Colliding Surface Instability for a High-Velocity Impact

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Abstract — The physical mechanisms of development of hydrodynamic instability during high-velocity impact are analyzed analytically and using numerical simulation. A new mechanism of development of instability based on the assumption of the existence of initial perturbations on the surfaces of the colliding plates is proposed. The progressive displacement of the maximum of the specific surface mixed mass from the shortwave to the longwave spectrum range and the self-similarity of the variation of the interface are established.

In solving topical problems of physics and mechanics, it is important to take the mechanisms of development of hydrodynamic instabilities and flow turbulization correctly into account [1]. The problem of inertial nuclear fusion and the realization of a demonstration experiment under the laboratory conditions ("breakeven") involve the need to study and overcome a series of hydrodynamic instabilities of differing nature such as the Richtmyer-Meshkov, Rayleigh-Taylor, and Kelvin-Helmholtz instabilities [2-6]. In recent years, much attention has been concentrated on multilayered targets in which each layer solves a certain functional problem and in the process of compression of the target interacts with the neighboring layers at a high relative velocity of up to 10^6 m/s.

Analogous shock-wave processes occur in explosion welding and hardening and in applying coatings and layers of various materials by means of explosives. In this case phenomena of instability of interfaces between plates during plane high-velocity impact have been experimentally detected [7, 8]. The underlying physical mechanisms of these effects can be quantitatively described only by solving multidimensional quasilinear and nonlinear systems of partial differential equations which depend on three spatial variables [9]. In a number of cases it is sufficient to use the Euler equations to reveal and explain the instability pattern observed experimentally [10-12].

In the present study, the development of hydrodynamic instability during high-velocity impact at relative velocities ranging from 10 to 100 km/s is investigated in the one- and two-dimensional axisymmetric spatial cases using analytic and numerical techniques. Processes with a complex vortex flow structure leading to a progressive increase in the mass of the mixed material with time are analyzed as functions of a series of physical parameters, namely, the thermodynarnic constants of the materials, the equations of state, and the interaction rates, and geometric parameters such as the scales of the initial perturbations for a given concave or convex shape of the colliding surfaces of the plates.

1. EXPERIMENTAL INVESTIGATION OF THE INSTABILITY

The instability of the interface between two materials was detected in experiments on explosion welding and hardening. In these experiments one of two or more plates (layers) was accelerated by a detonation wave propagating in an explosive applied to one of the plates. After acceleration to velocities of the order of 1 km/s, the plates interacted at certain angles and the interface turned out to be unstable and took a wavy shape [13]. Instability of the impact surface has also been observed in plane collision [7].

In studying the causes of the development of plane instability, the following experimental facts were established: (I) presence of melts in the weld; (2) the duration of the melted state of the interface was estimated at 5-10□s; (3) loss of stability was accompanied by cone-shaped metal splashing from the impacted plate toward the initially accelerated lead plate; (4) the perturbation amplitude was estimated on the basis of the distance from the base to the vertex of splash, the breakaway of part of the formations during plate separation being disregarded; (5) no instability was observed for thick lead plates (1 cm thick).

As the basic mechanism of development of instability, in [7] the conditions in which the density and pressure gradients are parallel and oppositely directed were considered. This corresponds to the Rayleigh-Taylor instability. During high-velocity impact these conditions can arise in the rarefaction wave arriving at the interface between the colliding plates from the direction of the free surface of the lead plate. In [7] the hydrodynamic instability was investigated experimentally. It was shown that: (1) the creation of perturbations on the free surface of the impacted plate and changes in its thickness have no effect on the nature of the development of the instability; (2) for constant velocity, an increase in the thickness of the projected lead plate leads to an increase in the geometric dimensions of the formations observed, doubling the thickness of the lead plate leads to the approximate doubling of the splash wavelength; (3) in experiments with different impacted plate materials and the same thickness and projection velocity of the lead plate the geometric dimensions of the splashes change.

