Investigation of gravitational turbulent mixing at sign-variable acceleration.

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We considered the modification of turbulent mixing model for the description of a separation at sign-variable acceleration. The model was obtained from the analysis of the equations for turbulent flows of concentration and density. $K\varepsilon$ model [1] was added with items and the equation, containing the coefficient of heterogeneity Γ.

The model was investigated in case of two incompressible fluids in a field of gravity. It is shown, that in an unstable case the dimensionless velocity of development of turbulent zone depends on the coefficient of heterogeneity Γ. This fact is a possible explanation of a distinction of experimental data. The stable phase begins, when the sign of acceleration changes. The zone of turbulent mixing decreases in this case. This fact is in the consent with experimental data [2] and results of direct numerical simulation [3]. Development of a mixing zone depends from Γ in this case. There is a full separation if Γ=1 and separation does not occur if Γ=0.

Introduction

The nonstationary flow of fluids at the availability of different density region is always unstable when the light substance accelerates the heavy one. At the contact boundary of these substances the gravitational turbulent mixing induced by the Rayleigh –Taylor instability develops. In this case the acceleration of the artificial gravitational field is direct from the heavy medium to the light one. If in the process of motion the contact boundary acceleration changes its sign, then boundary appears to be gravitationally stable. Since this instant of time the separation process of the heavy and light substances being found in the turbulent mixing zone begins to proceed.

A lot of experimental and theoretical works was devoted to studying of this question [2−5]. It was suggested the model of turbulent mixing for the description of a separation at sign-variable acceleration with using of these papers. The model was obtained from the analysis of the equations for turbulent flows of concentration and density. $K\varepsilon$ model [1] was added with items and the equation, containing the coefficient of heterogeneity Γ.

Model

Some assuming has been used at getting of the model:

1. $Re_t = \dfrac{u_t l}{\nu} \to \infty$,
2. $M_t = \dfrac{u_t}{c} << 1$,
3. Incompressible fluids.

The equations of fluid dynamics for average quantities may be written in the following form:

$$\frac{\partial\bar\rho}{\partial t}+\frac{\partial\bar\rho\overline{\overline{U}}_\alpha}{\partial x_\alpha}=0,$$

$$\frac{\partial\bar\rho\overline{\overline{C}}_k}{\partial t}+\frac{\partial\bar\rho\overline{\overline{U}}_\alpha\overline{\overline{C}}_k}{\partial x_\alpha}=\frac{\partial}{\partial x_\alpha}\left(\overline{\rho D\frac{\partial c_k}{\partial x_\alpha}}\right)-\frac{\partial}{\partial x_\alpha}\left(\bar\rho\overline{\overline{u_\alpha{}''c_k{}''}}\right),$$

$$\frac{\partial\bar\rho\overline{\overline{U}}_i}{\partial t}+\frac{\partial\bar\rho\overline{\overline{U}}_\alpha\overline{\overline{U}}_i}{\partial x_\alpha}=-\frac{\partial\bar P}{\partial x_i}-\frac{\partial}{\partial x_\alpha}\left(\bar\rho\overline{\overline{u_\alpha{}''u_i{}''}}\right),$$

where $\overline{\overline{\varphi}}=\frac{\overline{\rho\varphi}}{\bar\rho}$. This system of the equations includes both average quantities and correlations $\overline{\overline{u_\alpha{}''c_k{}''}}$ and $\overline{\overline{u_\alpha{}''u_i{}''}}$. That is why the system is not closed and it is necessary to use additional reasons for the solving of the system. Usually, the system is supplemented with equations for turbulent kinetic energy $k=\frac{\overline{\overline{(u_i{}'')^2}}}{2}$ and dissipation rate ε. According to [1] these equations may be written in the following form:

$$\frac{\partial\bar\rho k}{\partial t}+\frac{\partial\bar\rho\overline{\overline{U}}_\alpha k}{\partial x_\alpha}=-\bar\rho\overline{\overline{u_k{}''u_\alpha{}''}}\frac{\partial\overline{\overline{U}}_k}{\partial x_\alpha}-\overline{u_\alpha{}''}\frac{\partial\bar P}{\partial x_\alpha}-\bar\rho\varepsilon+\alpha_8\frac{\partial}{\partial x_\alpha}\left(\bar\rho D_t\frac{\partial k}{\partial x_\alpha}\right),$$

$$\frac{\partial\bar\rho\varepsilon}{\partial t}+\frac{\partial\bar\rho\overline{\overline{U}}_\alpha\varepsilon}{\partial x_\alpha}=-C_{3\varepsilon}\frac{\varepsilon}{k}\bar\rho\overline{\overline{u_k{}''u_\alpha{}''}}\frac{\partial\overline{\overline{U}}_k}{\partial x_\alpha}-C_{1\varepsilon}\frac{\varepsilon}{k}\overline{u_\alpha{}''}\frac{\partial\bar P}{\partial x_\alpha}-\bar\rho C_{2\varepsilon}\frac{\varepsilon^2}{k}+\alpha_{\varepsilon8}\frac{\partial}{\partial x_\alpha}\left(\bar\rho\frac{k^2}{\varepsilon}\frac{\partial\varepsilon}{\partial x_\alpha}\right).$$

