Discrete Wavelet Transform Techniques for Denoising, Pattern Detection and Compression of Turbulent Rayleigh-Taylor Mix Data

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Cal Tech
Pasadena, CA
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What Are Wavelets? Start @ (www.wavelets.org) surf (Mathsoft, amara, ...)

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Mallat, Meyer, Daubechies, Beylkin, Coifman, Strang, Sweldens, Jawerth...

- Wavelets are localized kernels or atoms in PHASE SPACE.
- You may think of them as basis functions with prescribed dilation and translation properties.
- They may or may not be orthonormal or have compact support or be differentiable everywhere, or be fractal, or have many zero momemts.
- Wavelets are like breathing wave packets which can home in on structures in phase space better than FT or WFT ever could.

$$\psi_{j,k}(x) = 2^{j/2}\,\Psi\left[2^j\left(x-\frac{k}{2^j}\right)\right];\; j,k \in Z$$

$$\Psi_n(x) = (-1)^n \frac{d^n}{dx^n}\left[\exp\left(-\kappa(x-x_c)^2/2\right)\right]$$

When the scale is decreased translation steps between wavelets should likewise be decreased

What is MRD or Multi-resolution Decomposition?

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- Multiresolution: Zoom in and out on a number of successively finer scales in a sequence of nested approximation subspaces $\{V_j\}_{j \text{ in } Z}$.
- In general, get an overcomplete basis set in $L_2(R)$. Approximate (or truncate) by bounding the scales of interest.

Scaling functions and the scaling equation:

Low pass filter

$$\varphi(x) = 2\sum_{k=0}^{2N-1} h_k \varphi(2x-k)$$

$$\sum_k h_k = 1 \qquad \int_{-\infty}^{\infty} \varphi(x)\,\mathrm{d}x = 1$$

The Wavelets:

High pass filter

$$\psi(x) = 2\sum_{k=0}^{2N-1} g_k \varphi(2x-k)$$

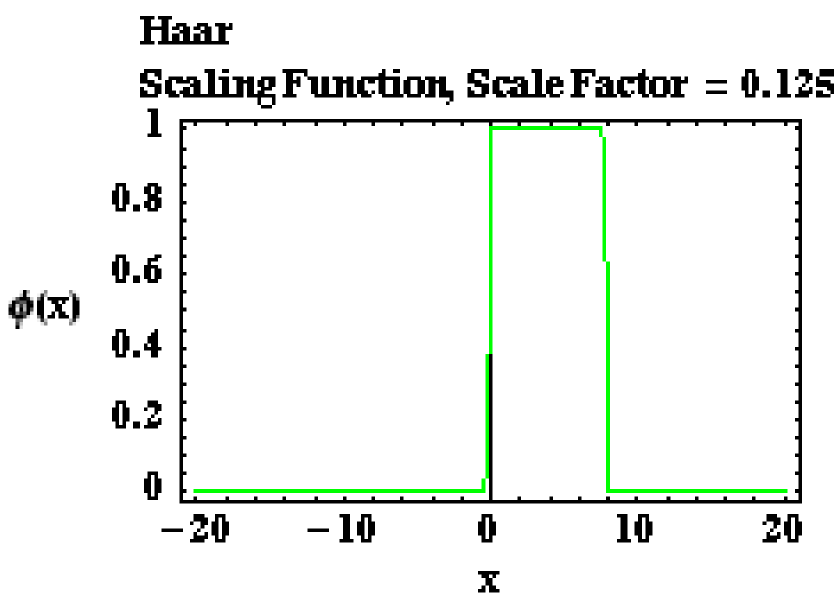
$$g_k = (-1)^k h_{2N-1-k}$$

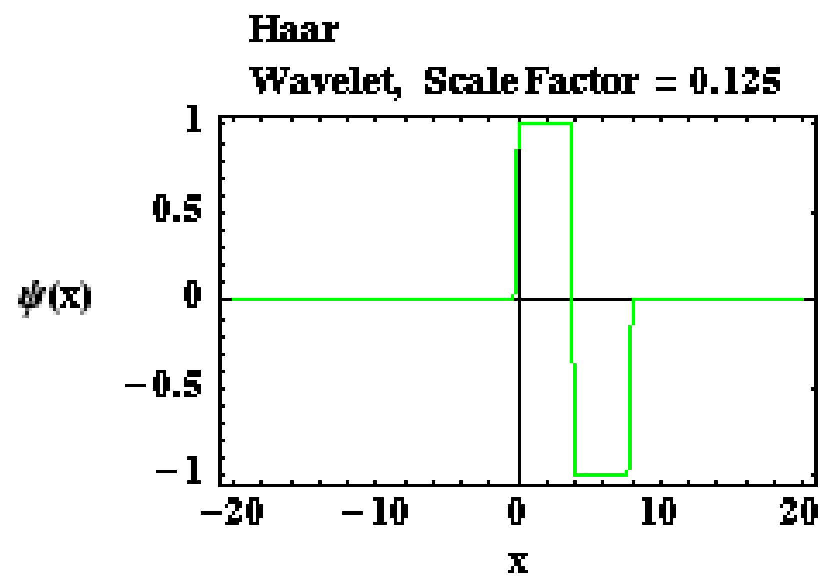
These filters decompose a sampled signal into 2 sub-sampled channels: the coarse approximation of the signal and the missing details at finer scales. The original signal can be reconstructed from these channels by interpolation.

What Are Discrete Wavelet Decompositions Good for?

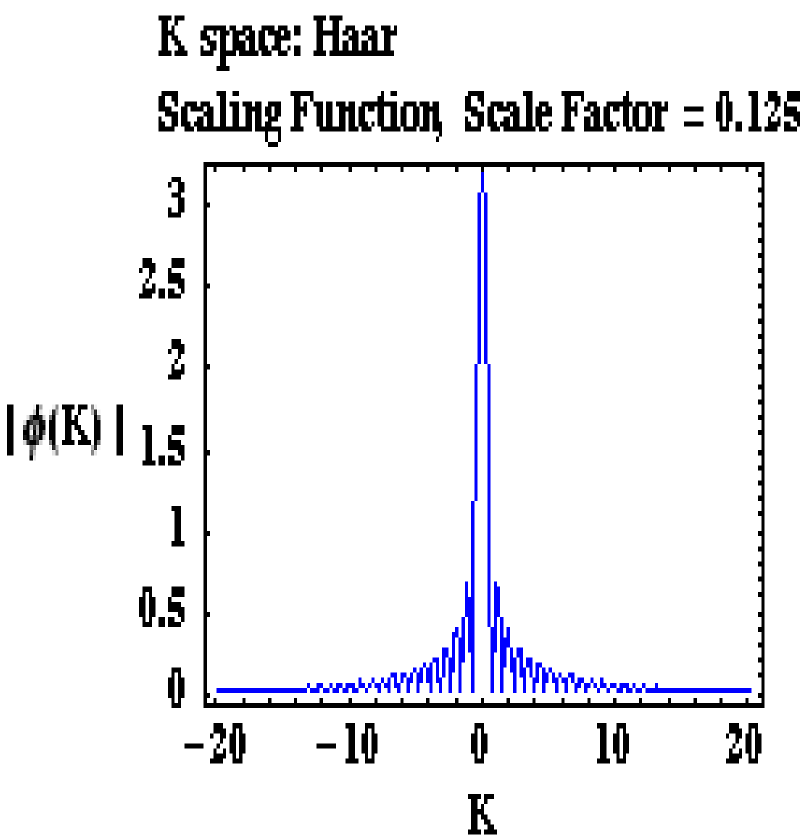
- Wavelet decompositions are very useful for the analysis of intermittent or bursty data.
- Spatial and scale localized information is efficiently represented.
- Because the trends you want are captured efficiently (get large coefficients in the expansion)very high quality denoising is possible.
- Similarly, pattern recognition and detection capability is enhanced.
- Compression is achieved where a few coefficients can represent what is needed in the data.
- All this depends on nonlinear (or largest coefficient)thresholding and not scale or level thresholding . The latter is rather reminiscent of what is done with Fourier expansions.

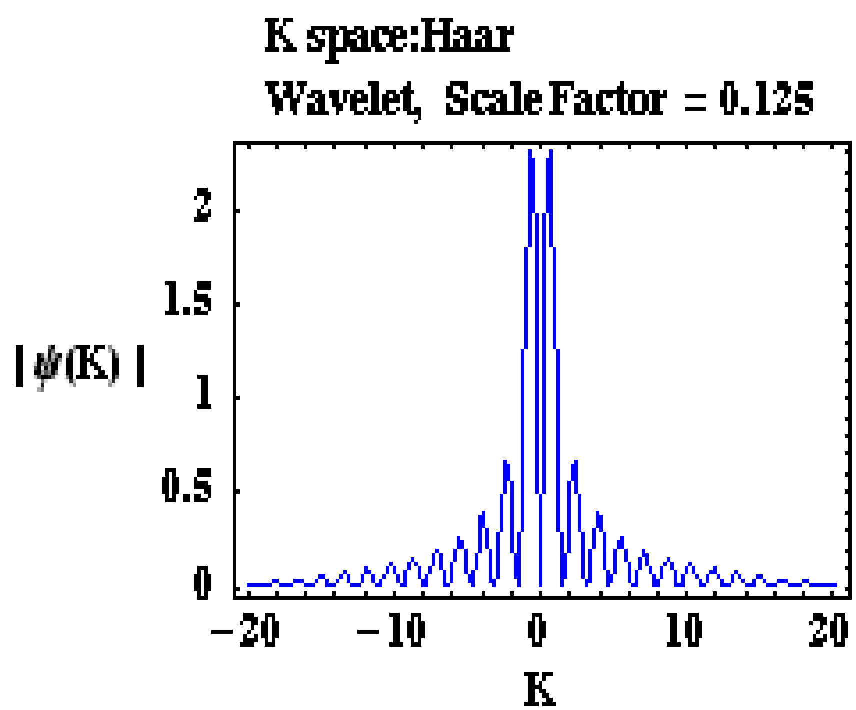
The Scaling Function and Wavelet for Haar or Daubechies 1 in X-Space





The Scaling Function and Wavelet for Haar or Daubechies 1 in K- Space





The Scaling Functions and Wavelets for Daubechies 2-6 in X-Space

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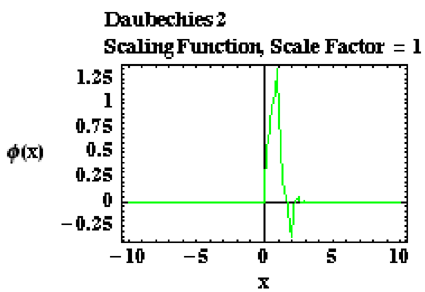
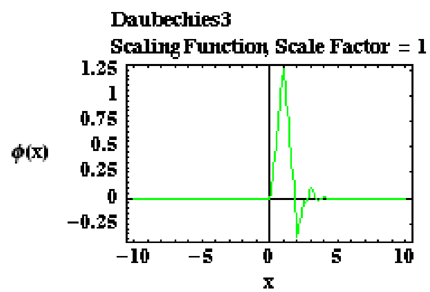
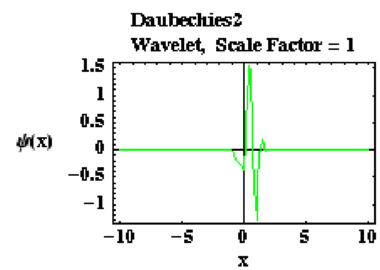
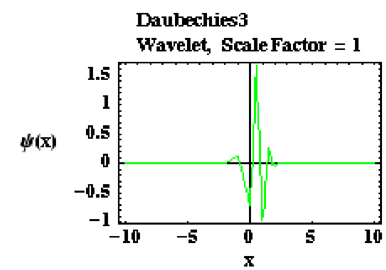
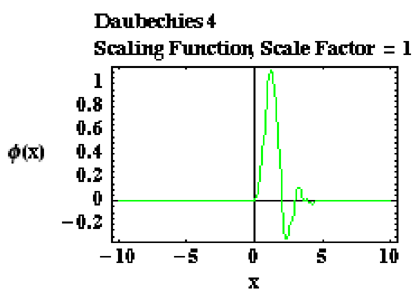
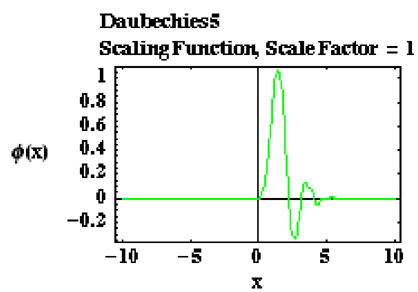


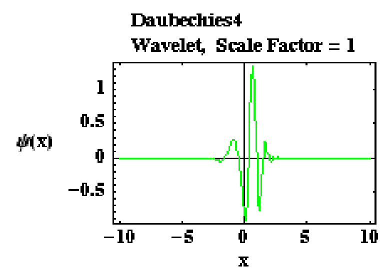


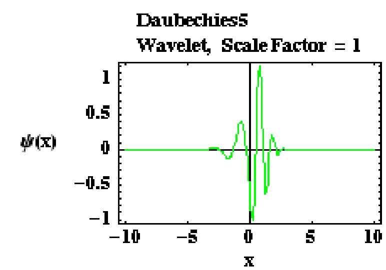


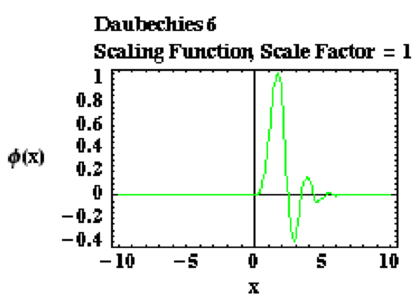


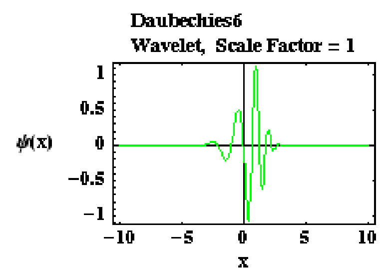




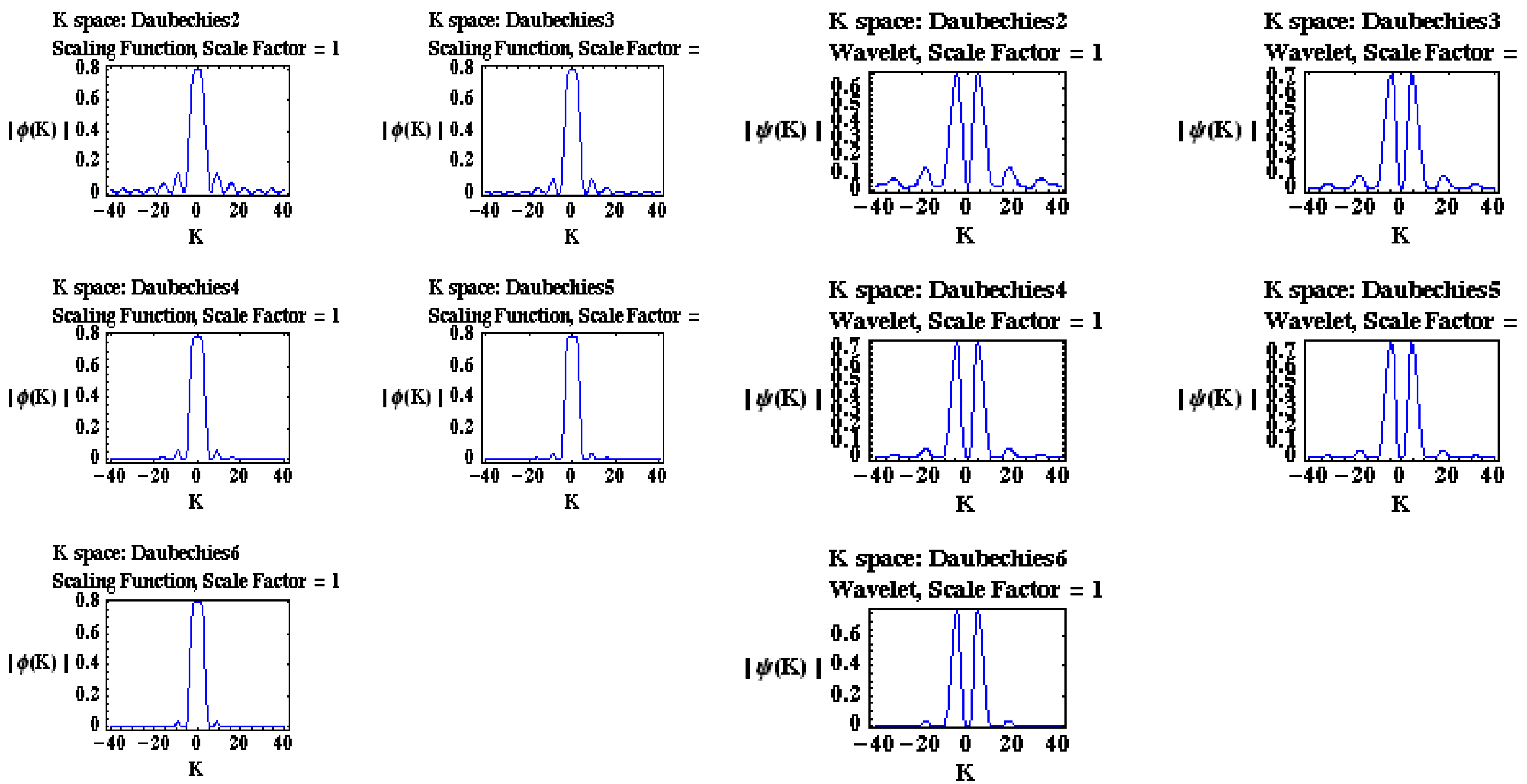








The Scaling Functions and Wavelets for Daubechies 2-6 in k-Space



Raw Thermocouple RT Strong Mix Data (30 cm Downstream, theta ~ 0.71) from Texas A&M

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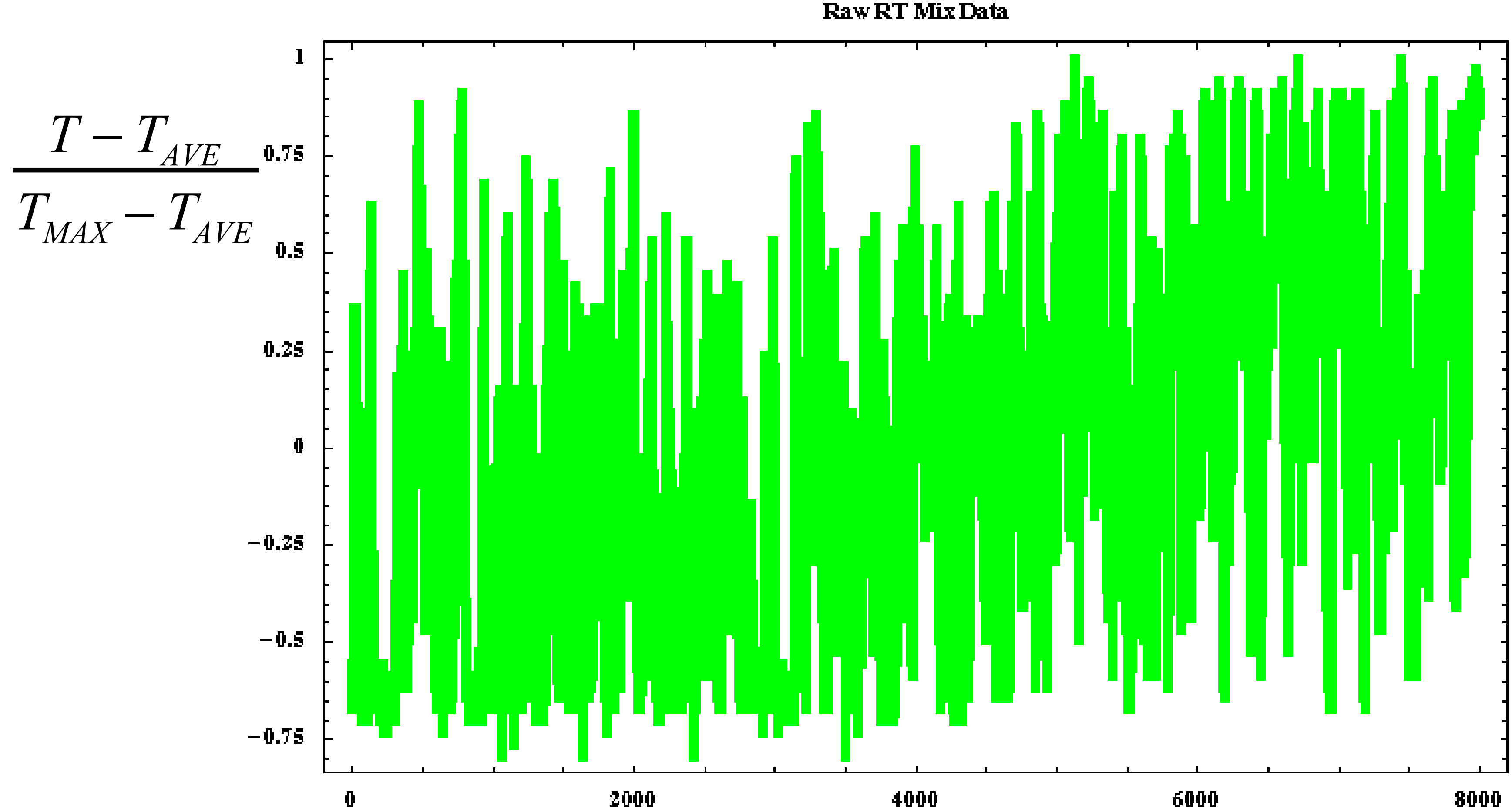

Time, arb. units (Delta t=0.012 sec, Sampling Rate = 85 Hz)

The Faded and Padded Version of the Data 8192 Points Long

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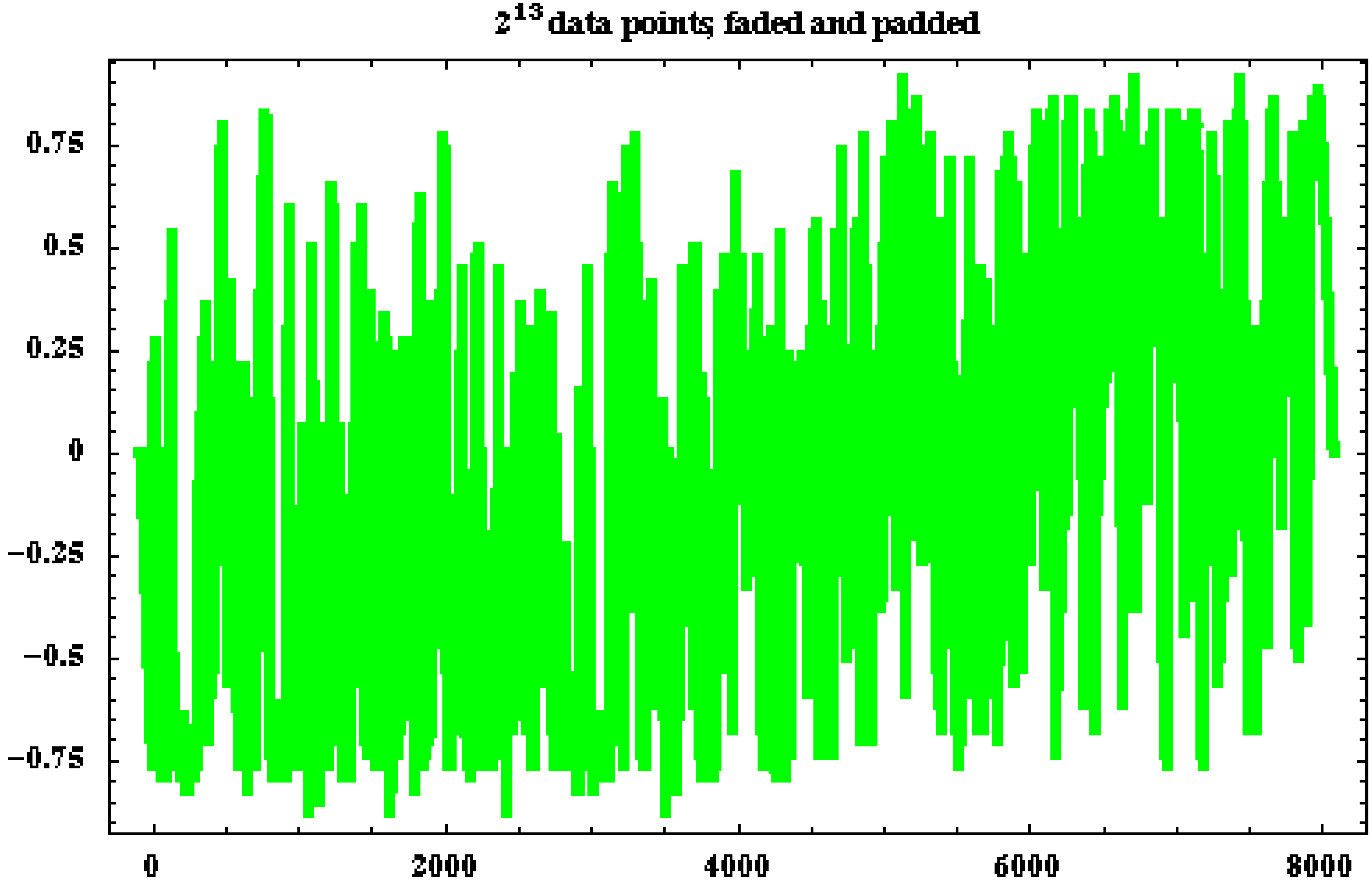

80 pts to fade
16 pts to pad per side

The Fourier Transform of the RT Mix Data

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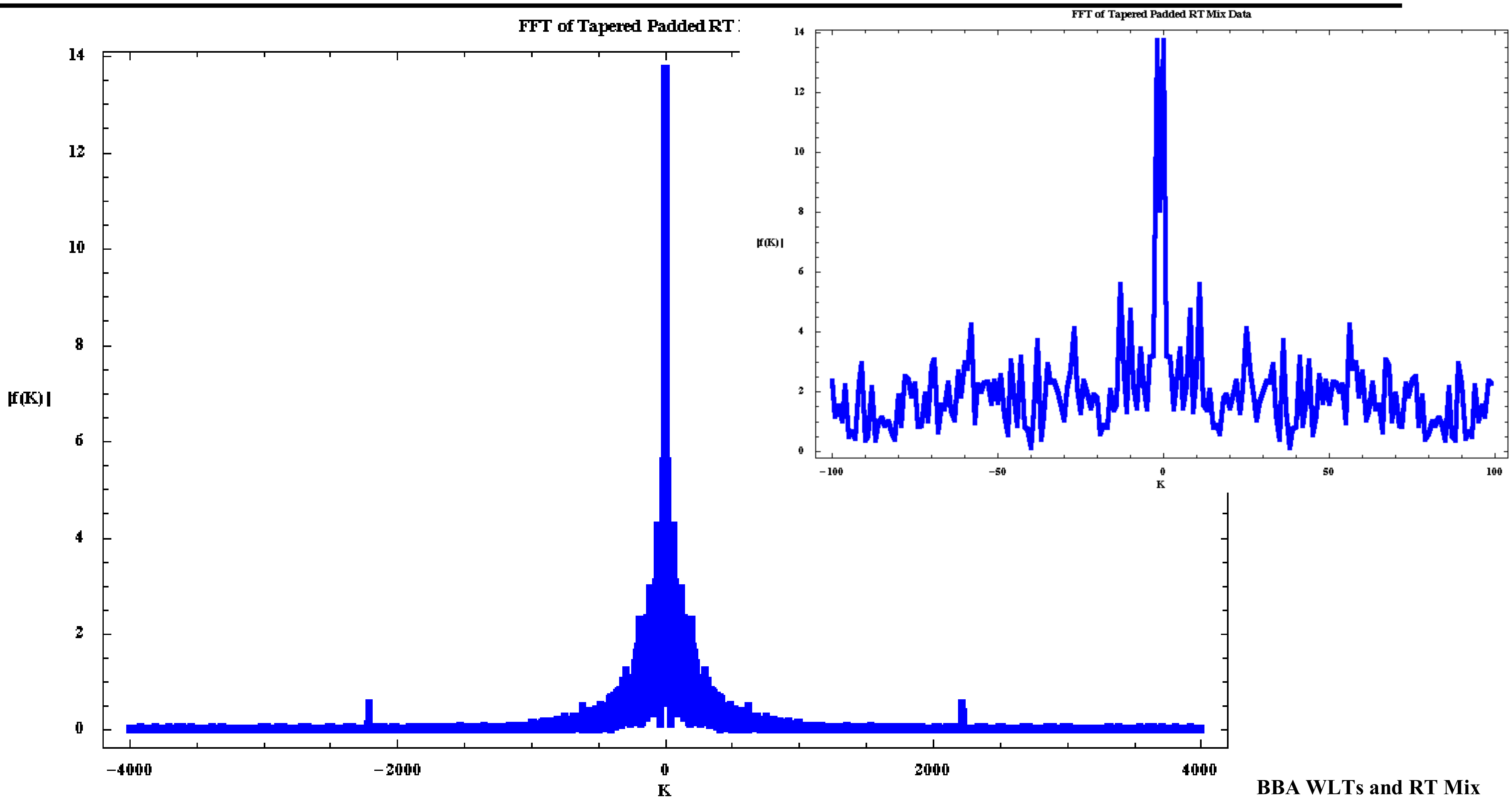
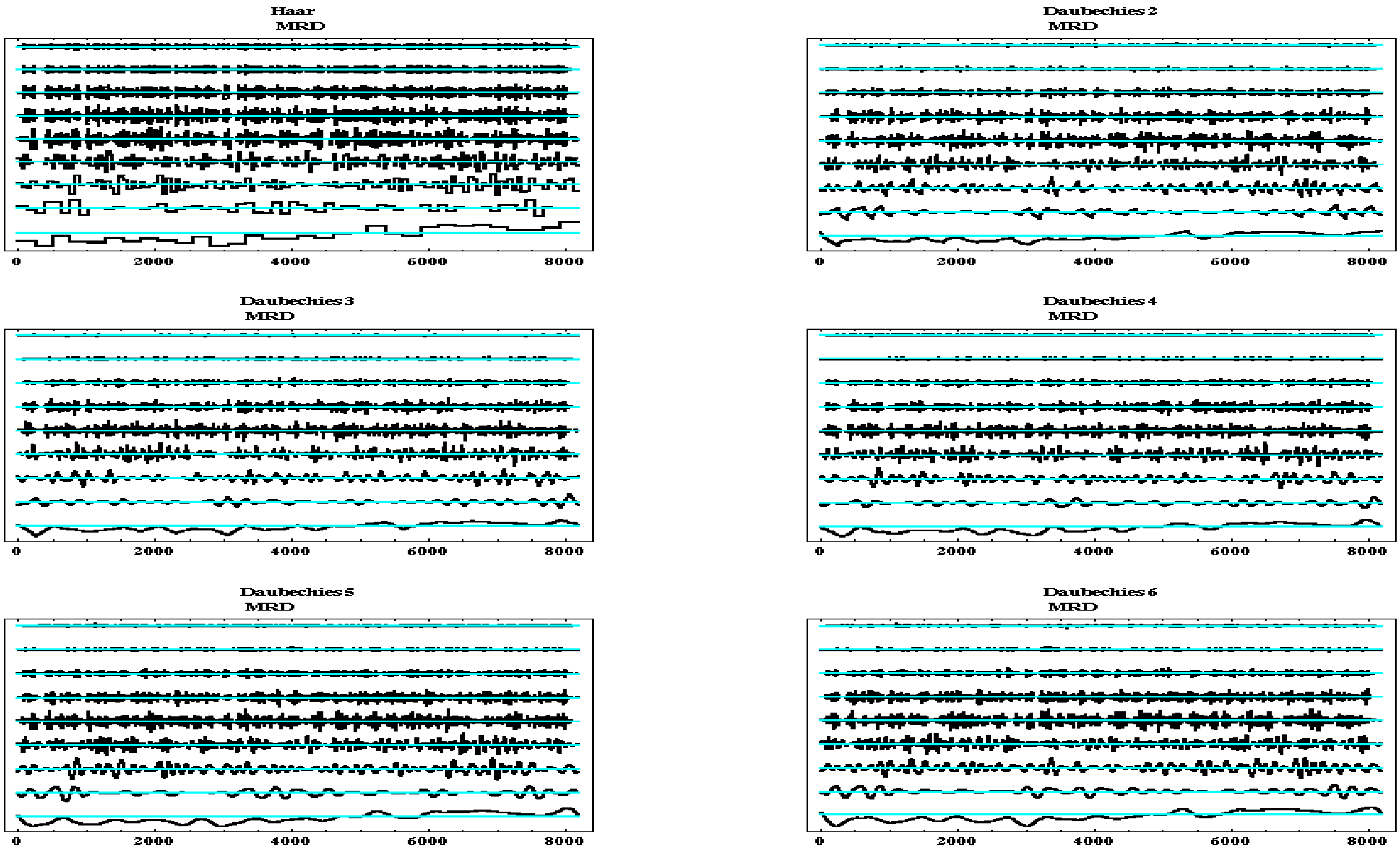
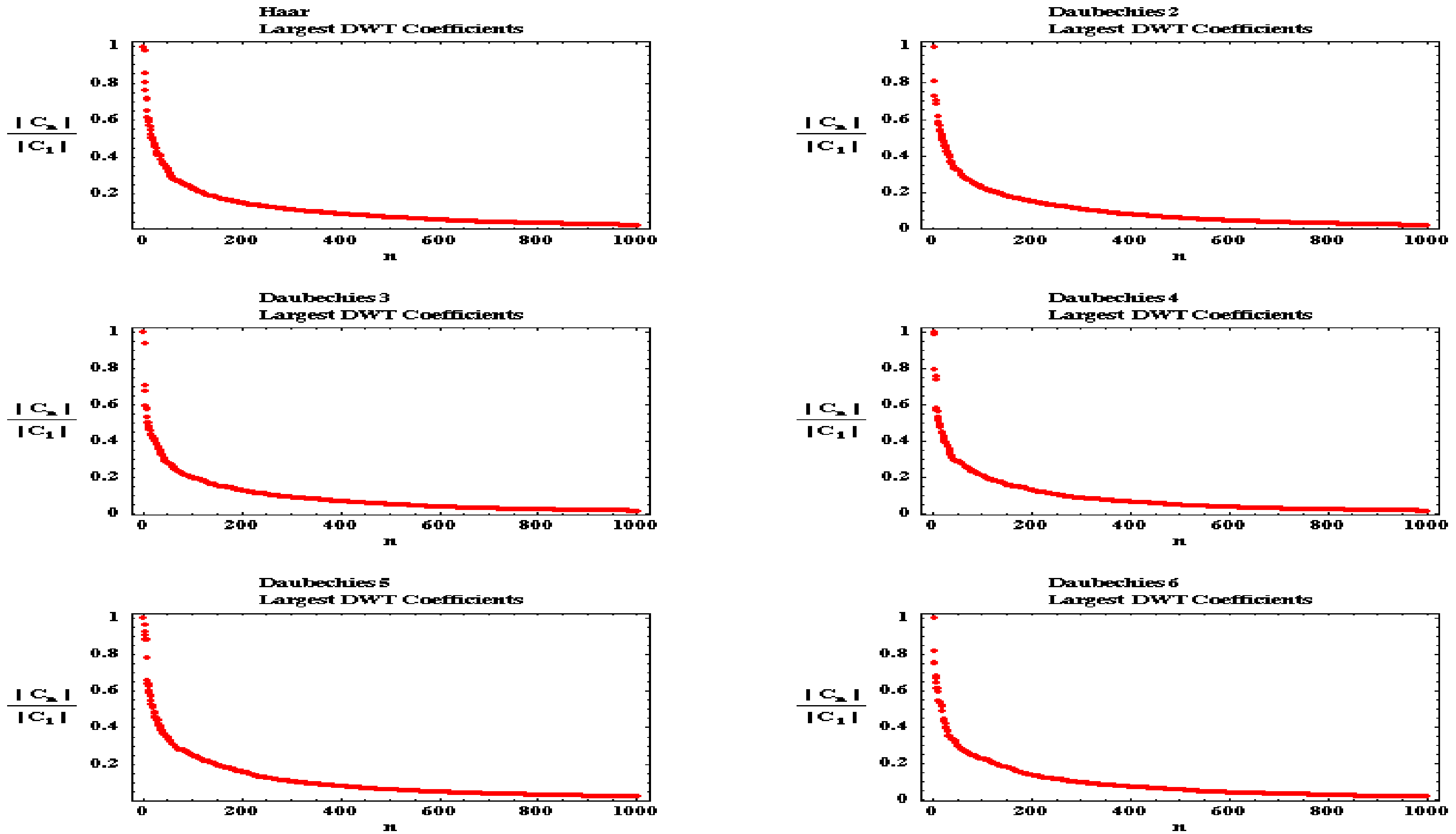