The relations observed experimentally may be explained otherwise than in [7] if we assume that during the acceleration of the lead plate in the detonation of the explosive or in the process of free flight the striking surface is deformed and takes a convex or concave shape.

Specifying initial perturbations on the free surface of the impacted plate and varying its thickness has no effect on the deformation of the projected plate and, consequently, does not affect the development of the instability. Thick lead plates are less subject to deformation during acceleration. This favours the radial growth of the initial perturbations and subsequently an increase in the splash dimension. From the assumption concerning the initial perturbations on the lead plate it follows that splashes on the steel plate are possible if the shape of the colliding lead surface is concave. The duration of the "pseudo-liquid" state of the plates corresponds to estimates of the time of propagation of the shock and rarefaction waves through the plate material and is confirmed by numerical calculations.

2. PHYSICAL DESCRIPTION OF THE COLLISION PROCESS

The experimental data mentioned above correspond to relative velocities of approximately 500 m/s. The reasons for the onset and development of instability were analyzed in [14]. It is possible that when the impact velocity increases these effects will be conserved or even increase.

The question of the equation of state can be clarified by considering the relative velocity range on which the plates collide. At pressures from 10□ to 10□ Pa (this corresponds to velocities of the order of 10 km/s) an equation of state of the Gruneisen type, which contains several empirical constants, is ordinarily used. At collision velocities of 100-1000 km/s fully ionized dense high-temperature plasma can be described by the equation of state of an ideal gas [2, 4]. In order to estimate the difference between using an equation of state of the Gruneisen type and that for an ideal gas we will consider the problem of one-dimensional breakdown of an arbitrary discontinuity (the Riemann problem) at a relative iron and lead plate collision velocity equal to 10 km/s.

In the ideal gas model we take the specific heat ratio □ = 5/3 as for a monatomic gas. At the initial instant, let the left half-space ($x < 0$) be occupied by iron at rest (□ = 7.9 g/cm^3 and p□= 3.04•10^7 dyn/cm^2). This region is denoted by the subscript "0". The right half-space ($x > 0$) is occupied by lead with □= 11.37 g/cm^3 and p□= 1.18• 10^7 dyn/cm^2 and has a velocity of 10^6 cm/s. It is denoted by the subscript "3". After collision, shock waves begin to propagate in the iron and the lead. The regions behind the shock waves in the iron and the lead are denoted by subscripts "1" and "2", respectively.

Three Rankine-Hugoniot relations on each shock front and the conditions of continuity of the pressure and velocity on the interface make it possible to write a system of eight nonlinear equations which must be supplemented with the equations of state of an ideal gas for iron and lead.

Solving this system, for example, as in [15], we obtain the values of the gas dynamic quantities behind the shock waves in regions 1 and 2: □ = 31.6 g/cm^3, u□= -5.454 • 10^5 cm/s, p□ = 3.133 • 10^{12} dyn/cm^2, D_0 = -7.272 • 10^5 cm/s, □$_2$ = 45.48 g/cm^3, u□ = -5.454 • I0^5 cm/s, p□= 3.133 • I0^{12} dyn/cm^2, and D□ = —3.939 • 10^5 cm/s. Here, □ are the densities, u□ are the mass velocities, p□ are the pressures, E□ are the total specific energies, □ are the internal specific energies, and D□ and D□are the shock wave velocities in the corresponding regions.

In what follows, we will use an equation of state of the Gruneisen type borrowed from [16] and assume that for the iron plate the specific heat ratio □= 3, the degree of maximum compression h□=2, □= 7.85 g/cm□, p□ = 0, and the speed of sound c□= 4.65 km/s. For the lead plate □ = 3, h□ = 2, □ = 11.346 g/cm□, p□= 0, c□ = 1.972 • 10^5 cm/s, and u□ = —10□ cm/s. For the notation of the regions we retain the subscripts adopted in the ideal gas model.

Writing the Rankine-Hugoniot relations on the shock fronts, the conditions of continuity of the pressure and velocity on the interface, and the Gruneisen-type equation of state for iron and lead, we arrive at the following system of eight algebraic equations in eight unknown gasdynamic quantities behind the shock fronts and their velocities where h□ is the degree of maximum compression and c□ is the initial speed of sound in the material.

