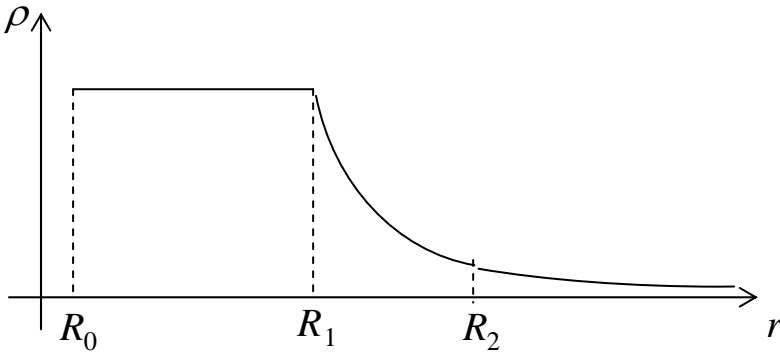
**Crab Nebula in  $H_{\alpha}$  lines. Filaments structure.**

Cambridge, UK

Edited by S.B. Dalziel

# Physical Model of SN expansion.

## 1) Initial Data:



Here:

$$\rho_1 = 1$$

$$\rho_2 = \left(\frac{R_1}{R_2}\right)^n$$

$$V = \frac{5}{3}$$

$$T_0 = 1.0$$

$$R_0 = 0.1$$

$$R_1 = 0.5$$

$$R_2 = 1.0$$

$$n = 10$$

$$m_{shell} \approx 1$$

$$\varepsilon = 1$$

$r \backslash f$	$\rho$	$u$	$T$
$0 \div R_0$ (PW)	pulsar wind		
$R_0 \div R_1$	$\rho_1$	$V \frac{r}{R_2}$	$T_0$
$R_1 \div R_2$	$\rho_1 \left(\frac{R_1}{r}\right)^n$		
$r > R_2$	$\rho_2 \left(\frac{R_2}{r}\right)^2$	0	$T_0$

## 2) Units:

$$\rho_0 = 10^{-16} \frac{g}{cm^3} \quad V_0 = 10^9 \frac{cm}{sec}$$

$$t_0 = \pi \cdot 10^7 \text{ sec (1 year)} \quad T_0 = 1 \text{ eV}$$

$$R_0 = Vt_0 = \pi \cdot 10^{16} \text{ cm} \approx 0.01 \text{ ps}$$

$$\text{energy: } \varepsilon_0 = \pi \cdot 10^{51} \text{ erg}$$

$$\text{mass: } m_0 = \pi \cdot 10^{33} \text{ g} \approx 1.5 m_\odot$$

## 3) Pulsar wind is given at

$r = R_0$  as (Jun,98)

mass flow:

$$\dot{m} = 0.25 \cdot 10^{-4} \left( 0.25 \cdot 10^{22} \frac{g}{sec} \right)$$

$$\text{energy flow: } L = 10^{-4} \left( 10^{40} \frac{erg}{sec} \right)$$

$$\text{velocity: } v = 2.2$$

## 4) Gasdynamics equations, $\gamma = 5/3$

(no magnetic field, no radiation, ideal fully ionized plasma)

Two types of computations:

4.1. 2D direct modelling (Yanilkin, “EGAK”,...).

## 4.2. 1D computation: gasdynamics + $k, \varepsilon$ – turbulent mixing model (Statsenko, Dolgolyova, ..., 2003).

Mass fraction equations:

$$\frac{d\alpha_i}{dt} = \frac{1}{\rho r^\theta} \frac{\partial}{\partial r} \left( r^\theta \rho (c_\alpha D + \xi_i) \frac{\partial \alpha_i}{\partial r} \right) + R_i \quad D = C_D k^2 / \varepsilon$$

Equations for turbulent energy and its dissipation

rate:

$$\frac{dk}{dt} = (G_1 + G_2) - \varepsilon \left( 1 + c_v \frac{v\varepsilon}{k^2} \right) + \frac{1}{\rho r^\theta} \frac{\partial}{\partial r} \left( r^\theta \rho (c_k D + v) \frac{\partial k}{\partial r} \right)$$