BBA WLTs and RT Mix
Cal Tech Pasadena CA
8th IW PCTM 12-11-01

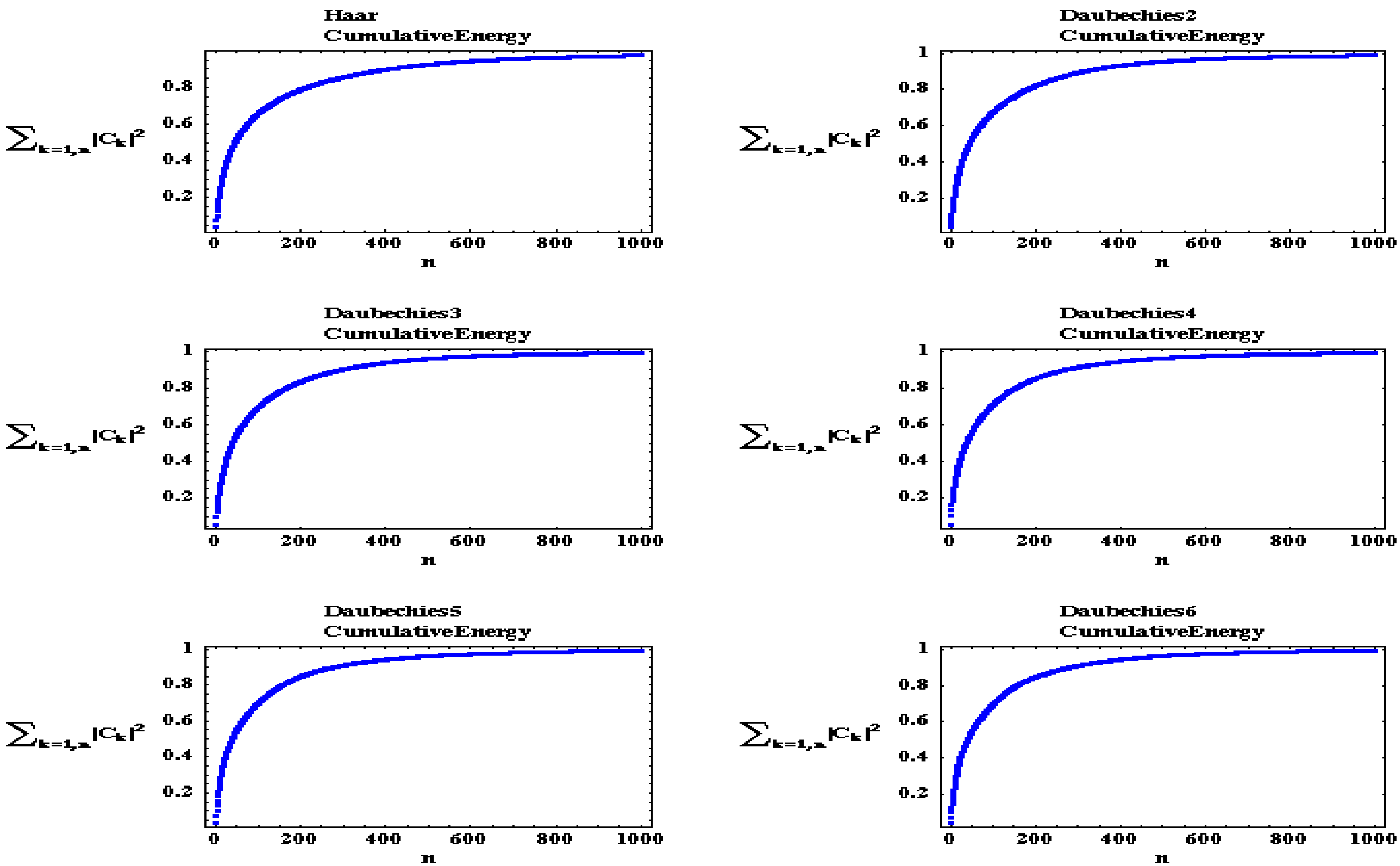
MRDs of the RT Mix Data in 6 Different Daubechies WLT Bases



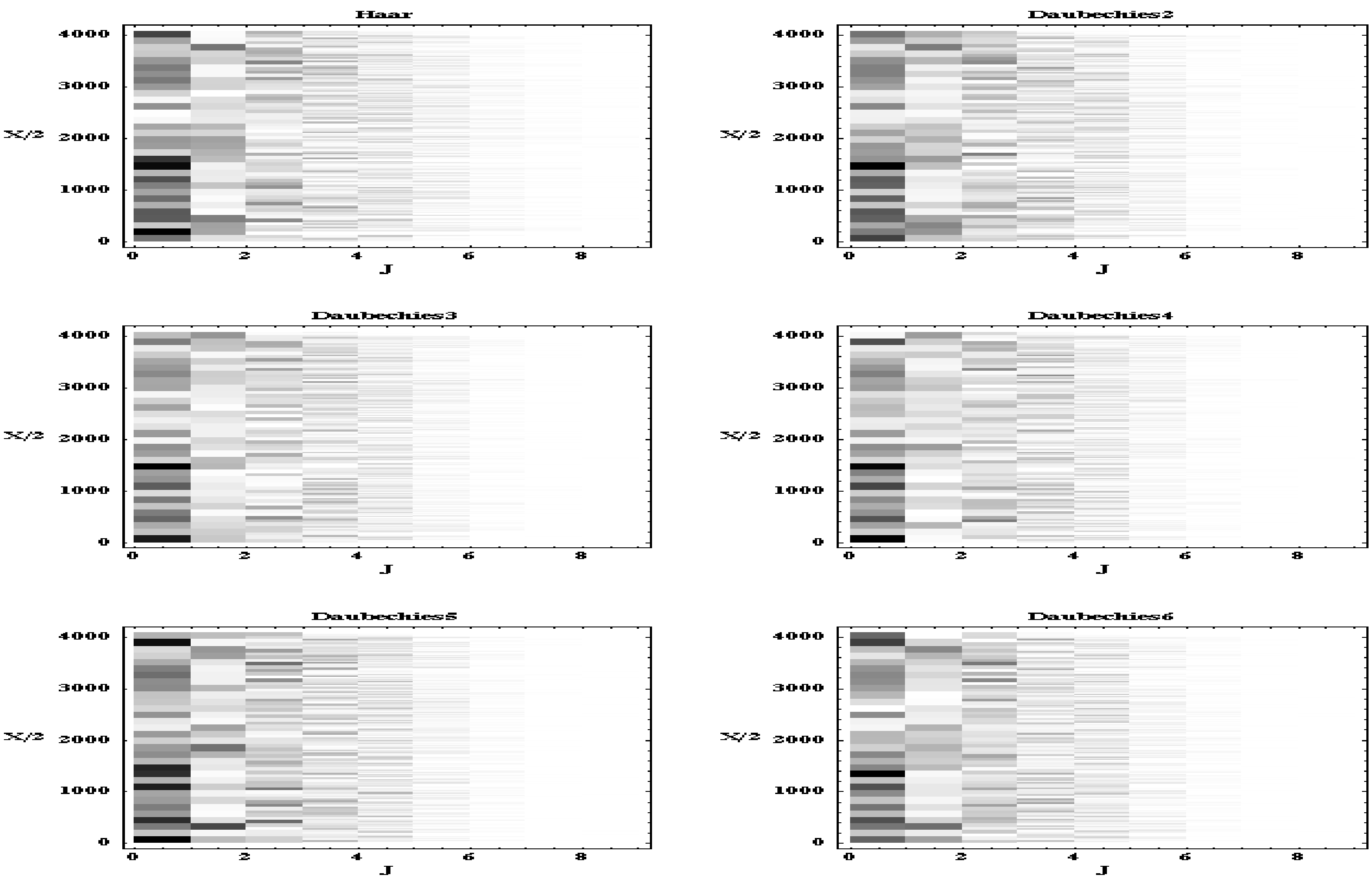
Decay Rate of Largest Coefficient vs Number of Coefficients Kept in 6 Different Daub WLT Decomps



Energy Accumulation Rate in Coefficient Space vs # of WLTs Kept in 6 Different Daub Decomps



Scaleograms: Waveleters Preferred Way of Judging Tiling in Scale-Translation Space



Least Square Error Incurred By Truncating the WLT Series at N of its Largest Coefficients

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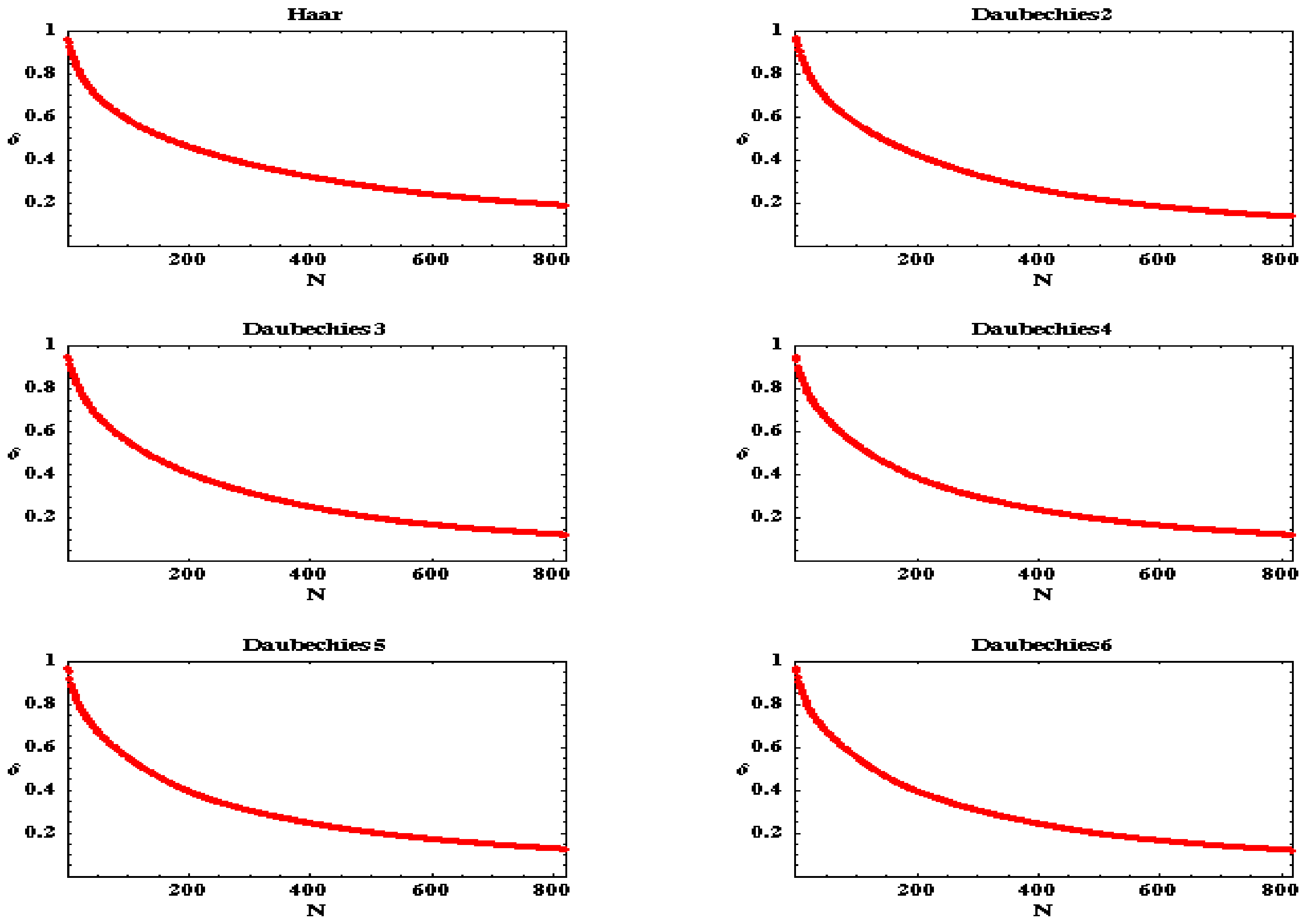

Least Square Error Incurred by Level Thresholding the DWT

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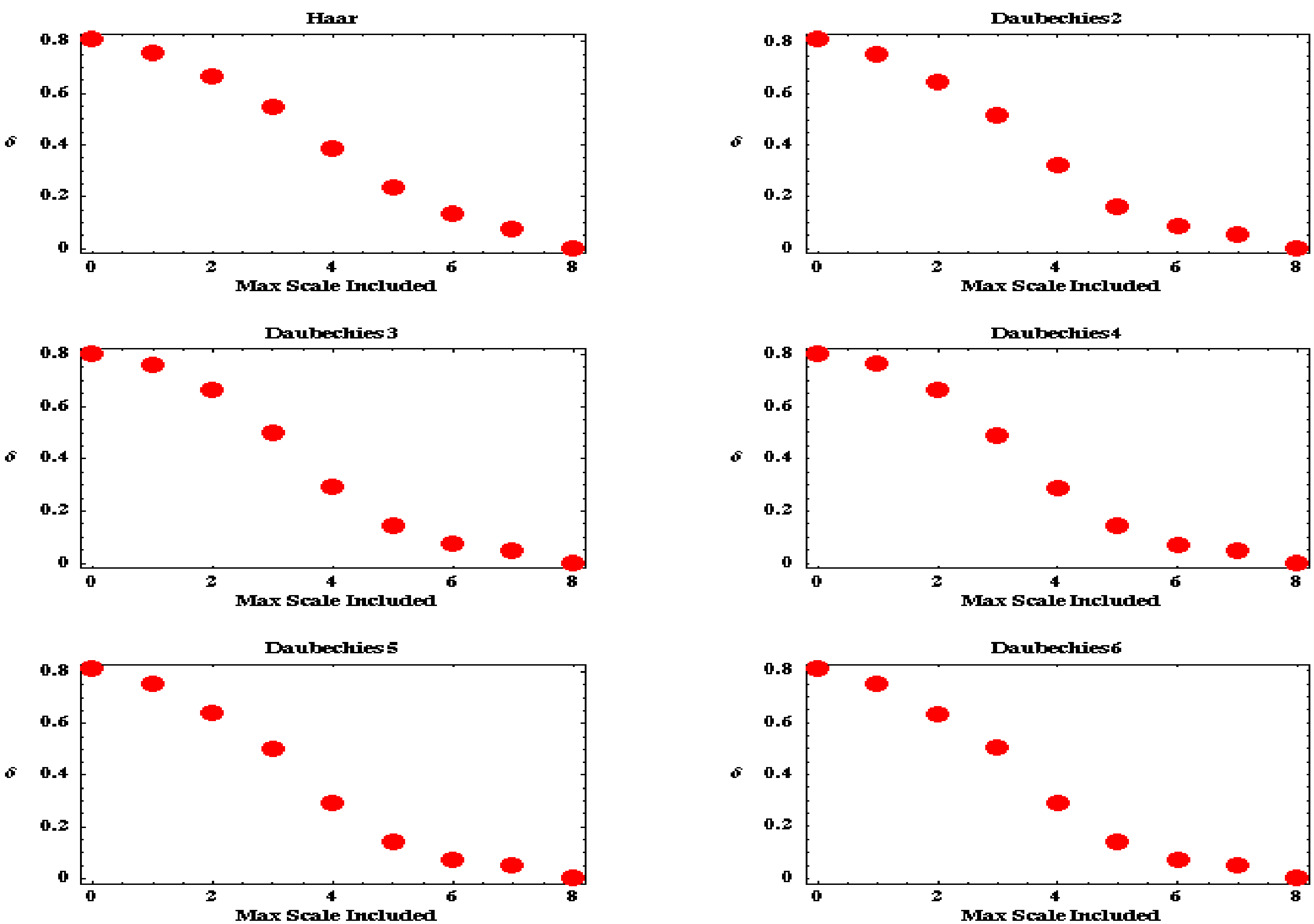
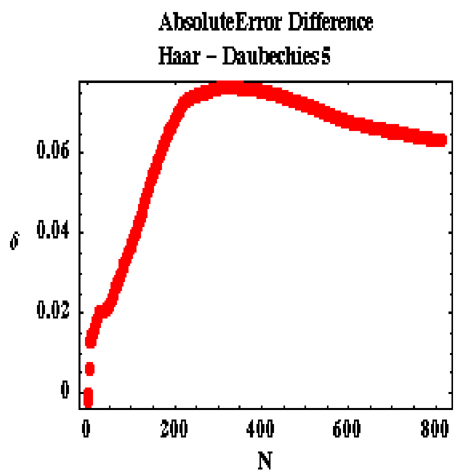
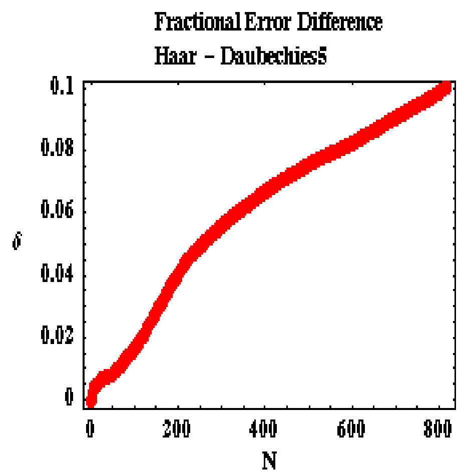
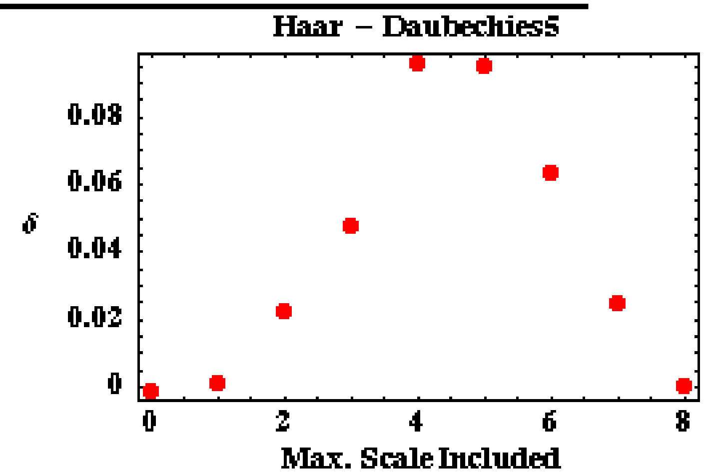
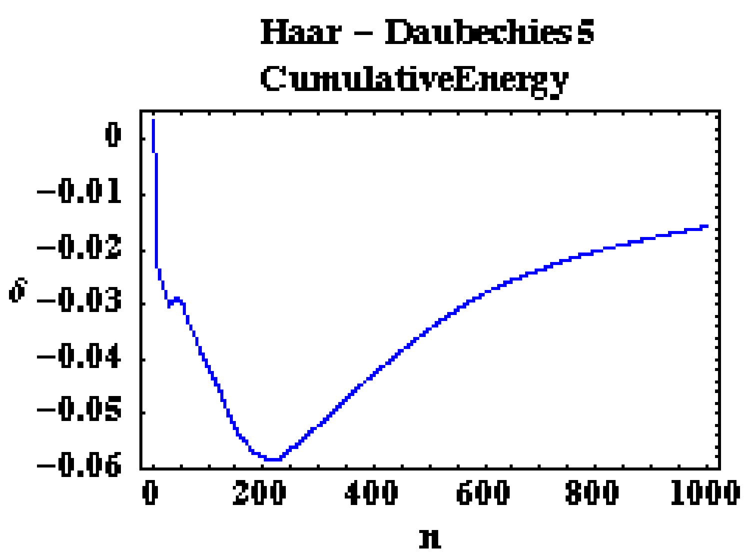


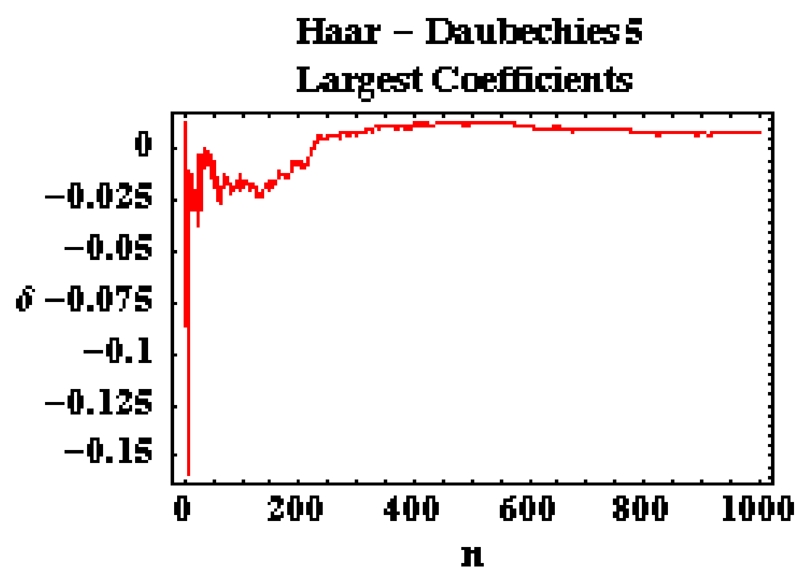
Daubechies 5 Does Much Better than Haar: 5 Quantitative Measures











Level by Level Decomposition of the RT Mix Data Using Daub5 WLTs

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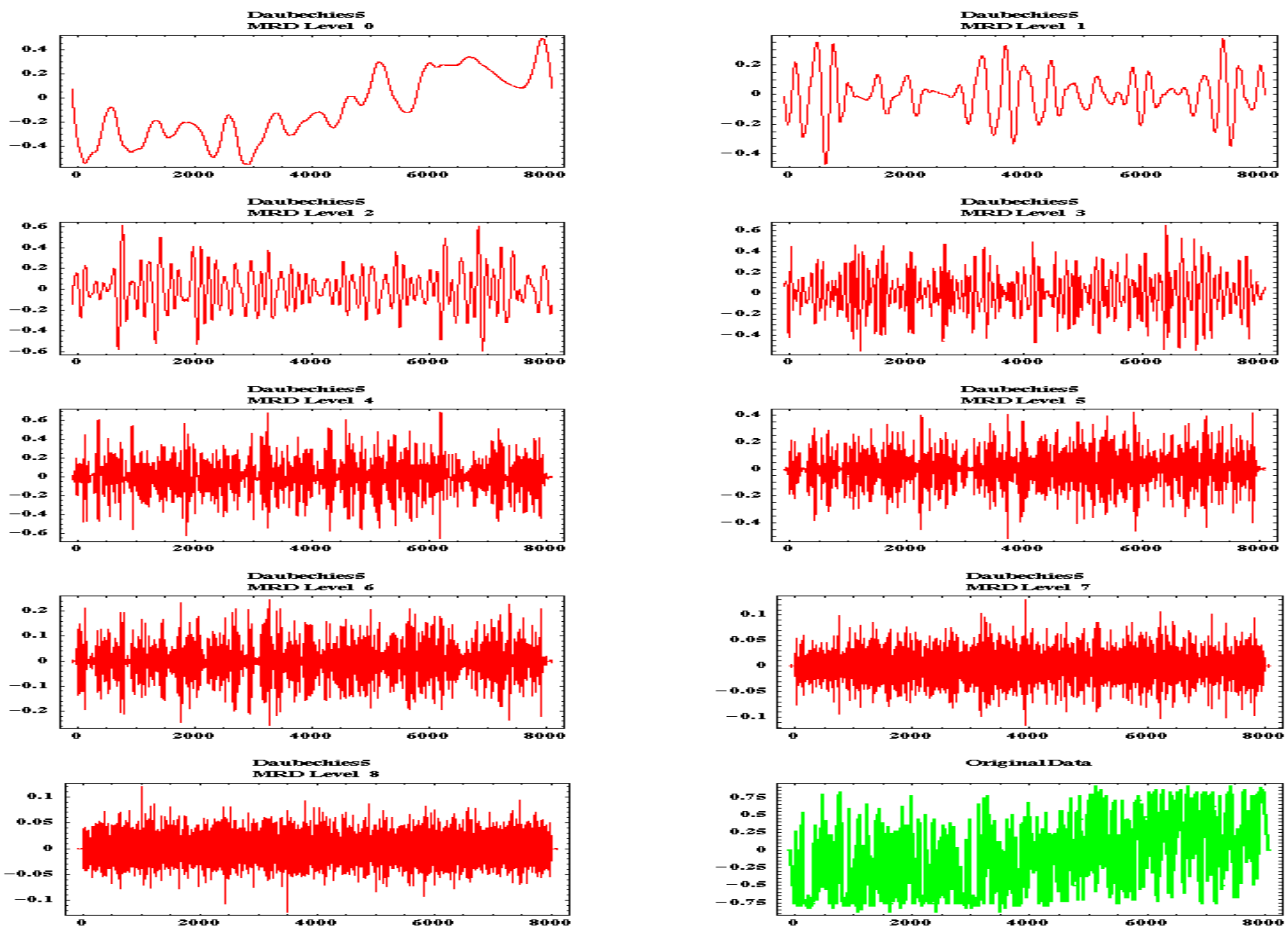
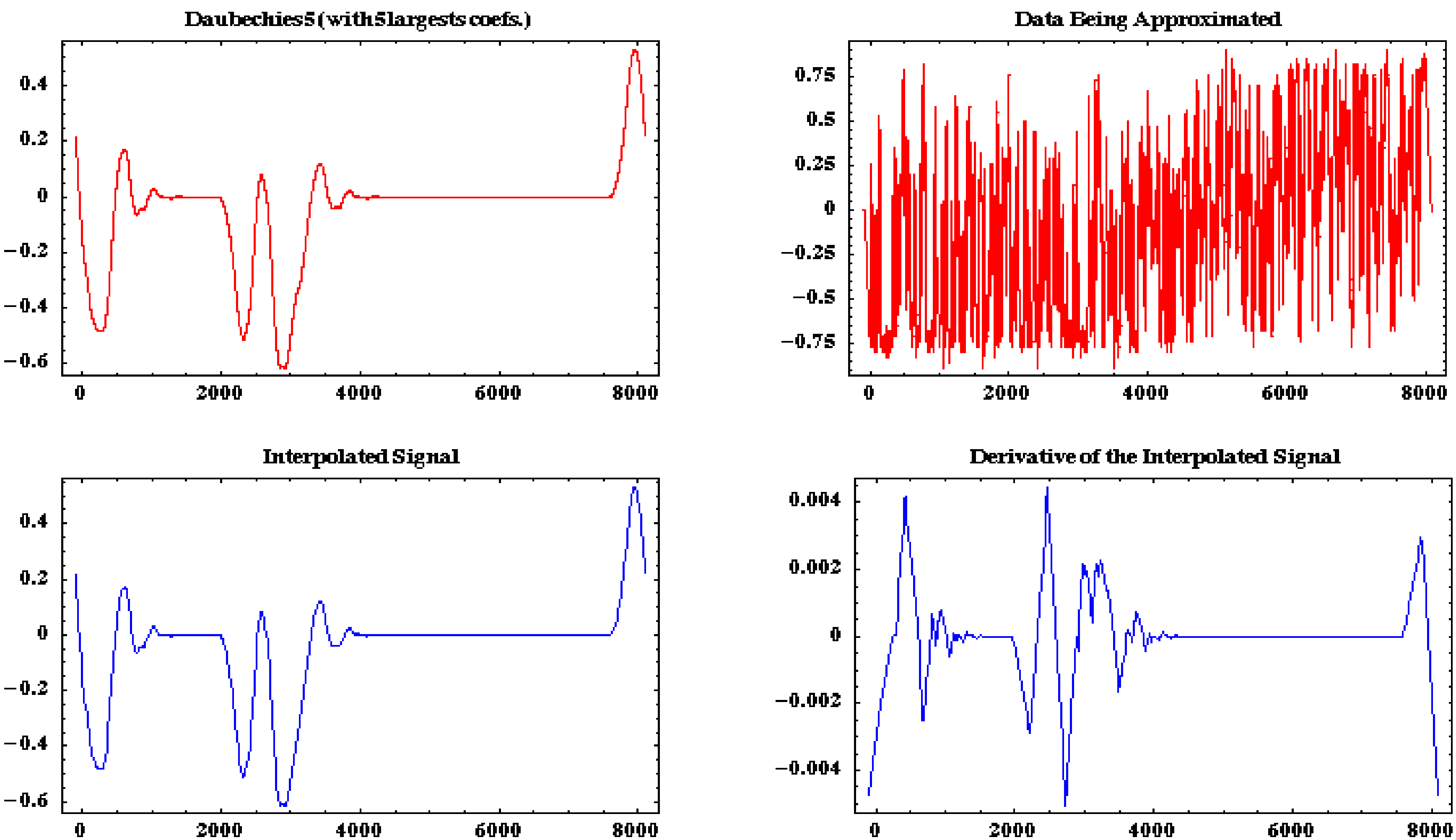


Reconstruction of the Data Using the 5 Largest WLT Coefficients



Reconstruction of the Data Using the 10 Largest WLT Coefficients

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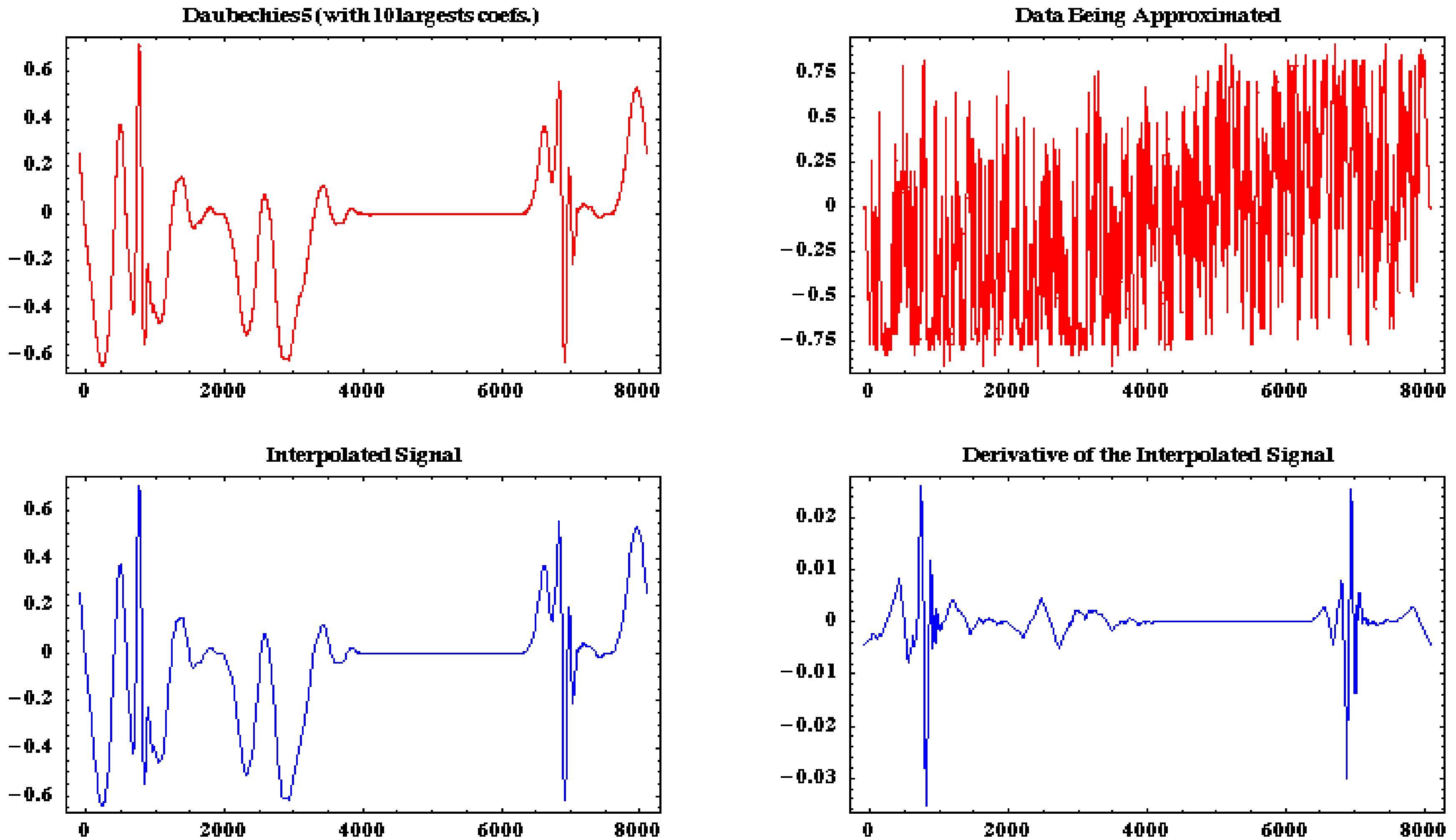
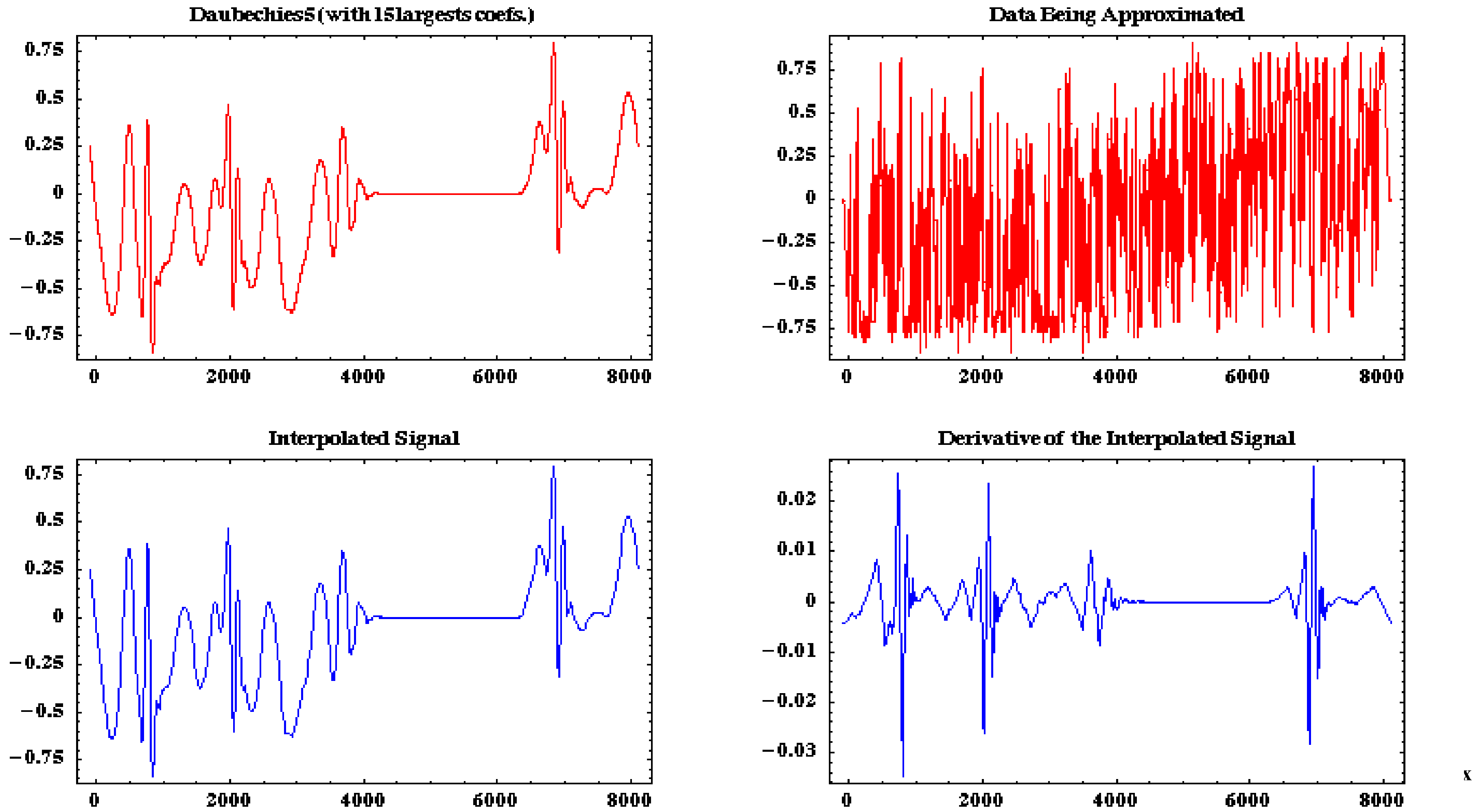
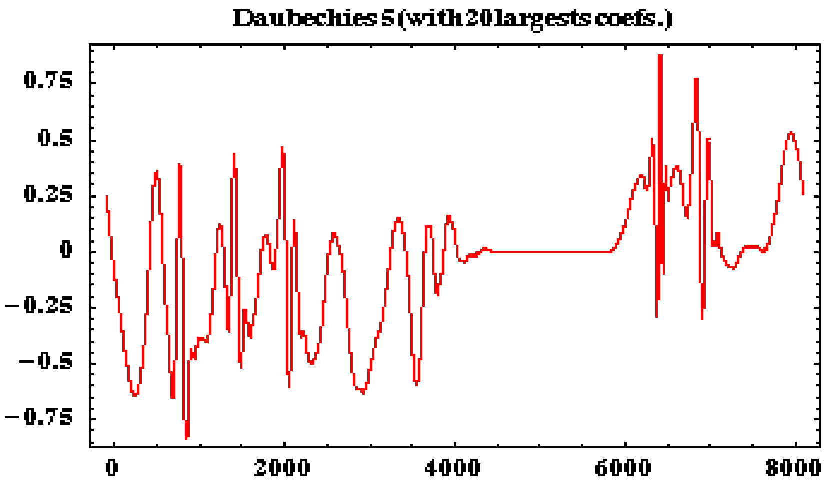
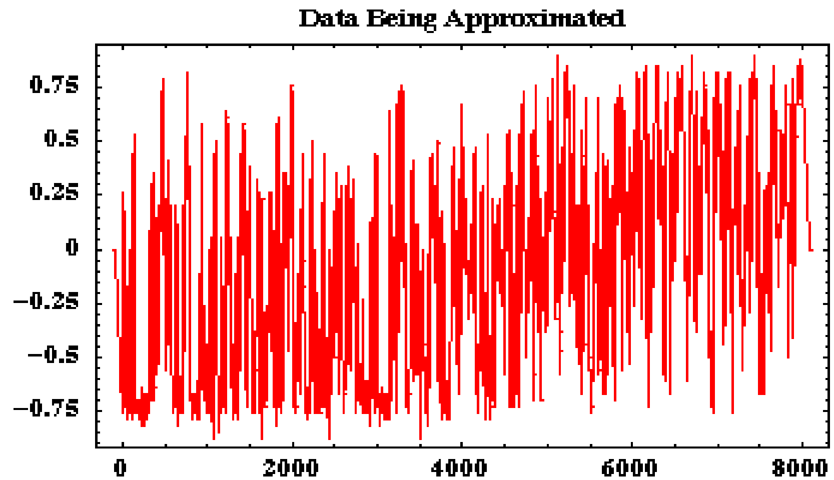


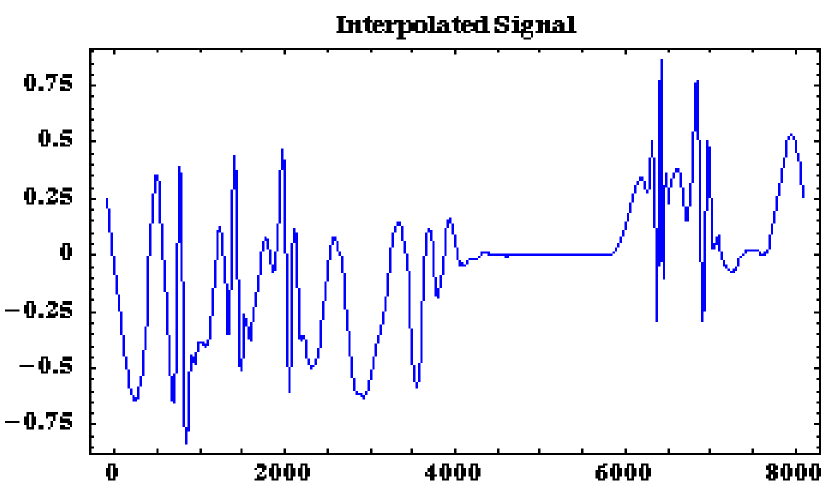
Reconstruction of the Data Using the 15 Largest WLT Coefficients

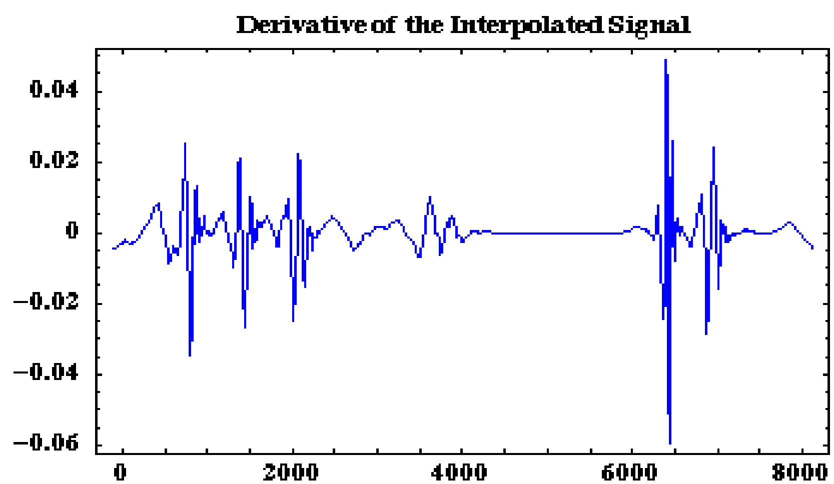


Reconstruction of the Data Using the 20 Largest WLT Coefficients









Reconstruction of the Data Using the 30 Largest WLT Coefficients

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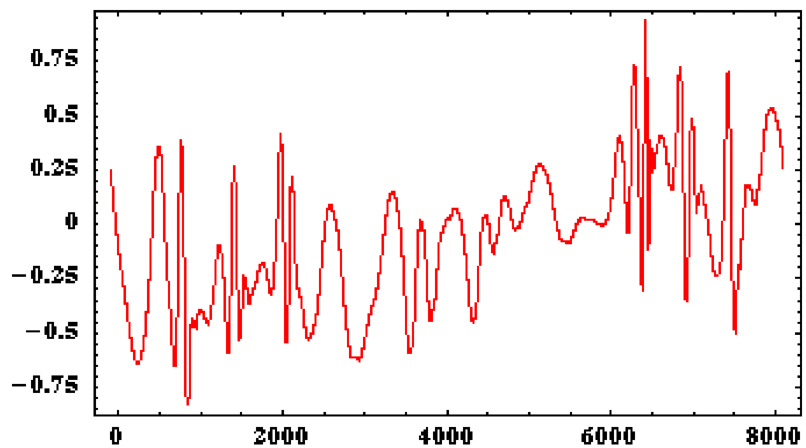
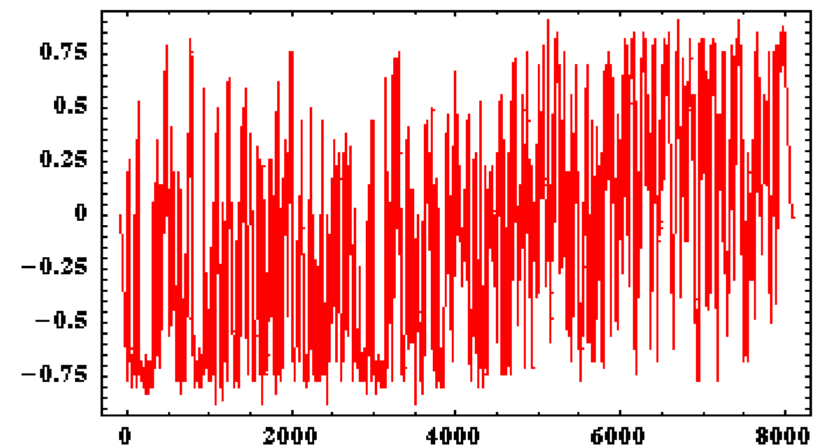
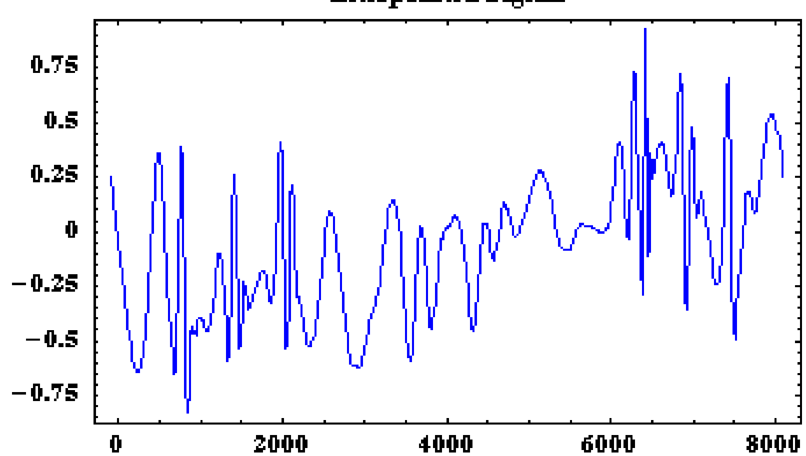
Daubechies5 (with 30 largests coefs.)



