9th International Workshop on the Physics of Compressible Turbulent Mixing

Linear stability analysis of self-similar ablation flows in ICF

Florian Abéguilé

CPHT – Ecole Polytechnique

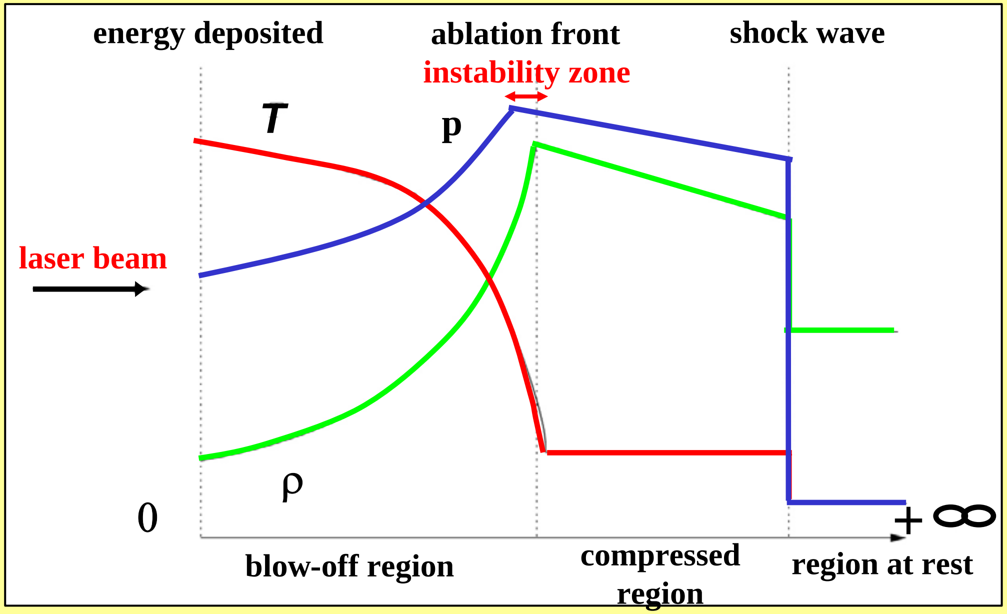
CEA - Bruyères-le-Châtel

co-workers : Carine Boudesocque-Dubois, Jean-Marie Clarisse & Serge Gauthier

Ablation Front Instability (AFI)

- **semi-infinite slab of perfect gas heated by a laser**
- **non linear heat conduction**

$$\vec{\varphi} = -\chi\, \rho^{\mu}\, T^{\nu}\, \nabla T$$



Approach 1

stationary flow

- ✓ **Ishizaki & al. (1997): blow-off region omitted**
- ✓ **Goncharov (1999): discontinuous ablation front**

MODE ANALYSIS

- ✓ **Goncharov: SBM analysis**

Approach 2

self-similar solutions

exact solution of the ablation flow

- ✓ **Marshak (1958), Saillard (1983)**

COMPLETE STABILITY ANALYSIS

- ✓ **Boudesocque-Dubois (2000)**

Mean flow equations

Gas dynamic equations with non linear heat conduction

$$\begin{cases} \dfrac{\partial}{\partial t}\left(\dfrac{1}{\overline{\rho}}\right) - \dfrac{\partial \overline{v}_x}{\partial m} = 0, & \text{mass conservation} \\ \dfrac{\partial}{\partial t}\left(\overline{v}_x\right) + \dfrac{\partial \overline{p}}{\partial m} = 0, & \text{momentum conservation} \\ \dfrac{\partial}{\partial t}\left(\dfrac{1}{2}\overline{v}_x^2 + \overline{\mathcal{E}}\right) + \dfrac{\partial}{\partial m}\left(\overline{p}\,\overline{v}_x + \overline{\varphi}_x\right) = 0, & \text{energy conservation} \end{cases}$$

$$\begin{aligned} \overline{p} &= \mathcal{B}_p \left(\frac{t}{t_*}\right)^{1/(\nu-1)}, \\ \overline{\varphi}_x &= \mathcal{B}_\varphi \left(\frac{t}{t_*}\right)^{(3/2)/(\nu-1)}, \end{aligned} \quad \text{for } m = 0$$