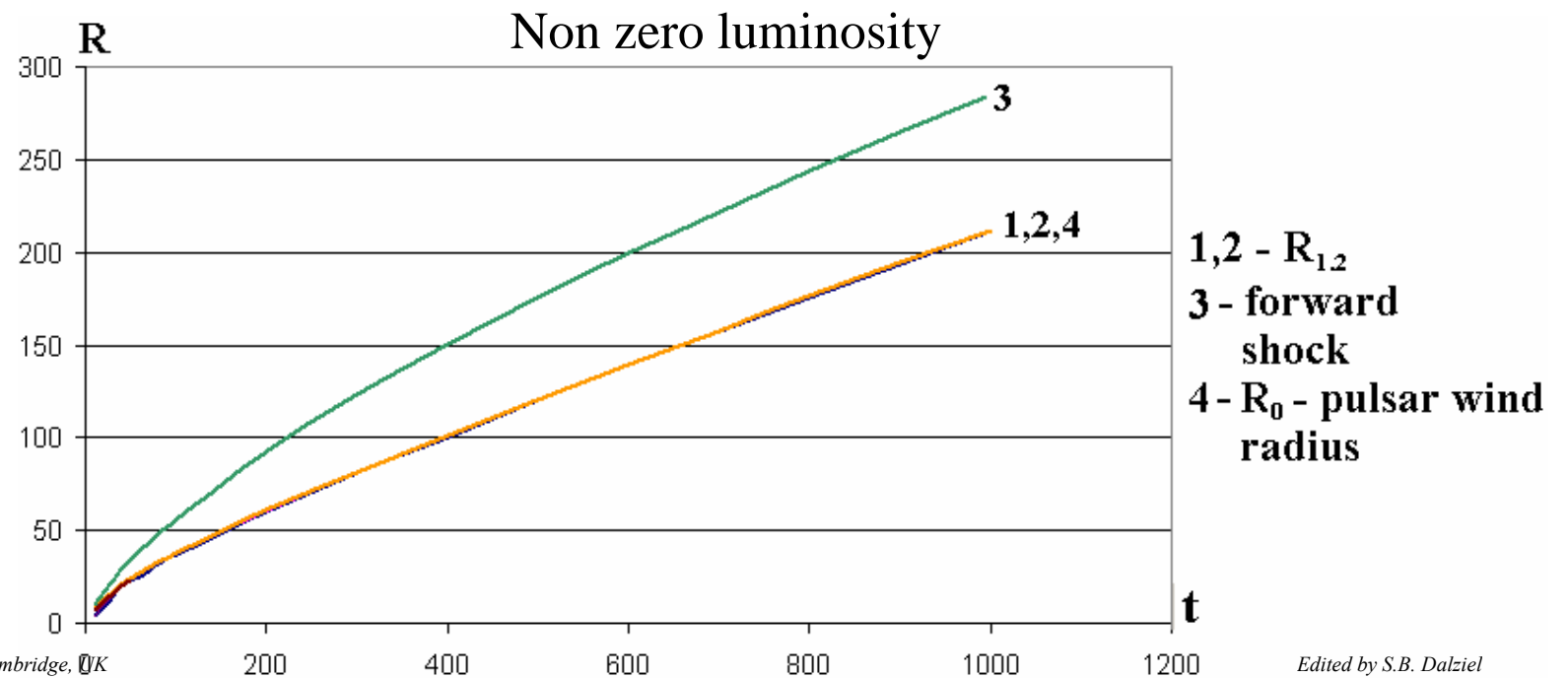
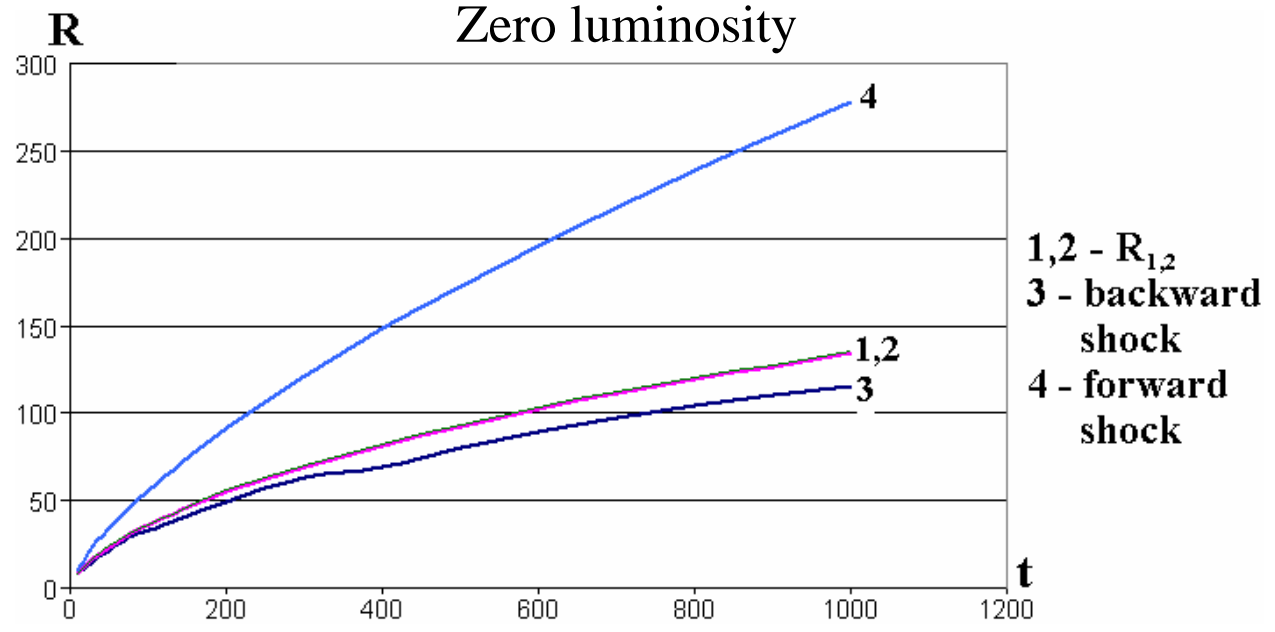
$$\frac{d\varepsilon}{dt} = \frac{\varepsilon}{k} (c_{\varepsilon 1} G_1 + c_{\varepsilon 2} G_2 - c_{\varepsilon 3} \varepsilon \left( 1 + c_v \frac{v\varepsilon}{k^2} \right)) + \frac{1}{\rho r^\theta} \frac{\partial}{\partial r} \left( r^\theta \rho (c_\varepsilon D + \tilde{v}) \frac{\partial \varepsilon}{\partial r} \right)$$

where:

$$G_1 = D \left\{ 2 \left[ \left( \frac{\partial u}{\partial r} \right)^2 + \theta \left( \frac{u}{r} \right)^2 \right] - \frac{2}{3} (\text{div} \bar{u})^2 \right\} - \frac{p_T}{\rho} \text{div} \bar{u}$$