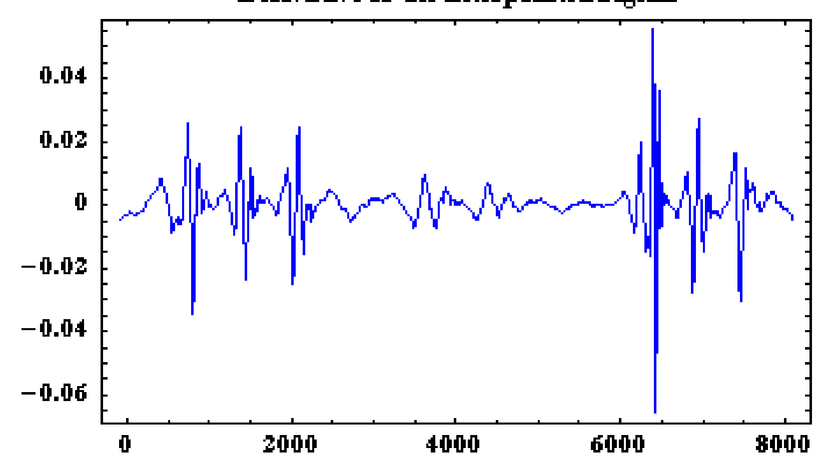
Data Being Approximated



Interpolated Signal



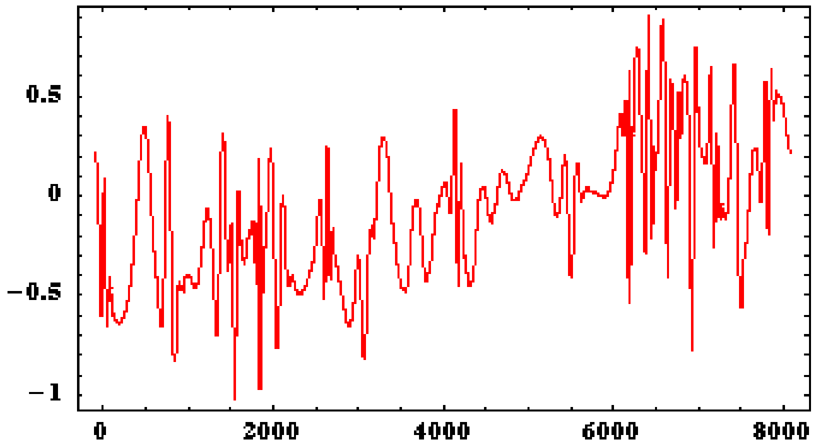
Derivative of the Interpolated Signal



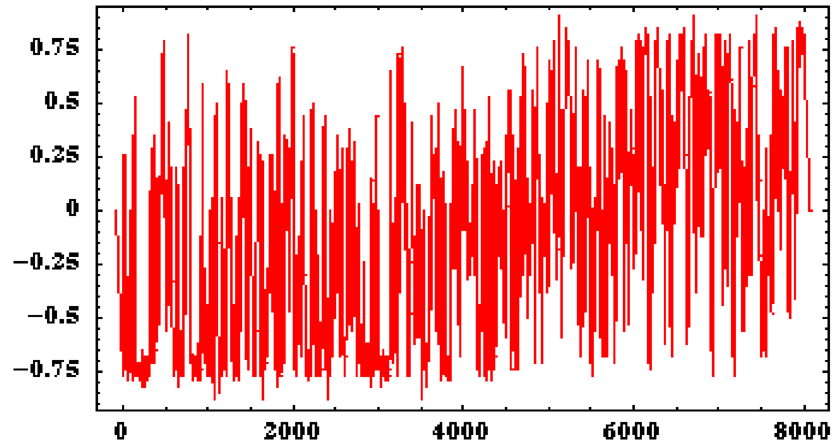
x

Reconstruction of the Data Using the 50 Largest WLT Coefficients

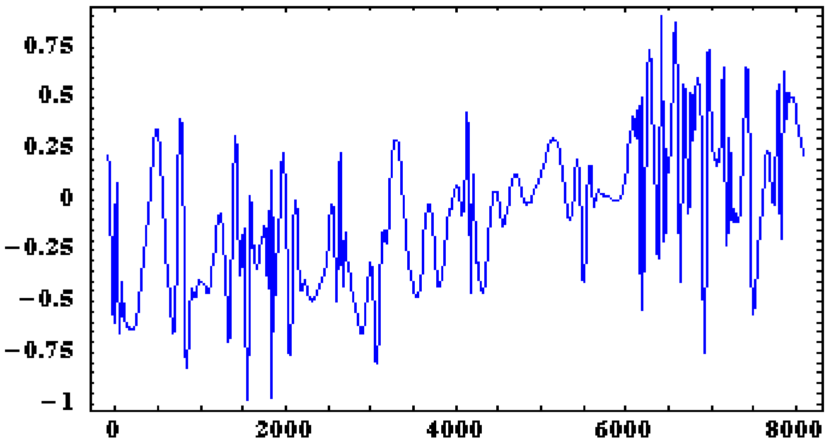
Daubechies5 (with 50 largests coefs.)



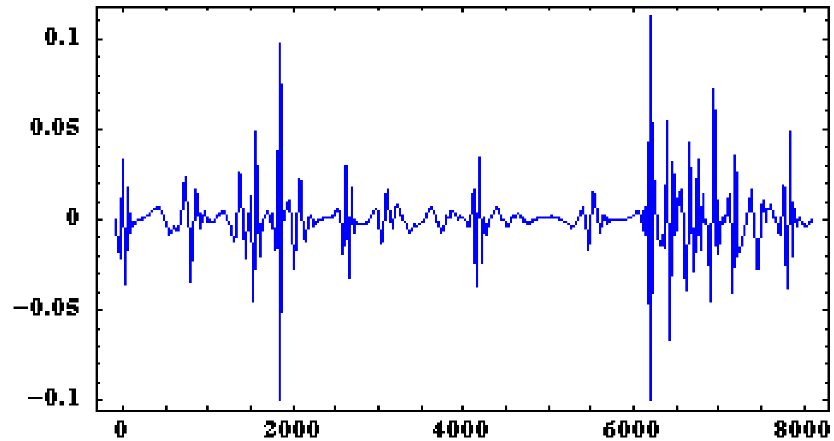
Data Being Approximated



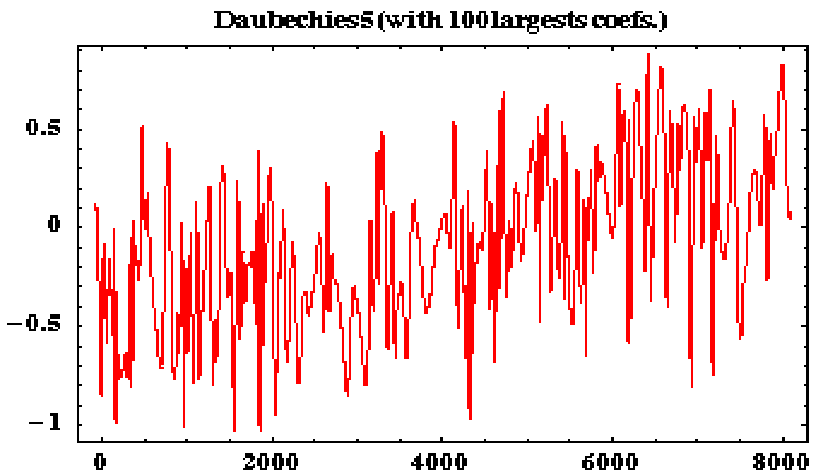
Interpolated Signal

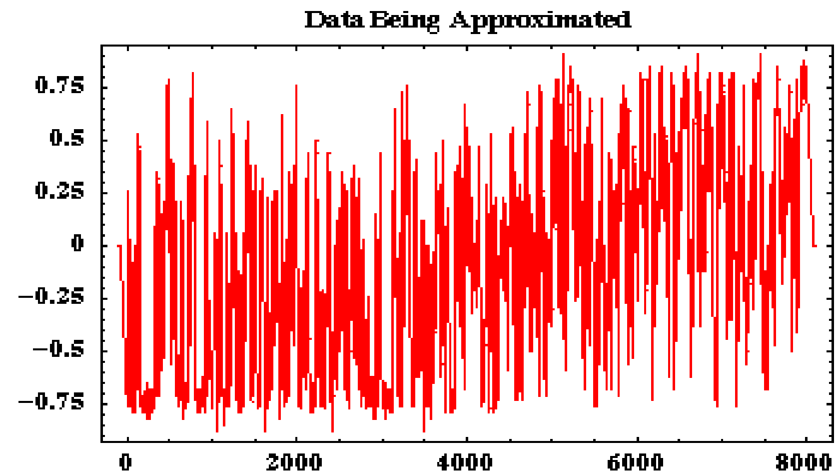


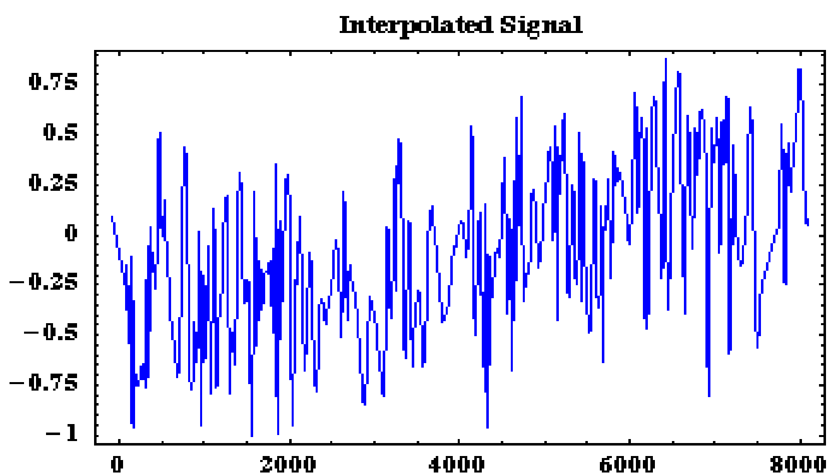
Derivative of the Interpolated Signal

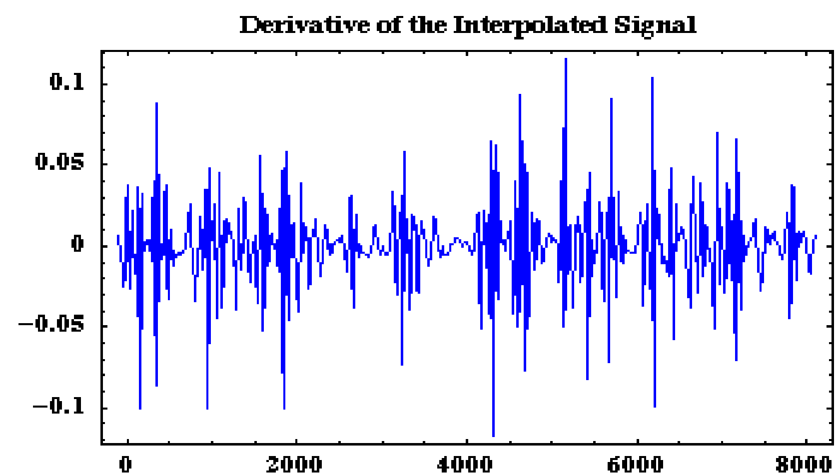


Reconstruction of the Data Using 100 Largest WLT Coefficients









Reconstruction of the Data Using 200 Largest WLT Coefficients

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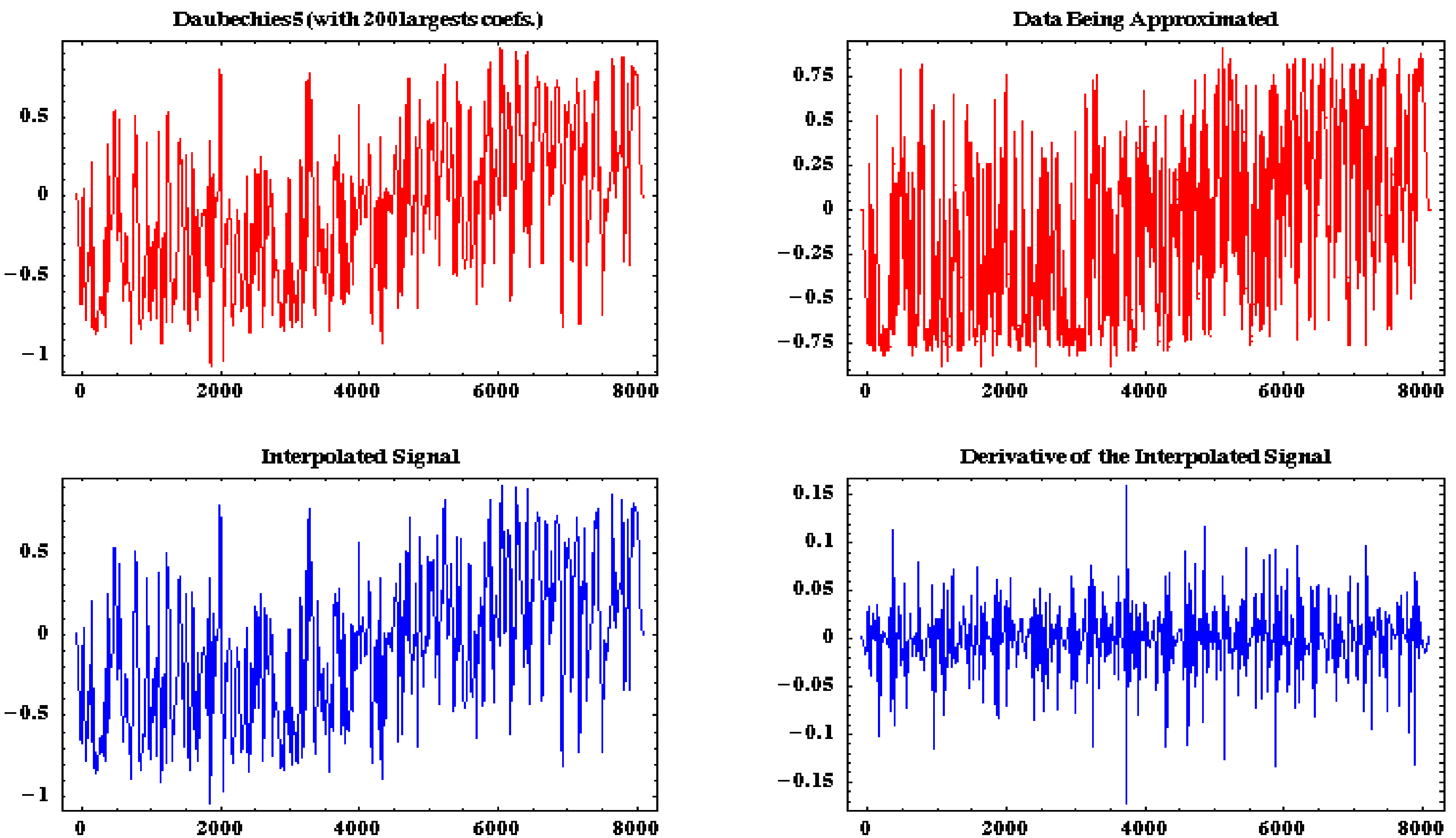

Reconstruction of the Data Using 400 Largest WLT Coefficients

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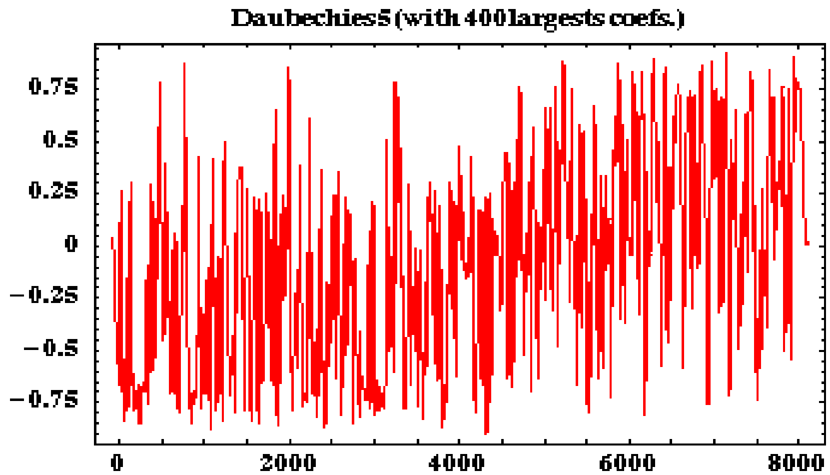

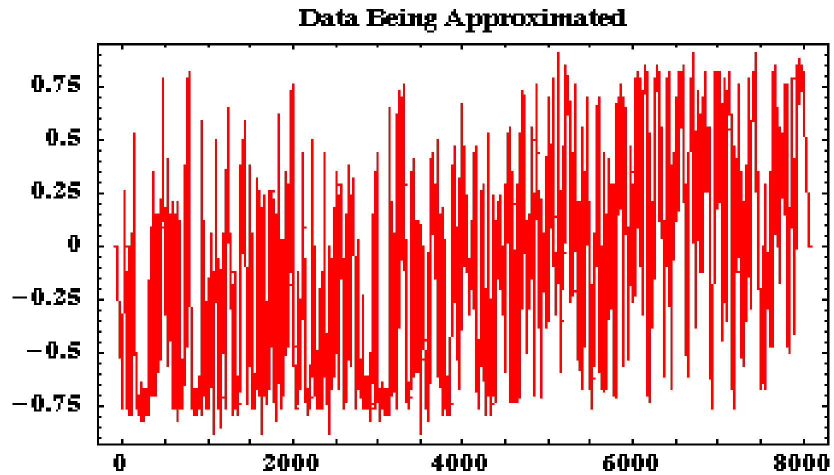

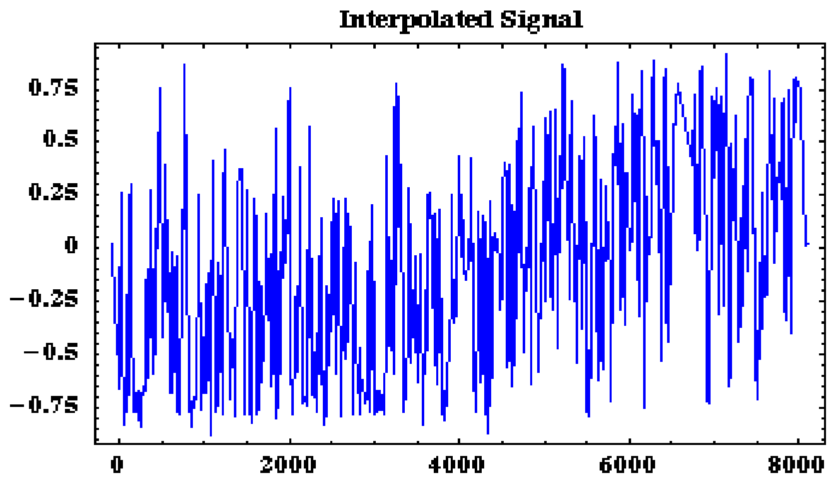

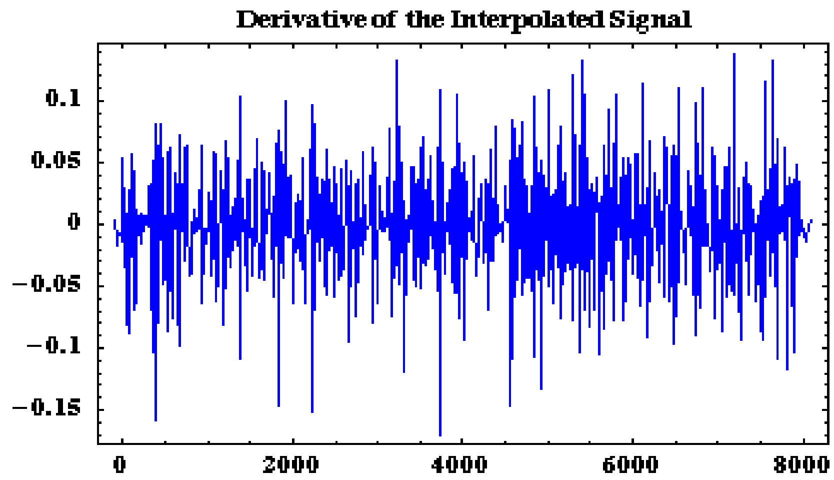

Reconstruction of the Data Using Up to 0.75 times the Largest WLT Coefficient

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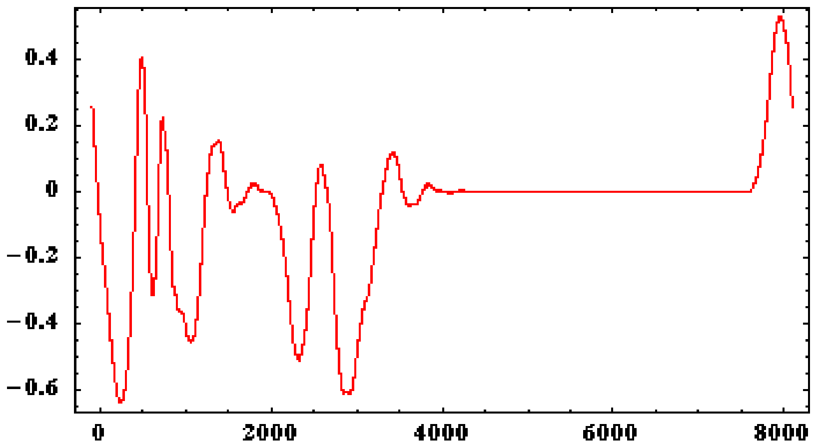
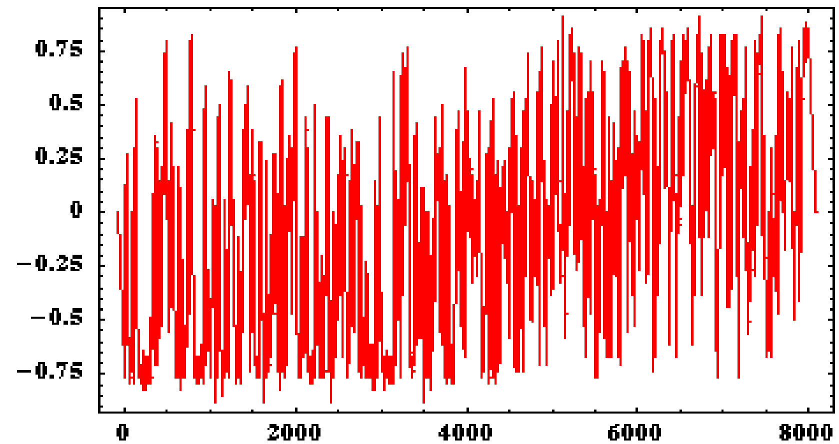
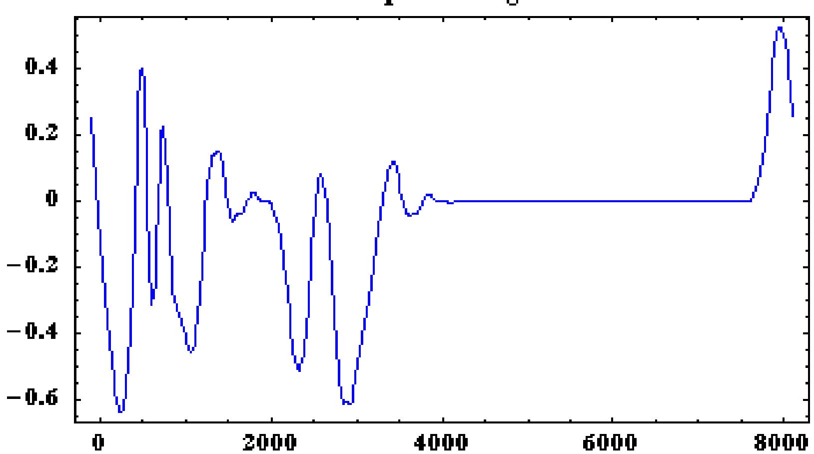
Daubechies5 (Threshold = 0.75∗ Largest Coeff.)



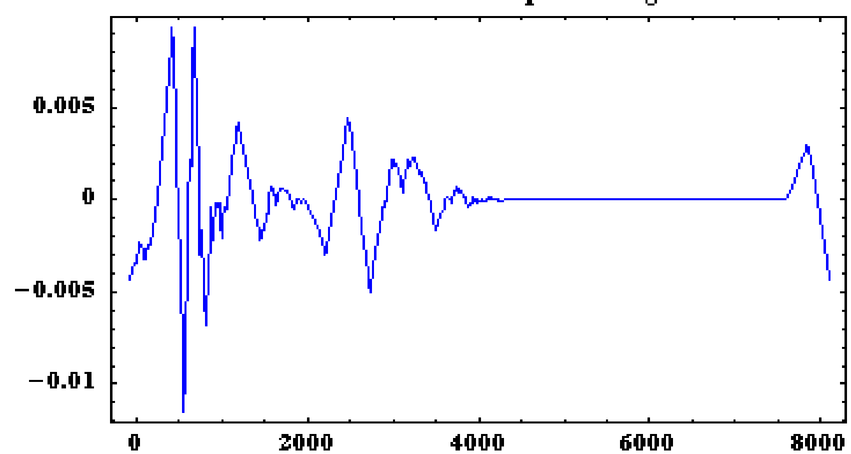
Data Being Approximated



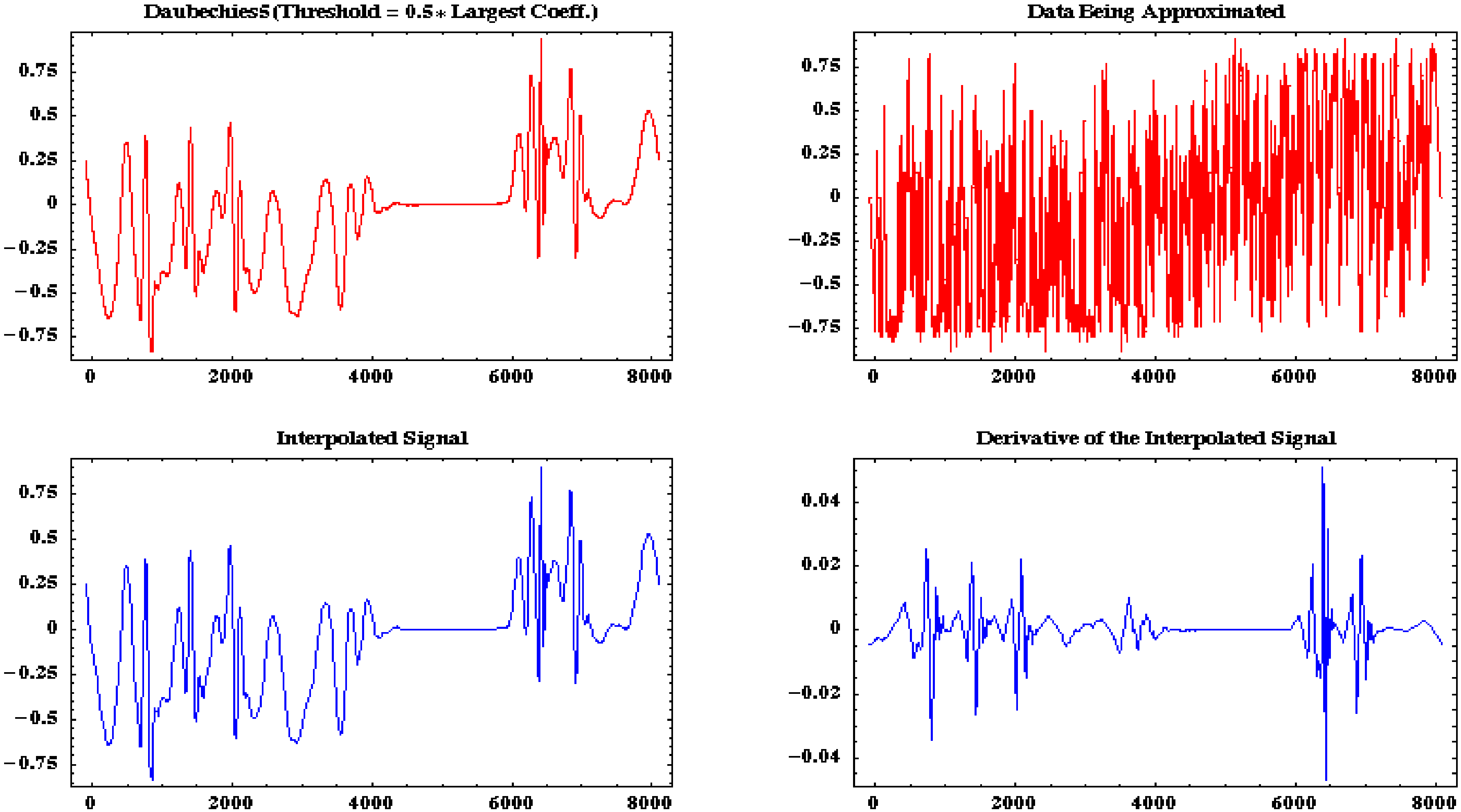
Interpolated Signal



Derivative of the Interpolated Signal

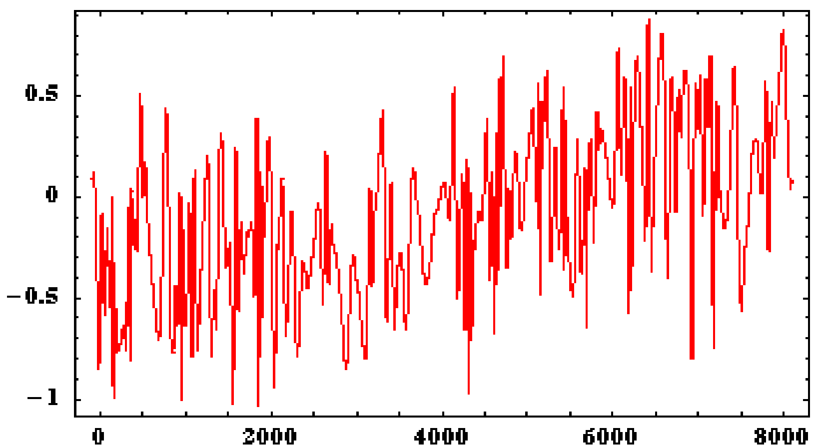


Reconstruction of the Data Using Up to 0.5 times the Largest WLT Coefficient

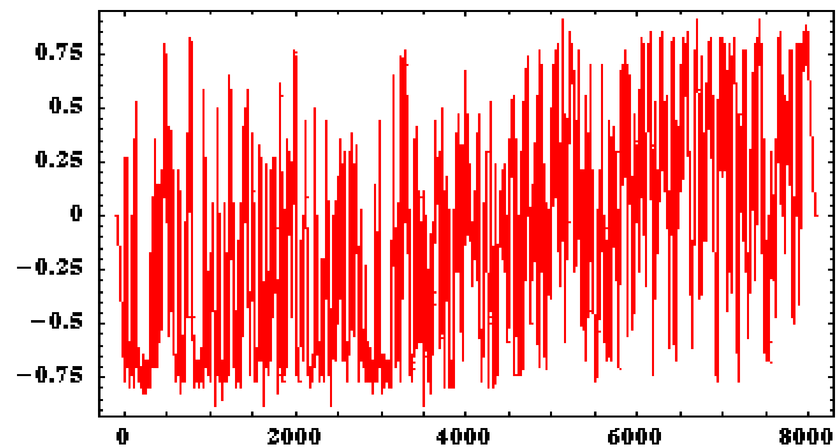


Reconstruction of the Data Using Up to 0.25 times the Largest WLT Coefficient

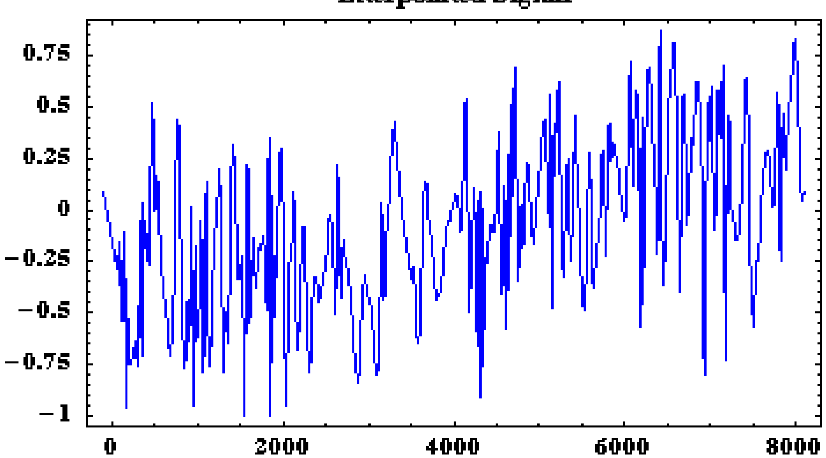
Daubechies5 (Threshold = 0.25 * Largest Coeff.)