It is necessary to write equations for $\overline{\overline{u_\alpha{}''c_k{}''}}$ and $\overline{u_\alpha{}''}$ for closing of the model. The gradient hypothesis was used for this purpose in recent papers [1]. But we'll write transport equations for these items:

$$\bar\rho\frac{d\overline{u_i{}''}}{dt}-\bar\rho\overline{u_i{}''}\frac{\partial\overline{u_\alpha{}''}}{\partial x_\alpha}-\bar\rho\frac{\partial}{\partial x_\alpha}\left(\frac{\overline{\rho' u_\alpha{}''u_i{}''}}{\bar\rho}\right)=-\bar\rho\overline{u_\alpha{}''}\frac{\partial\overline{\overline{U}}_i}{\partial x_\alpha}+\overline{\overline{u_\alpha{}''u_i{}''}}\frac{\partial\bar\rho}{\partial x_\alpha}-b\frac{\partial\bar P}{\partial x_i}-2\frac{k}{D_t}\overline{u_i{}''},\quad(1)$$

$$\bar\rho\frac{d\overline{\overline{u_i{}''c_k{}''}}}{dt}=-\bar\rho\left(\overline{\overline{u_i{}''u_\alpha{}''}}\frac{\partial\overline{\overline{C}}_k}{\partial x_\alpha}+\overline{\overline{u_\alpha{}''c_k{}''}}\frac{\partial\overline{\overline{U}}_i}{\partial x_\alpha}\right)+\rho\overline{c_k{}''}g-2\frac{k}{D_t}\bar\rho\overline{\overline{u_i{}''c_k{}''}}.\quad(2)$$

Let's get algebraic expressions for the turbulent flows of the concentrations and density after neglecting of the items in the left part of the equations (1, 2).

$$\overline{u_i{}''}=\frac{1}{2}\frac{D_t}{k}\left(\overline{\overline{u_\alpha{}''u_i{}''}}\frac{1}{\bar\rho}\frac{\partial\bar\rho}{\partial x_\alpha}+bg_i\right),\quad(3)$$

$$\overline{\overline{u_i{}''c_k{}''}}=-\frac{1}{2}\frac{D_t}{k}\left(\overline{\overline{u_\alpha{}''u_i{}''}}\frac{\partial\overline{\overline{C}}_k}{\partial x_\alpha}-\overline{c_k{}''}g_i\right),\quad(4)$$

where $b=\frac{\overline{\rho'\rho'}}{\bar\rho}$, $\overline{c_k{}''}=-\frac{\overline{\rho c_k{}''}}{\bar\rho}$. As the liquids are not compressible we can write

$$b=\frac{\overline{\rho'\rho'}}{\bar\rho^2}=\frac{\overline{f_1{}'f_1{}'}(\rho_1-\rho_2)^2}{\bar\rho^2}=\Gamma\frac{(\rho_1-\rho_2)^2\,\bar f_1\bar f_2}{\bar\rho^2}=\Gamma\frac{(\bar\rho-\rho_2)(\rho_1-\bar\rho)}{\bar\rho^2},\text{ where }\Gamma=\frac{\overline{f_1{}'f_1{}'}}{\bar f_1\bar f_2}.$$

$$\overline{c_1''} \approx -\overline{f_1'\rho'}\frac{\rho_1}{\bar\rho^2} = -\Gamma\frac{\rho_1\left(\bar\rho-\rho_2\right)\left(\rho_1-\bar\rho\right)}{\bar\rho^2\left(\rho_1-\rho_2\right)}.$$

$f_i -$ volume concentration.

A measure of the concentration fluctuations is given by Γ. The quantity Γ lies in the range 0 to1. If there were no dissipation of density fluctuations, Γ would be unity. It is easy to see, that at $\Gamma=0$ the equations (3, 4) correspond the gradient hypothesis.

It is necessary to write an equation for Γ for closing of the system. According to [6] the equation may be written in the following form:

$$\frac{d}{dt}\left(\Gamma B_0\right) = \frac{1}{\bar\rho^2}\frac{\partial}{\partial x_j}\bar\rho^2\left\{D_t\frac{\partial}{\partial x_j}\left(\Gamma B_0\right)+\overline{u_j''}\Gamma B_0\right\} - \Gamma B_0\frac{\partial\overline{u_j''}}{\partial x_j} + \frac{2}{n-1}\overline{u_j''}\frac{1}{\bar{\bar\rho}}\frac{\partial\bar\rho\bar{\bar{C}}_1}{\partial x_j} - \bar{D},$$

where $B_0 = \dfrac{\bar{C}_1\left(1-\bar{C}_1\right)}{n}$, $n = \dfrac{\rho_1}{\rho_2}$-the density ratio, $\bar{D} = k_b\dfrac{\varepsilon}{k}\Gamma B_0$.