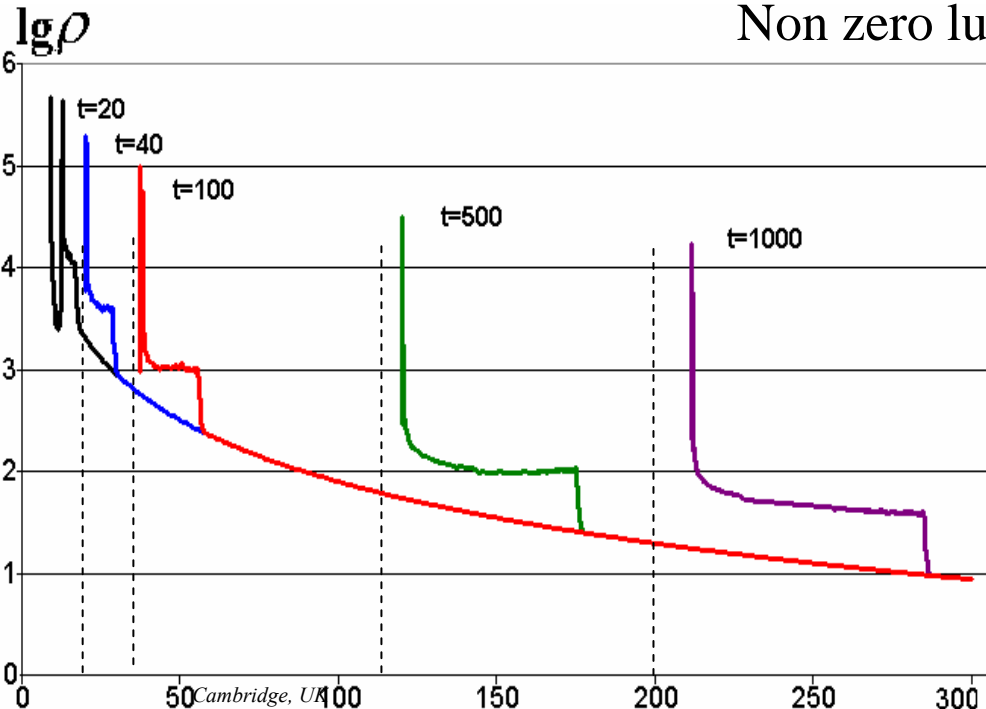
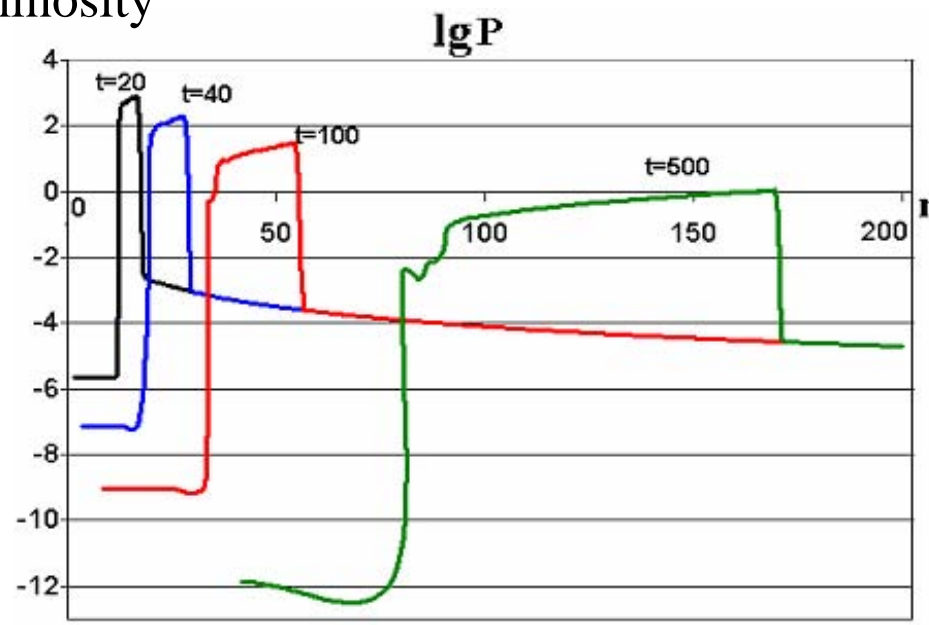
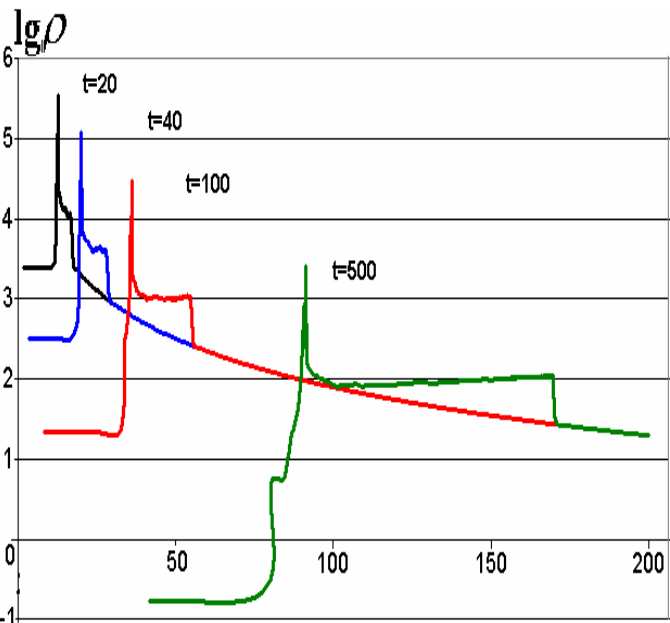
$$G_2 = \frac{c_\alpha D}{\rho} \frac{\partial P}{\partial r} \left( \frac{1}{\gamma} \frac{\partial \ln P}{\partial r} - \frac{\partial \ln \rho}{\partial r} \right)$$