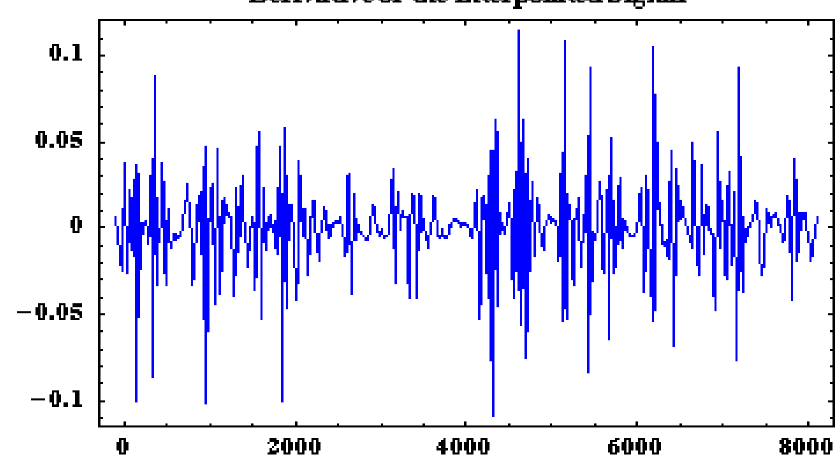
Data Being Approximated



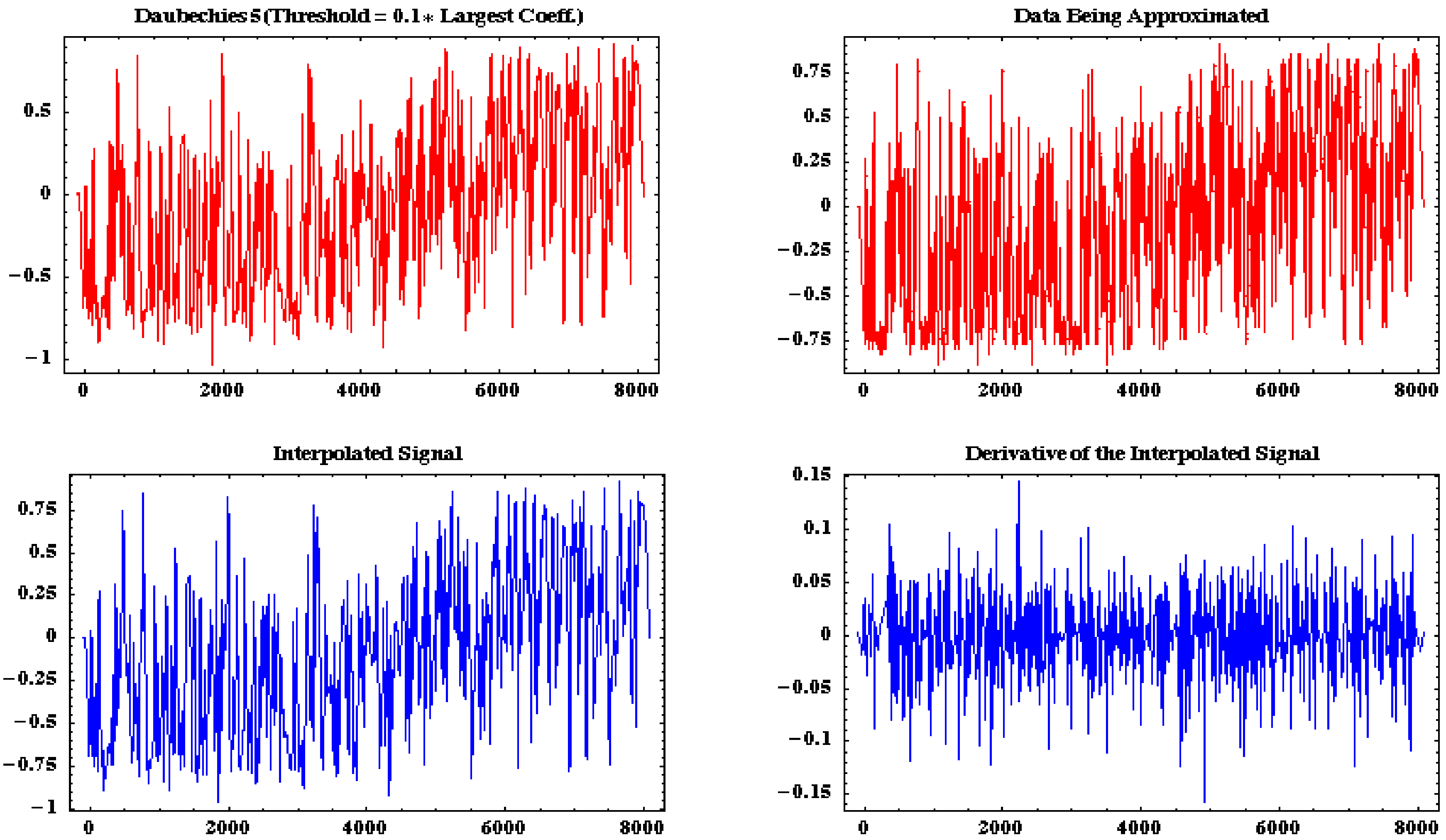
Interpolated Signal



Derivative of the Interpolated Signal



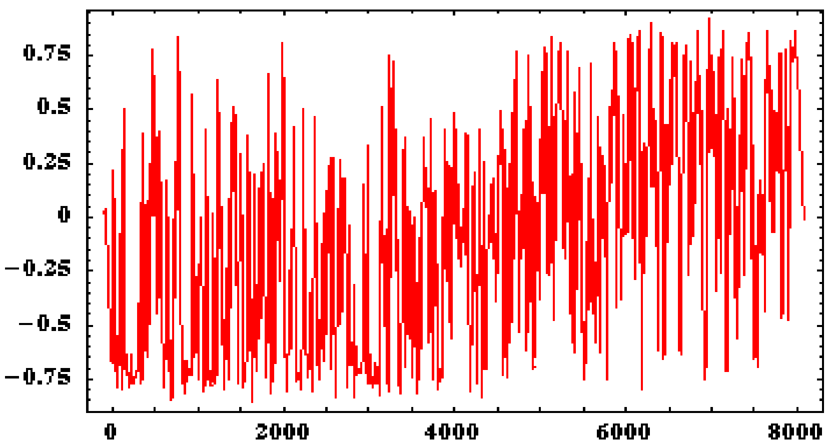
Reconstruction of the Data Using Up to 0.1 times the Largest WLT Coefficient



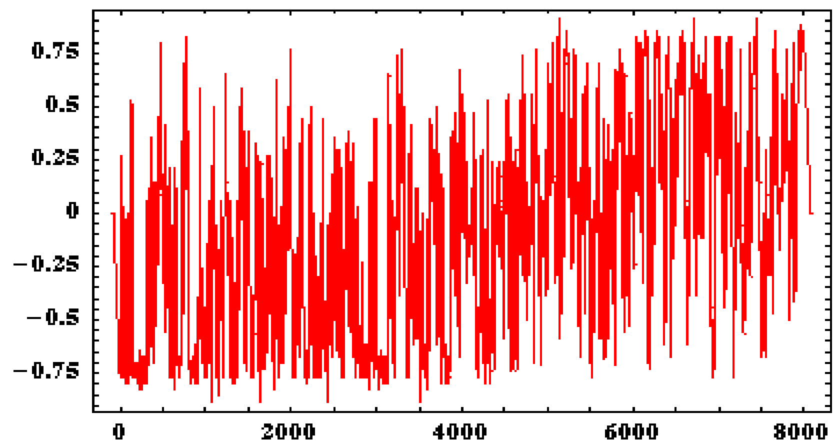
Reconstruction of the Data Using Up to 0.05 times the Largest WLT Coefficient

Polymath Research Inc.

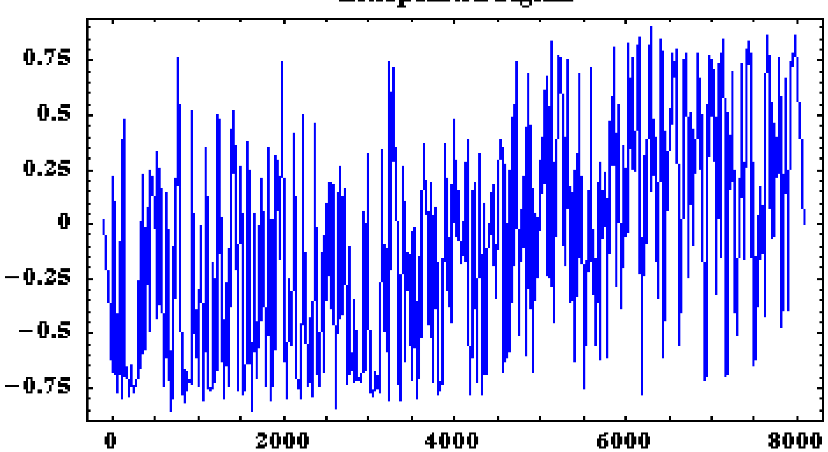
Daubechies 5 (Threshold = 0.05 * Largest Coeff.)



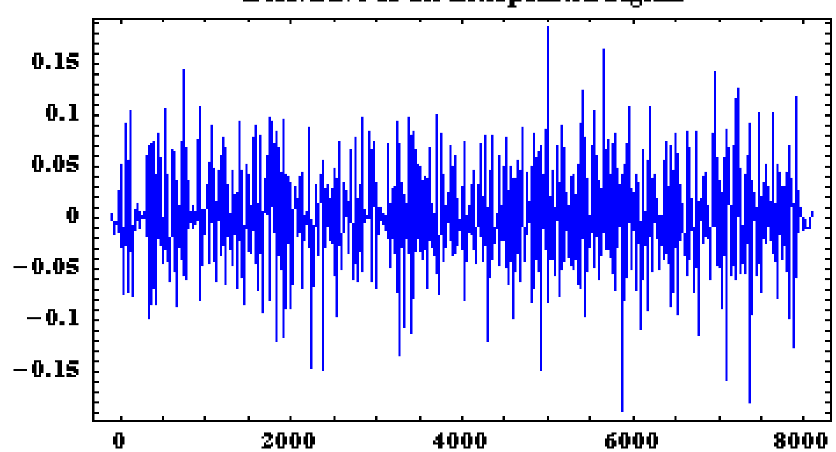
Data Being Approximated



Interpolated Signal

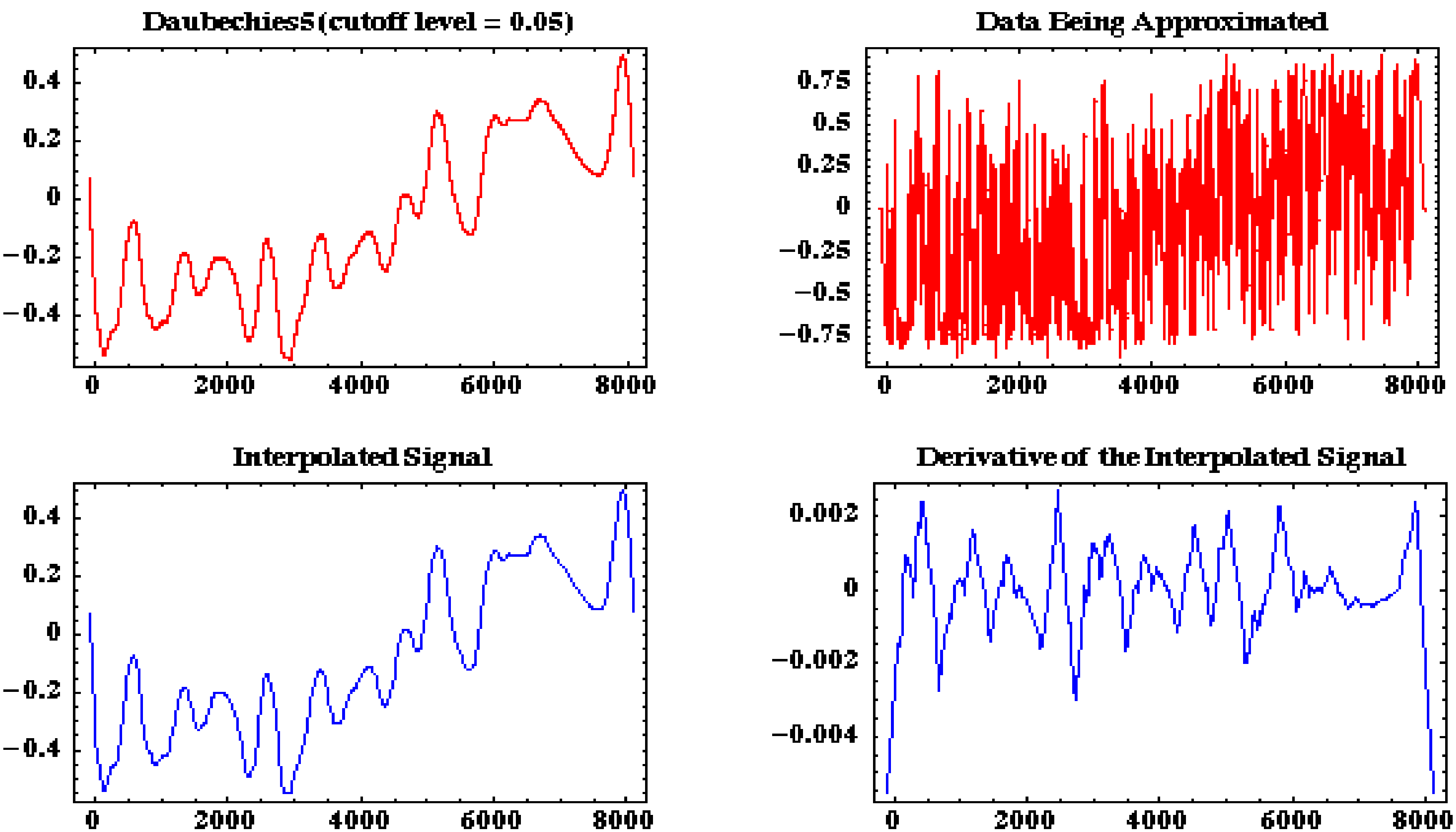


Derivative of the Interpolated Signal



Reconstruction of the Data Using the First (of 10) Level of the MRD

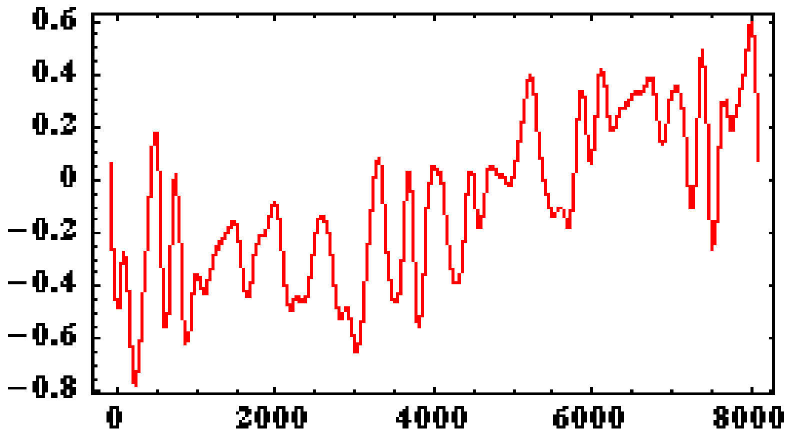
Polymath Research Inc.



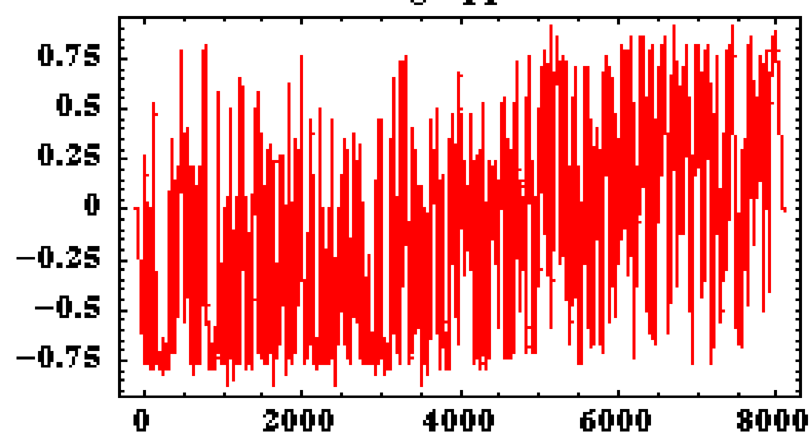
Reconstruction of the Data Using the First Two (of 10) Levels of the MRD

Polymath Research Inc.

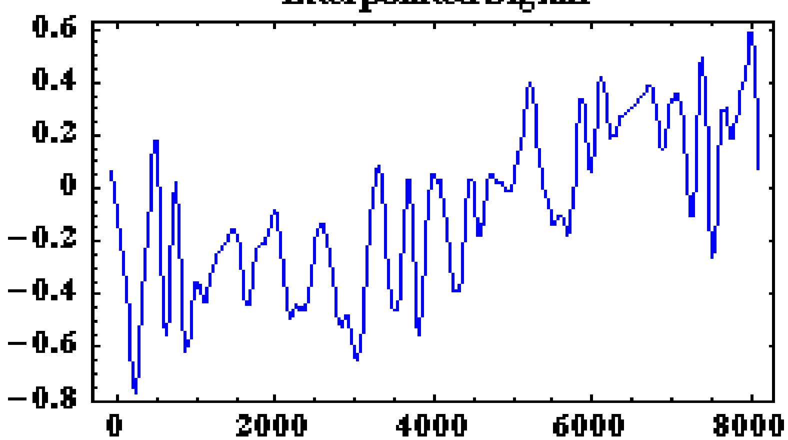
Daubechies 5 (cutoff level = 0.5)



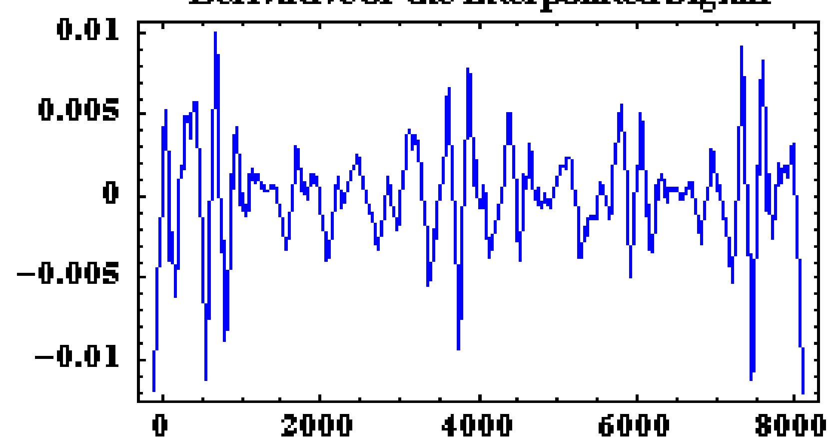
Data Being Approximated



Interpolated Signal

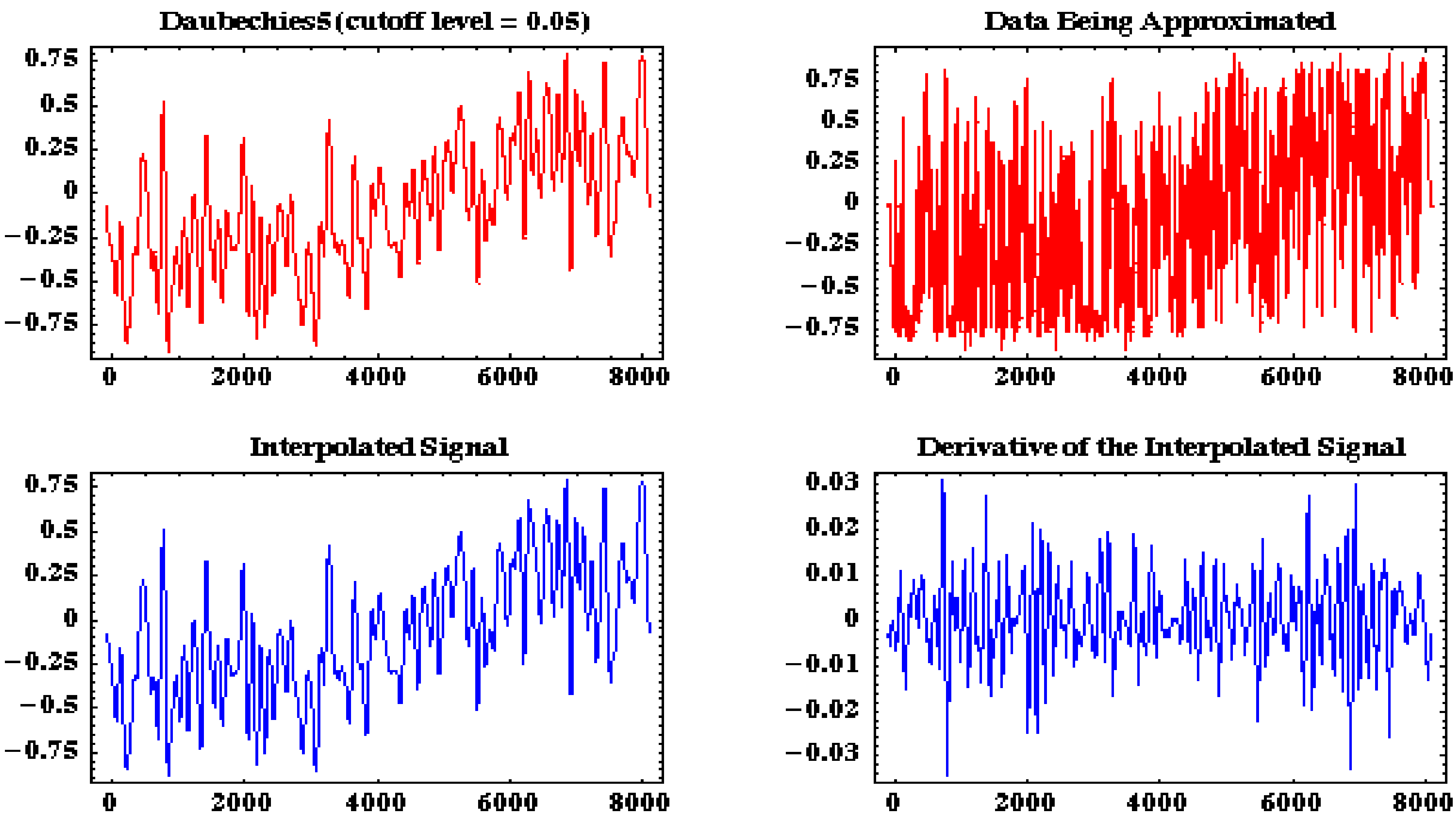


Derivative of the Interpolated Signal

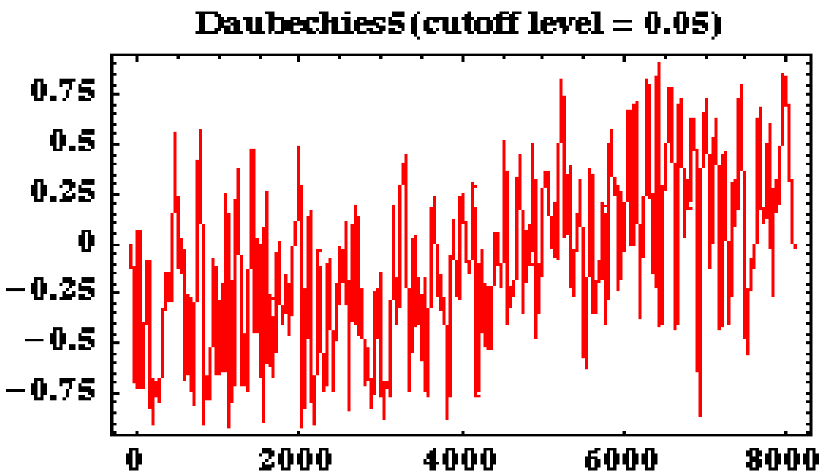


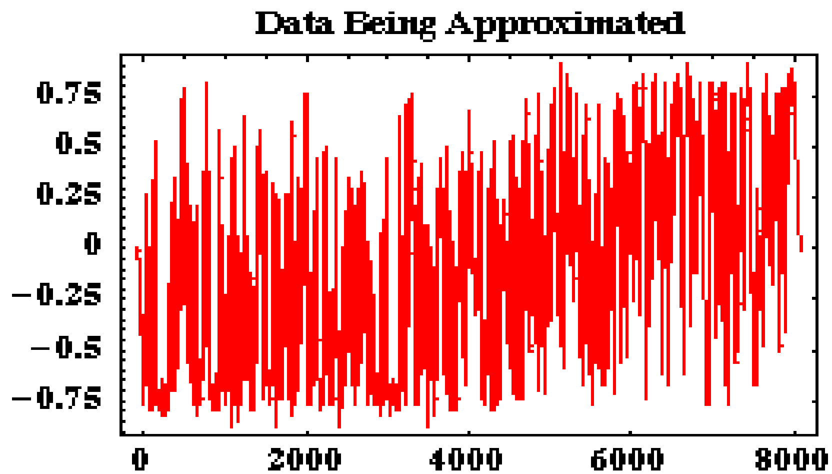
Reconstruction of the Data Using the First Three (of 10) Levels of the MRD

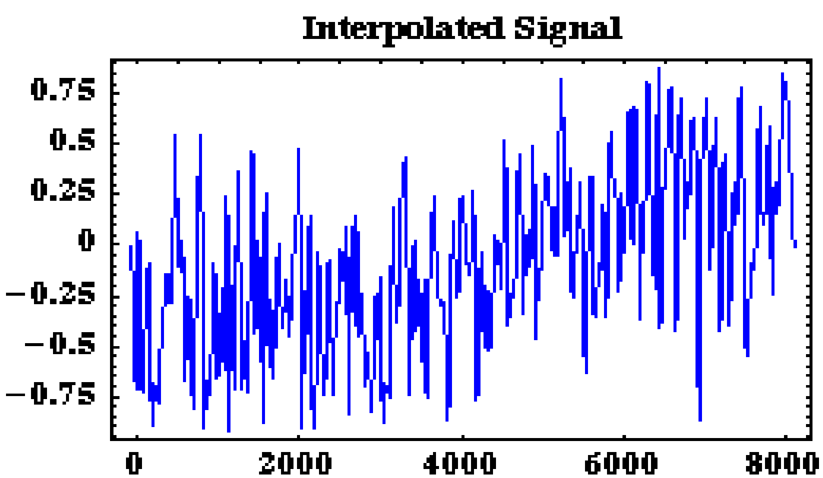
Polymath Research Inc.

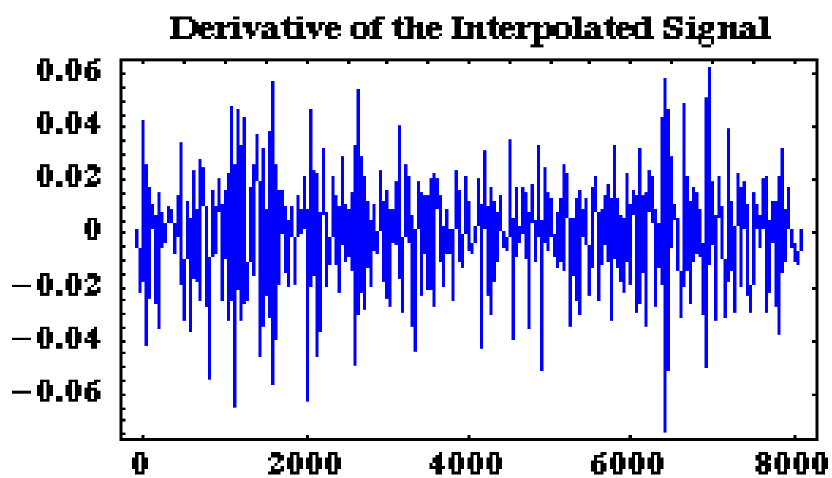


Reconstruction of the Data Using the First Four (of 10) Levels of the MRD

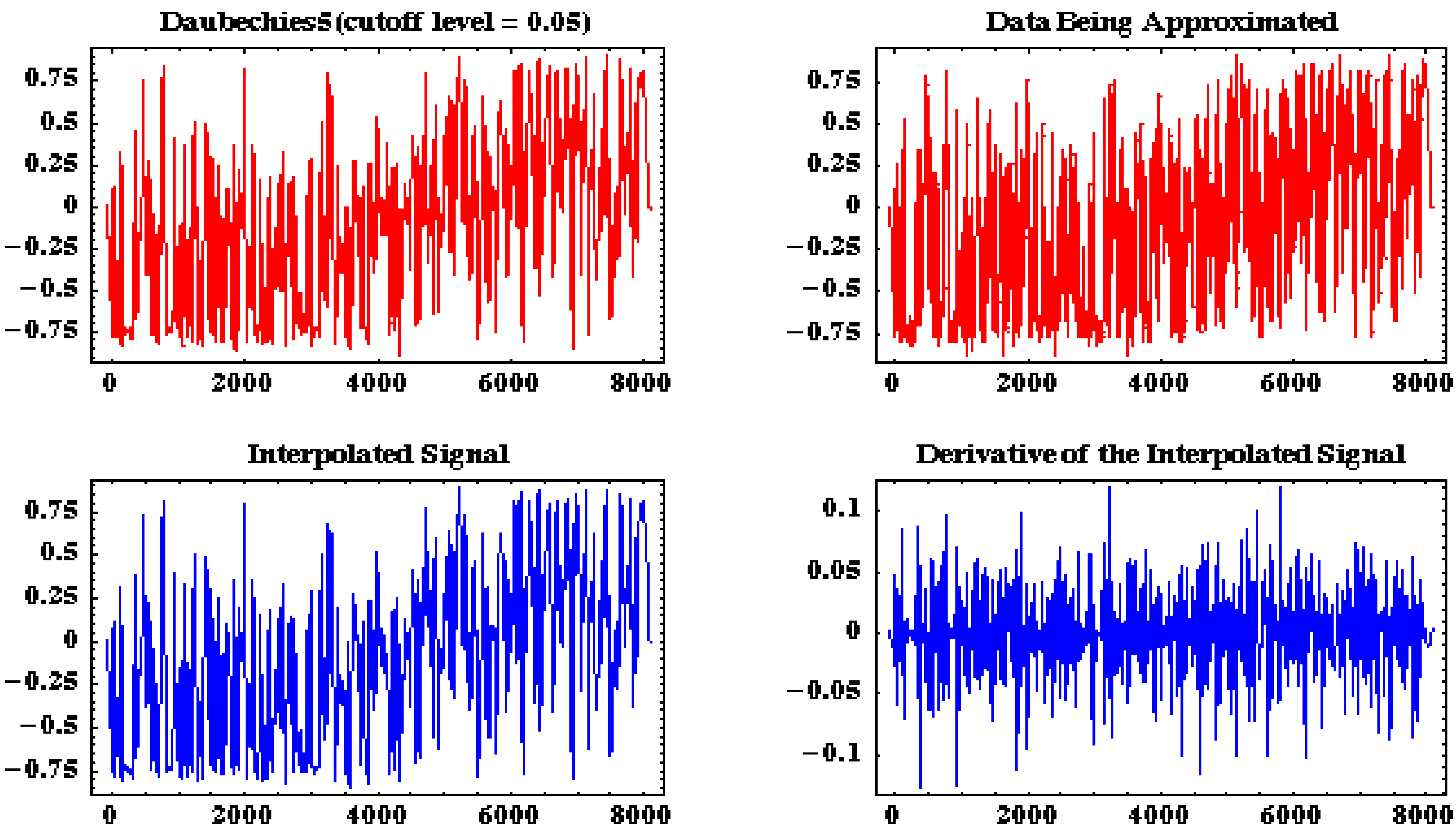






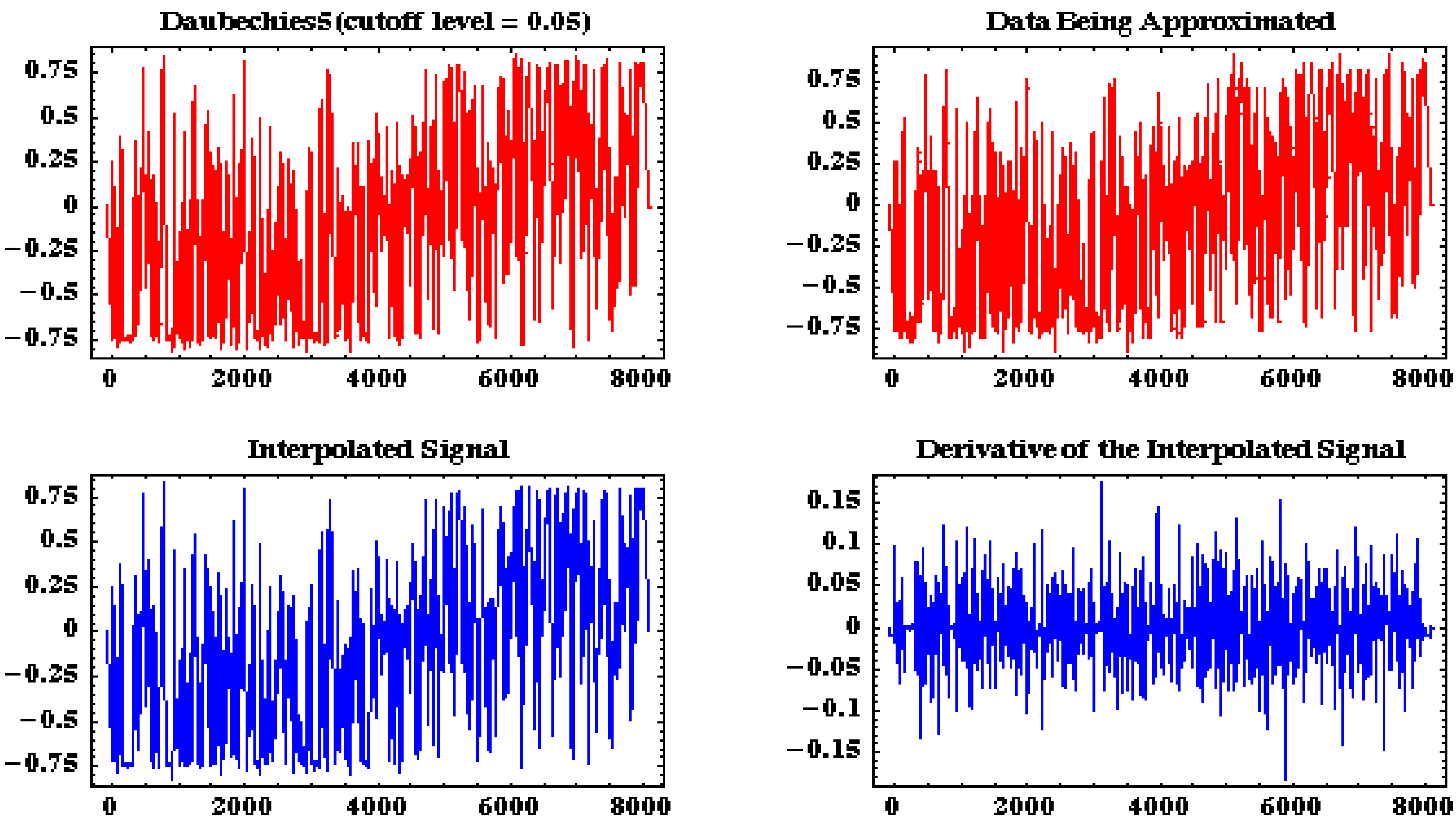


Reconstruction of the Data Using the First Five (of 10) Levels of the MRD



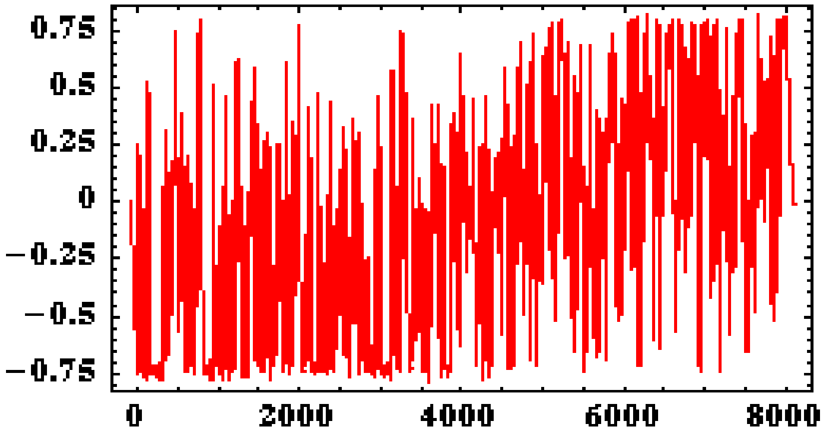
Reconstruction of the Data Using the First Six (of 10) Levels of the MRD

Polymath Research Inc.