□ Taking into account that at the initial instant p□ =p□= 0, we can reduce the system of nonlinear equations to the following sixth-order algebraic equation:

□

$$0.7933Y^6 - 56.79Y^5 + 22.3Y^4 + 8.466\,Y^3 + 0.8128Y^2 + 0.02767Y + 0.0003086=0$$

□ Y=

Solving the equation, we find that □=13.76 g/cm^3, u□ =-5.3•10^5 cm/s, p□ = 5.18•I0^{12} dyn/cm^2, D□=-1.239•10^6 cm/s, □= 21.8 g/cm^3, u□= -5.3•10^5 cm/s, p□ = 5.18•10^{12} dyn/cm^2, and D□=-2.46•10^4 cm/s.

For an ideal gas and Gruneisen-type equation of state the mass velocities behind the shock fronts differ by 3%. In [14] it was shown that, as compared with the one-dimensional case, the absolute values of the mass velocities and the values of deviations of the mass fluxes behind the shock fronts and in the neighborhood of the interface are of determining significance for the development of instability. Therefore, given the insignificant difference in the mass velocities and the same initial perturbation geometry we can expect the process of development of instability to take place similarly for both the ideal gas model and the more exact equation of state of the Gruneisen type. In what follows, we will use the ideal gas model in the numerical calculations.

3.MATHEMATICAL MODEL AND FORMULATION OF THE PROBLEM

In order to study the mixing and the deformation of the interface during high-velocity impact it is convenient to take a basis coordinate system moving together with the impact surface.

If in the chosen coordinate system the velocity of the impinging plate is equal to V□ then, using the one-dimensional solutions of the problem of plate impact obtained in the previous section, we can determine the propagation time of a shock wave traveling through the material of a sample of thickness □ at a velocity D:

The shocked layer thickness Δ can be expressed in terms of the shock wave velocity and time t□

After the shock wave reaches the free surface of the plate, a rarefaction wave begins to propagate along the plate. The velocity of the rarefaction wave coincides with the speed of sound in the shocked layer. Thus, the time of propagation of the rarefaction wave from the free surface to the interface is determined by the ratio of the thickness □ of the shocked layer to the speed of sound C in it:

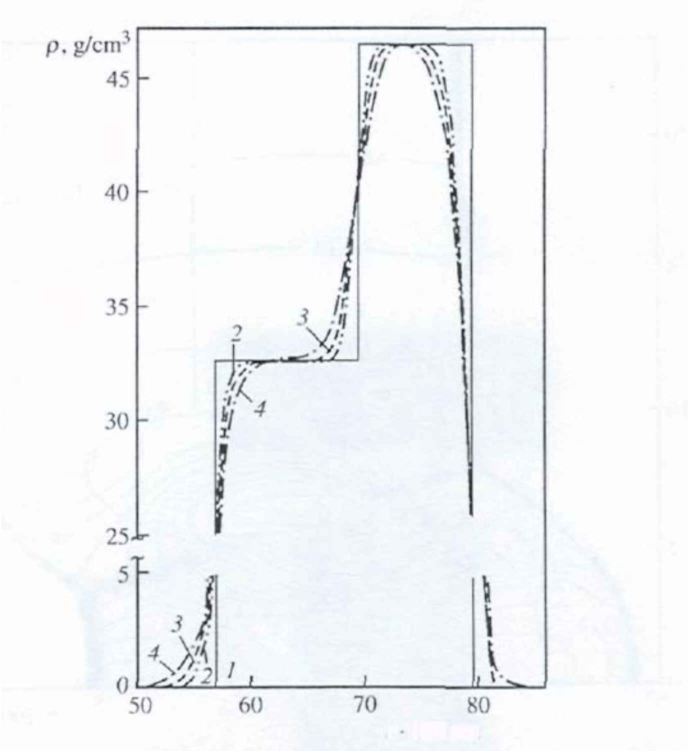
The total time of the process from the instant of plate collision to the arrival of the rarefaction wave at the interface is equal to t□+t□.