# SN Remnants dynamics

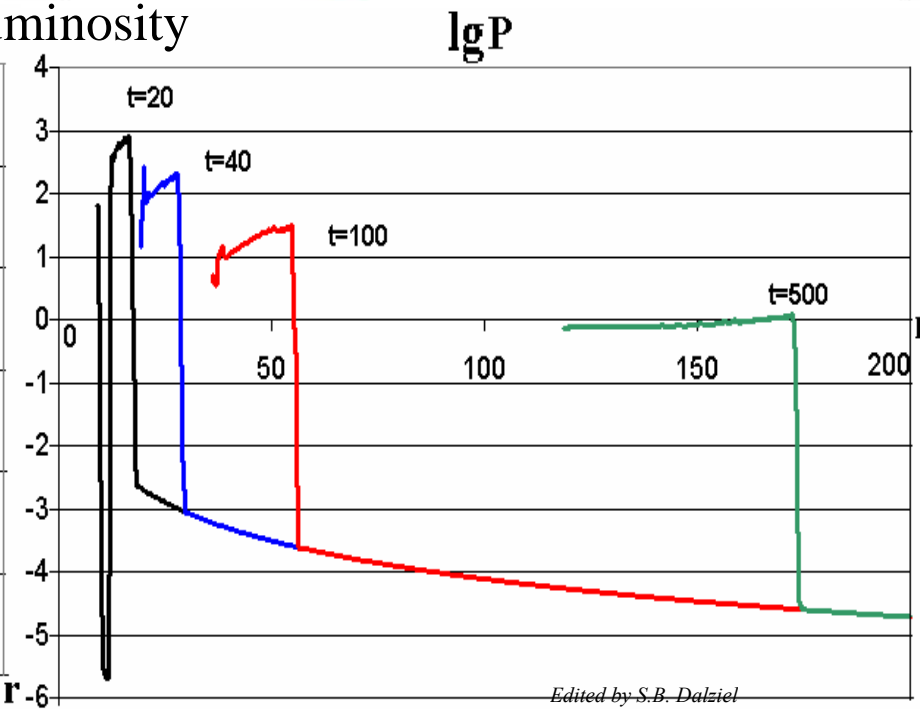


# Gasdynamical profiles. No mixing.

## Zero luminosity



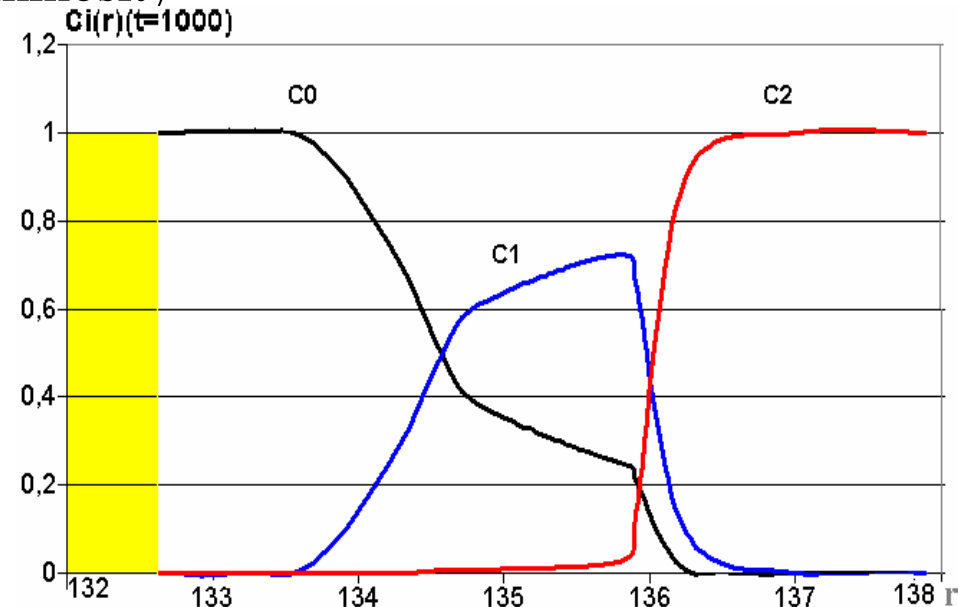
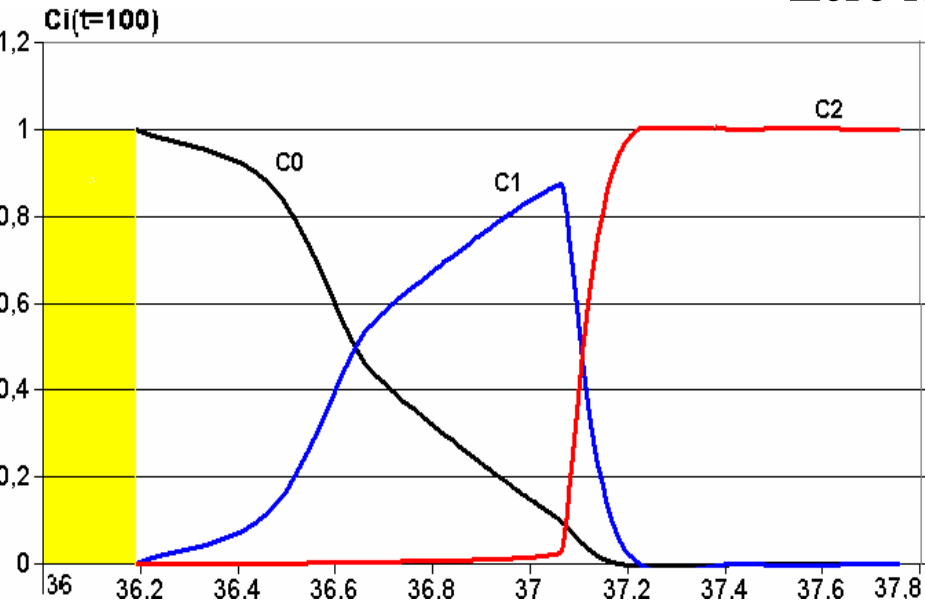
## Non zero luminosity