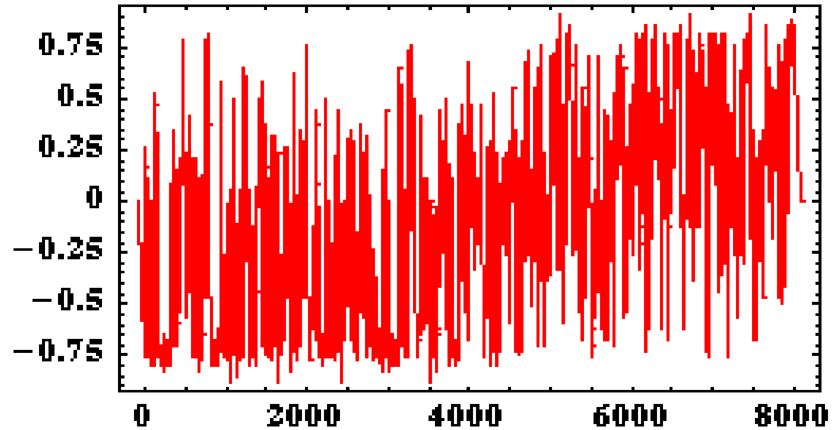


Reconstruction of the Data Using the First Seven (of 10) Levels of the MRD

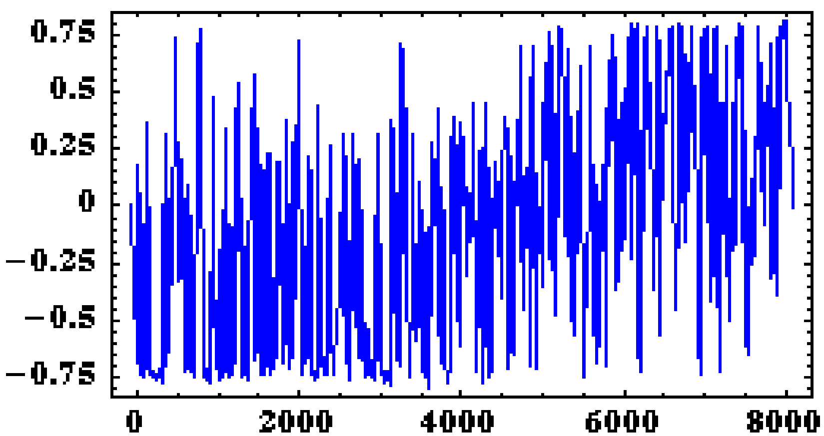
Daubechies5 (cutoff level = 0.05)



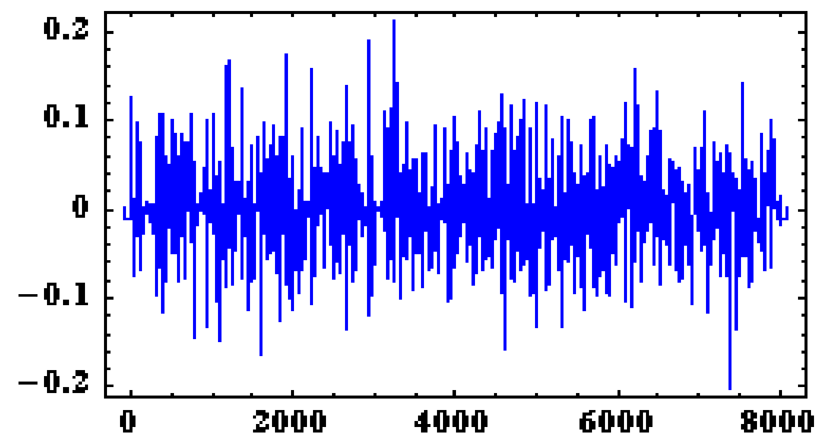
Data Being Approximated



Interpolated Signal



Derivative of the Interpolated Signal

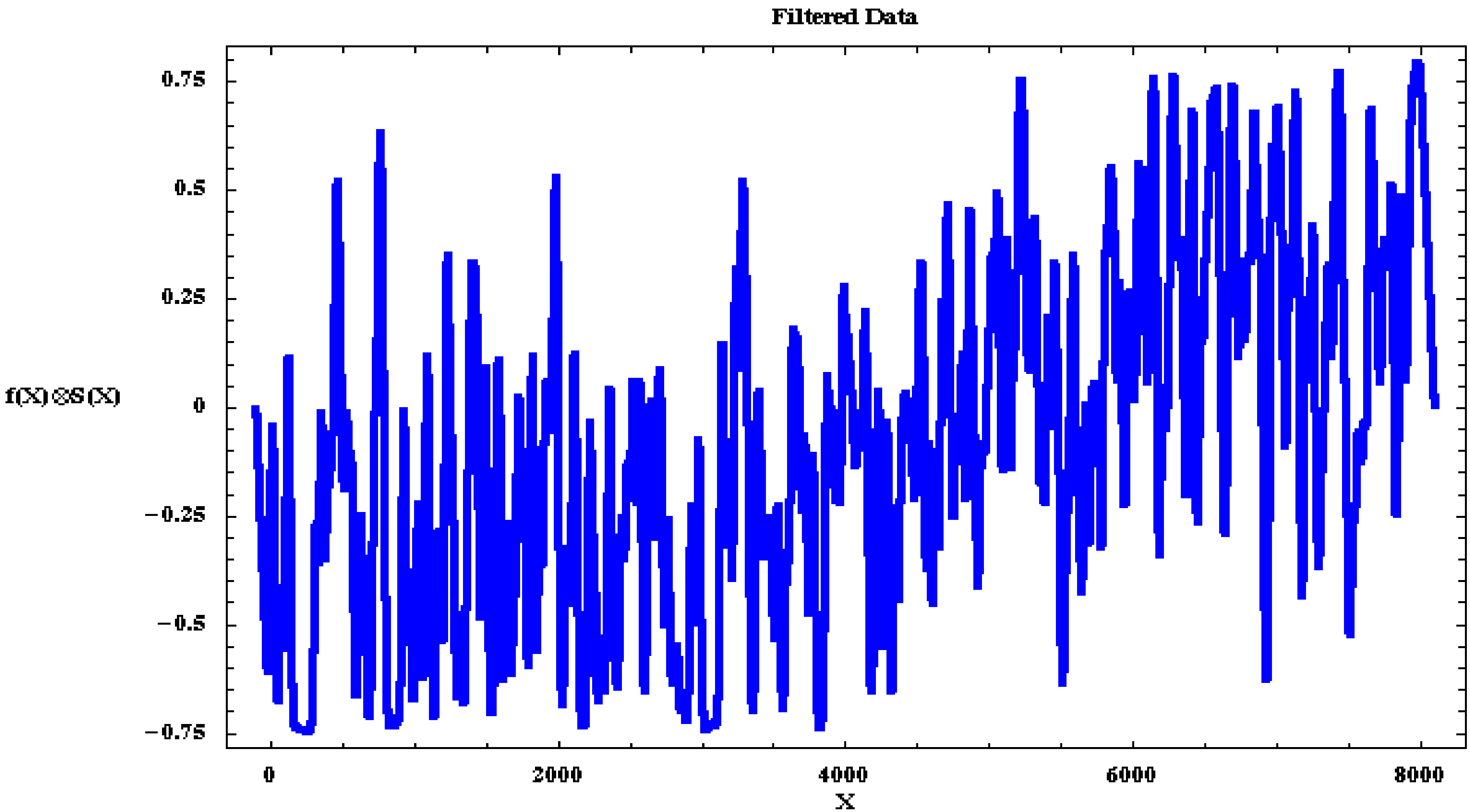


Conclusions on Raw RT Mix Data Analysis Using DWT

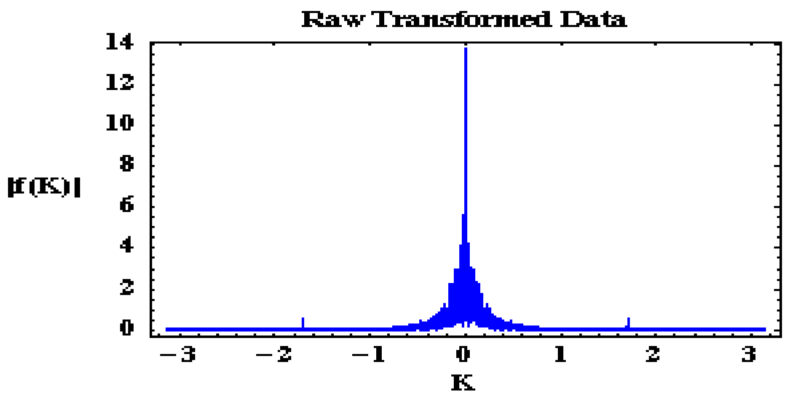
Polymath Research Inc.

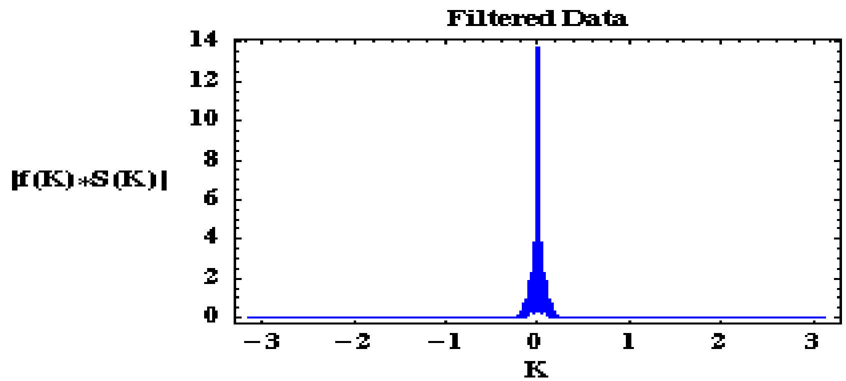
- Compression of around a factor of 20 seems likely with full data set.
- Will see what low pass filtering will do to initial data and its subsequent WLT analysis.
- Looks like 25% of the largest coefficients are enough to reconstruct the clean parts of the data.
- We should compare different stages of evolution of RT Mix in terms of their optimum WLT representations.
- Significant dynamical degrees of freedom vs insignificant ones which vary more slowly or not at all or randomly might be obtainable if we keep at it!

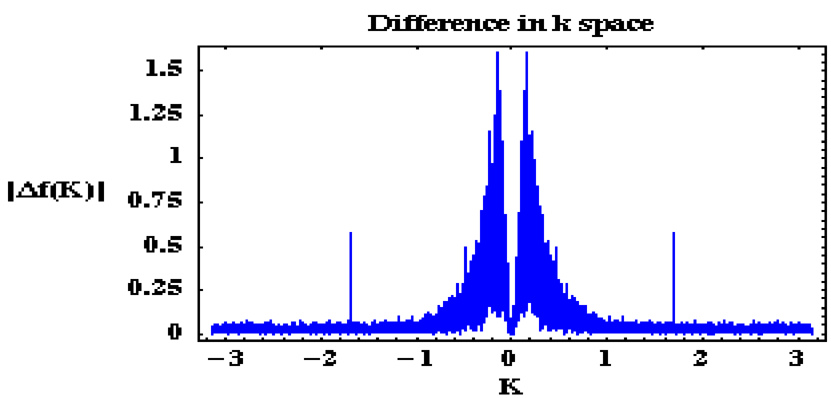
Low Pass Filtered RT Mix Data



The Filtering Has This Form and Effect in k-Space







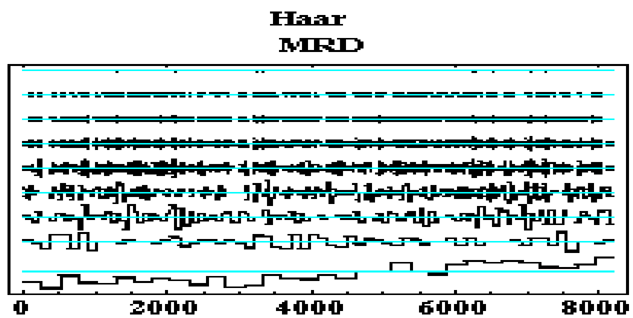
Filter was of the form:

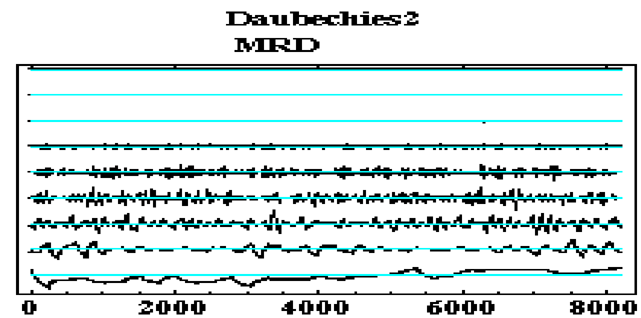
$$S(k) = \exp\left[-\left(\frac{k}{k_{width}}\right)^{2\alpha}\right]$$

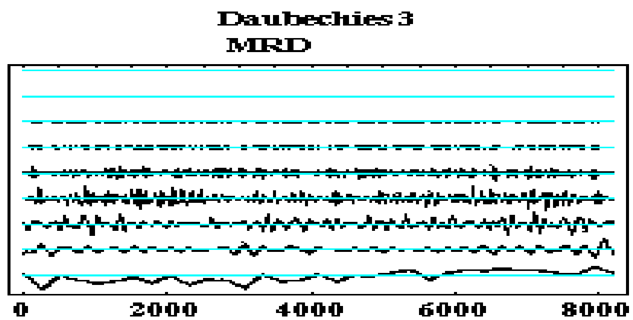
Where α=5 and k_{width} = 400

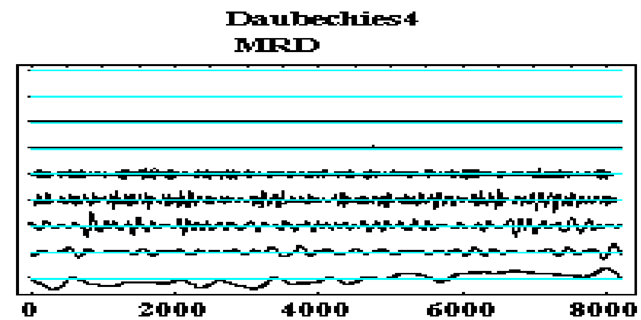
MRDs of the LP Filtered RT Mix Data in 6 Different Daubechies WLT Bases

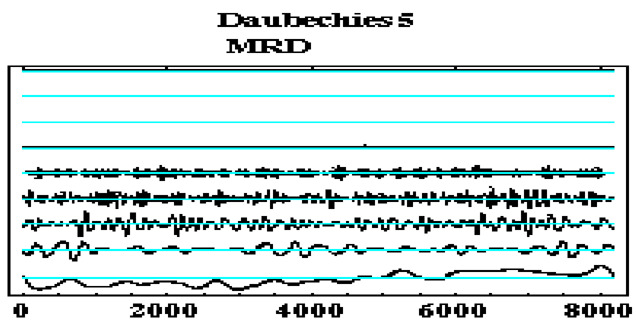
Polymath Research Inc.

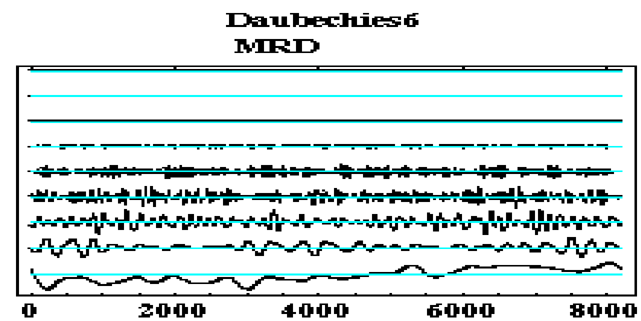




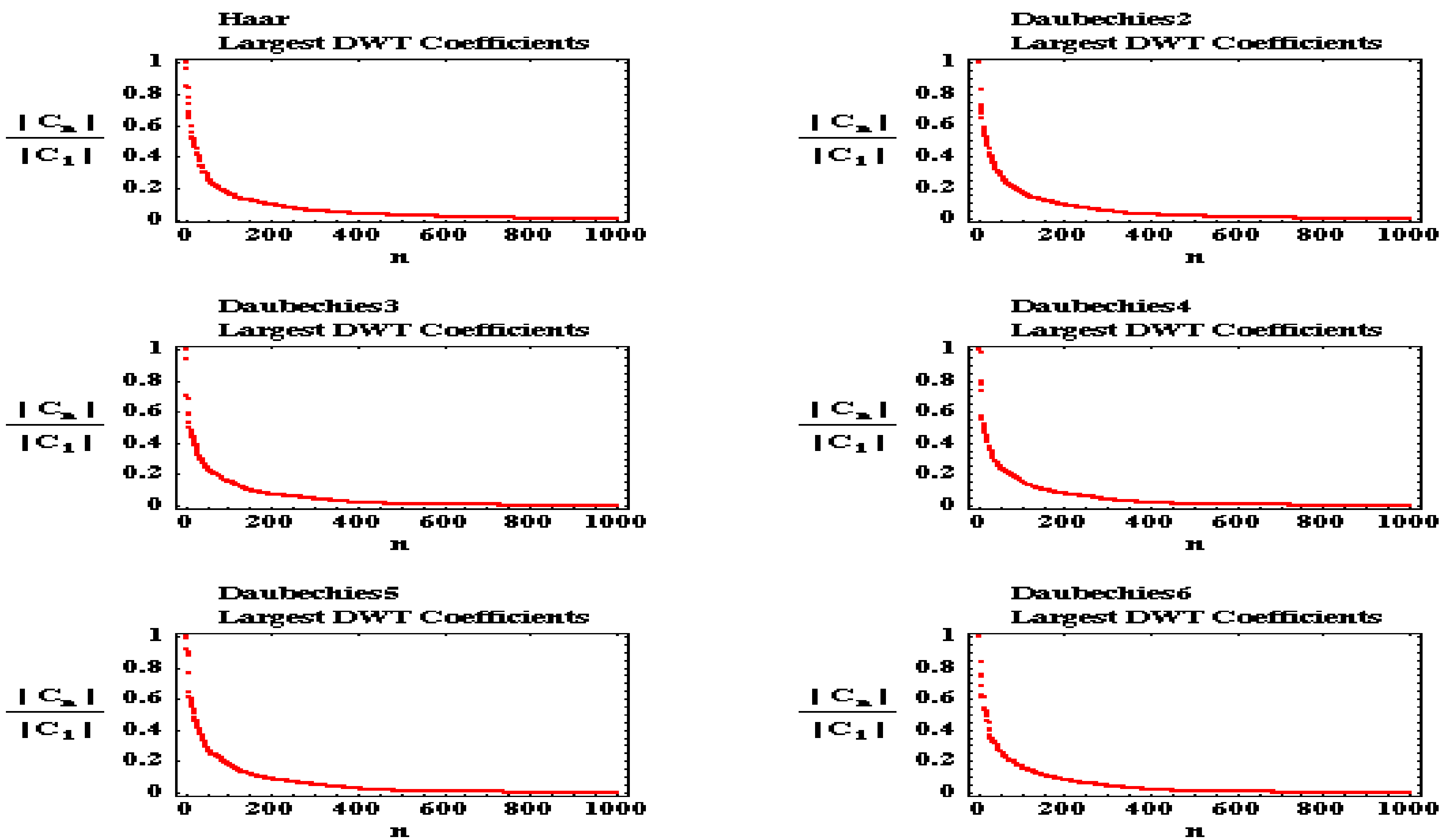






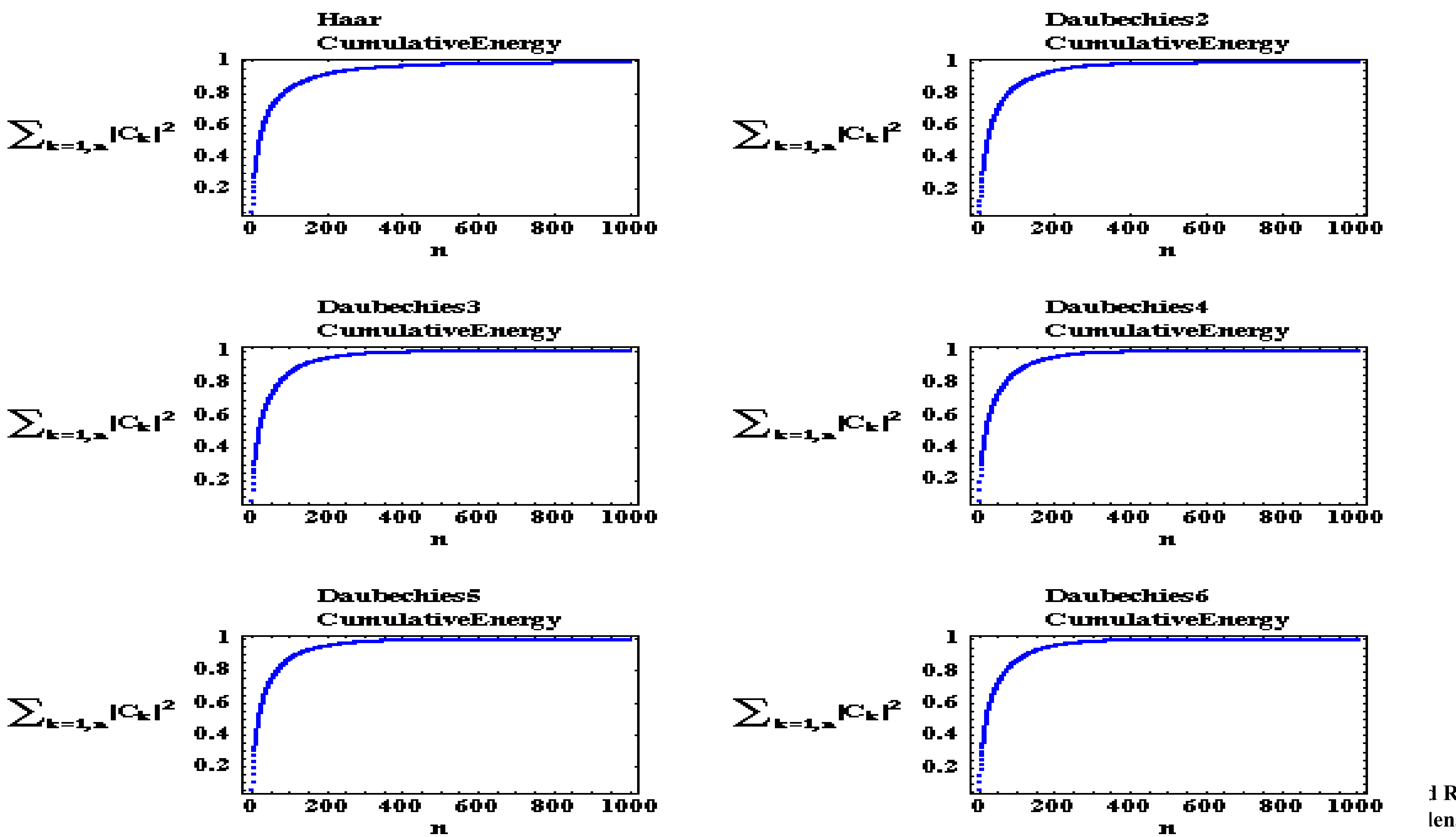


Decay Rate of Largest Coefficient vs Number of Coeffs Kept in LPF RT Mix Data



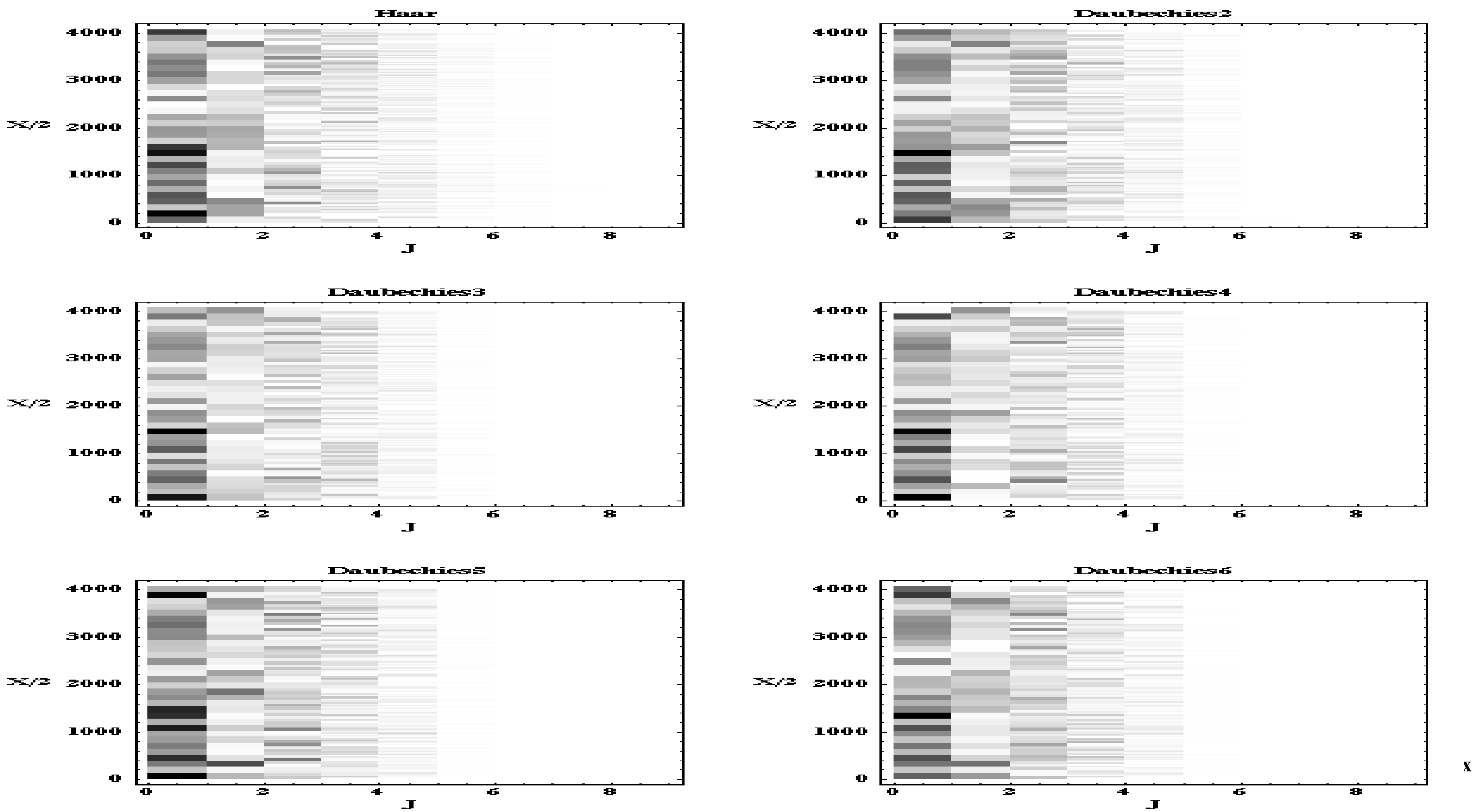
Energy Accumulation Rate in Coefficient Space vs # of WLTs Kept for LPF RT Mix Data

Polymath Research Inc.

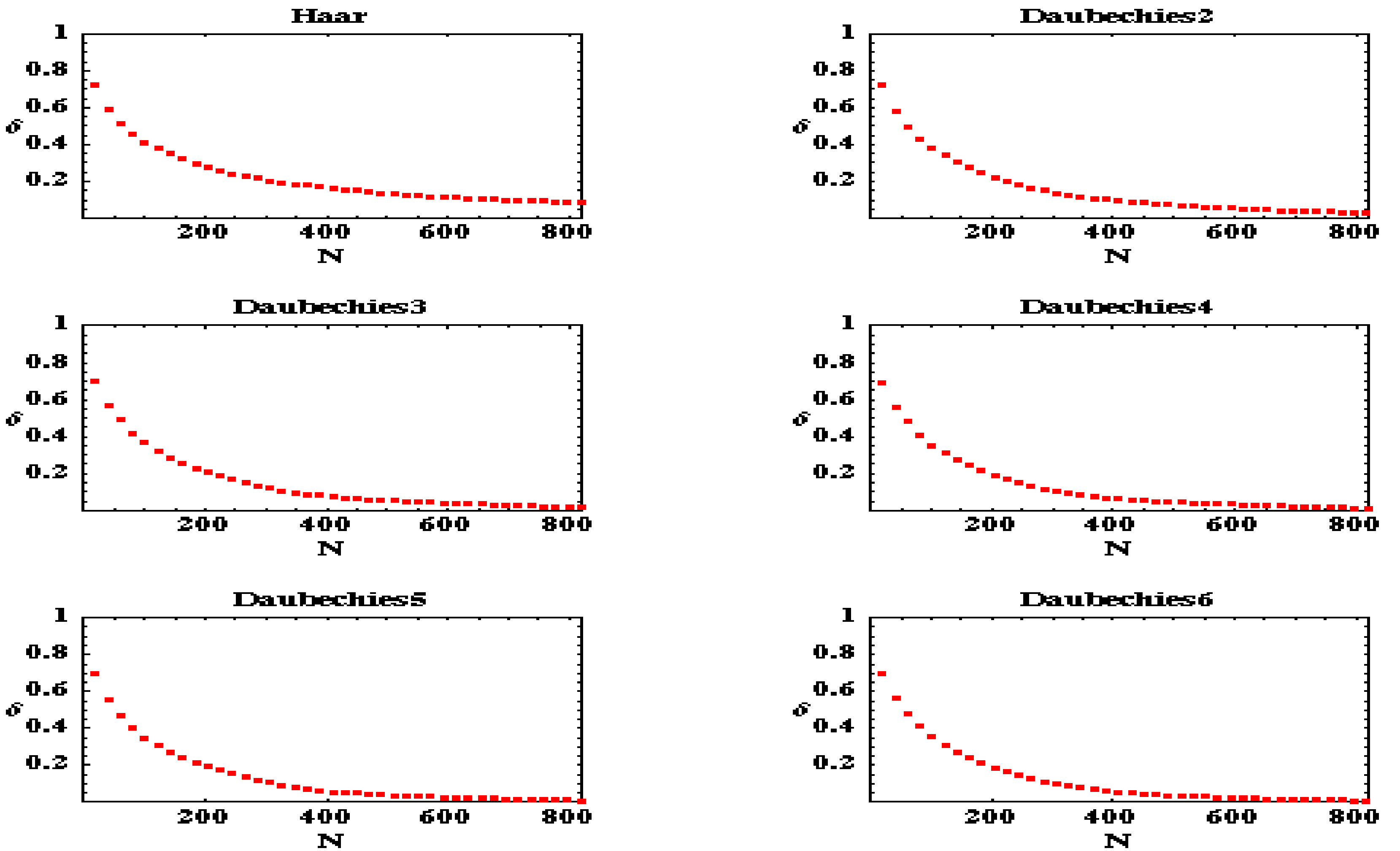


Scaleograms: Waveleters Preferred Way of Judging Tiling in Scale-Translation Space for LPF RT Mix

Polymath Research Inc.

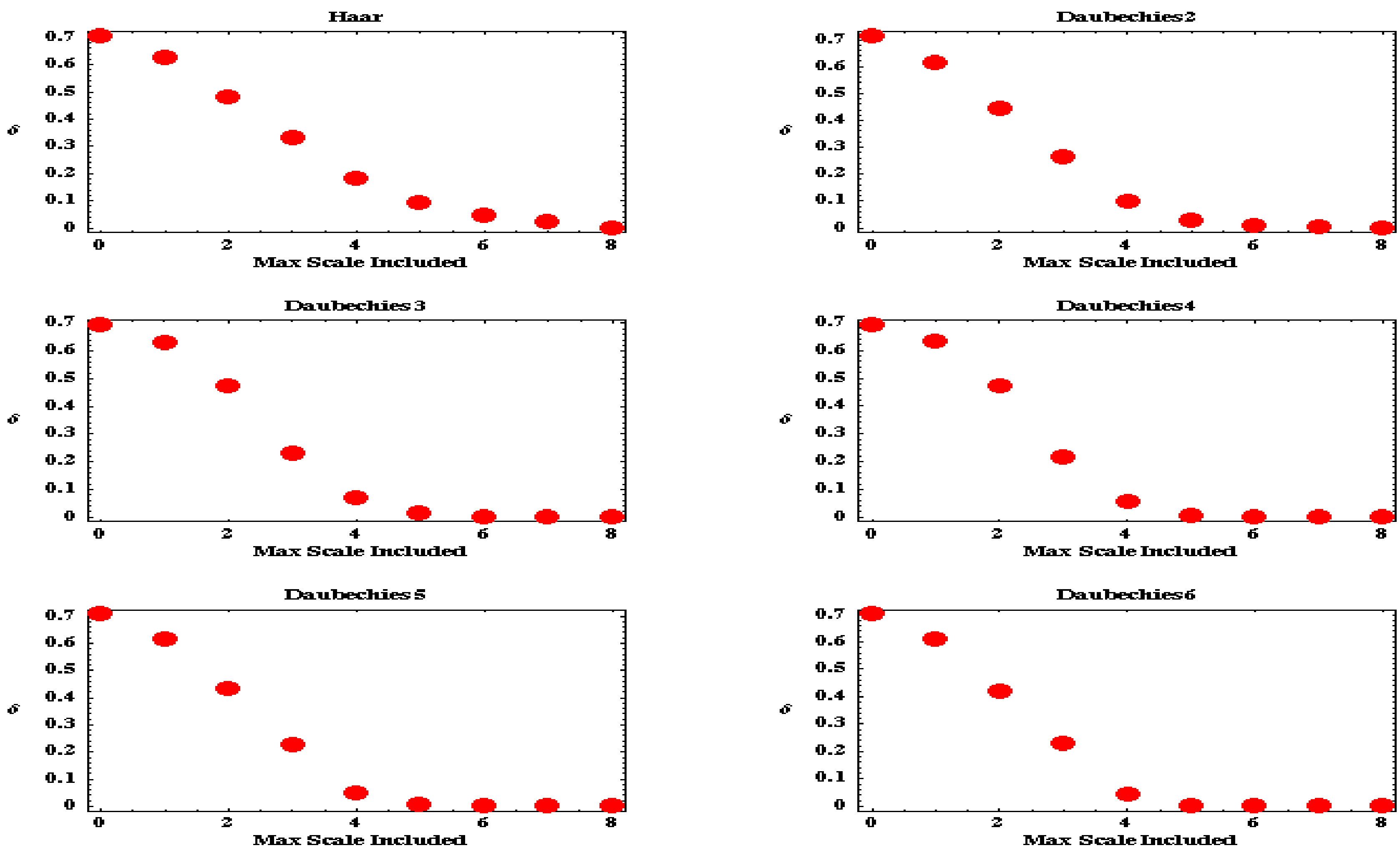


Least Square Error Incurred By Truncating the WLT Series at N of its Largest Coeffs LPF RT Mix Data

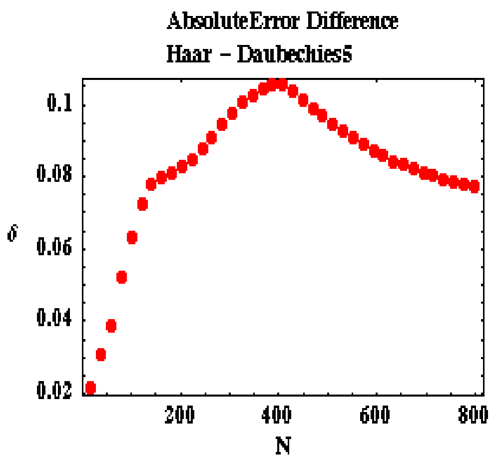


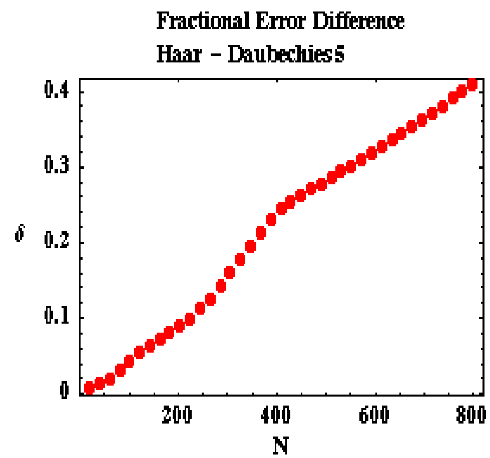
Least Square Error Incurred by Level Thresholding the DWT of LPF RT Mix Data

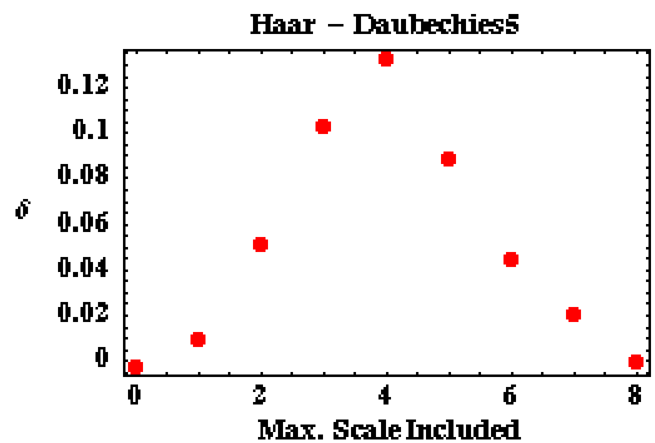
Polymath Research Inc.