Finally, the set of the equation may be written in the following form:

$$\begin{aligned}
&\frac{\partial\bar\rho}{\partial t} + \frac{\partial\bar\rho\bar{\bar{U}}_\alpha}{\partial x_\alpha} = 0\\
&\frac{\partial\bar\rho\bar{\bar{C}}_k}{\partial t} + \frac{\partial\bar\rho\bar{\bar{U}}_\alpha\bar{\bar{C}}_k}{\partial x_\alpha} = -\frac{\partial}{\partial x_\alpha}\left(\bar\rho\overline{\overline{u_\alpha''c_k''}}\right)\\
&\frac{\partial\bar\rho\bar{\bar{U}}_i}{\partial t} + \frac{\partial\bar\rho\bar{\bar{U}}_\alpha\bar{\bar{U}}_i}{\partial x_\alpha} = -\frac{\partial\bar{P}}{\partial x_i} - \frac{\partial}{\partial x_\alpha}\left(\bar\rho\overline{\overline{u_\alpha''u_i''}}\right)\\
&\frac{\partial\bar\rho k}{\partial t} + \frac{\partial\bar\rho\bar{\bar{U}}_\alpha k}{\partial x_\alpha} = -\bar\rho\overline{\overline{u_k''u_\alpha''}}\frac{\partial\bar{\bar{U}}_k}{\partial x_\alpha} - \overline{u_\alpha''}\frac{\partial\bar{P}}{\partial x_\alpha} - \bar\rho\varepsilon + \alpha_8\frac{\partial}{\partial x_\alpha}\left(\bar\rho D_t\frac{\partial k}{\partial x_\alpha}\right)\\
&\frac{\partial\bar\rho\varepsilon}{\partial t} + \frac{\partial\bar\rho\bar{\bar{U}}_\alpha\varepsilon}{\partial x_\alpha} = -C_{3\varepsilon}\frac{\varepsilon}{k}\bar\rho\overline{\overline{u_k''u_\alpha''}}\frac{\partial\bar{\bar{U}}_k}{\partial x_\alpha} - C_{1\varepsilon}\frac{\varepsilon}{k}\overline{u_\alpha''}\frac{\partial\bar{P}}{\partial x_\alpha} - \bar\rho C_{2\varepsilon}\frac{\varepsilon^2}{k} + \alpha_{\varepsilon 8}\frac{\partial}{\partial x_\alpha}\left(\bar\rho\frac{k^2}{\varepsilon}\frac{\partial\varepsilon}{\partial x_\alpha}\right)\\
&\overline{u_i''} = \frac{1}{2}\frac{D_t}{k}\left(\overline{\overline{u_i''u_\alpha''}}\frac{1}{\bar{\bar\rho}}\frac{\partial\bar\rho}{\partial x_\alpha} + \Gamma\frac{\left(\bar\rho-\rho_2\right)\left(\rho_1-\bar\rho\right)}{\bar\rho^2}g_i\right)\\
&\overline{\overline{u_i''c_1''}} = -\frac{1}{2}\frac{D_t}{k}\left(\overline{\overline{u_i''u_\alpha''}}\frac{\partial\bar{\bar{C}}_1}{\partial x_\alpha} + \Gamma\frac{\rho_1\left(\bar\rho-\rho_2\right)\left(\rho_1-\bar\rho\right)}{\bar\rho^2\left(\rho_1-\rho_2\right)}g_i\right)\\
&\frac{\partial\left(\Gamma B_0\right)}{\partial t} + \bar{\bar{U}}_j\frac{\partial\left(\Gamma B_0\right)}{\partial x_j} = \frac{1}{\bar\rho^2}\frac{\partial}{\partial x_j}\bar\rho^2\left\{D_t\frac{\partial}{\partial x_j}\left(\Gamma B_0\right)+\overline{u_j''}\Gamma B_0\right\} - \Gamma B_0\frac{\partial\overline{u_j''}}{\partial x_j} + \frac{2}{n-1}\overline{u_j''}\frac{1}{\bar{\bar\rho}}\frac{\partial\bar\rho\bar{\bar{C}}_1}{\partial x_j} - \bar{D}
\end{aligned}\qquad(5)$$