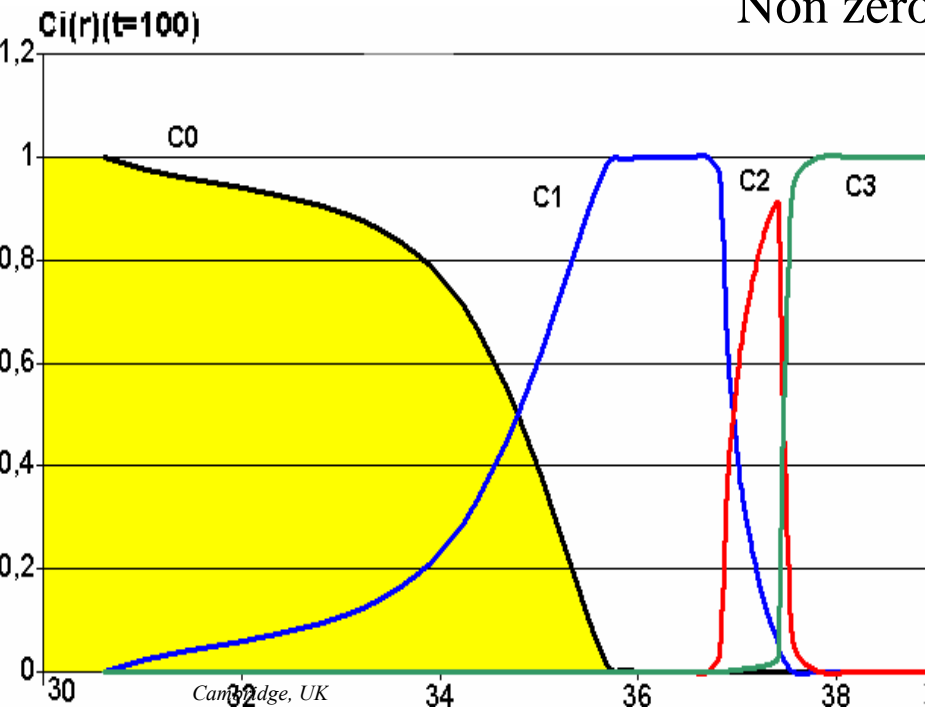


# Concentrations in the 1D computations.

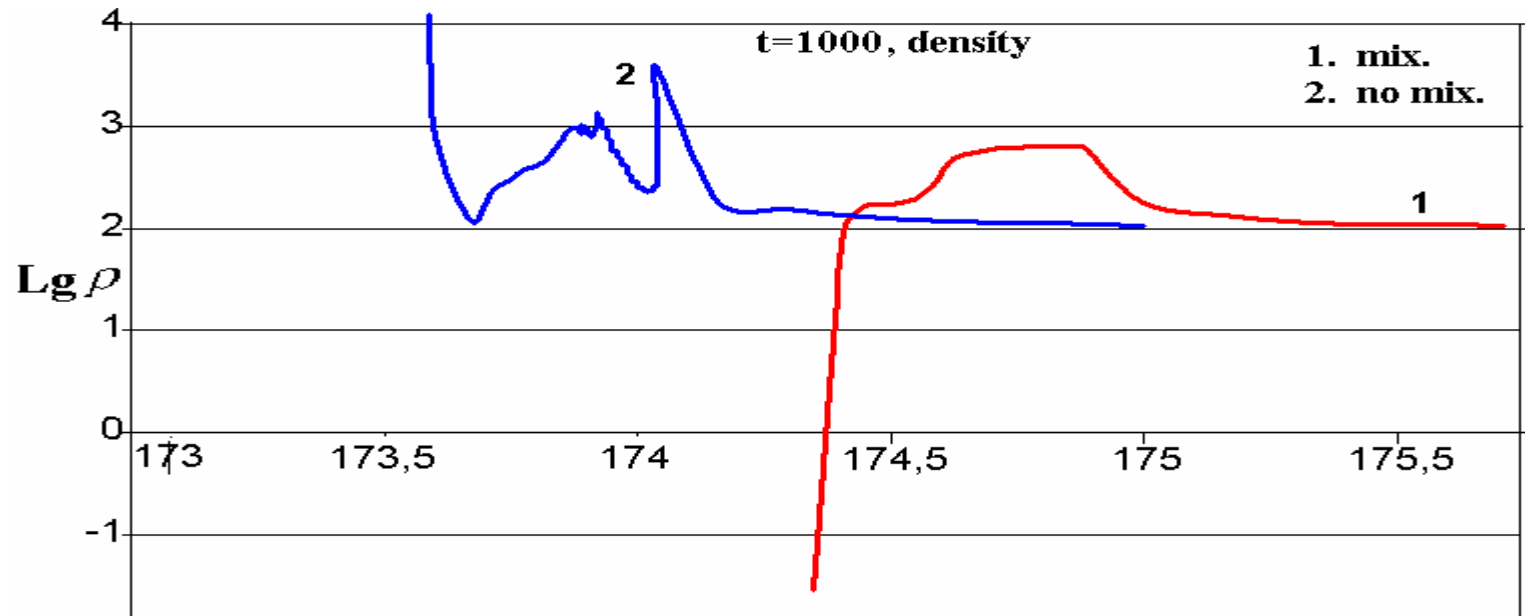
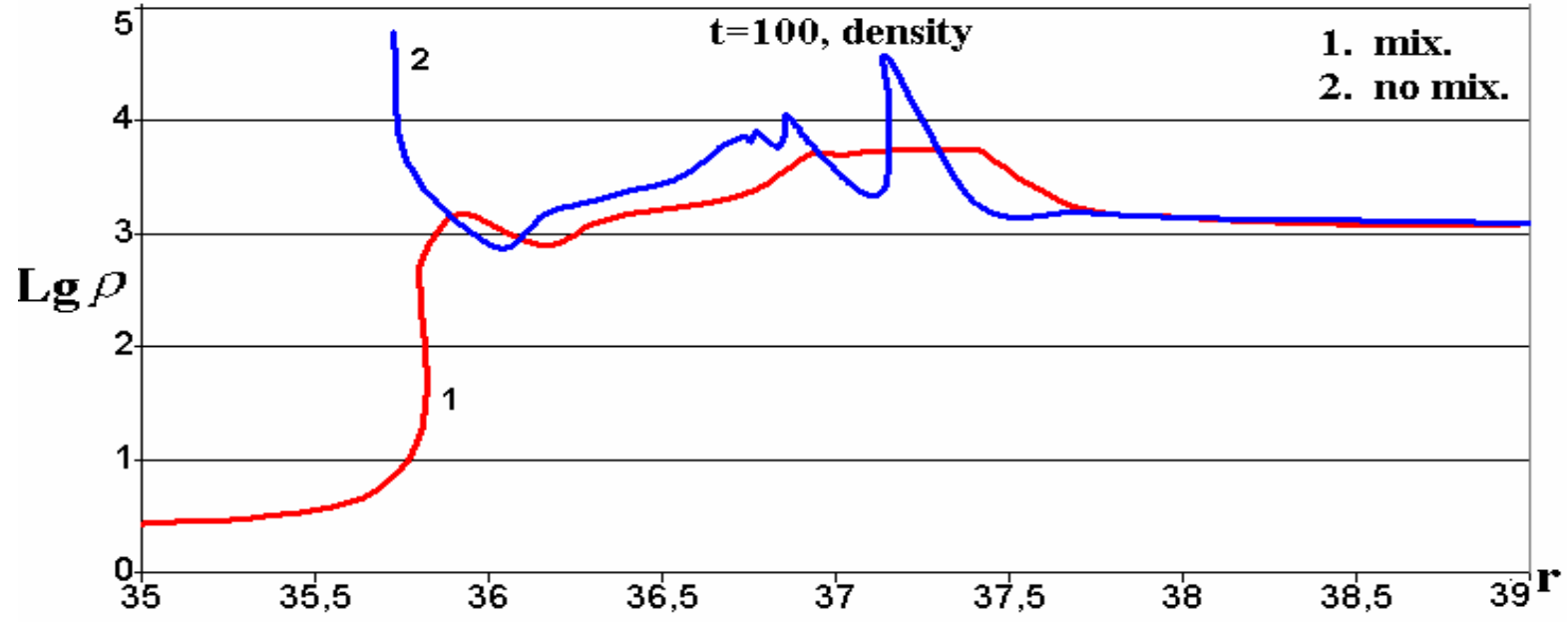
## Zero luminosity