Self-similar formulation $\xi = \dfrac{m}{t^\alpha}$ $\alpha = \dfrac{2\nu - 1}{2\nu - 2}$

$$\begin{aligned} \overline{\rho}(m,t) &= && \overline{G}(\xi), \\ \overline{v}_x(m,t) &= t^{\alpha-1} && \overline{V}_x(\xi), \\ \overline{T}(m,t) &= t^{2(\alpha-1)} && \overline{\Theta}(\xi), \\ \overline{\varphi}_x(m,t) &= t^{3(\alpha-1)} && \overline{\Phi}_x(\xi). \end{aligned}$$

$$\frac{d\mathsf{Y}}{d\xi} = \mathcal{F}(\xi, \mathsf{Y}) \qquad \mathsf{Y} = \left(\overline{G}, \overline{V}_x, \overline{\Theta}, \overline{\Phi}_x\right)^\top$$

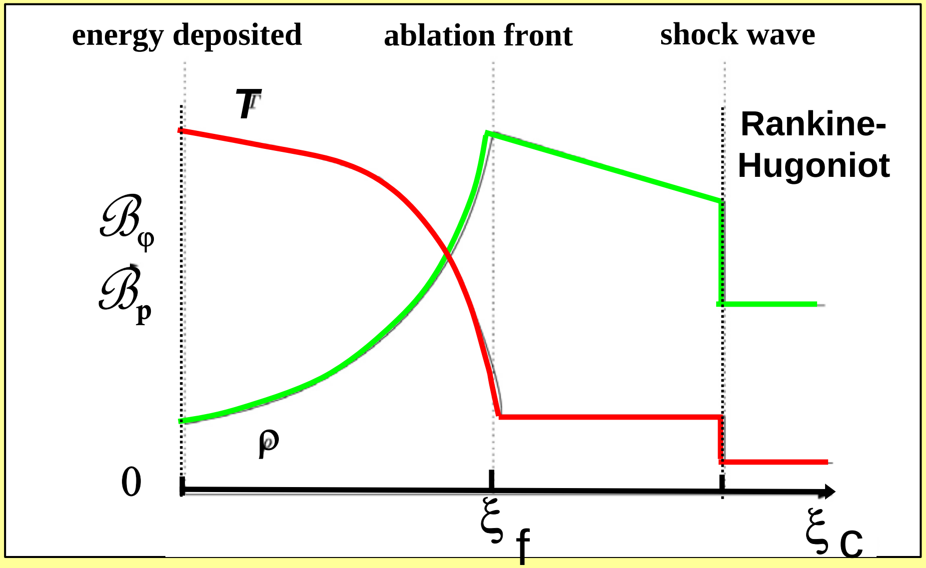
α depends only on ν ⇒ **Self-similar flow governed by thermal diffusivity**

Mean flow numerical resolution

ODE system with boundary conditions

$$\frac{dY}{d\xi} = \mathcal{F}(\xi, Y) \qquad Y = \left(\overline{G}, \overline{V}_x, \overline{\Theta}, \overline{\Phi}_x\right)^\top$$