The mathematical model is constituted by the following system of Euler equations written in terms of the variables r,z of the cylindrical coordinate system:

(3.1)

where □ is the density, □ is the mass concentration of one of components, U is the radial velocity component, V is the axial velocity component, E is the total specific energy, □ is the internal specific energy, p is the pressure, and □ is a constant of the adiabatic process.

The z axis is directed along the initial velocity of the plates, the r axis lies in the collision plane. In this case the z-axis is the axis of symmetry for the initial perturbations on the colliding surfaces of the plates and for the problem as a whole. The initial conditions corresponding to the gas dynamic quantities in the low-density material were maintained on the boundaries of the integration domain at z= const up to the arrival of the shock waves and at subsequent instants of time the initial conditions were replaced by the values corresponding to the parameters behind the shock fronts in the vapor surrounding the plate obtained from the one-dimensional approximation. The radial dimensions of the integration domain were such that the perturbations associated with the mass flows from the axis of symmetry could not reach the boundaries; therefore, on these boundaries flux conservation conditions were imposed.



r 100cm

Fig. 1. Density for the one-dimensional analytic (curve 1) and numerical solutions for successively doubled grids at the instant of time 0.69 □s (curves 2—*4* correspond to grids with a step of 5, 10, and 20 □m, respectively)

4. ONE-DIMENSIONAL TEST CALCULATIONS

The programs used in the numerical simulation are based on the mathematical method in which the input system of equations (3.1) is reduced to characteristic form and then some code is employed for its finite-difference approximation, as in [17]. In this case the scheme must be divergent. Sources originating in the finite-difference approximation which violate this property are not permitted even for solutions with discontinuity jumps of up to several orders.

For testing the computational properties of the programs designed we compared the analytic results with the data of the numerical calculations with reference to the problem of discontinuity breakdown (the Riemann problem) (see Fig. 1).

5.PHYSICAL MECHANISMS OF INSTABILITY DEVELOPMENT

Let a perturbed domain of radius R on the interface be bounded in space and have an amplitude □ and the rest of the colliding surface be plane. In the initial moments of time the plane parts of the plates will collide in accordance with the analytic solution for the breakdown of a one-dimensional discontinuity considered in Section 2. The interaction on the perturbed parts of the interface will cause distortion of the fronts of the shocks formed.

In the inertial coordinate system in which the material ahead of the shock front is at rest, the mass velocity component, tangential to the front, is equal to zero behind the shock wave since the tangential momentum components are conserved across the shock front. In other words, behind the shock the velocity is directed at right angles to its front, i.e., in the region of the initial perturbation there will be a deviation of the mass flows as compared with the one-dimensional variant. The further the shock wave propagates from the origin, the greater the mass of material which begins to move in a direction different from that initially given.

As a result, the flows deflected from the initial direction interact both with each other and with the material which continues to move in the initial direction. Hence the pressure increases in the interaction domain and part of the kinetic energy is transformed into internal energy.