Approximate integration

Let's consider Rayleigh –Taylor instability in case of mixing two incompressible liquids. The liquids are in the gravitational field with acceleration g, directed from heavy substance to light one. According to [5] let us assume that the turbulent kinetic energy k, the dissipation rate ε and the coefficient of heterogeneity Γ are constant in whole turbulent mixing zone and depend only on time. The system may be written in the following form after simplification:

$$\frac{d\bar{k}}{dt} + \bar{\varepsilon} = 2\frac{c_\mu}{Sc}\frac{\bar{k}^2 Ag}{\bar{\varepsilon}L} + \frac{1}{3}\frac{c_\mu}{Sc}\frac{\bar{k}}{\bar{\varepsilon}}\Gamma g^2 A^2, \tag{6}$$

$$\frac{d\bar{\varepsilon}}{dt} + C_{2\varepsilon}\frac{\bar{\varepsilon}^2}{\bar{k}} = 2C_{1\varepsilon}\frac{c_\mu}{Sc}\bar{k}\frac{1}{L}Ag + \frac{1}{3}C_{1\varepsilon}\frac{c_\mu}{Sc}\Gamma A^2 g^2, \tag{7}$$

$$\frac{dL}{dt} = 6c_1\frac{c_\mu}{Sc}\frac{\bar{k}^2}{\bar{\varepsilon}L} + 2\frac{c_\mu}{Sc}\frac{\bar{k}}{\bar{\varepsilon}}\Gamma Ag, \tag{8}$$

$$\frac{d\Gamma}{dt} = 6c_1\frac{c_\mu}{Sc}\frac{\bar{k}^2}{\bar{\varepsilon}L^2}(1-\Gamma) + 2\frac{c_\mu}{Sc}\frac{\bar{k}}{L\bar{\varepsilon}}Ag\Gamma(1-\Gamma) - k_b\frac{\bar{\varepsilon}}{\bar{k}}\Gamma. \tag{9}$$

The system contains constants which should be taken from experimental and theoretical data. There is $C_{2\varepsilon} = 1.7$ for realization of Kolmogorov's law $L \sim t^{2/7}$. Let us consider $c_\mu = 0.09$ and $Sc = 0.5$. In order to provide realization of law $L=2\alpha Agt^2$ (g=const) $C_{1\varepsilon} = 1.04$. At the same time it was supposed, that coefficients Γ=0.18 and α=0.04, that corresponds to mixable substances [6].

As we can see from the equations for turbulent flows of concentration and density (3, 4), the mass transfer may be carried out by two ways. There are turbulent diffusion and the mass transfer due to buoyancy – drag forces. Influence of buoyancy forces on growth rate of turbulent zone was shown by means of the equations (6-9). Dependence of growth rate of turbulent zone from coefficient of heterogeneity Γ is shown on Fig. 1.

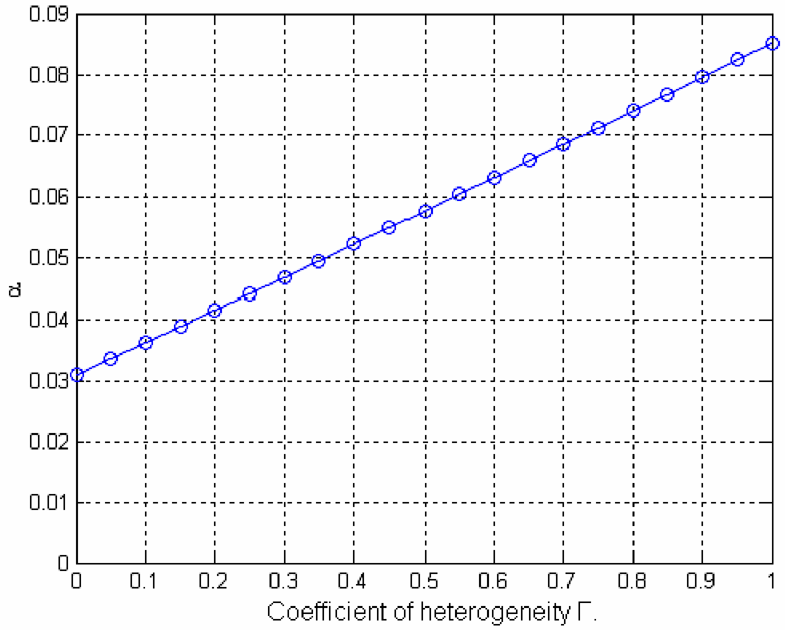

Fig.1. Dependence of growth rate of turbulent zone from coefficient of heterogeneity Γ

Sign − variable acceleration

The system (6-9) was solved numerically at sign-variable acceleration and different self-similar Γ. The value of self-similar Γ depends from constant k_b in equation (9) (fig.2).