Numerical resolution : shooting method and relaxation method

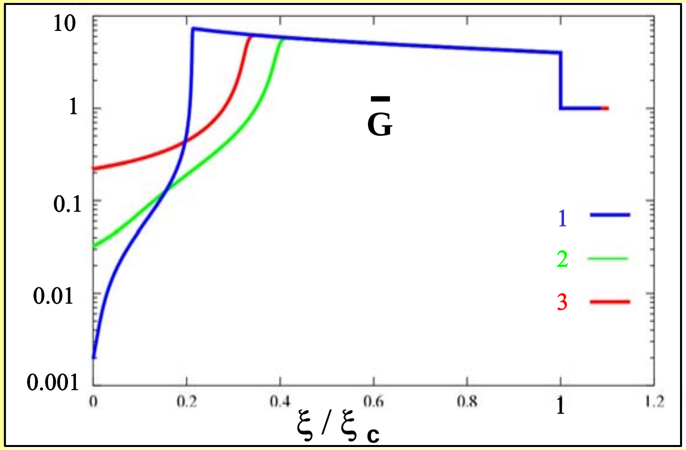


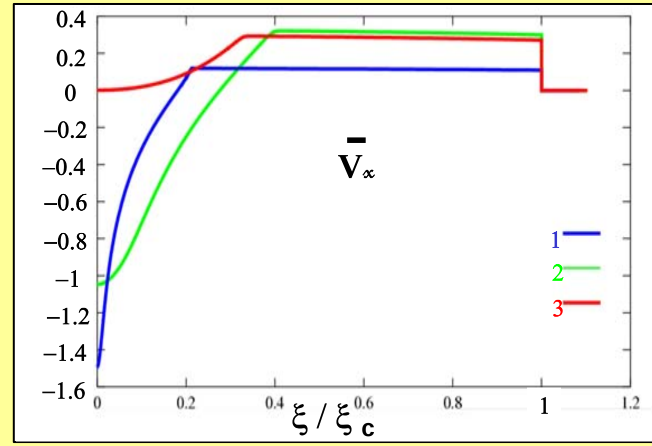
First step: Shooting method in finite-difference

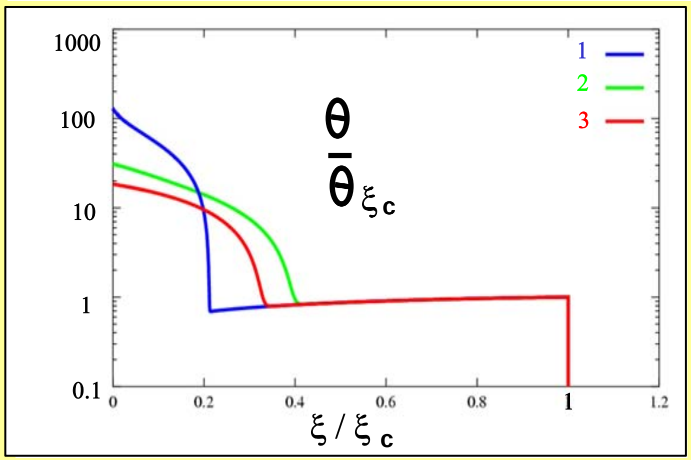
Second step: Relaxation process (dynamical multidomain Chebyshev pseudo-spectral method[1,2])

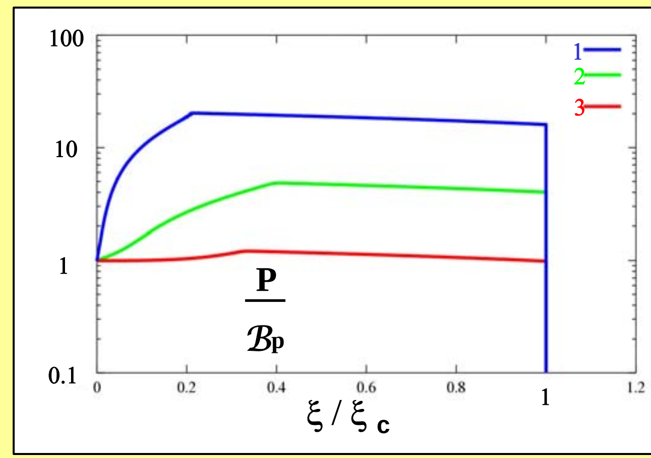
1 Renaud & Gauthier, 1998. 2 Le Creurer's talk (tuesday)