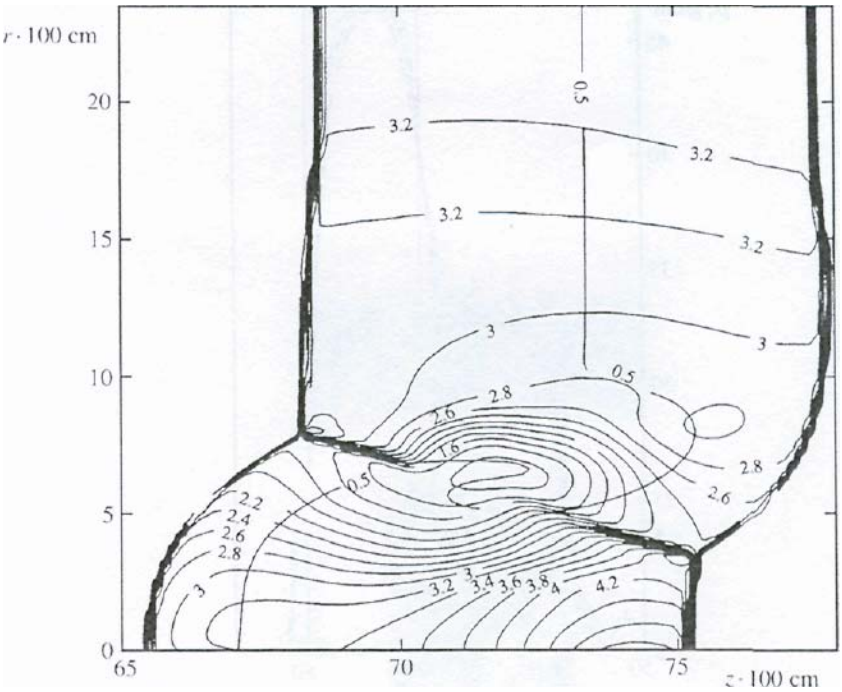


Fig. 2. Isobars in Mbar and location of the 0.5 mass concentration isoline at the instant of time 0.345 □s

We will now direct our attention to a variant in which the initial perturbation is specified on the impinging plate and has a convex shape of radius R= 750 □m and an amplitude □ = 750 □m. The relative velocity of plate collision is 10 km/s. The left impacted iron plate of thickness 5 mm will be called plate A and the right lead plate of thickness 4 mm will be called plate B. In Fig. 2 we have plotted both the isobars and the mass concentration isolines at the instant of time 0.345 □s. The curves formed by the merged isobars correspond to the location of the shocks in plates A and B. We note the presence of two zones of higher pressure as compared with the one-dimensional case. One of them is located in the neighborhood of the axis of symmetry in plate B and the other on the boundary of the perturbation domain between the shock waves and the interface (Fig. 2). Their appearance is due to the deviation and interaction of the flows in the region of distortion of the shock fronts. A zone of lower pressure is located between the fronts.

As a result of the numerical simulation, it was found that with time the first two domains themselves become sources of secondary compression waves or shock waves when the stagnation pressure in the domain exceeds the dynamic pressure. The secondary waves, overtaking the shock waves formed in the plate collision and interacting with them, tend to straighten and flatten the shock fronts. Thus, the conditions favoring deviation of the flows behind the curved shock fronts are eliminated. The pressure increase due to the interaction between the deflected flows ceases in the domains considered. However, the flow along the interface is conserved but travels in opposite on different sides of the interface.

In Fig. 3 we have plotted the isobars and the 0.5 mass concentration isoline at the instant of time 0.69 □s when in the one-dimensional case the shock waves approach the free surfaces of the plates. Clearly, the process of deformation of the interface continues and it becomes "mushroom-shaped". The dimensions of the higher-pressure zones increase but the maximum pressure in them falls. It is worth noting the presence of a stable lower-pressure zone at the point of contact between the "cap" and the "stem" of the mushroom-shaped structure.

Then rarefaction waves, in which the pressure, density and temperature gradually decrease, begin to propagate through the shocked material and a velocity directed toward the departing shock waves is generated in the inertial system moving with the plane interface. In the experiments, after the temperature falls below the melting point, deformation of the interface ceases and thus an instantaneous flow pattern is fixed