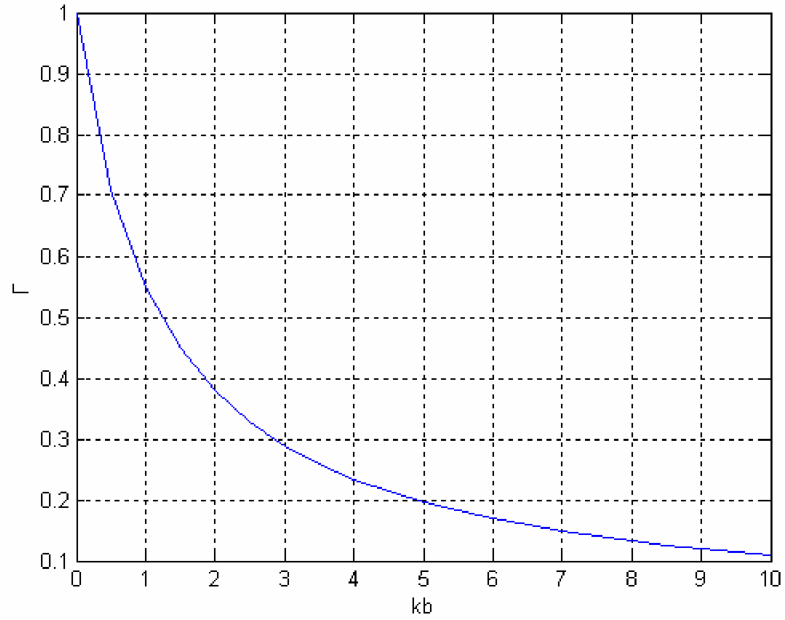

Fig.2. The dependence of self-similar Γ from constant k_b.

Initial conditions:

$$n = \frac{\rho_2}{\rho_1} = 2.95$$

$$g = \begin{cases} g_0 > 0, & 0 \le t < t_0 \\ g_1 < 0, & t_0 \le t < t_1 \end{cases}$$

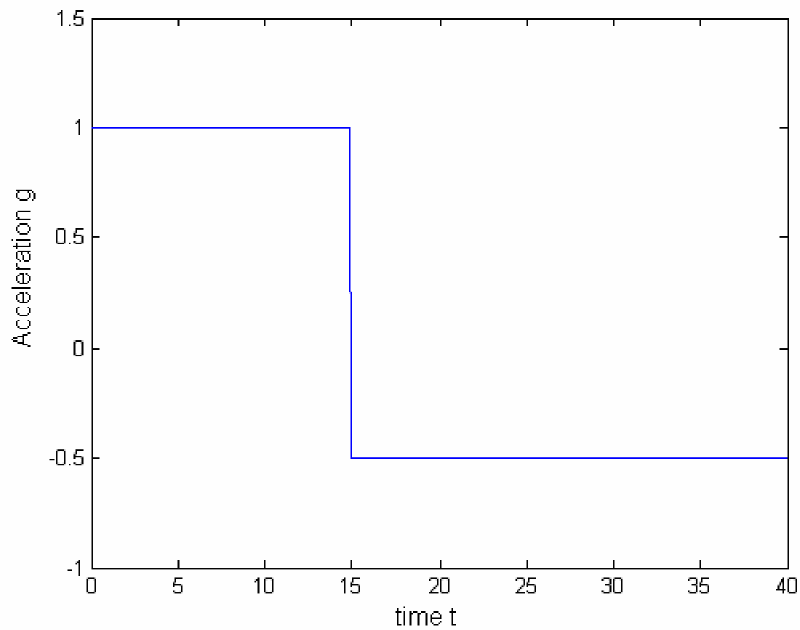

Fig.3. The dependence of g(t).