Mean flow results









	$\mathcal{B}_P$	$\mathcal{B}_\varphi$	ξ_f	ξ_c	M_a	Fr_a
1	$0.3.10^{-1}$	$2.6.10^{-1}$	0.12	0.300	0.139	3.88
2	$0.1.10^{-2}$	$2.6.10^{-2}$	$0.23.10^{-1}$	0.110	0.063	10.68
3	1.10^{-1}	1.10^{-1}	$0.09.10^{-1}$	0.272	0.108	2.94

Modification of parameters ➡

Modification of relative thickness

Modification of compressibility

Linear perturbation equations

Three-dimensional linear perturbations

$$\frac{\partial}{\partial t}\left(\frac{\rho}{\overline{\rho}}\right) + \frac{\partial}{\partial m}\left(\overline{\rho} v_x\right) + \nabla_\perp \cdot \vec{v}_\perp = 0,$$

mass conservation

$$\frac{\partial v_x}{\partial t} + \overline{\rho} v_x \frac{\partial \overline{v}_x}{\partial m} + \frac{\partial p}{\partial m} - \frac{\rho}{\overline{\rho}} \frac{\partial \overline{p}}{\partial m} = 0,$$

x-momentum conservation

$$\frac{\partial \vec{v}_\perp}{\partial t} + \frac{1}{\overline{\rho}} \nabla_\perp p = \vec{0},$$

transverse-momentum conservation

$$\frac{\partial \mathcal{E}}{\partial t} + \overline{\rho} v_x \frac{\partial \overline{\mathcal{E}}}{\partial m} = \overline{p}\left(\frac{\rho}{\overline{\rho}} \frac{\partial \overline{v}_x}{\partial m} - \frac{\partial v_x}{\partial m}\right) - p \frac{\partial \overline{v}_x}{\partial m} + \frac{\rho}{\overline{\rho}} \frac{\partial \overline{\varphi}_x}{\partial m} - \frac{\partial \varphi_x}{\partial m} - \frac{1}{\overline{\rho}}\left(\overline{p} \nabla_\perp \cdot \vec{v}_\perp + \nabla_\perp \cdot \vec{\varphi}_\perp\right)$$

energy conservation

$$\varphi_x = -\overline{\rho}^{(\mu+1)} \overline{T}^{\nu} \left(\frac{\partial T}{\partial m} + \left((\mu+1)\frac{\rho}{\overline{\rho}} + \nu \frac{T}{\overline{T}}\right) \frac{\partial \overline{T}}{\partial m}\right),$$

$$\vec{\varphi}_\perp = -\overline{\rho}^{\mu} \overline{T}^{\nu} \nabla_\perp T.$$

Incompletely parabolic system with time-dependent boundary conditions

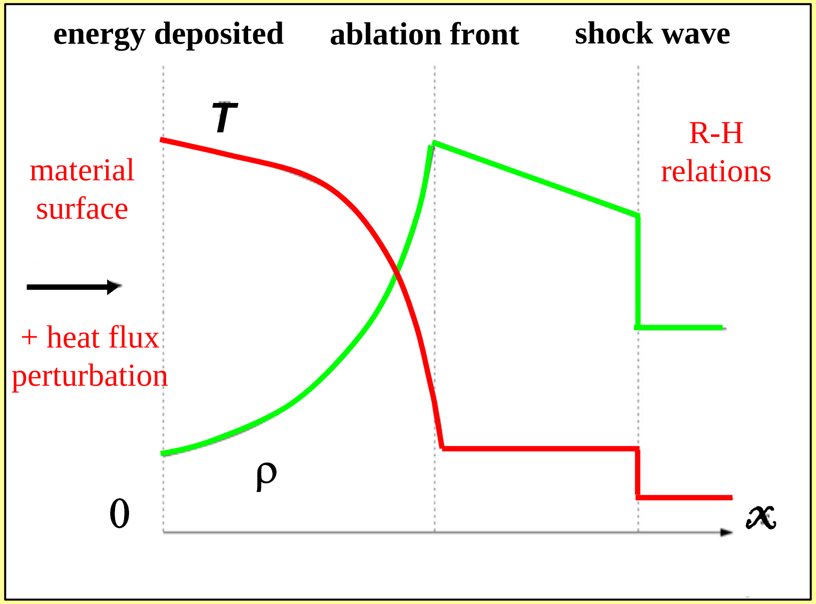
Linear perturbations numerical resolution