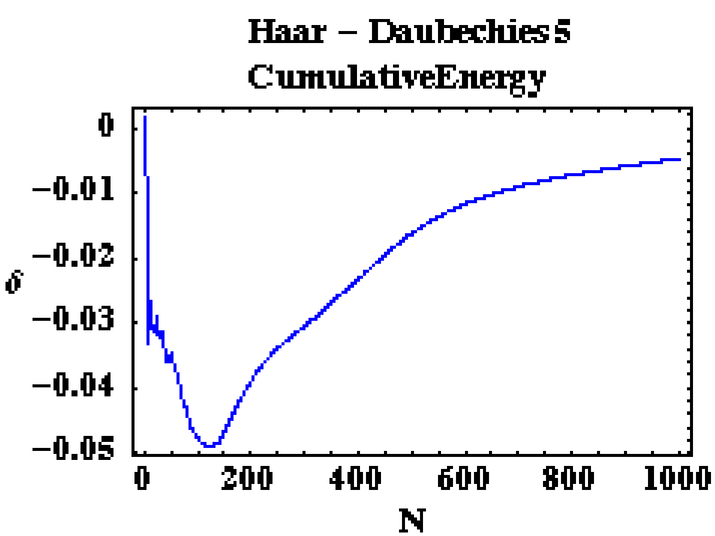


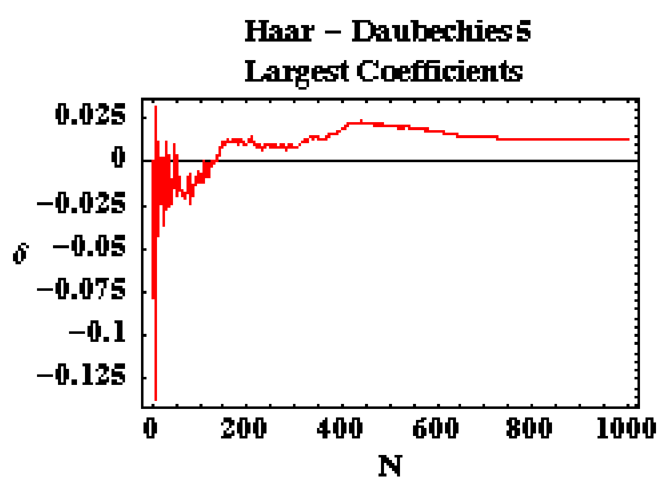
Daubechies 5 Does Much Better than Haar: 5 Quantitative Measures for LPF RT Mix Data



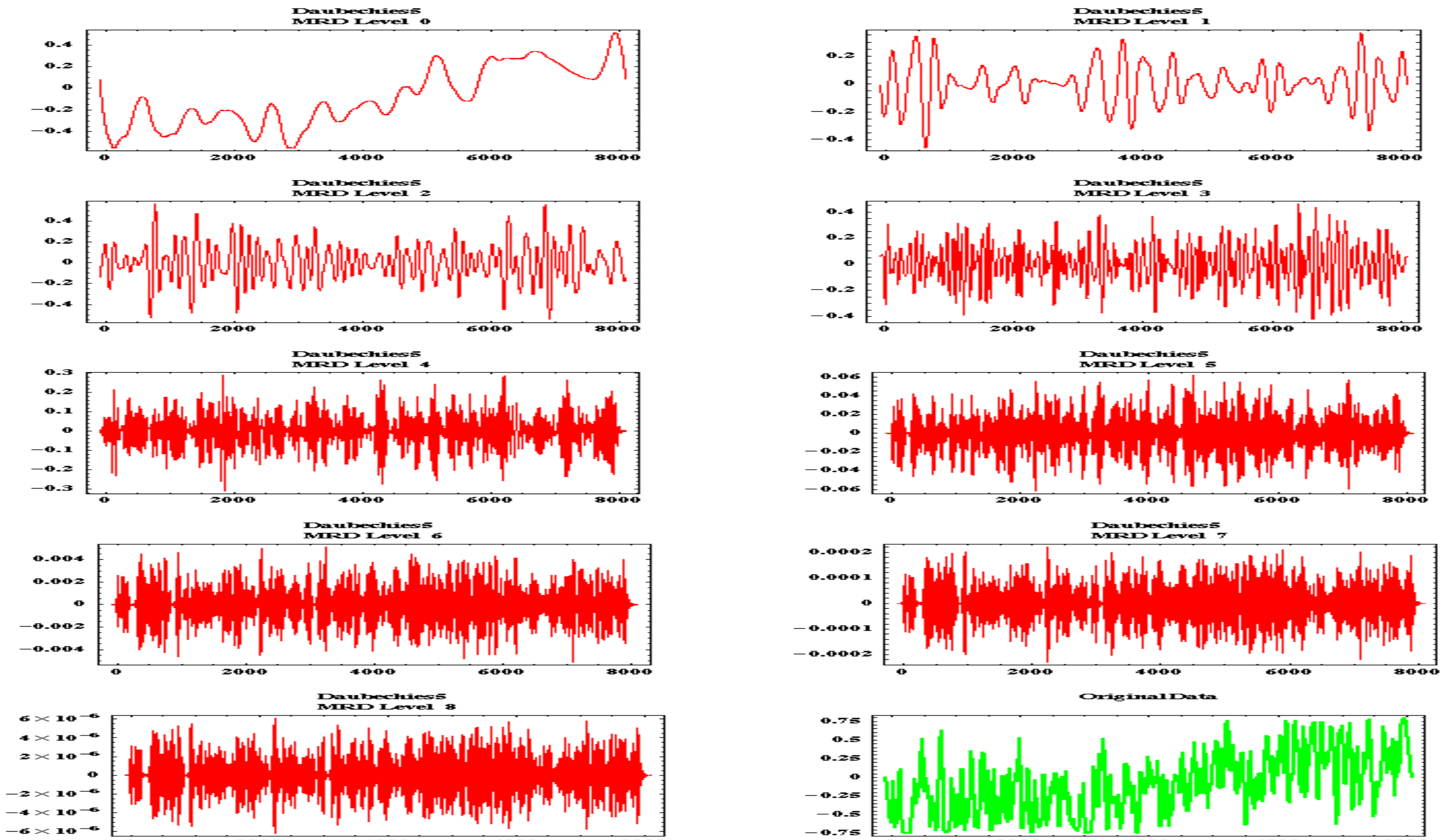






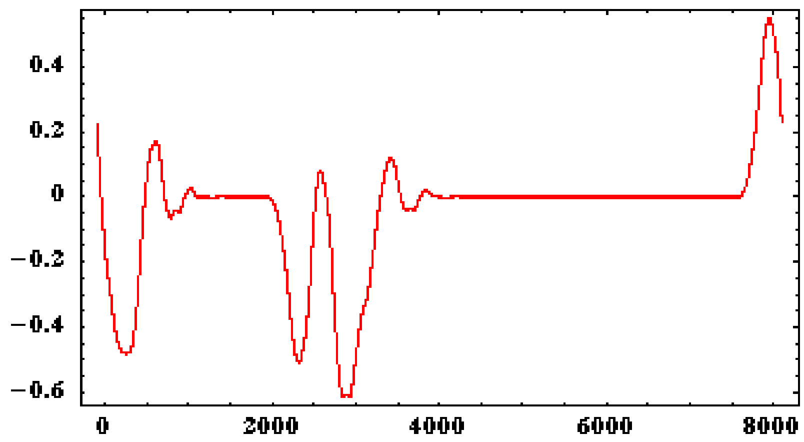


Level by Level Decomposition of the LPF RT Mix Data Using Daub5 WLTs

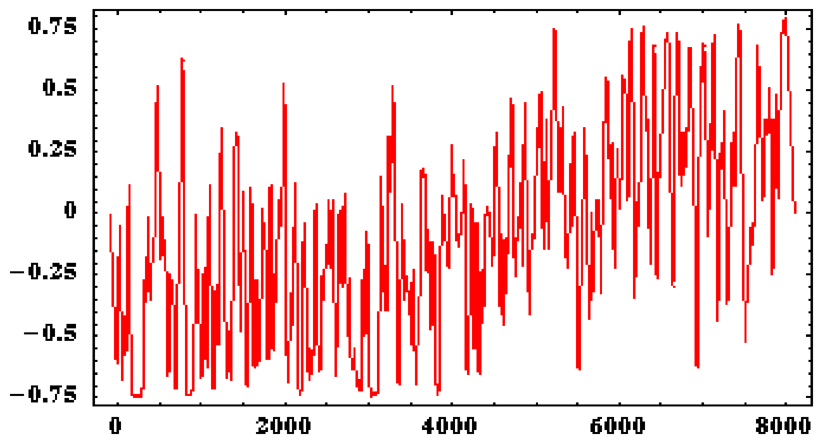


Reconstruction of the LPF Data with 5 Largest WLT Coeffs

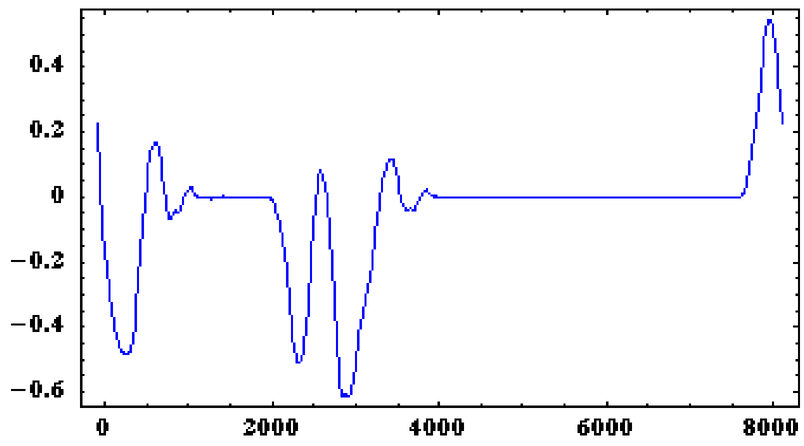
Daubechies5 (with5largests coefs.)



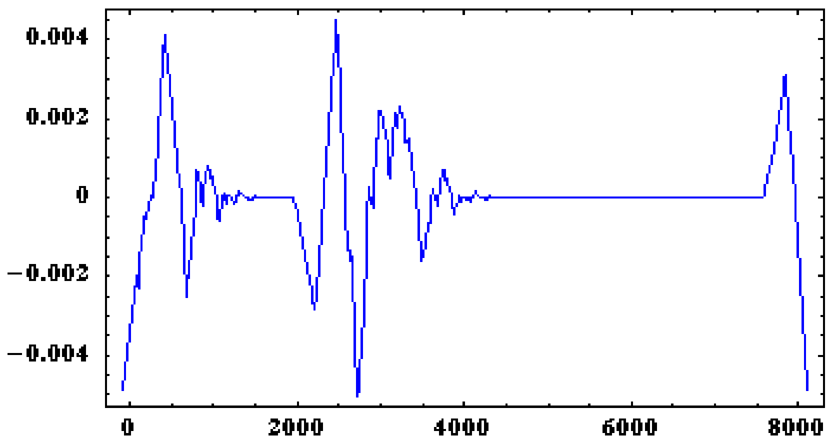
Data Being Approximated



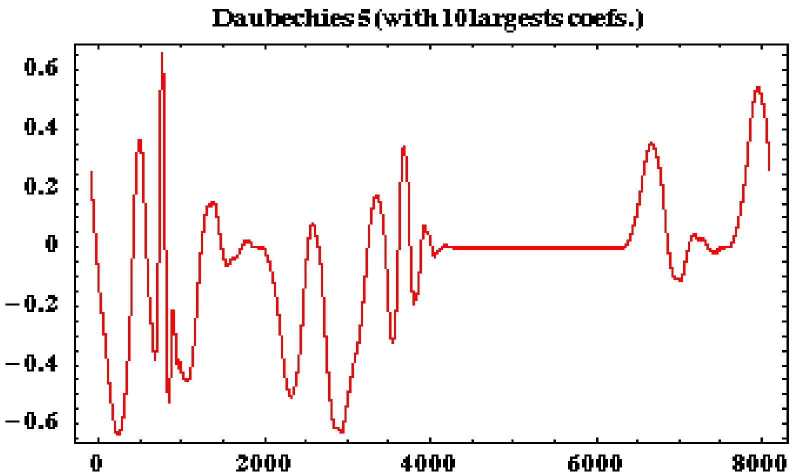
Interpolated Signal

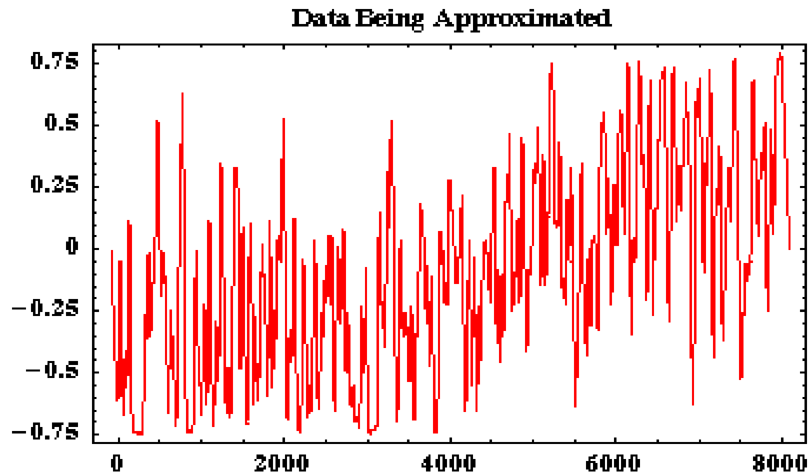


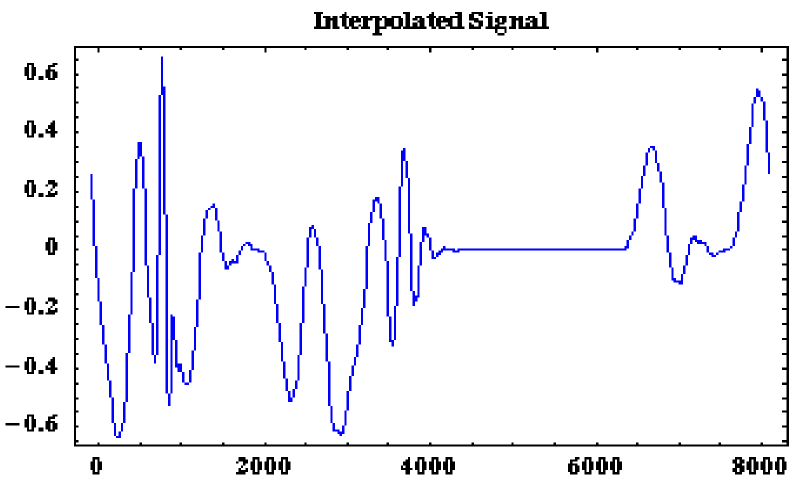
Derivative of the Interpolated Signal

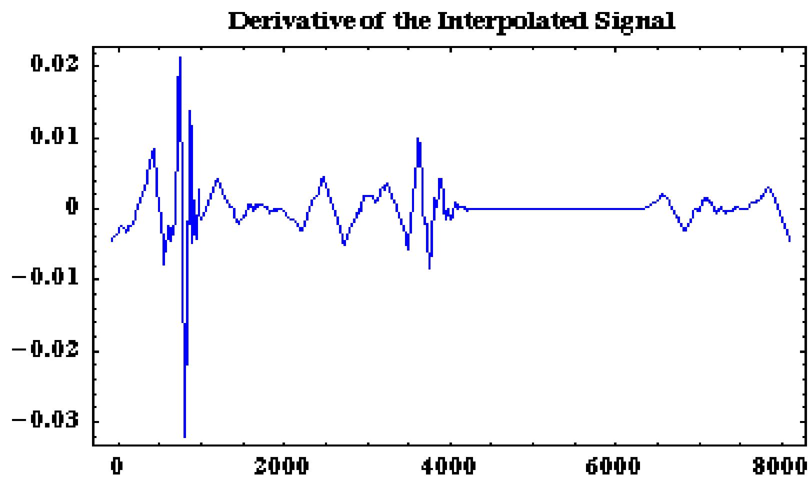


Reconstruction of the LPF Data with 10 Largest WLT Coeffs



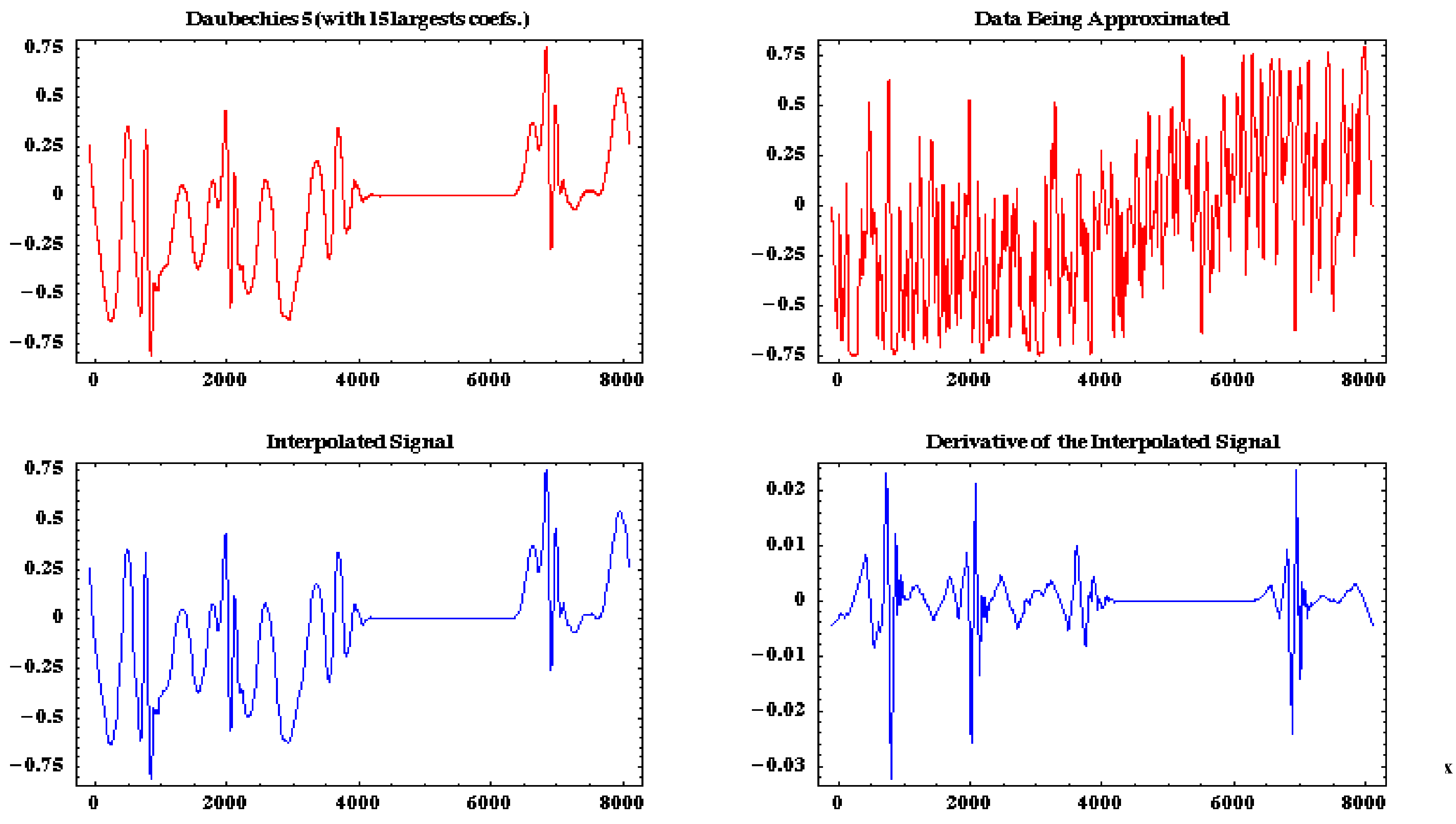






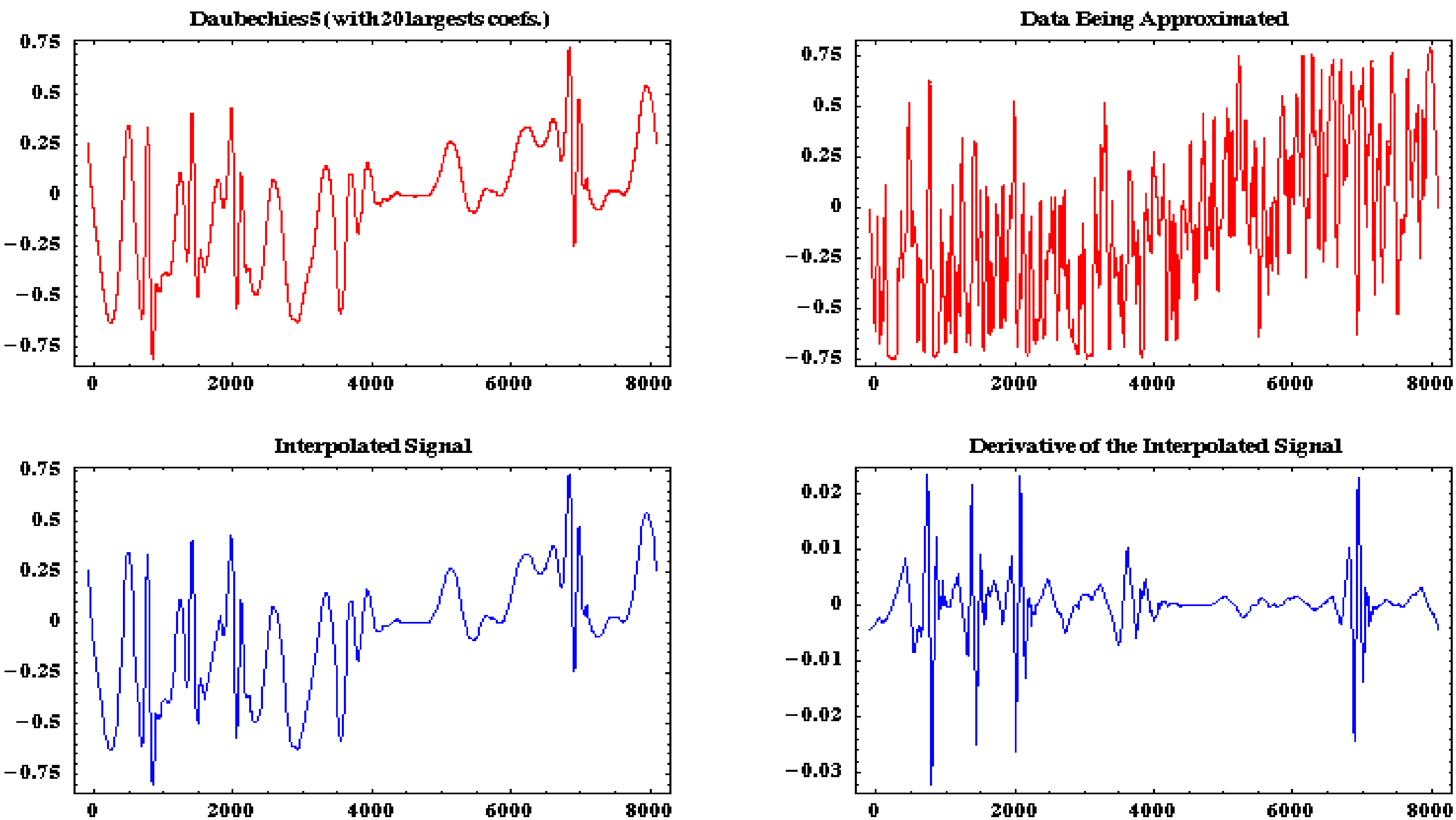
Reconstruction of the LPF Data with 15 Largest WLT Coeffs

Polymath Research Inc.

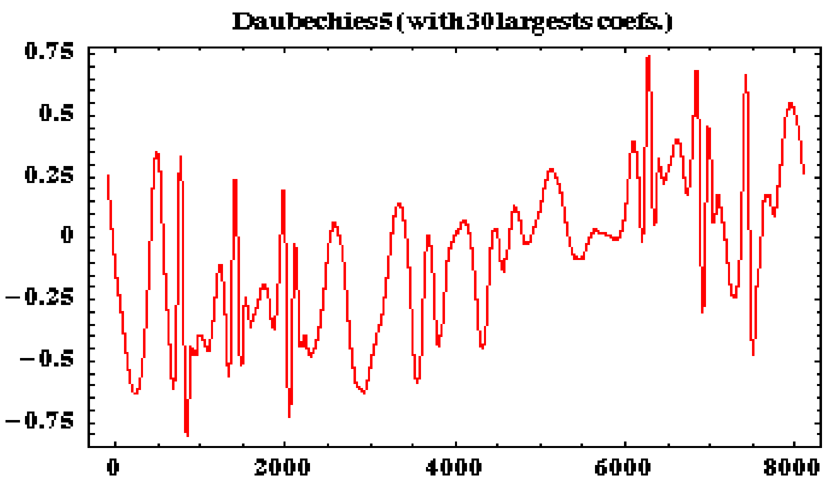


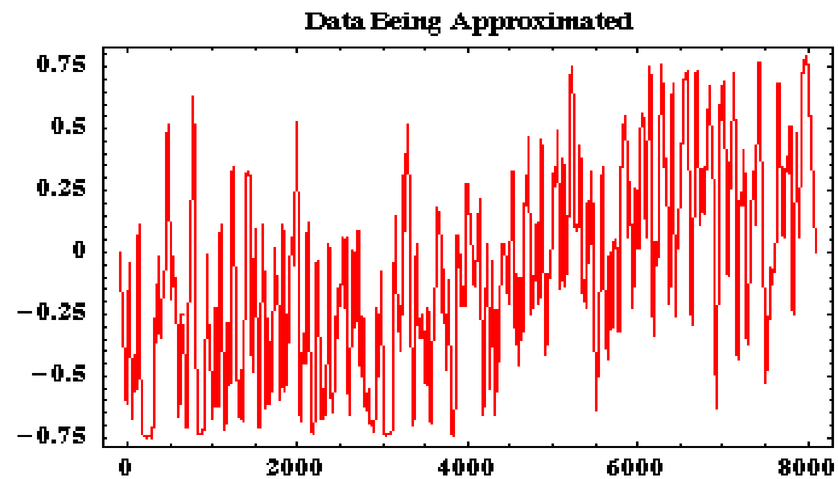
Reconstruction of the LPF Data with 20 Largest WLT Coeffs

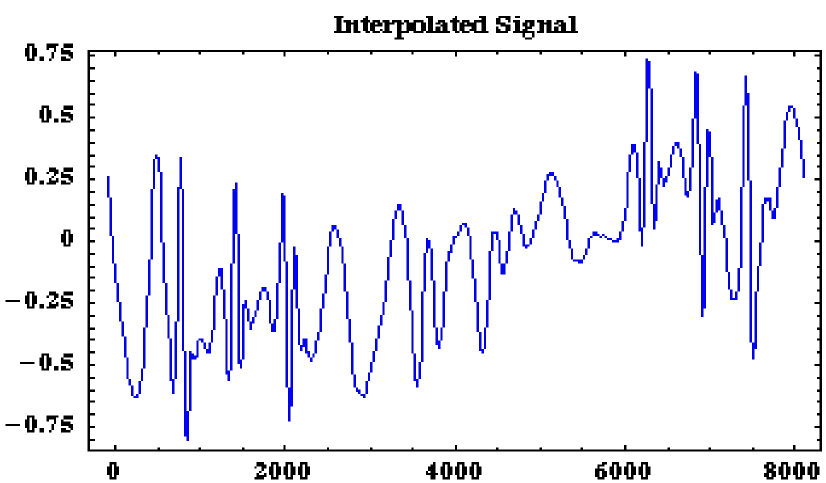
Polymath Research Inc.

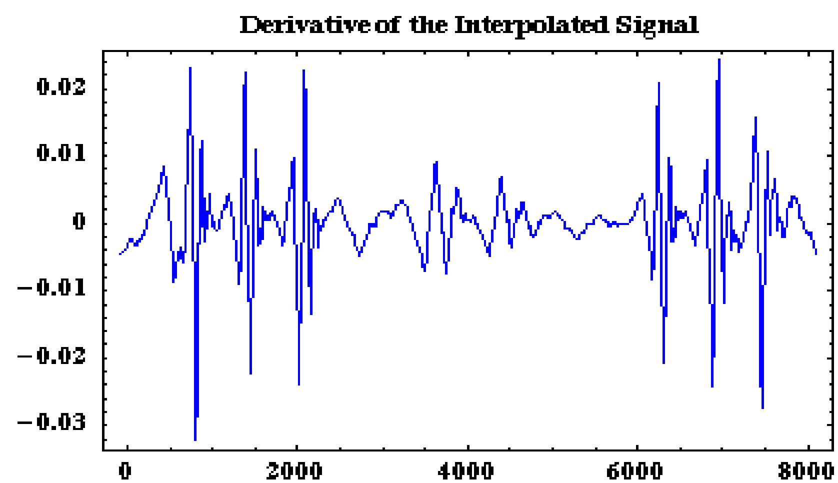


Reconstruction of the LPF Data with 30 Largest WLT Coeffs

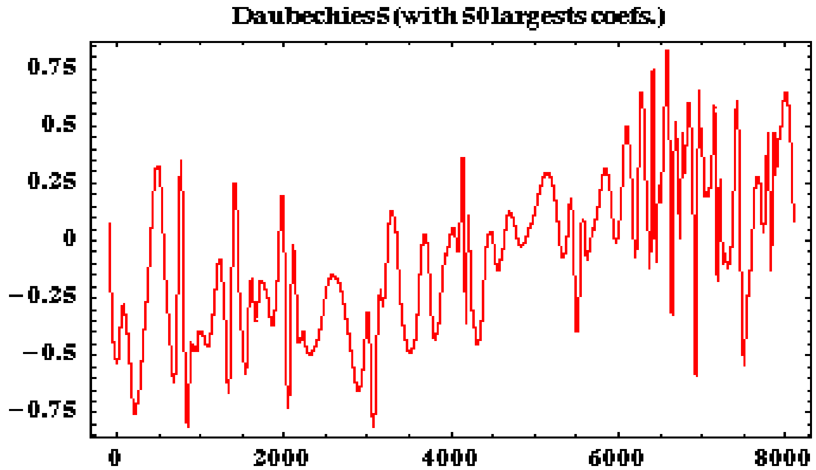


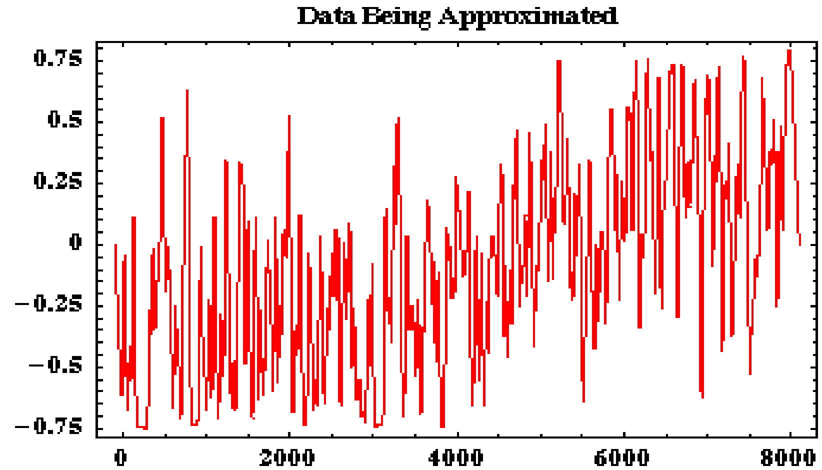


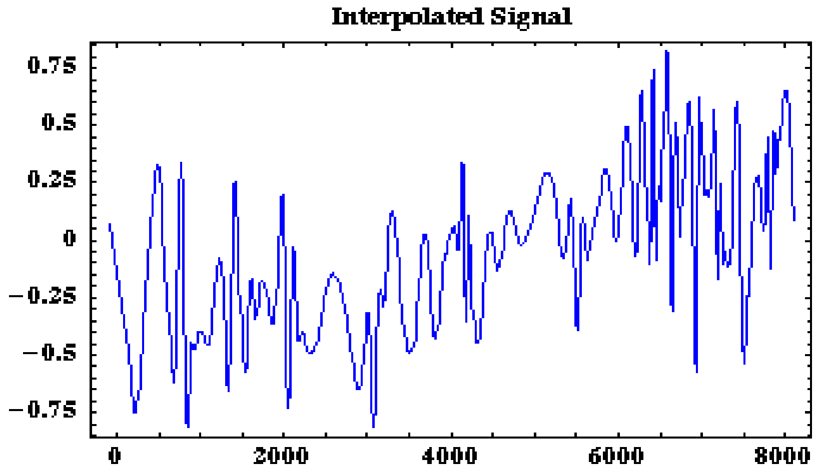


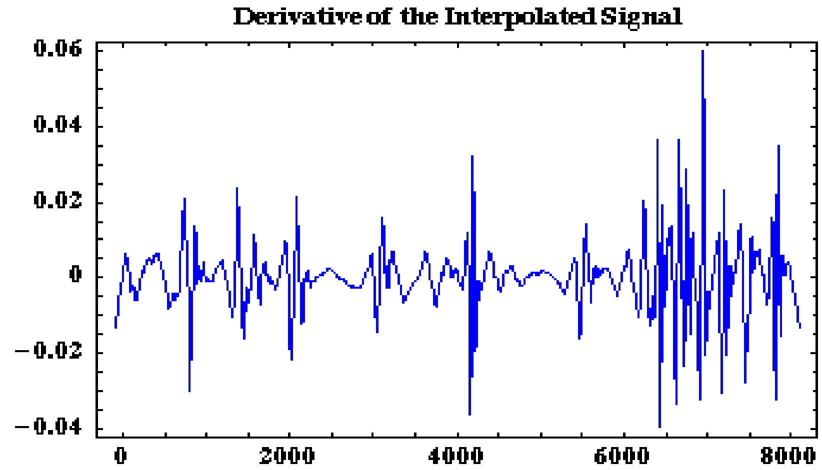


Reconstruction of the LPF Data with 50 Largest WLT Coeffs



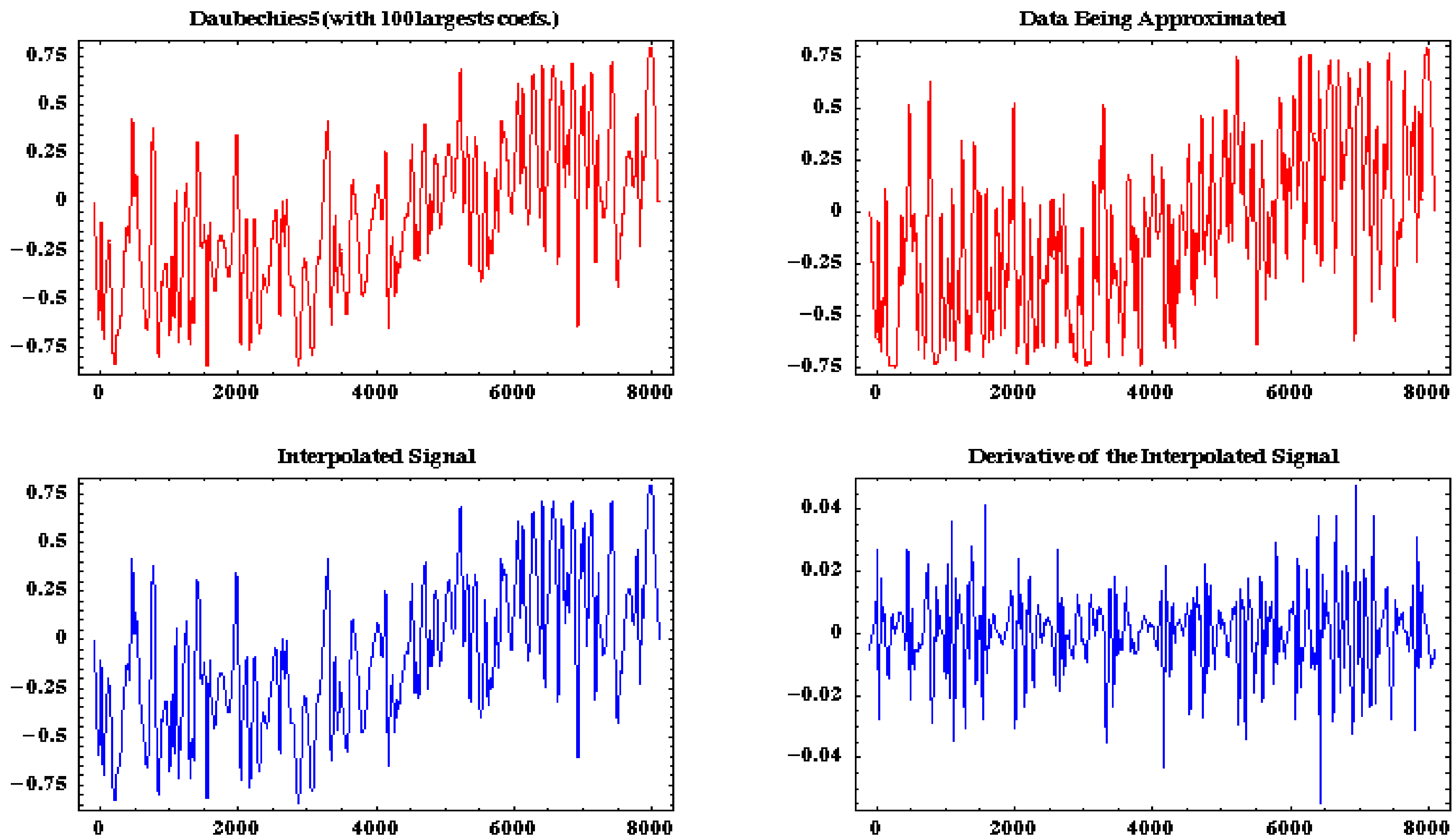






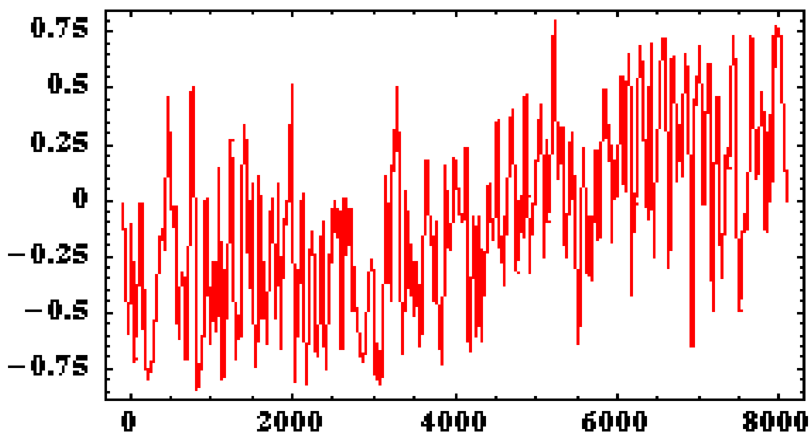
Reconstruction of the LPF Data with 100 Largest WLT Coeffs

Polymath Research Inc.

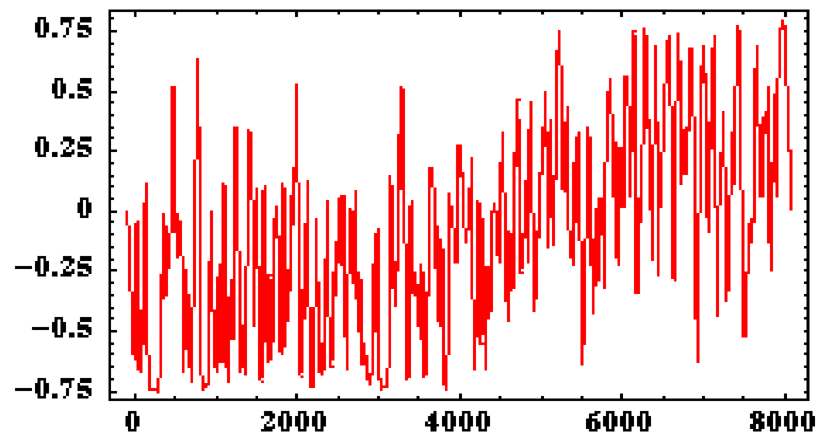


Reconstruction of the LPF Data with 200 Largest WLT Coeffs

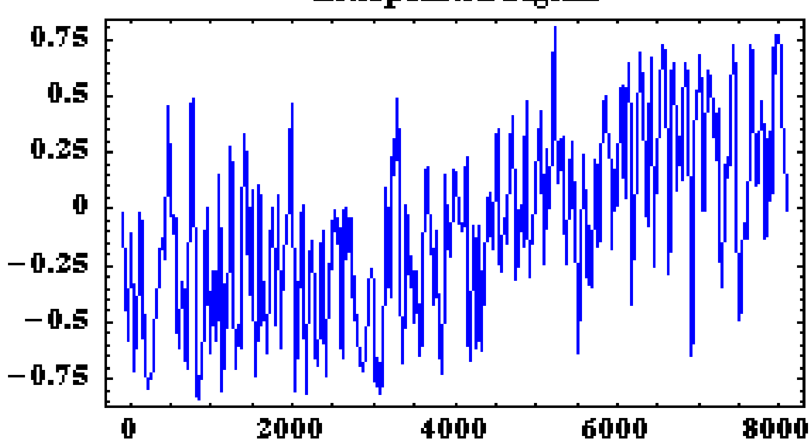
Daubechies5 (with 200 largests coefs.)