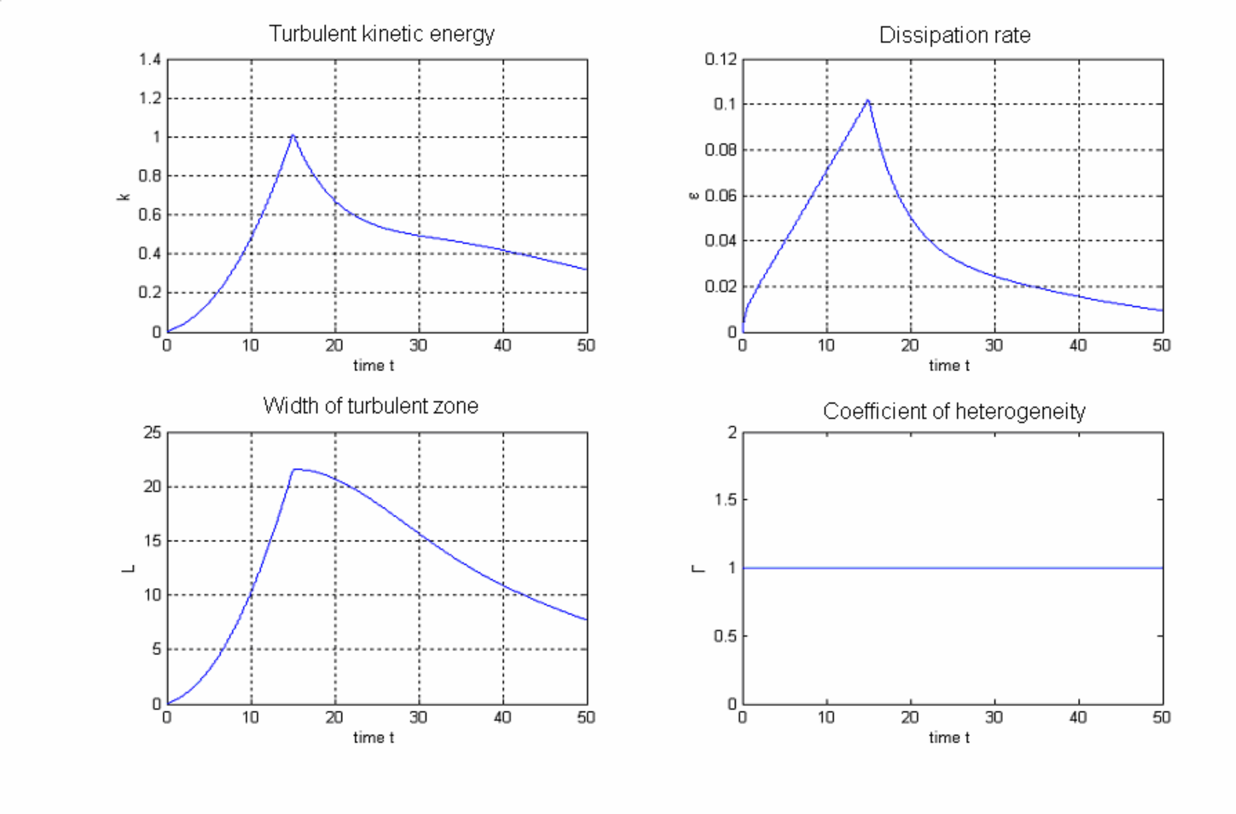

Fig.4. The dependence of *k*, *ε* and *L* from time at Γ->1.

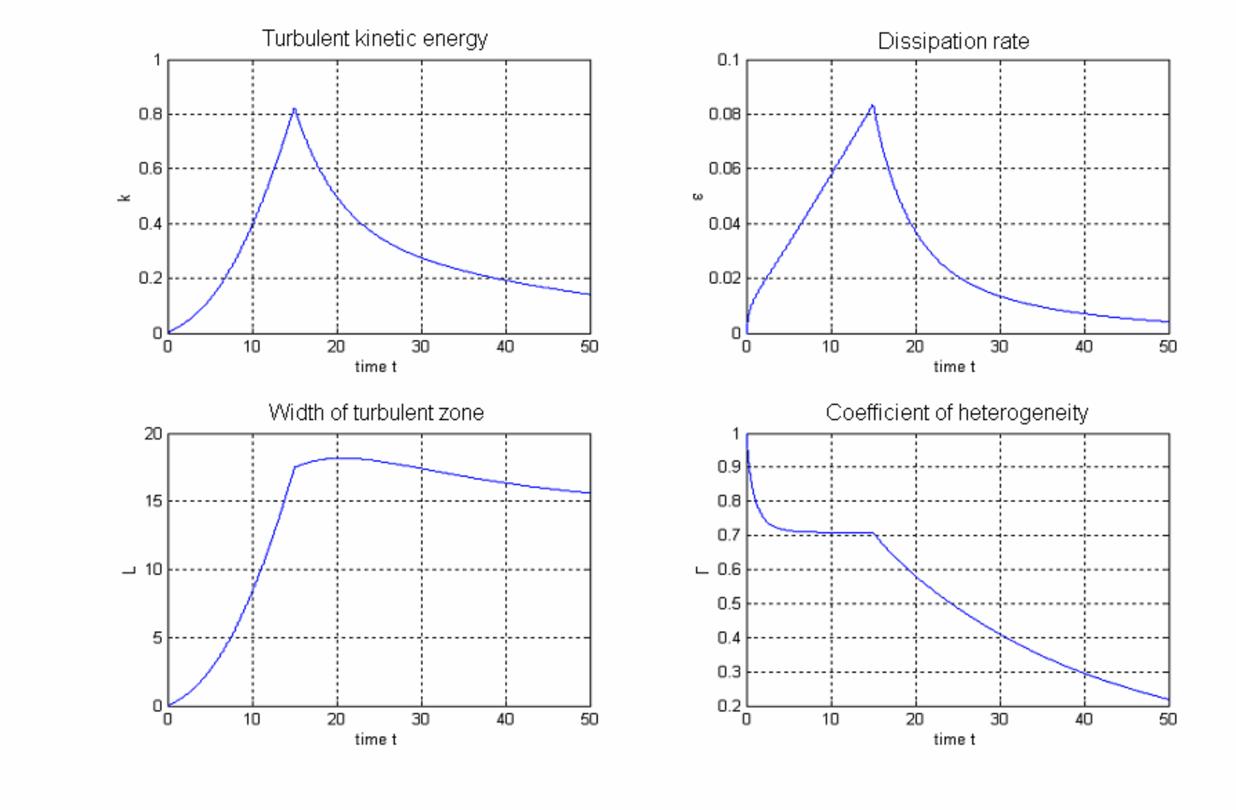

Fig.5. The dependence of *k*, *ε* and *L* from time at Γ->0.7.