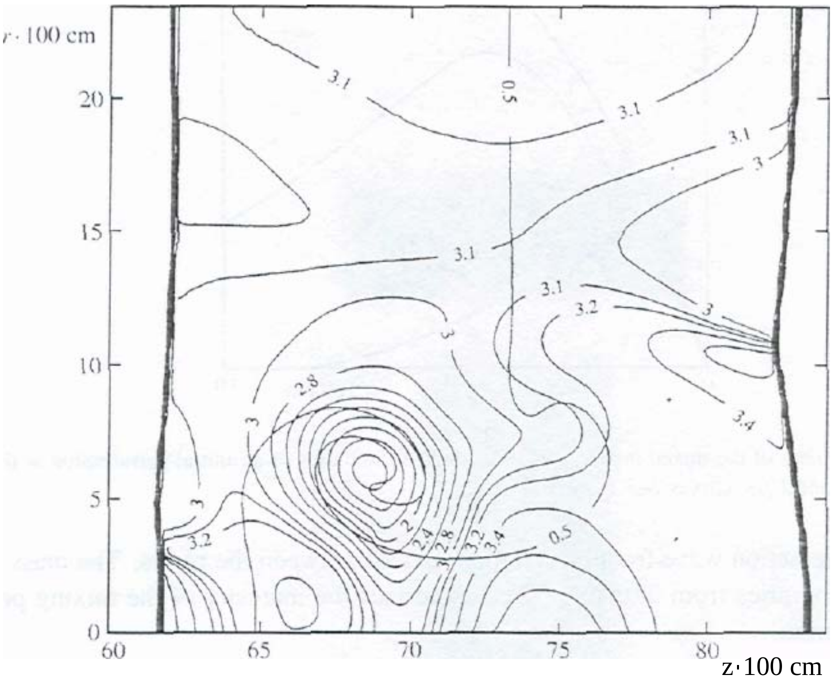


Fig. 3. Isobar distribution in Mbar and 0.5 mass concentration isoline at the instant of time 0.69 □s

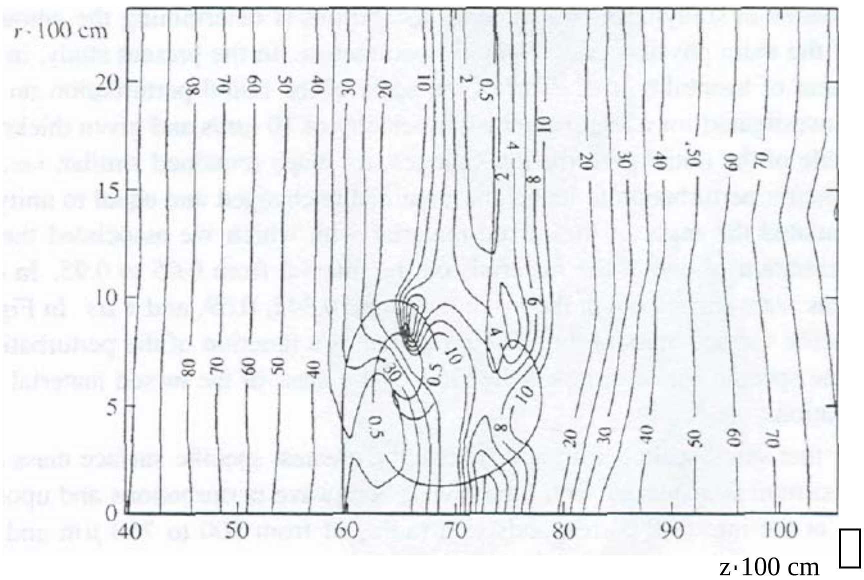


Fig. 4. Isolines of the absolute value of the velocity in and the 0.5 mass concentration isoline at the instant of time