## Non zero luminosity



# Turbulent mixing effect. 1D computation.

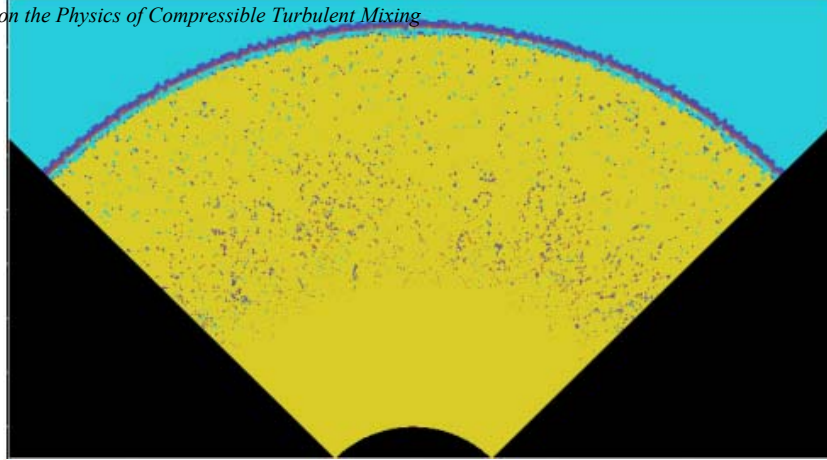


# 2D computations:

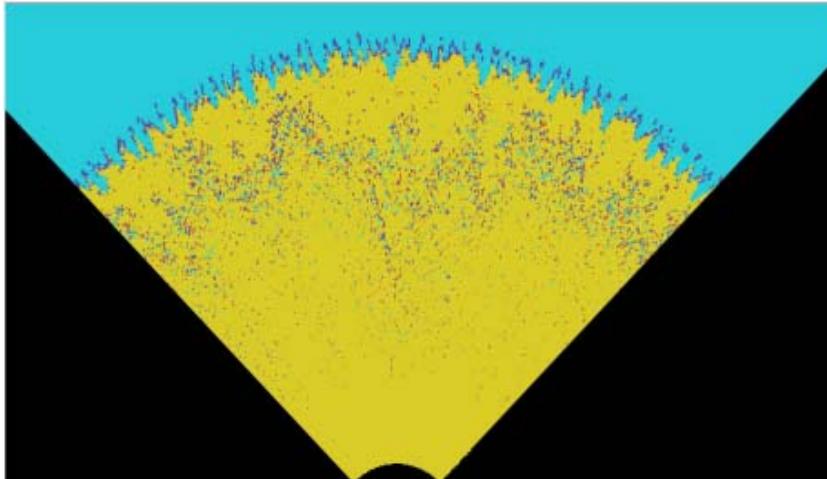
1. Initial data – as in the 1D task,  
but there are random perturbations in density  
(1% level, everywhere).
2. No semiempirical models.
3. Spherical grid, 800\*400 cells.
4. Angle averaging of computed data.