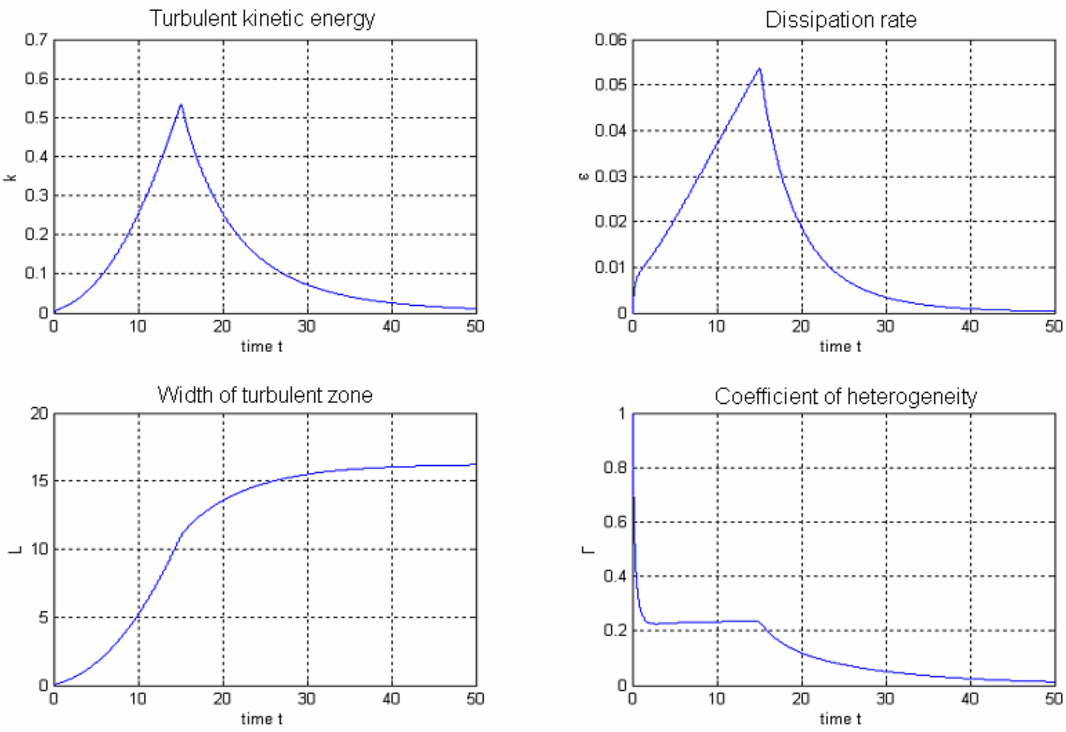

Fig.6. The dependence of *k*, *ε* and *L* from time at Γ->0.25.

We will compare our model against a turbulent Rayleigh-Taylor instability numerical experiment by D.L. Youngs [3]. In this experiment the initial density distribution was given by $\rho = \rho_1 = 3$ for x<0, $\rho = \rho_2 = 1$ for x>0. For the gravitational field:

$$g = \begin{cases} +g_1 & 0 < t < 1.5 \\ -g_1 & 1.5 < t < 3 \\ +g_1 & 3 < t \end{cases}, \text{ where } \frac{\rho_1 - \rho_2}{\rho_1 + \rho_2} g_1 = 1.$$

In fig.7, 8 are shown the dependence of width of the turbulent zone and the coefficient of heterogeneity from time.

Conclusions

It is shown, that in an unstable case the dimensionless velocity of development of turbulent zone depends on the coefficient of heterogeneity *Γ*. This fact is a possible explanation of a distinction of experimental data. At sign – variable acceleration the stable phase begins, when the sign of acceleration changes. The zone of turbulent mixing decreases in this case. This fact is in the consent with experimental data [2] and results of direct numerical simulation [3]. Development of a mixing zone depends from *Γ* in this case. There is a full separation if *Γ*=1 and separation does not occur if *Γ*=0.