Numerical method

Hyperbolic system [3] **&** **Parabolic equation**

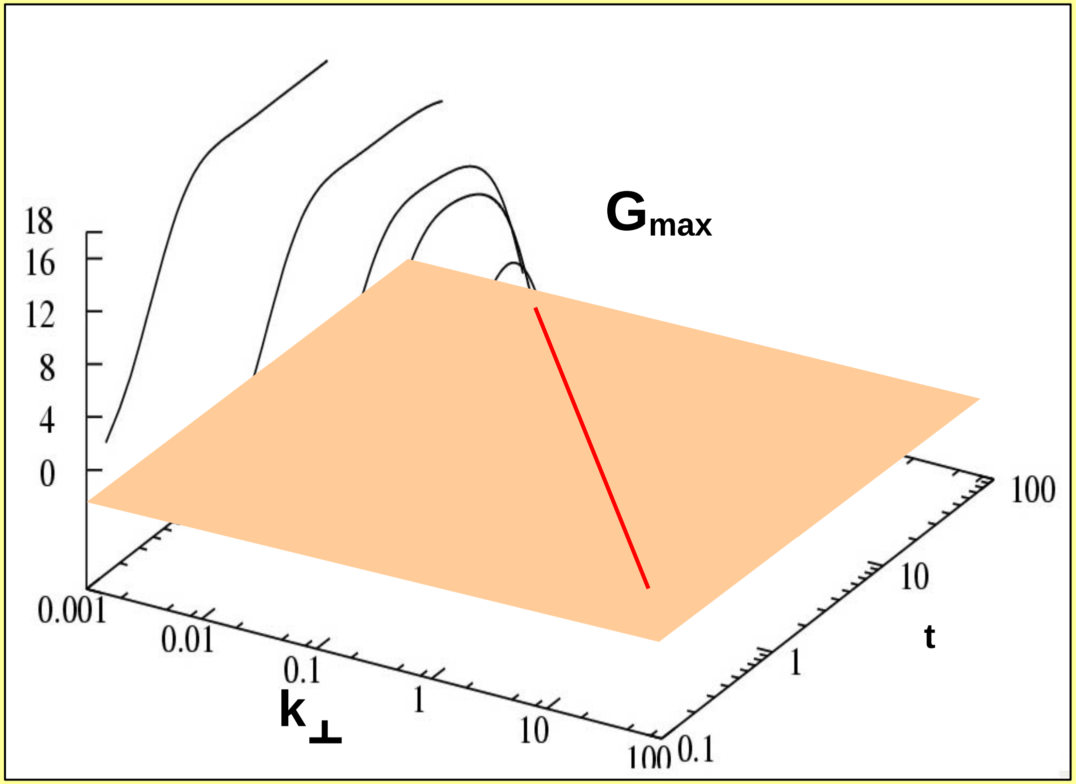
- **dynamical multidomain Chebyshev collocation method with three step Runge-Kutta scheme (explicit & complete)**
- **time-dependent boundary conditions: characteristics method**
  influence matrix technique[4]