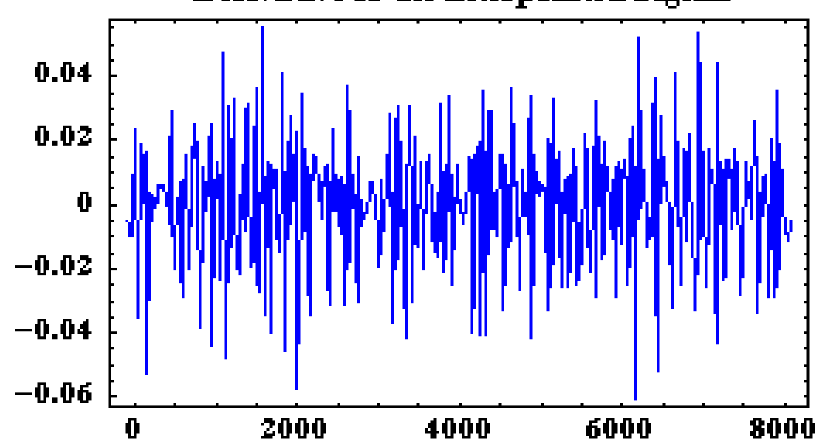
Data Being Approximated



Interpolated Signal

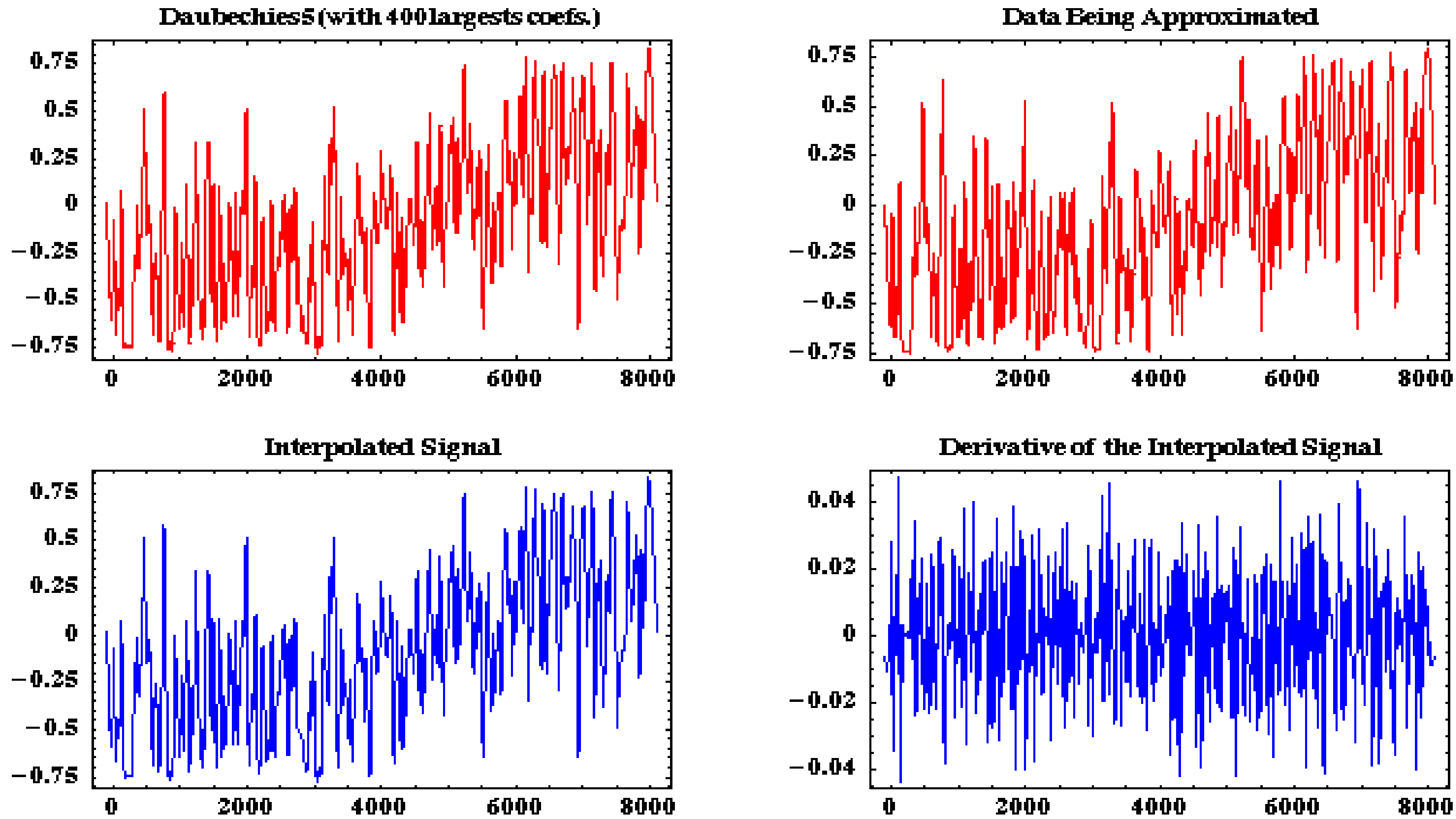


Derivative of the Interpolated Signal

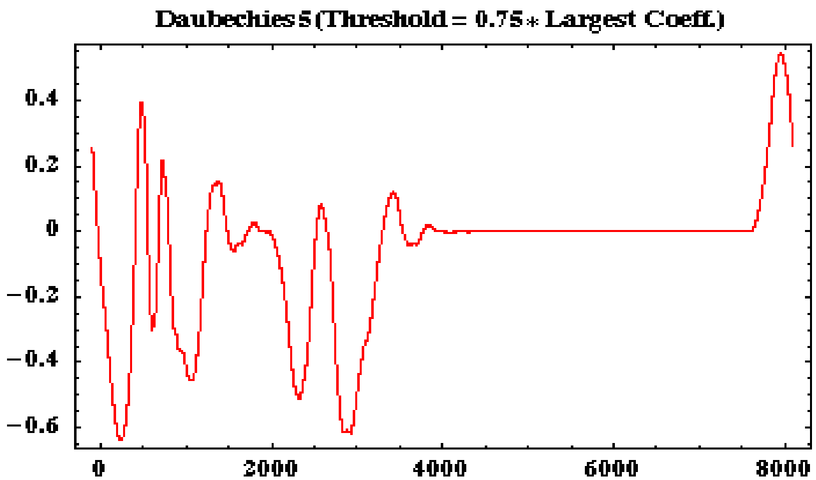


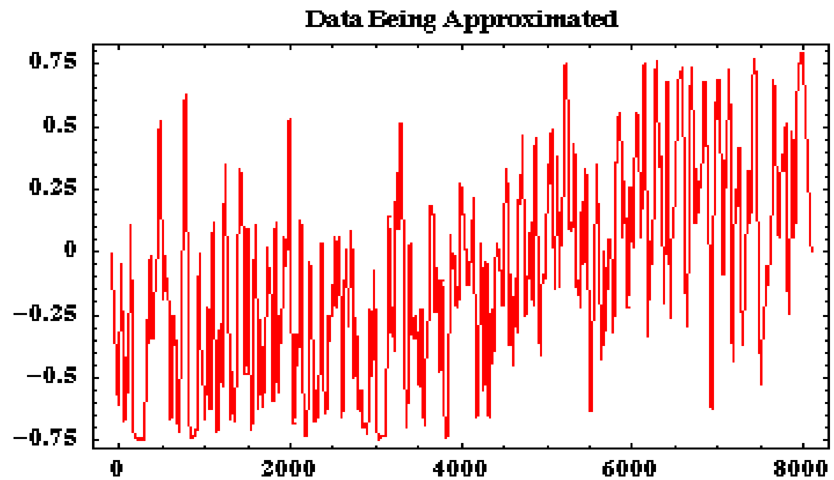
Reconstruction of the LPF Data with 400 Largest WLT Coeffs

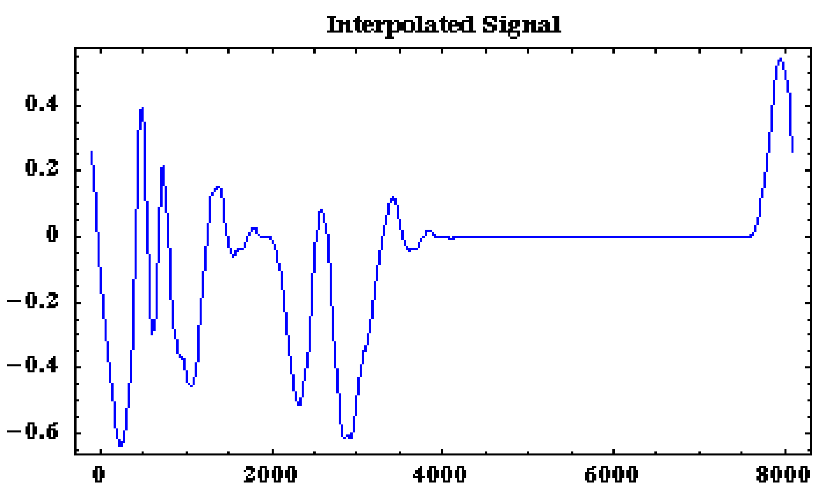
Polymath Research Inc.

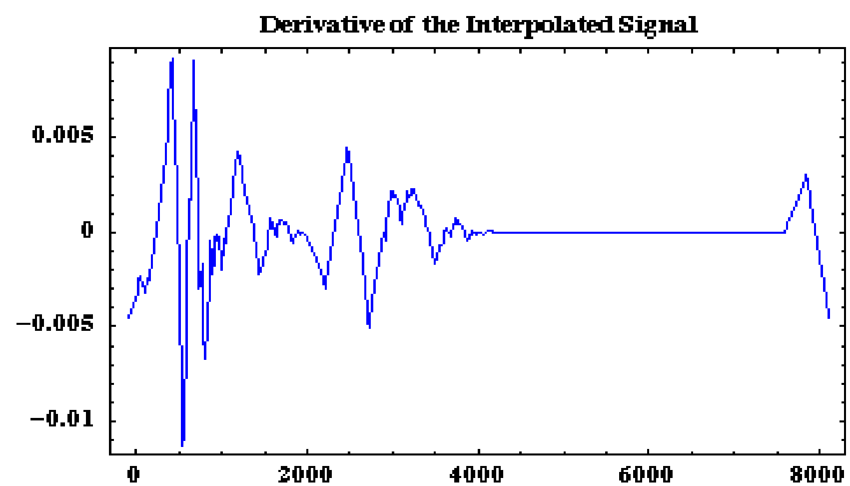


Recons. of the LPF Data Using Up to 0.75 x the Largest WLTs



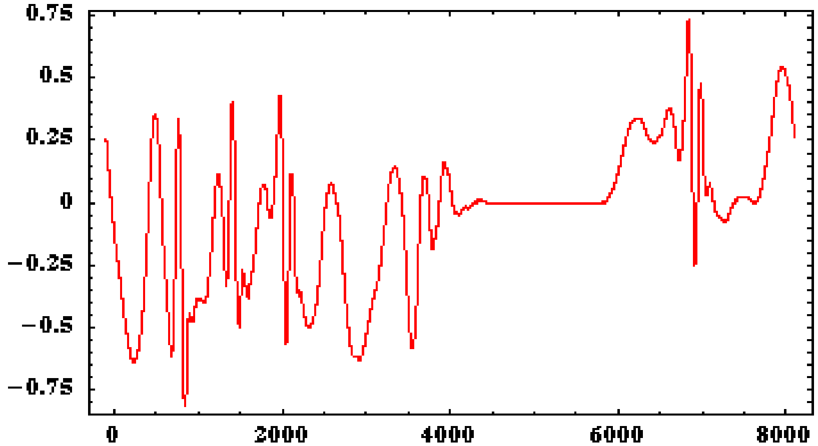




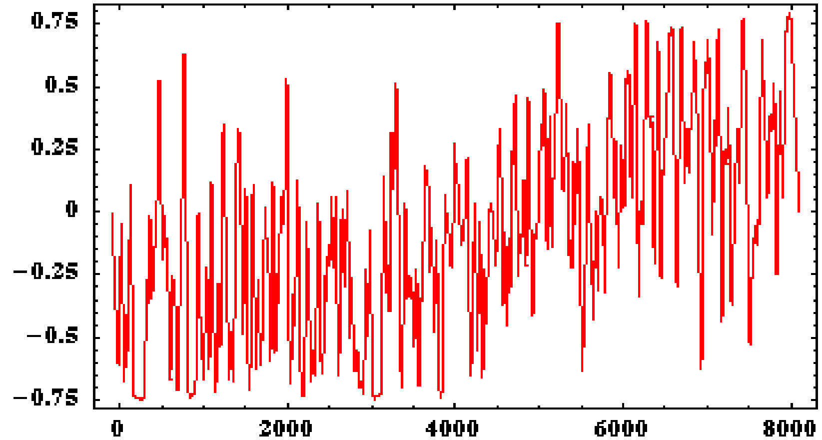


Recons. of the LPF Data Using Up to 0.5 x the Largest WLTs

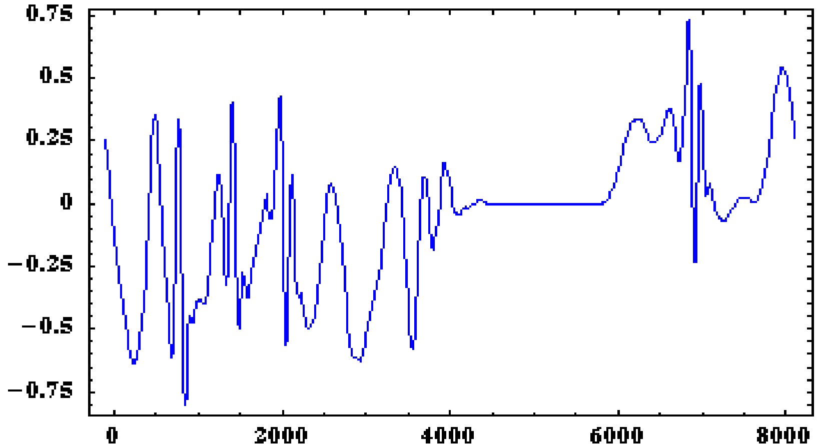
Daubechies 5 (Threshold = 0.5 * Largest Coeff.)



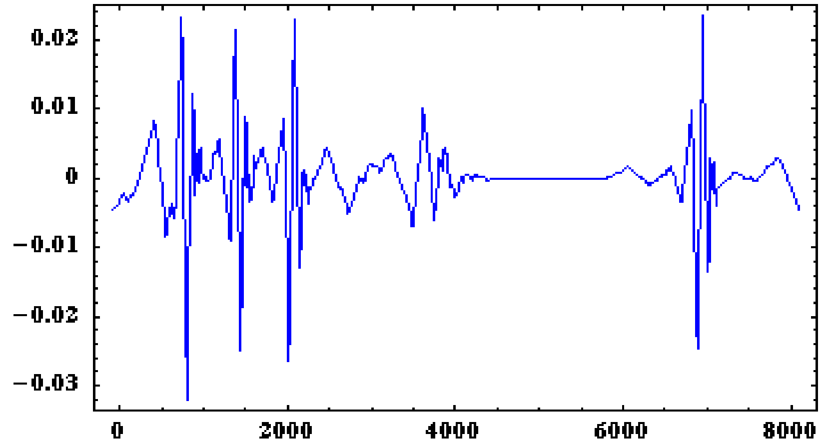
Data Being Approximated



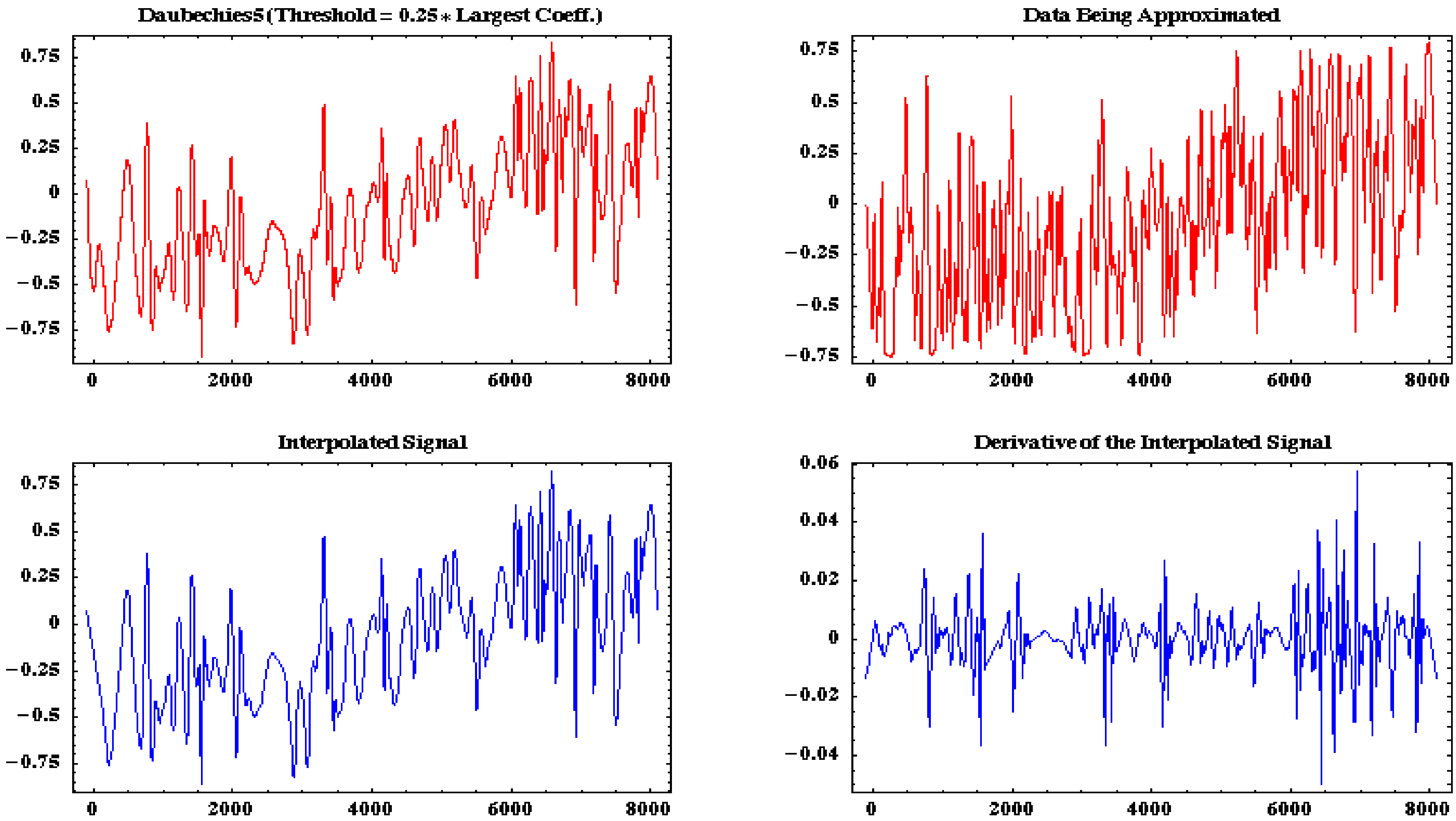
Interpolated Signal



Derivative of the Interpolated Signal

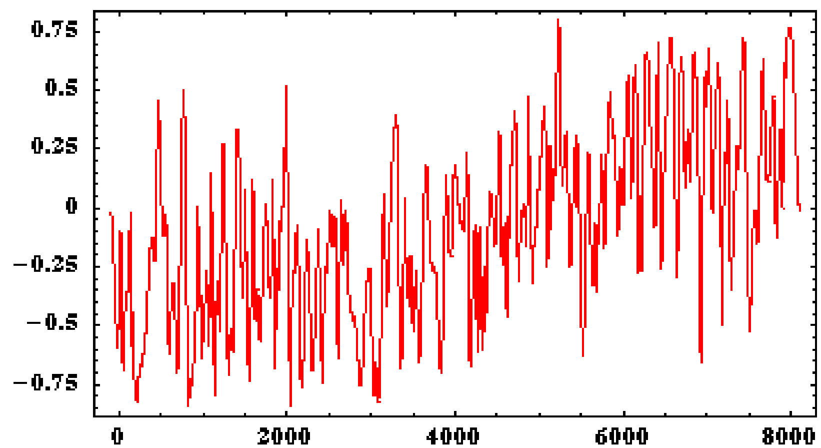


Recons. of the LPF Data Using Up to 0.25 x the Largest WLTs

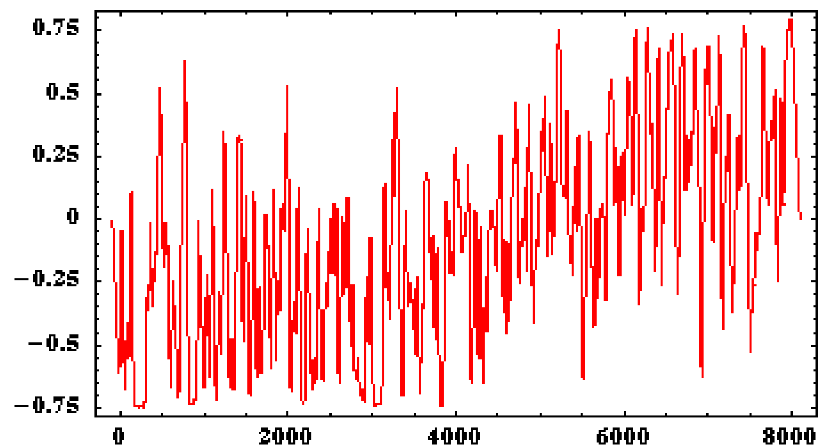


Recons. of the LPF Data Using Up to 0.1 x the Largest WLTs

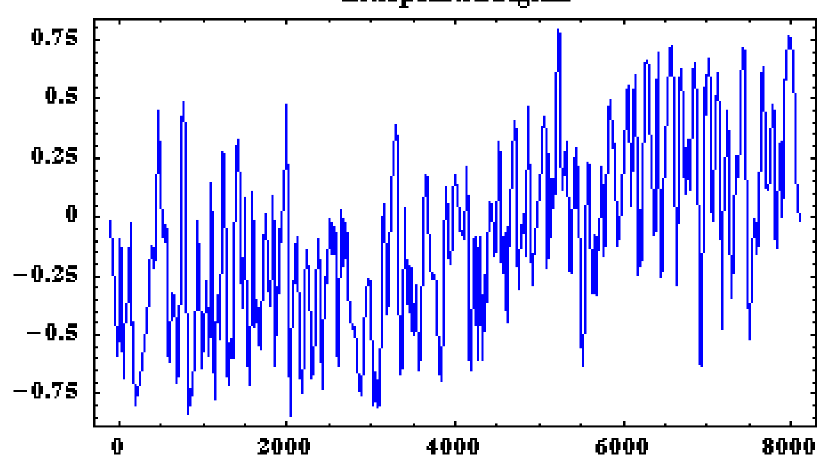
Daubechies5 (Threshold = 0.1 * Largest Coeff.)



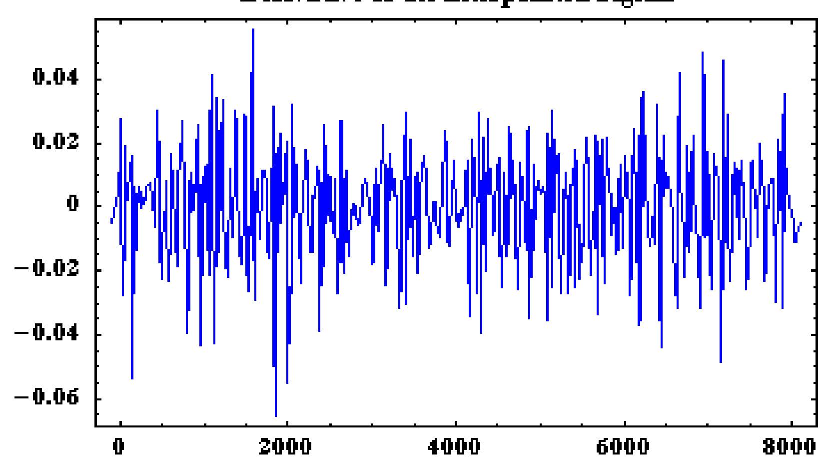
Data Being Approximated



Interpolated Signal



Derivative of the Interpolated Signal



Recons. of the LPF Data Using Up to 0.05 x the Largest WLTs

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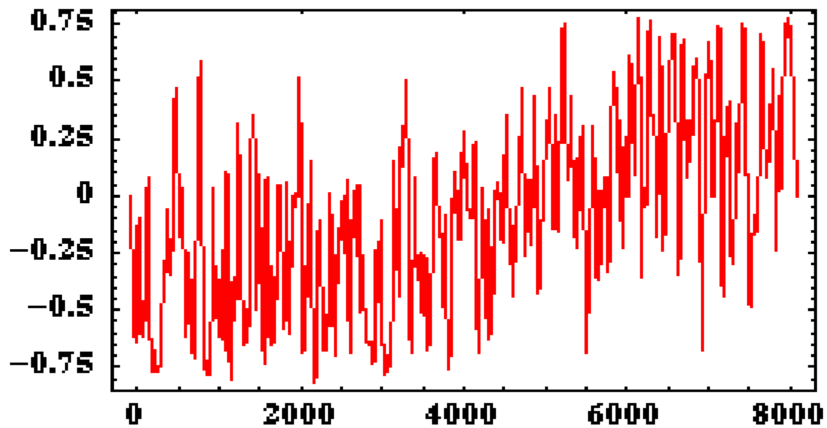
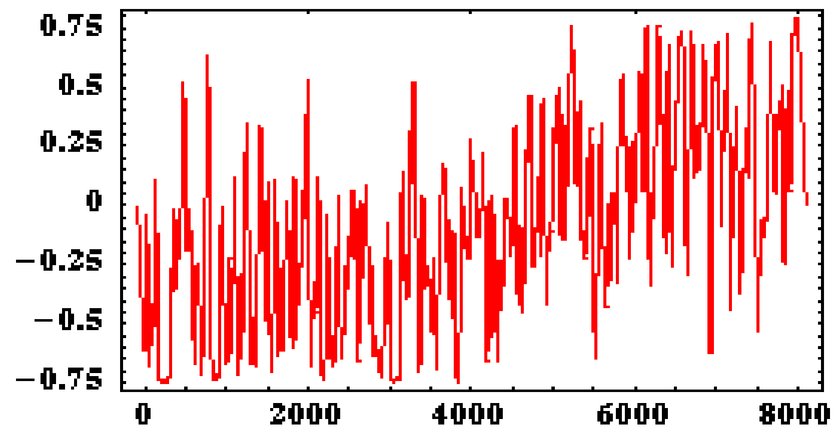
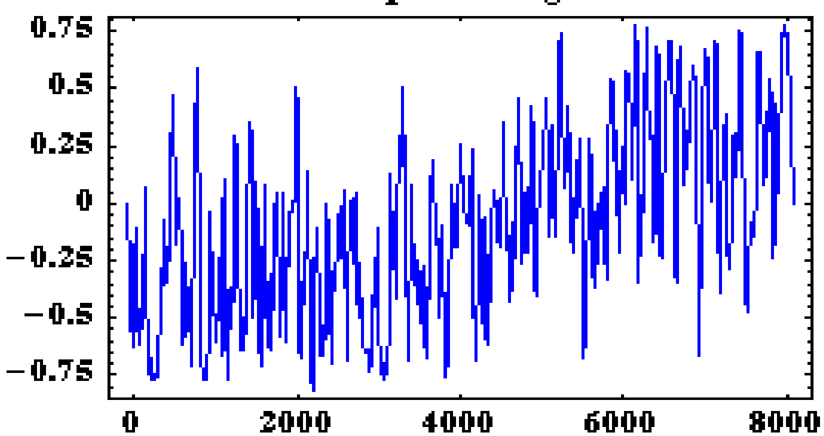
Daubechies5 (Threshold = 0.05 * Largest Coeff.)



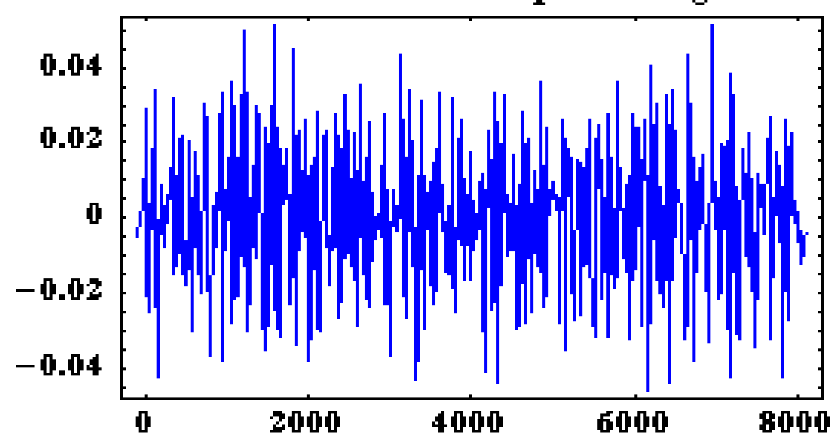
Data Being Approximated



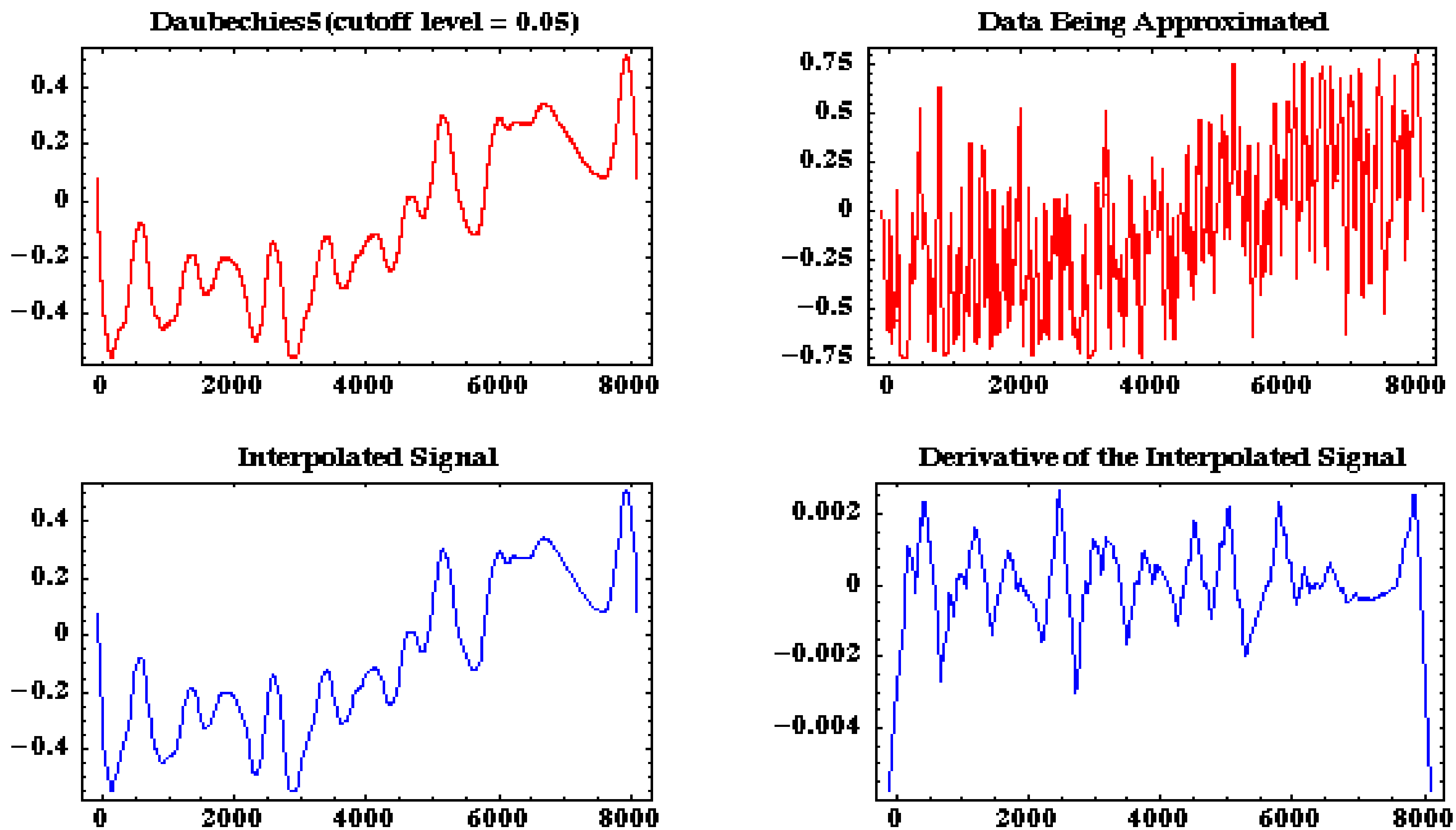
Interpolated Signal



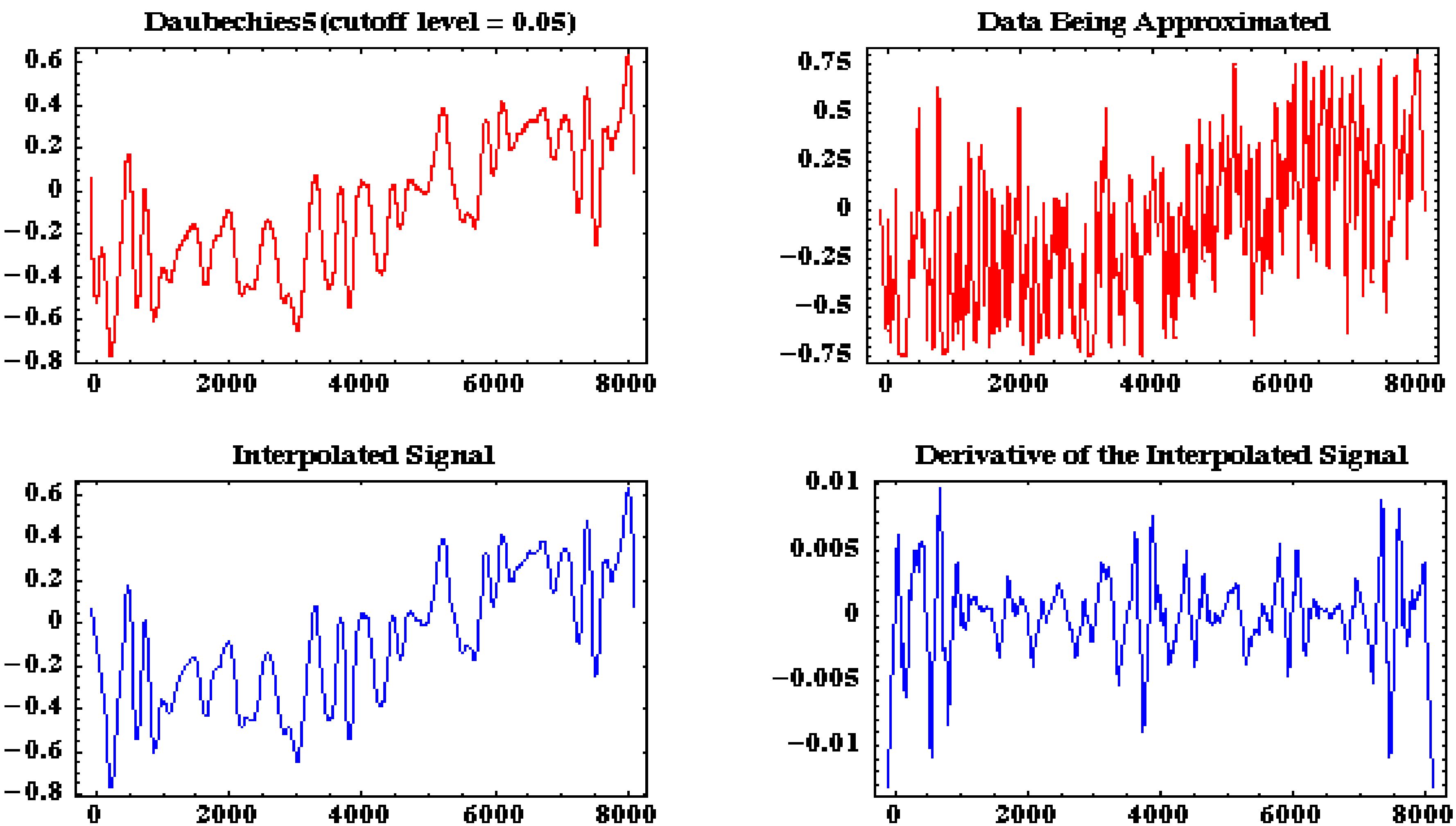
Derivative of the Interpolated Signal



Reconstruction of the LPF Data Using the First MRD Level

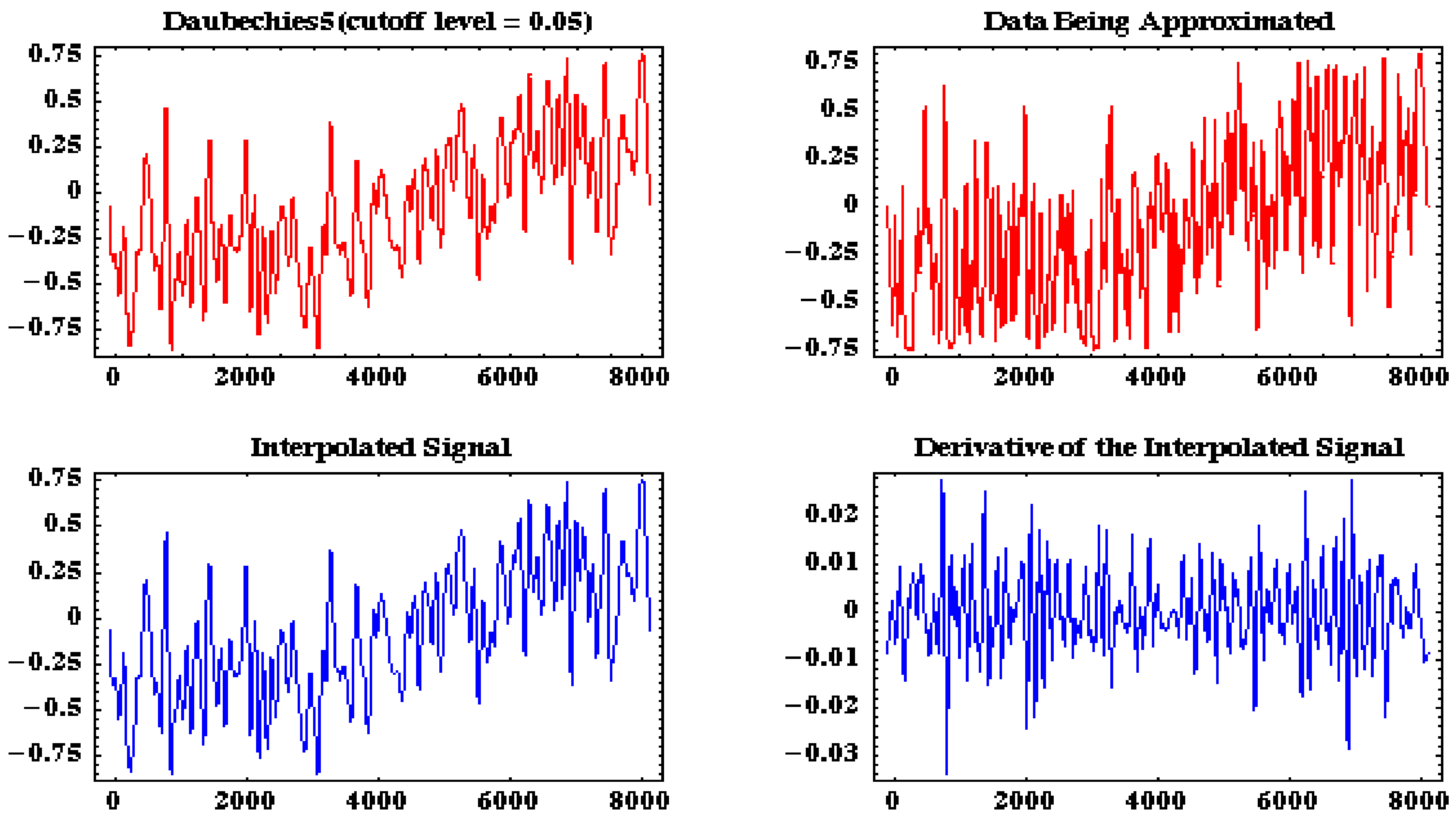


Reconstruction of the LPF Data Using the First 2 MRD Levels



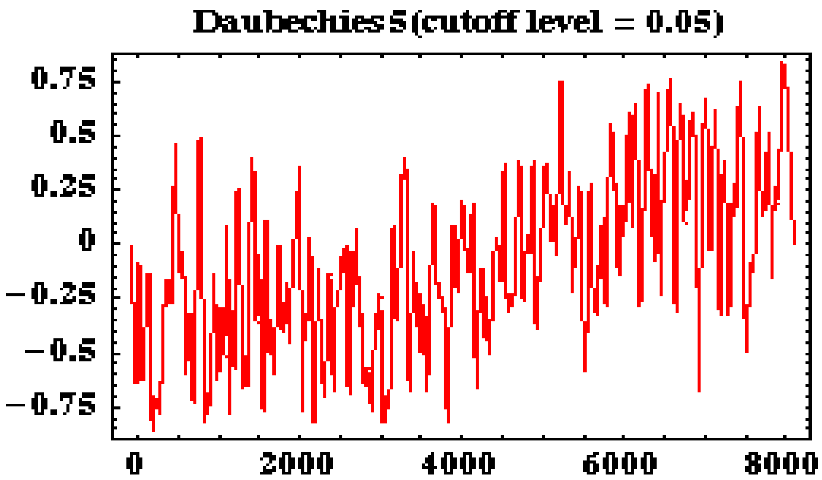
Reconstruction of the LPF Data Using the First 3 MRD Levels

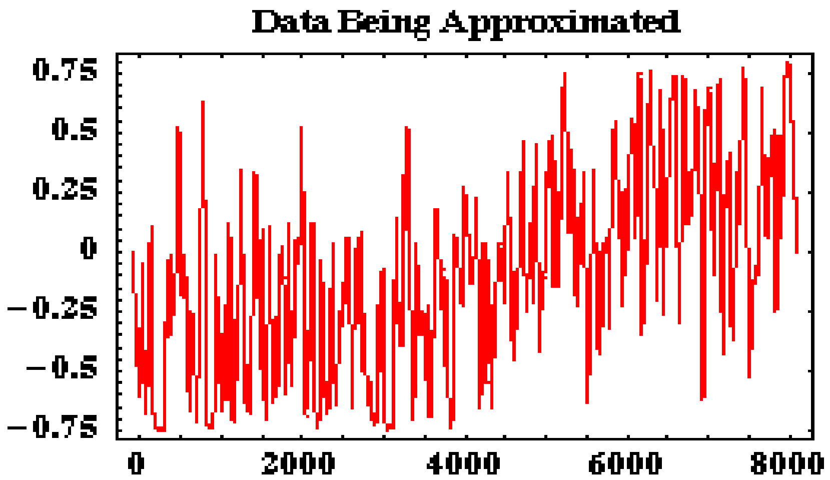
Polymath Research Inc.