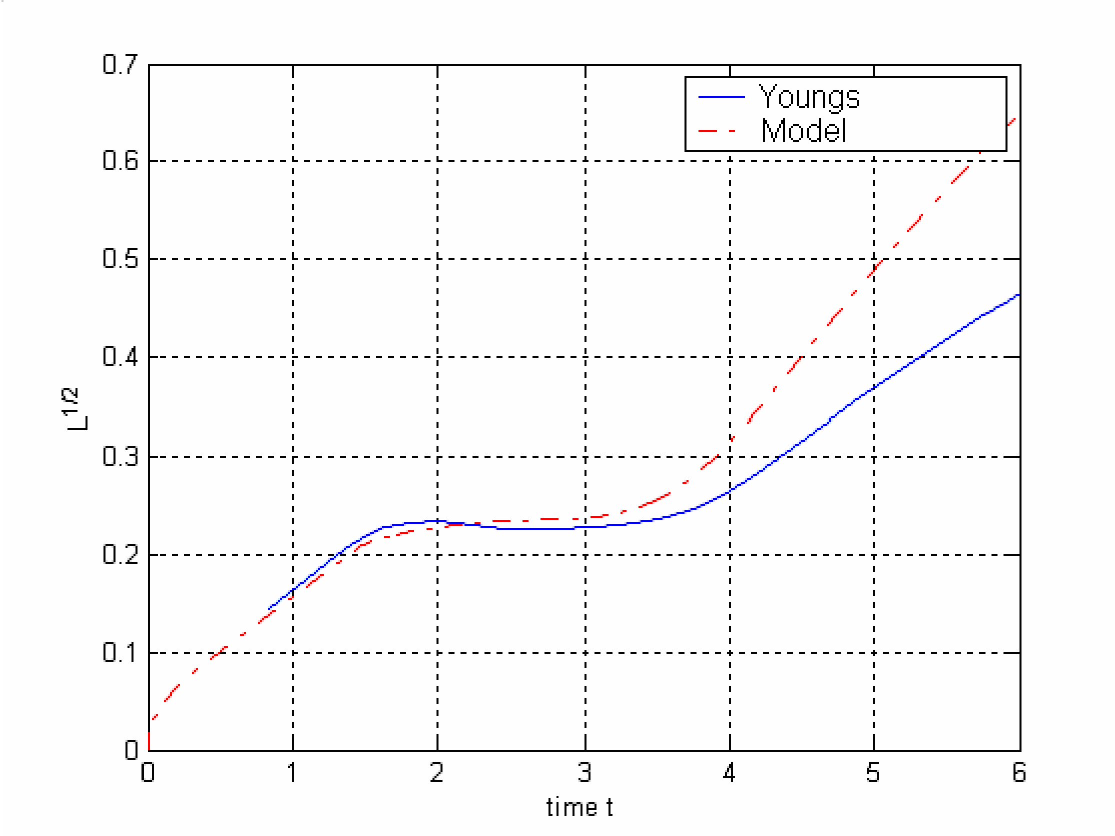

Fig.7. The dependence of *L* from time.

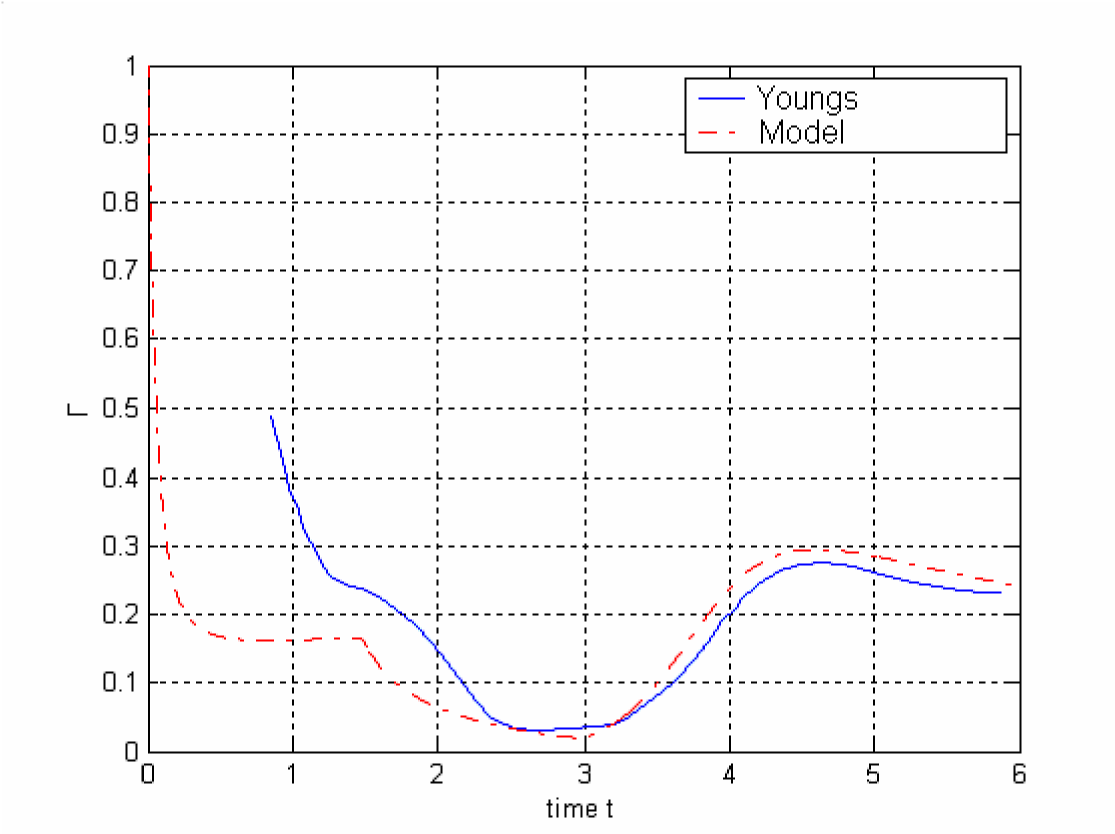

Fig.8. The dependence of *Γ* from time.

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