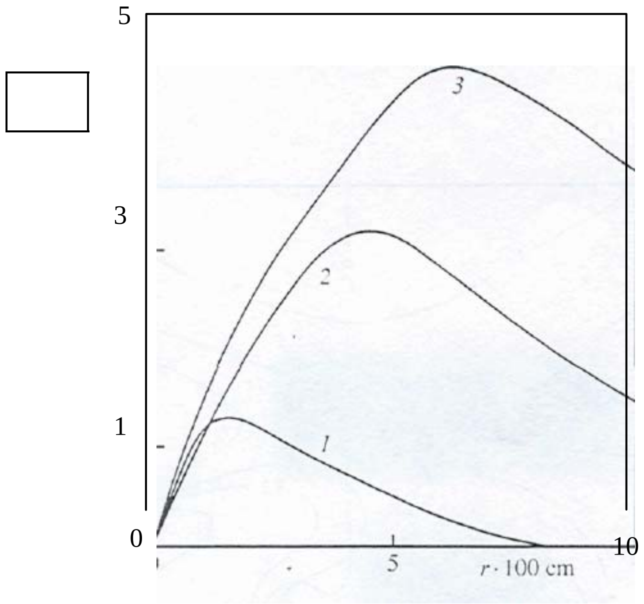


Fig. 5. Specific surface mass of the mixed material as a function of the radius of an initial perturbation at the instants of time 0.345 □s, 0.69 □s, and 1 □s (curves 1-3 respectively)

This occurs after the rarefaction wave fronts reach the interface between the plates. The mass concentration of mixed material which varies from 0.05 to 0.95 characterizes the intensity of the mixing process and the degree of flow turbulization.

In Fig. 4 we have plotted the isolines of the absolute value of the velocity and the 0.5 mass concentration isoline 1 □s after the arrival of the rarefaction waves at the interface. As compared with the previous instant of time shown in Fig. 3, the perturbation amplitude has increased even more strongly and sections with greater curvature have developed on the interface.

6. RESULTS OF THE NUMERICAL SIMULATION

One of the main problems in studying hydrodynamic instabilities is determining the amount of mixed material as a function of the main physical and geometric parameters. In the present study, in considering the process of development of instability, the effect of the scale of the initial perturbation on the mass of the mixed material was investigated for a relative collision velocity of 10 km/s and given thicknesses of the two plates. When the scale of the initial perturbation changes, its shape remained similar, i.e., the ratio of the radius of the axisymmetric perturbation to its height remained unchanged and equal to unity. At certain instants of time we calculated the mass of the mixed material with which we associated the part of the plates with a mass concentration of one of the materials on the interval from 0.05 to 0.95. In each variant considered the calculations were carried out at the instants of time 0.345, 0.69, and 1 μs. In Fig. 5 we have plotted graphs of the specific surface mass of the mixed material as a function of the perturbation radius at these instants of time. The specific surface mass is the ratio of the mass of the mixed material to the plane area of the initial perturbation.

From Fig. 5 it is clear that small-scale perturbations have the greatest specific surface mass at the initial instants of time. The maximum is displaced with time toward longwave perturbations and upon the arrival of the rarefaction waves at the interface corresponds to a radius of from 500 to 750 μm and amounts to approximately 5 g/cm^2.

In a series of calculations the initial collision velocity was increased to 100 km/s. The time of development of instability from the instant of initial contact between the plates to the instant of arrival of the rarefaction waves at the interface fell to 0.1μs for the same plate thicknesses. In this case the shape and location (Fig. 6) of the 0.5 mass concentration isoline remained practically the same. This makes it possible to assume that there exists a self-similar variable equal to the ratio of the plate thickness to the product of the

Fig. 6. Isolines of the absolute value of the velocity in 10^4 cm/s and the 0.5 mass concentration isoline at the instant of time 0.1 μs

relative collision velocity and the duration of the process. In Fig. 6 we have plotted isolines of the absolute value of the velocity, analogous to those plotted in Fig. 4, at the instant 0.1 μs for a collision velocity of 100 km/s.

Summary. A new physical mechanism of development of instability from initial perturbations of the shape of the interacting surfaces of the plates is proposed. The determining elements of this mechanism are: the deviation of the mass flows behind the curved sections of the shock waves and the interaction of the secondary compression waves and shock waves with the initial shock waves. A quantitative dependence of the specific surface mixed mass of the materials of the colliding plates on the radius of an initial axisymmetric perturbation of given shape is obtained. The maximum specific surface mixed mass is displaced with time from the short to the long-wave part of the spectrum. The dependence of the specific surface mass of the mixed material on a dimensionless quantity equal to the ratio of the plate thickness to the product of the relative impact velocity the duration of the process from the instant of initial contact of the plates to the instant of arrival of the rarefaction waves at the interface between the two materials is shown to be self-similar.

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