Boundary conditions

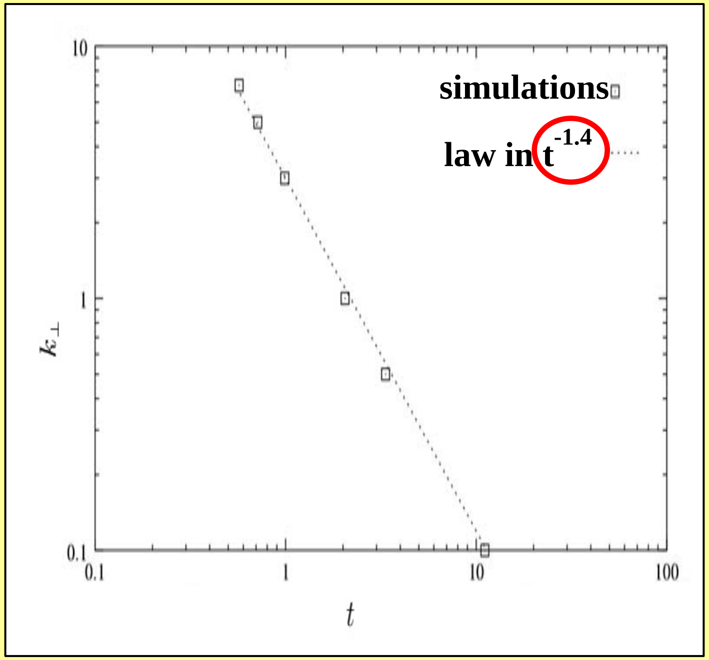


3 Boudesocque-Dubois & al., 2003. **4 Pulicani, 1989.**

Self-similar ablation flow stability analysis 1.