# 2D computations. Concentrations field.

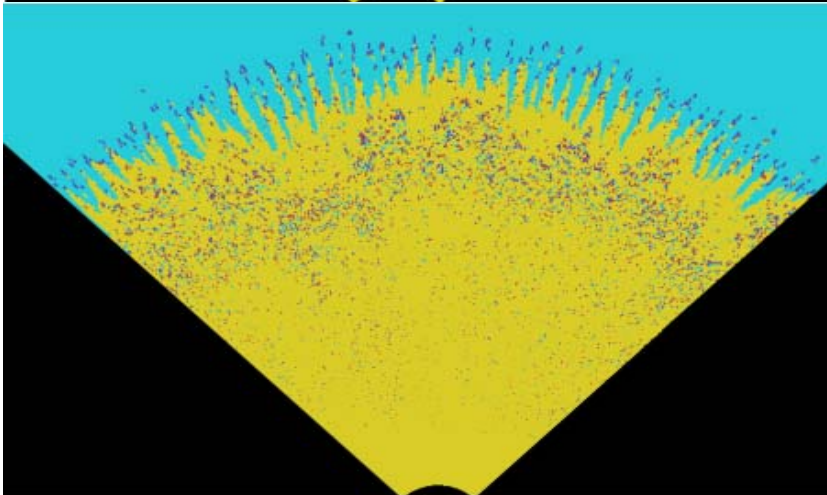
t = 40




t = 80



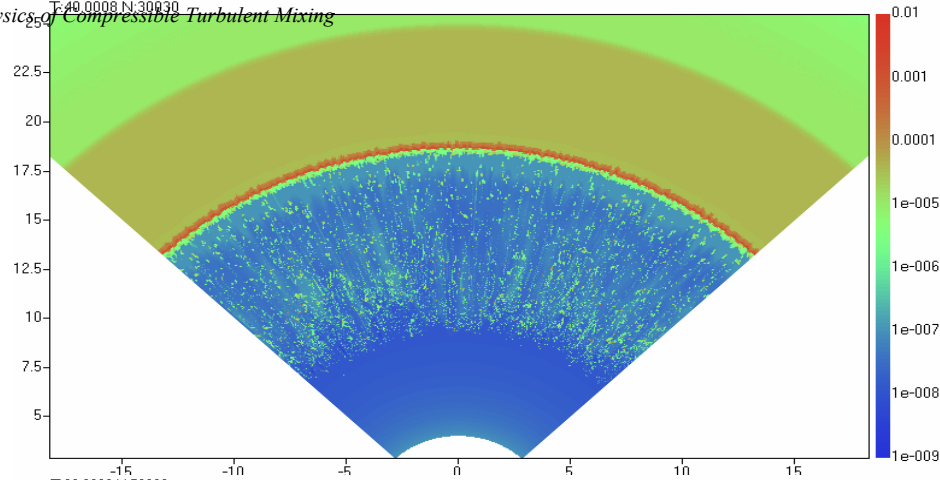
t = 130



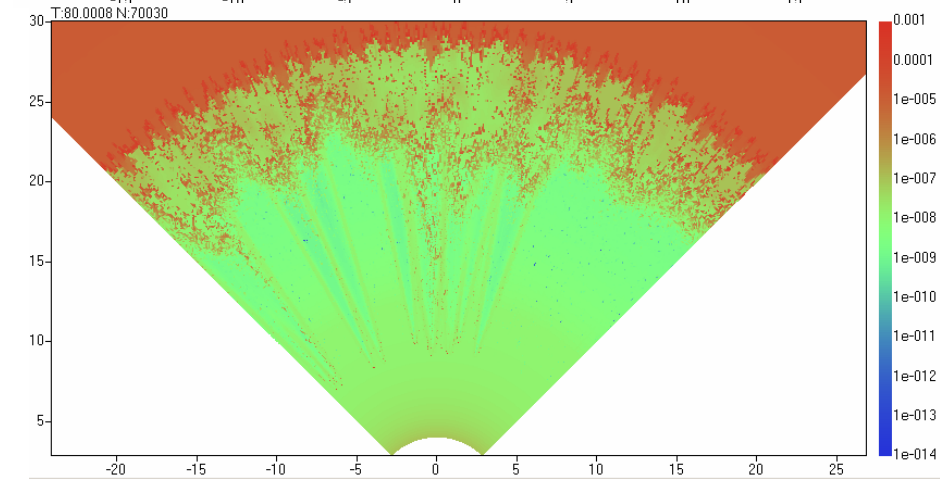
-  - pulsar wind
-  - shell
-  - star wind

# 2D computations. Density fields.

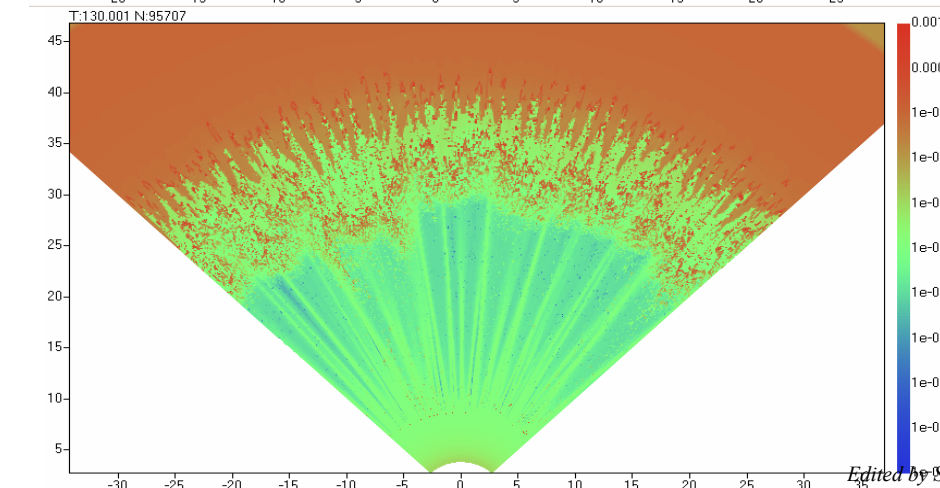
t = 40



t = 80



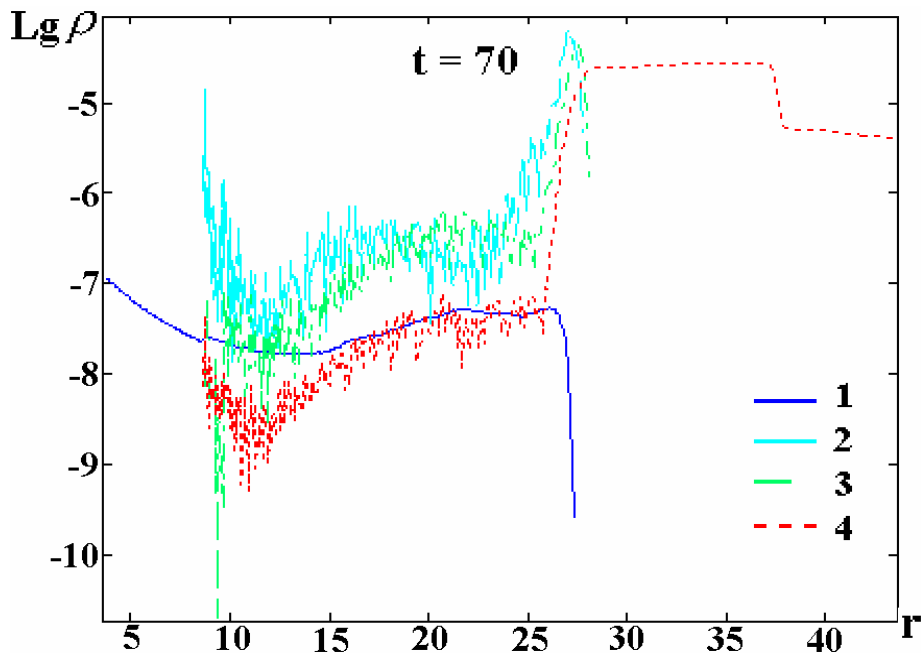
t = 130



# 2D computations.

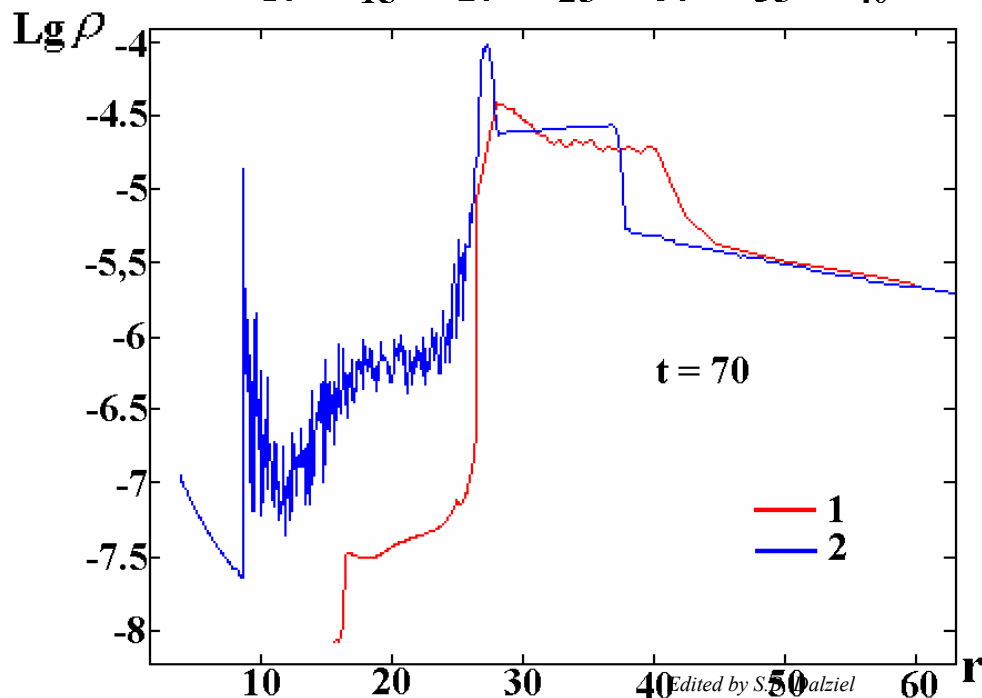
Partial density profiles:

1. Pulsar wind
2. He shell
3. H shell
4. Star wind



Total density profiles:

1. 1D computations
2. 2D computations



# Summary.

1. Both types of our computations give good agreement between computed size of wind region and observed size of Crab Nebula. This is additional support of pulsar nature of Crab Nebula.
2. Both types of our computations give effect of turbulent mixing of different shells in this SN remnants.  
In particular there is a mixing of pulsar wind and other shells, but masses of mixed shell in 1D and 2D computations are very different. Besides, mixed mass from the 1D computations appears to be small comparing to observations, temp of mixing from the 2D – appears to be reasonable.
3. This work shows use data of Crab Nebula observation for turbulent mixing models development.  
Further investigations are planned.