amplification sheet

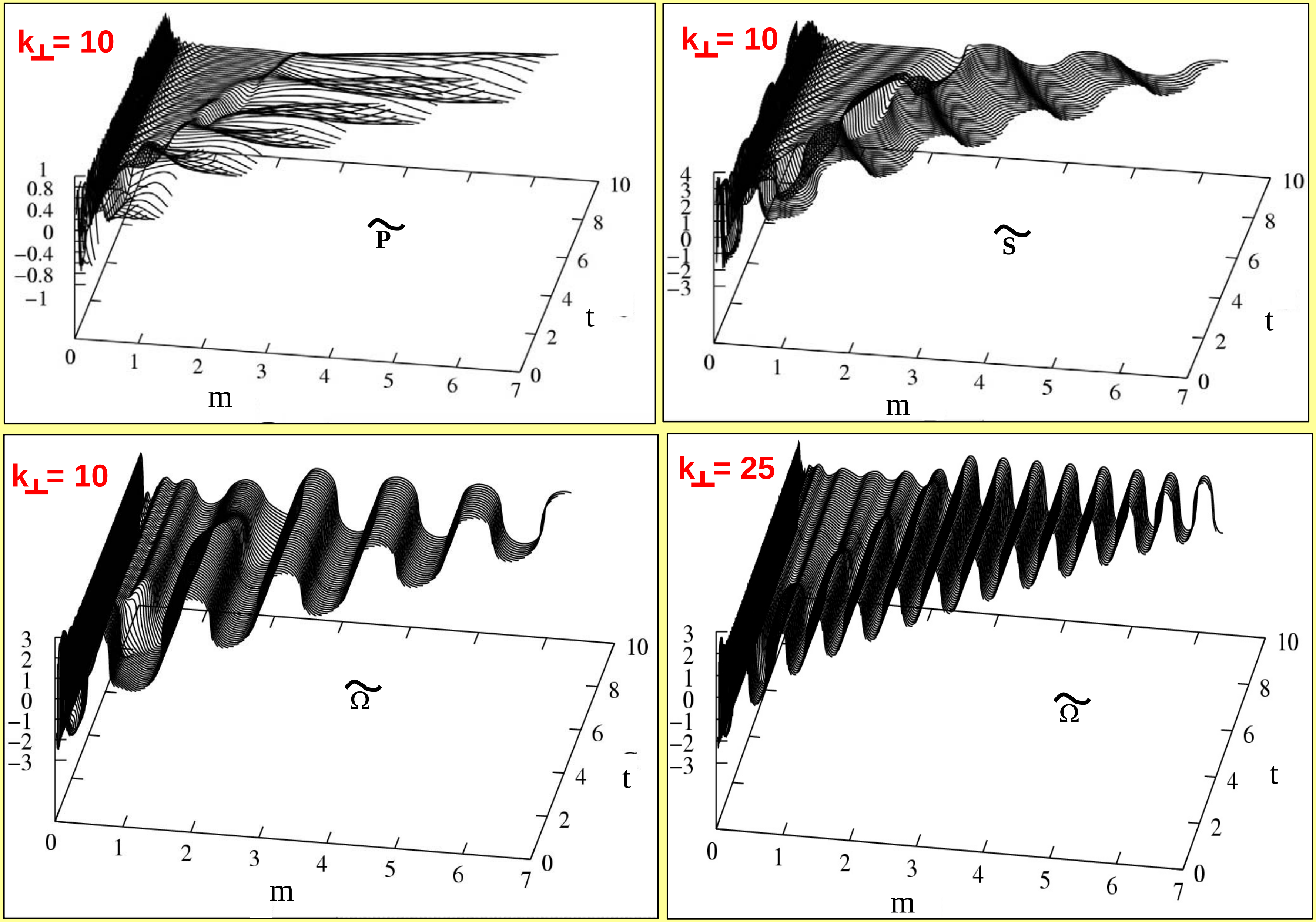


threshold wavenumber

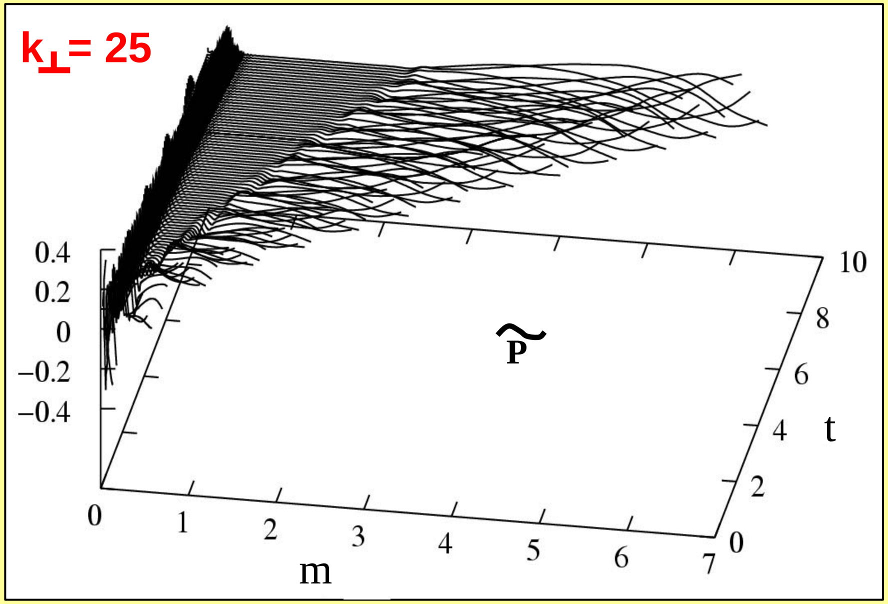
- Maximum reached for zero transverse wavenumber
- Plateau for small wavenumbers
- Attenuation for large wavenumbers followed by oscillatory regime in time
- Perturbations seem to persist although wavenumber increases

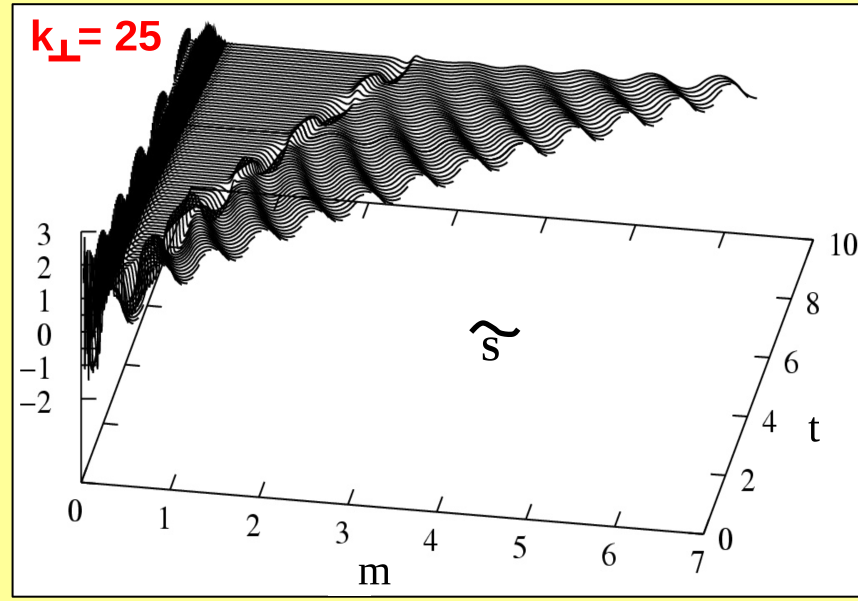
➡ Perturbation attenuation for large wavenumbers is essentially governed by thermal diffusivity

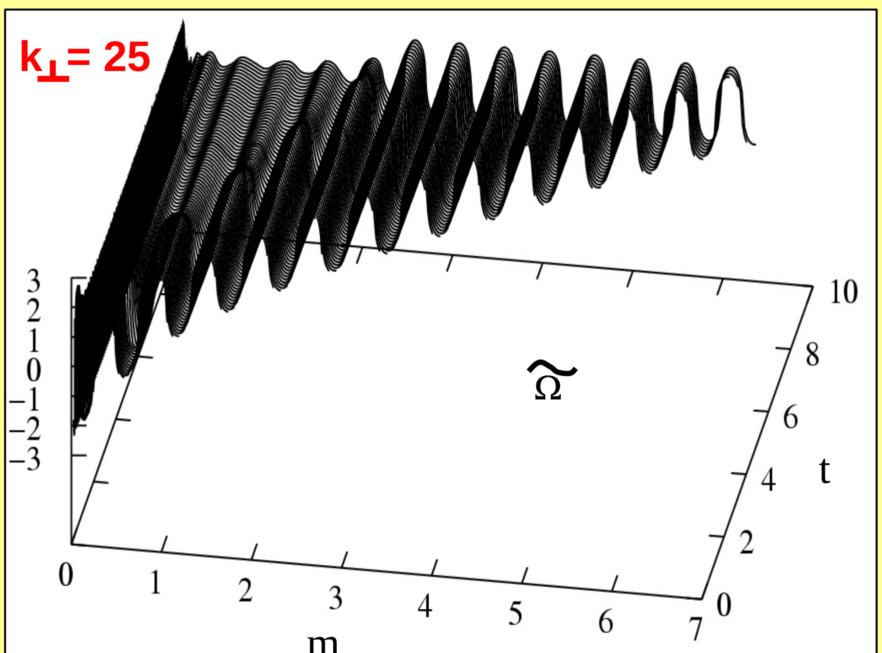
Space-time structures of perturbations 1.



Space-time structures of perturbations 2.







Perturbations convection from the rippled shock towards the ablation front

Conclusions

Stability analysis of EXACT / UNSTEADY / COMPRESSIBLE ablation flow is now possible

First trail : Main results

- MAXIMUM amplification for ZERO transverse wavenumber
- 2 regimes : AMPLICATION REGIME / ATTENUATION REGIME
- Attenuation governed by THERMAL DIFFUSION
- Absence of CUT-OFF ?

Second trail : Acoustic part on the stability (in progress)

- Perturbations convection from the rippled shock towards the ablation front
- Couplage between the ablation front and the shock