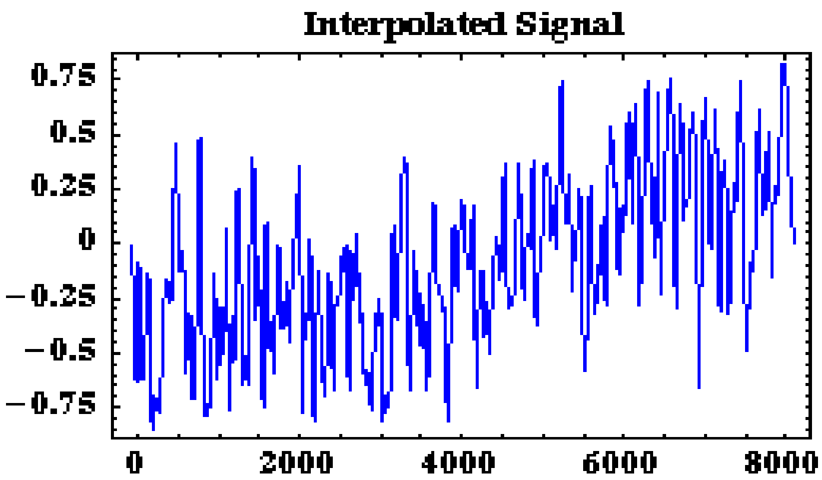


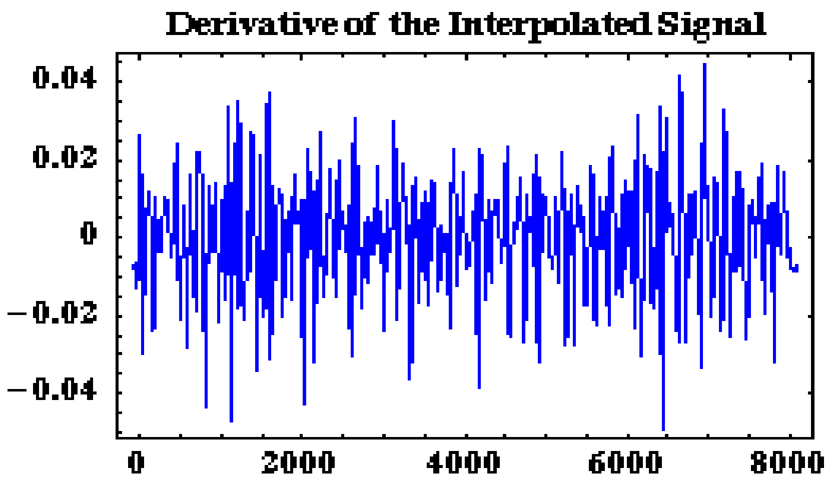
Reconstruction of the LPF Data Using the First 4 MRD Levels

Polymath Research Inc.

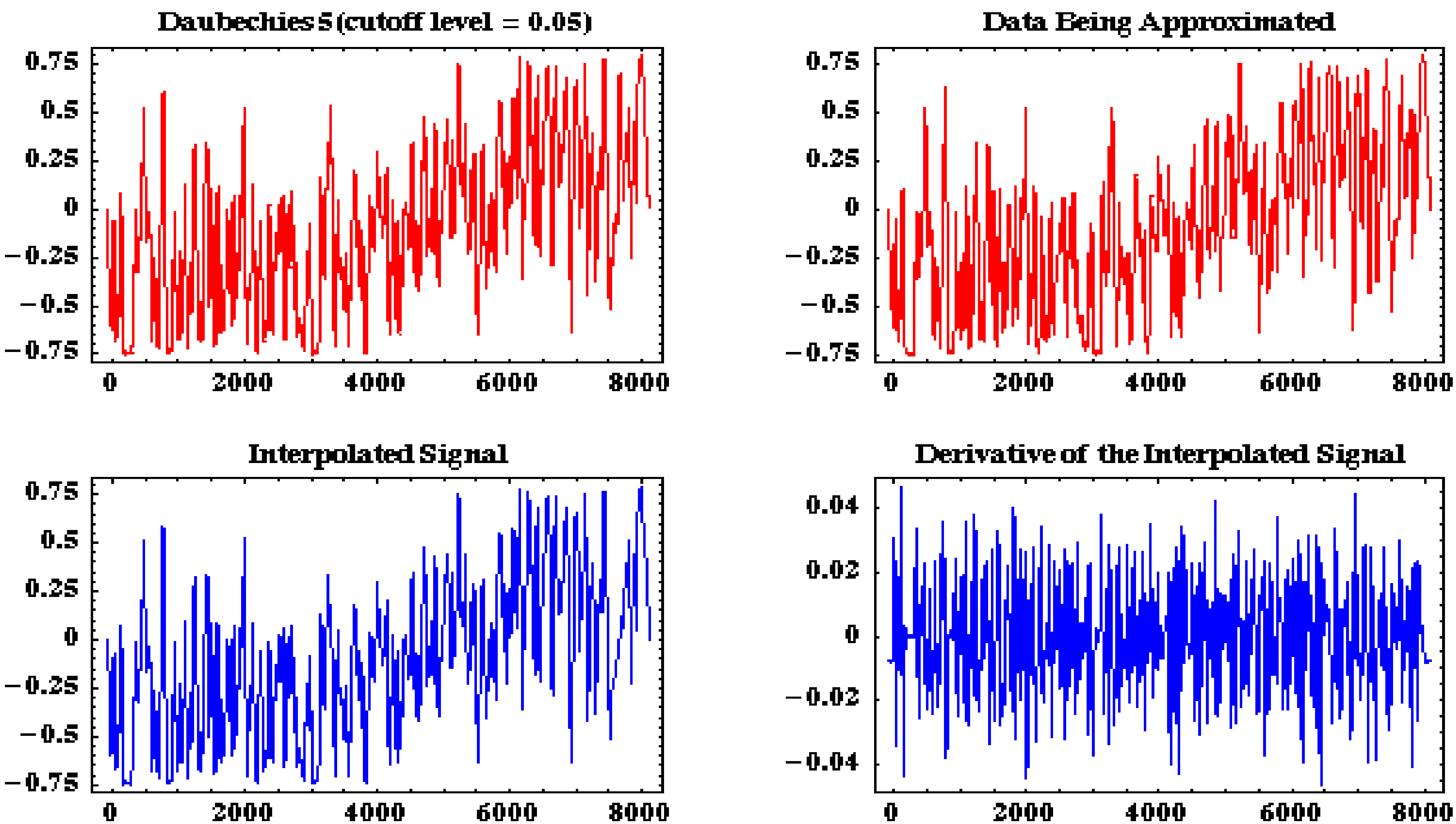








Reconstruction of the LPF Data Using the First 5 MRD Levels



Conclusions Regarding the WLT Analysis of the LPF RT Mix Data

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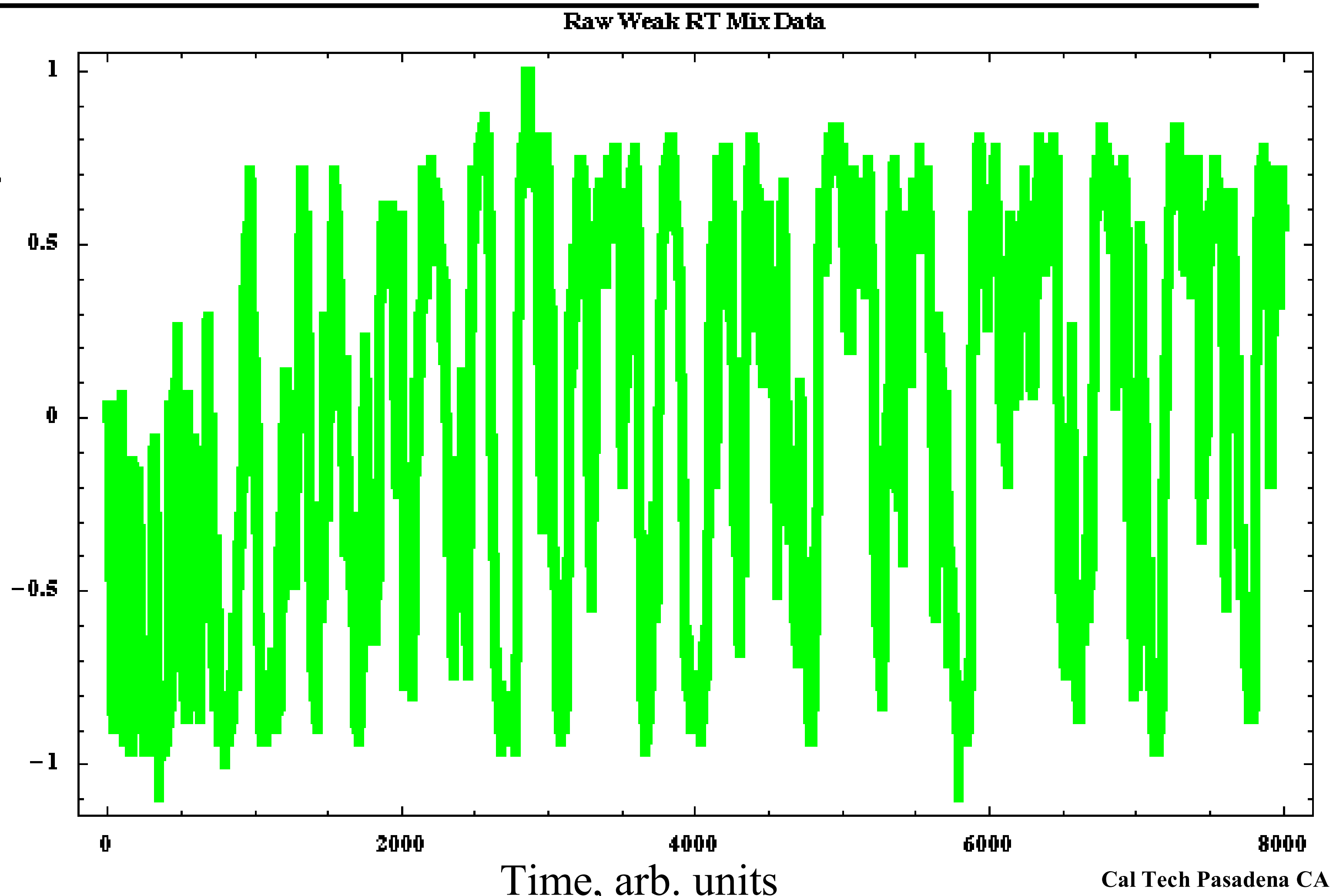
- Far better compression and denoising is achieved once a modest amount of initial low pass filtering is done on the data.
- Note the extremely small contributions levels 5 and above make to the MRD while with the unfiltered data that contribution was of order 1 or 0.1
- Far cleaner structures are observable in levels 1, 2 and 3, periodic correlations in time, or so it seems to the eye!
- The reconstruction with largest wavelets kept shows long patches of flatness surrounded by localized structures which could be indicative of the correlation properties of the data.
- More to come!

Raw RT Weak Mix Data (2 cm Downstream, Theta = 0.7) from Texas A&M

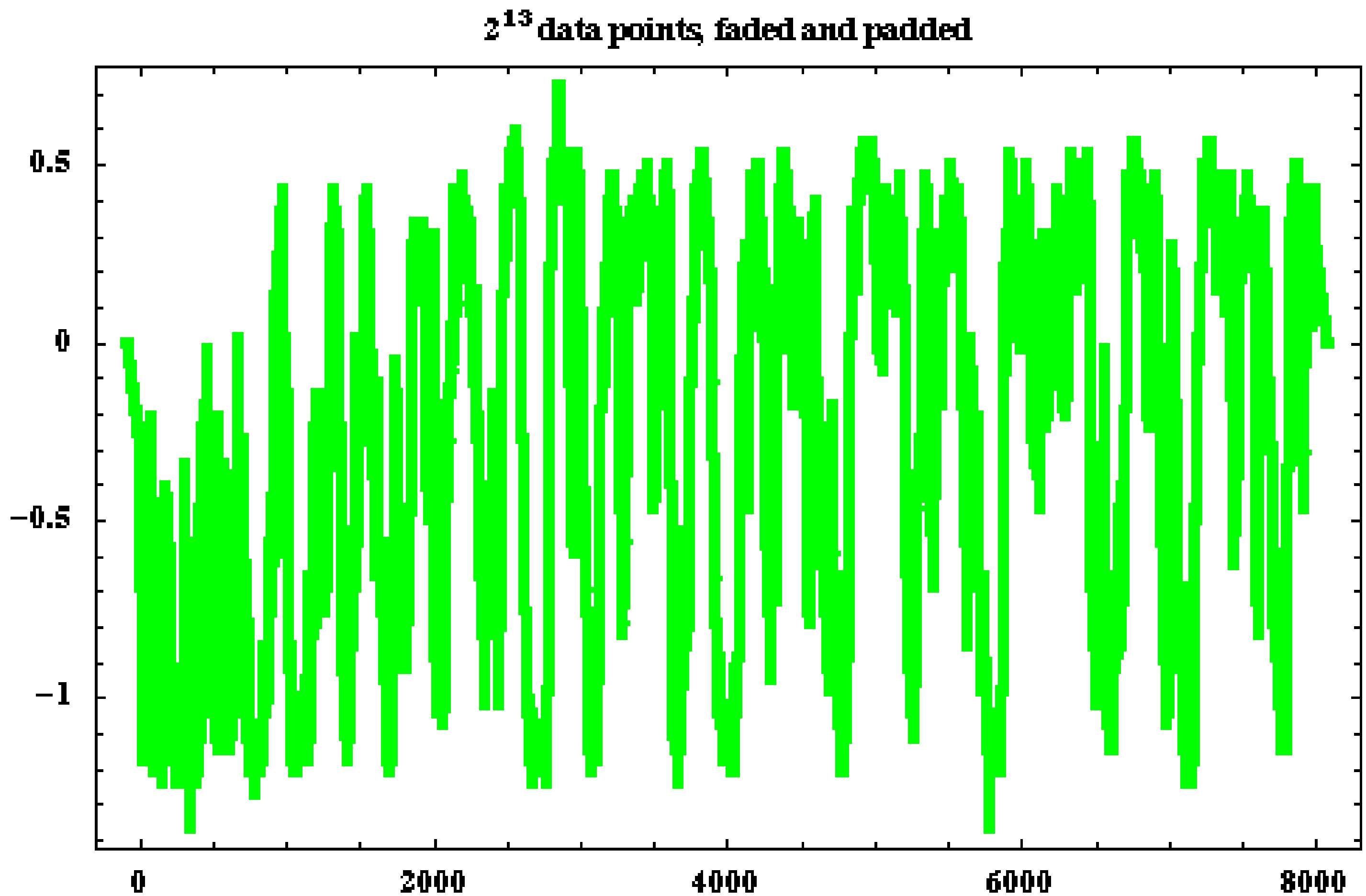
Polymath Research Inc.

$$\frac{T - T_{AVE}}{T_{MAX} - T_{AVE}}$$

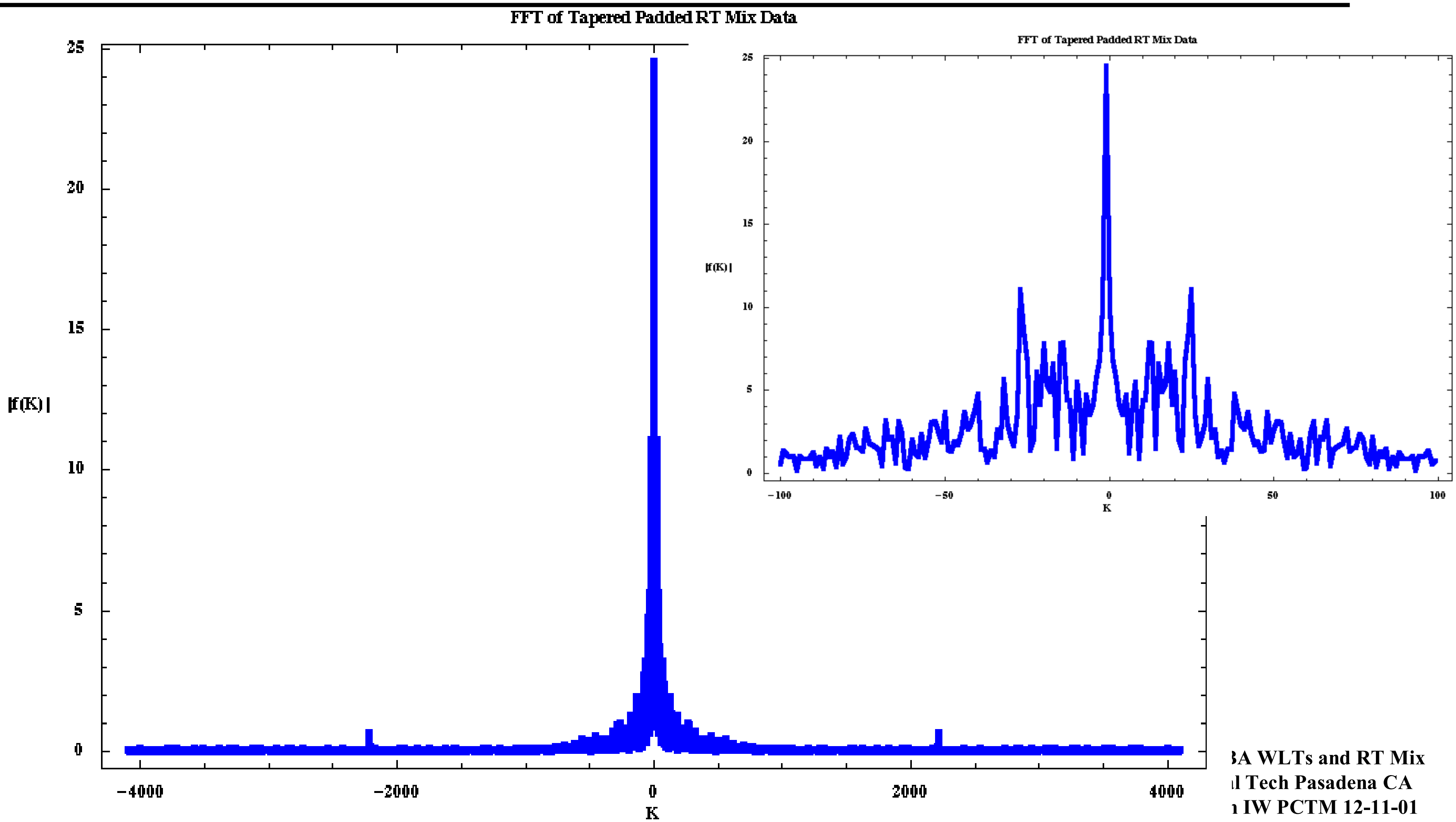


The Faded and Padded Version of the RT Weak Mix Data: 8192 Points

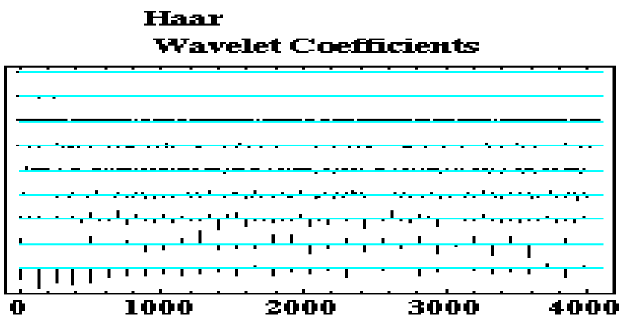
Polymath Research Inc.

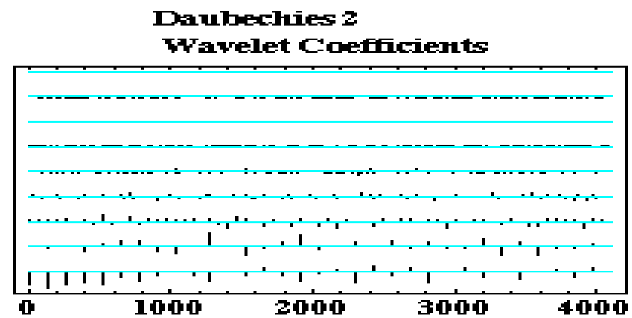


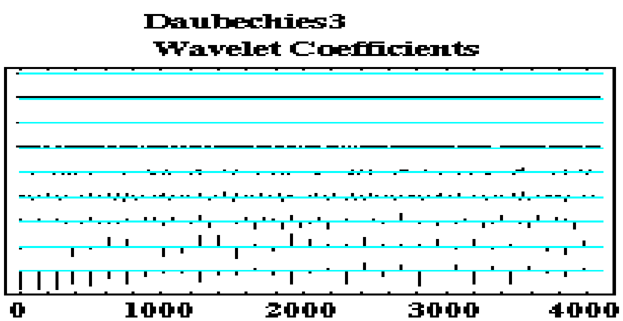
The Fourier Transform of the RT Weak Mix Data

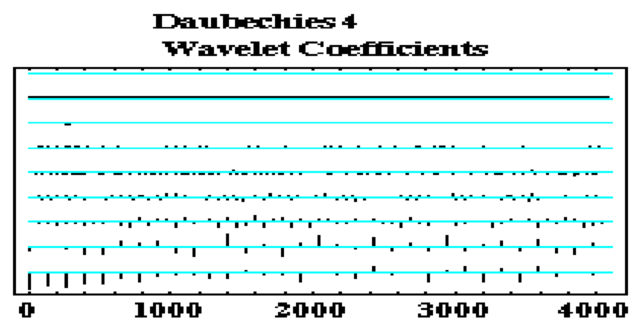


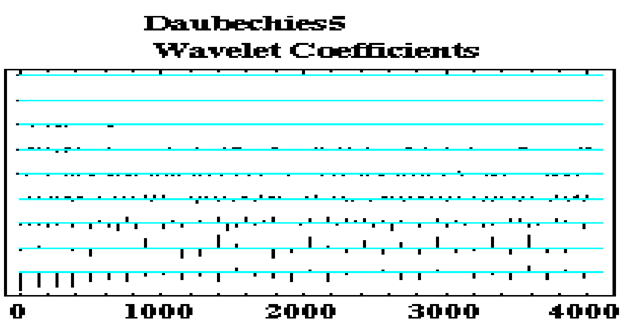
MRD Coefficients of the RT Weak Mix Data in 6 Different Daubechies WLT Bases

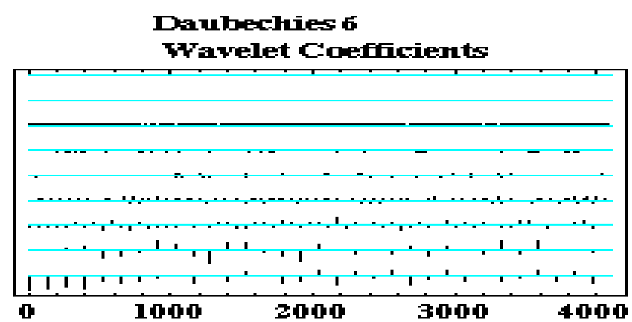




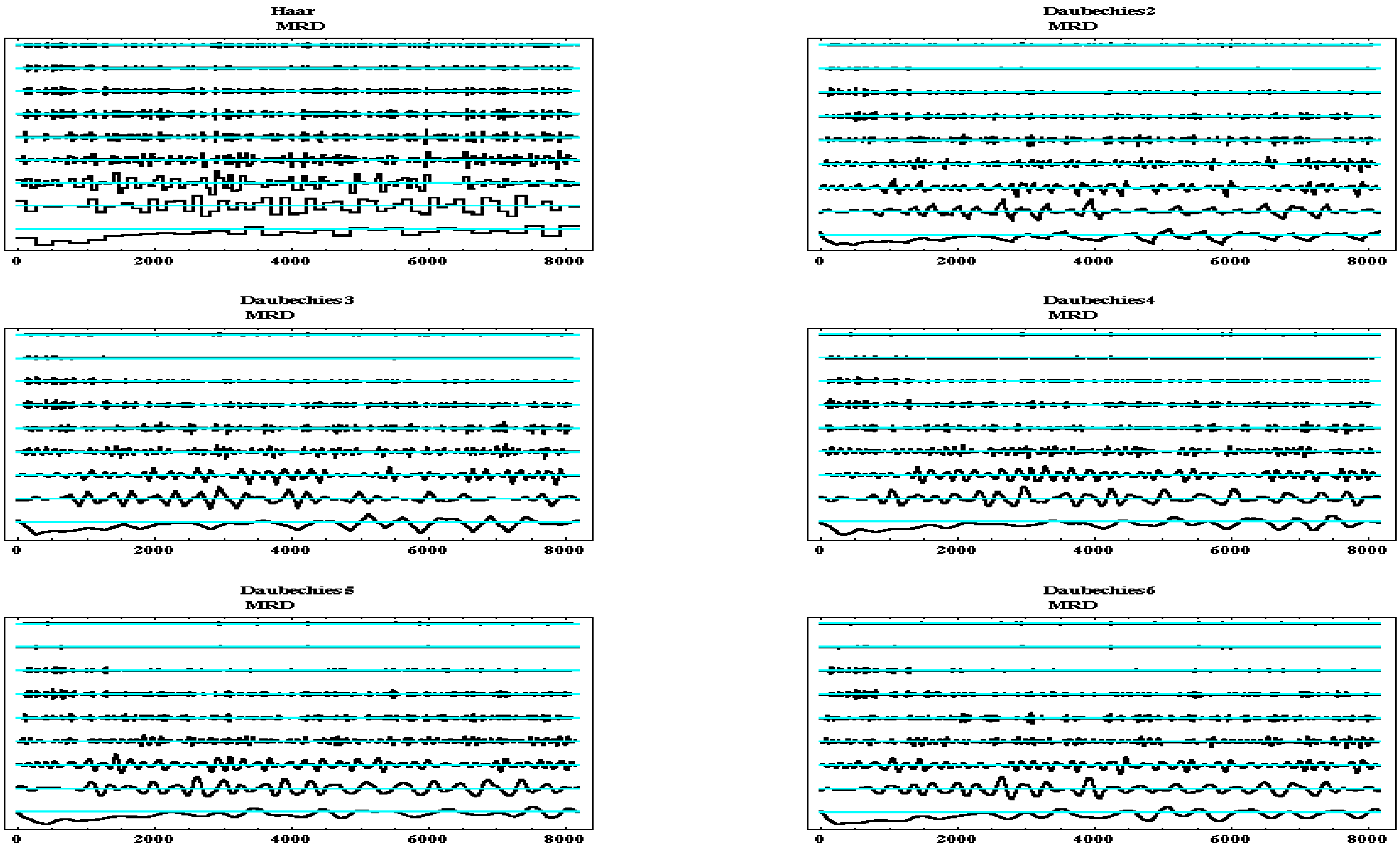






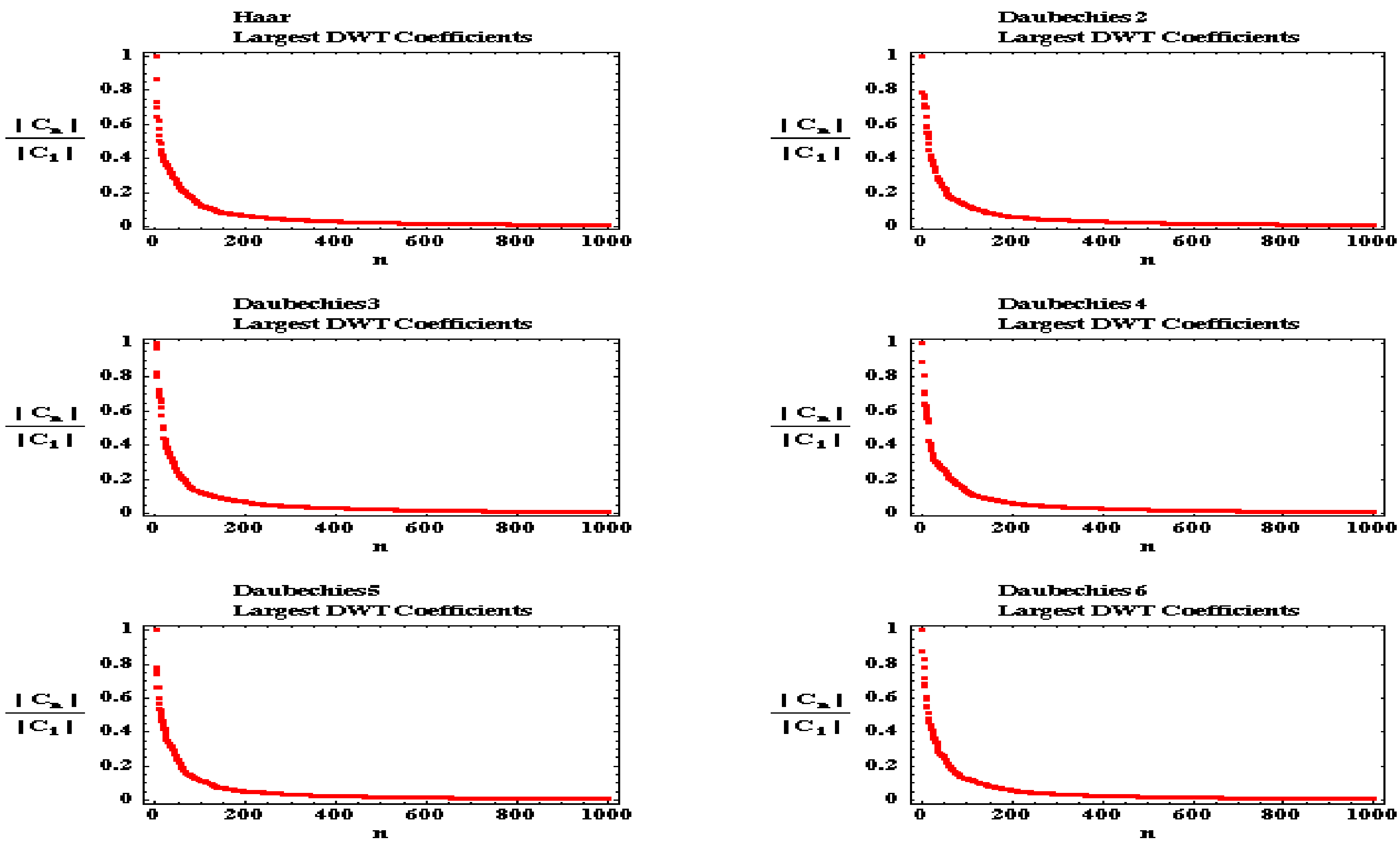


Actual MRDs of the RT Weak Mix Data in 6 Different Daubechies WLT Bases



Decay Rate of Largest Coefficient vs Number of Coefficients Kept in 6 Different Daub WLT Decomps

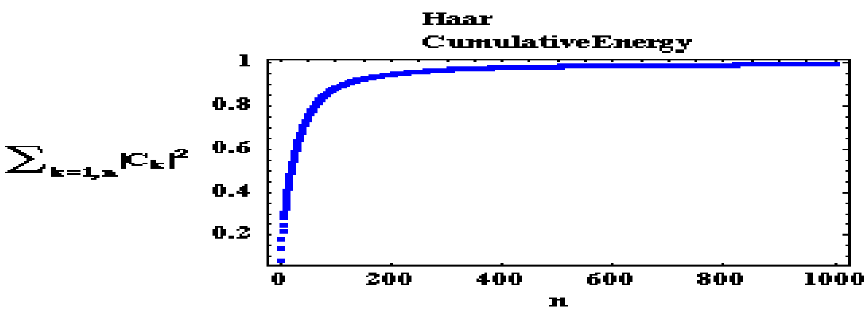
Polymath Research Inc.

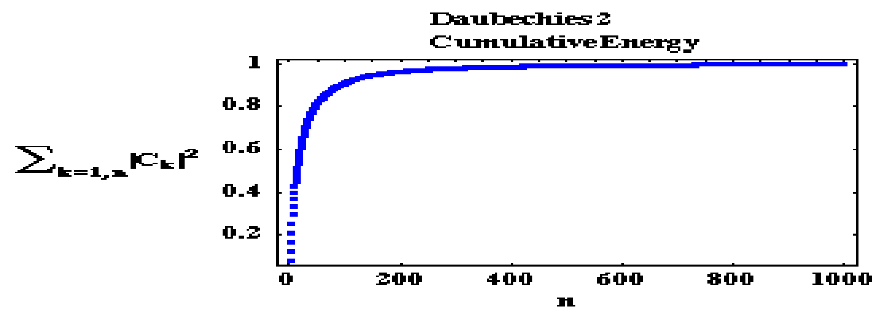


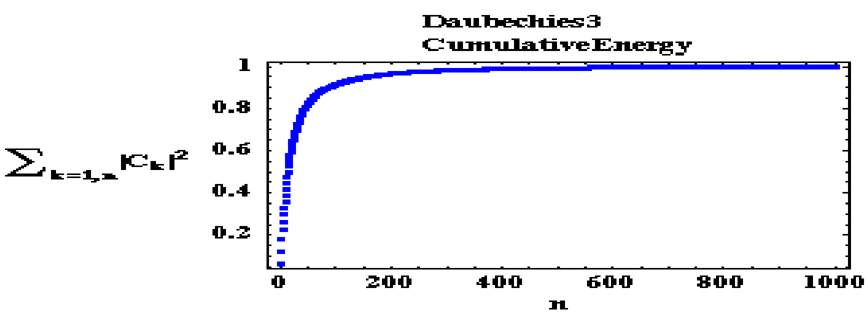
x

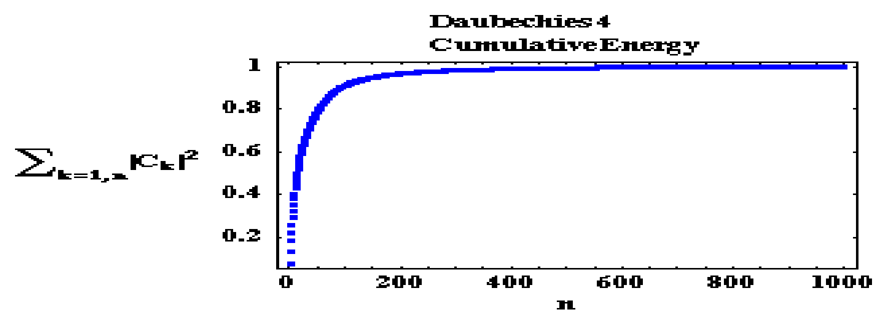
Energy Accumulation Rate in Coefficient Space vs # of WLTs Kept in 6 Different Daub Decomps

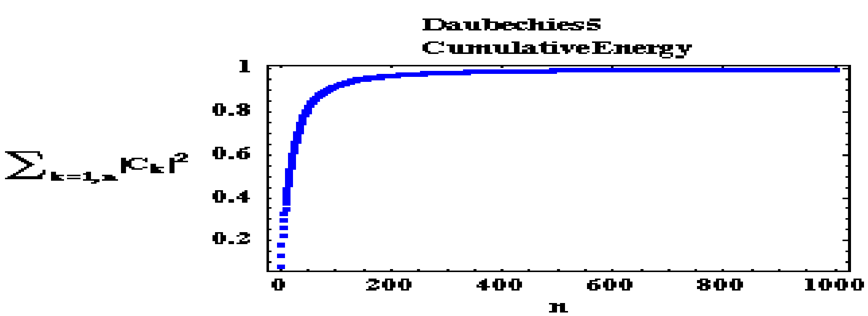
Polymath Research Inc.

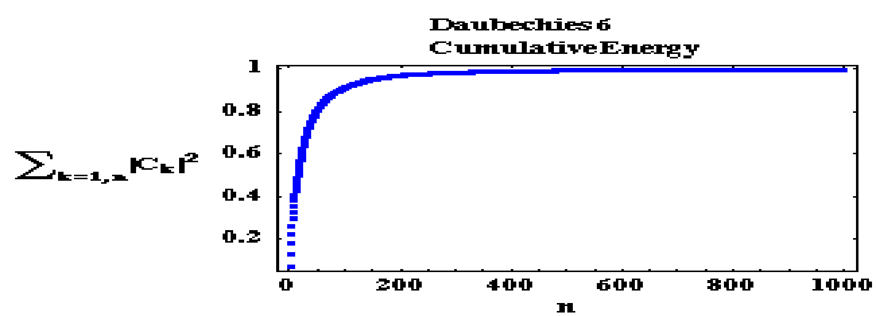




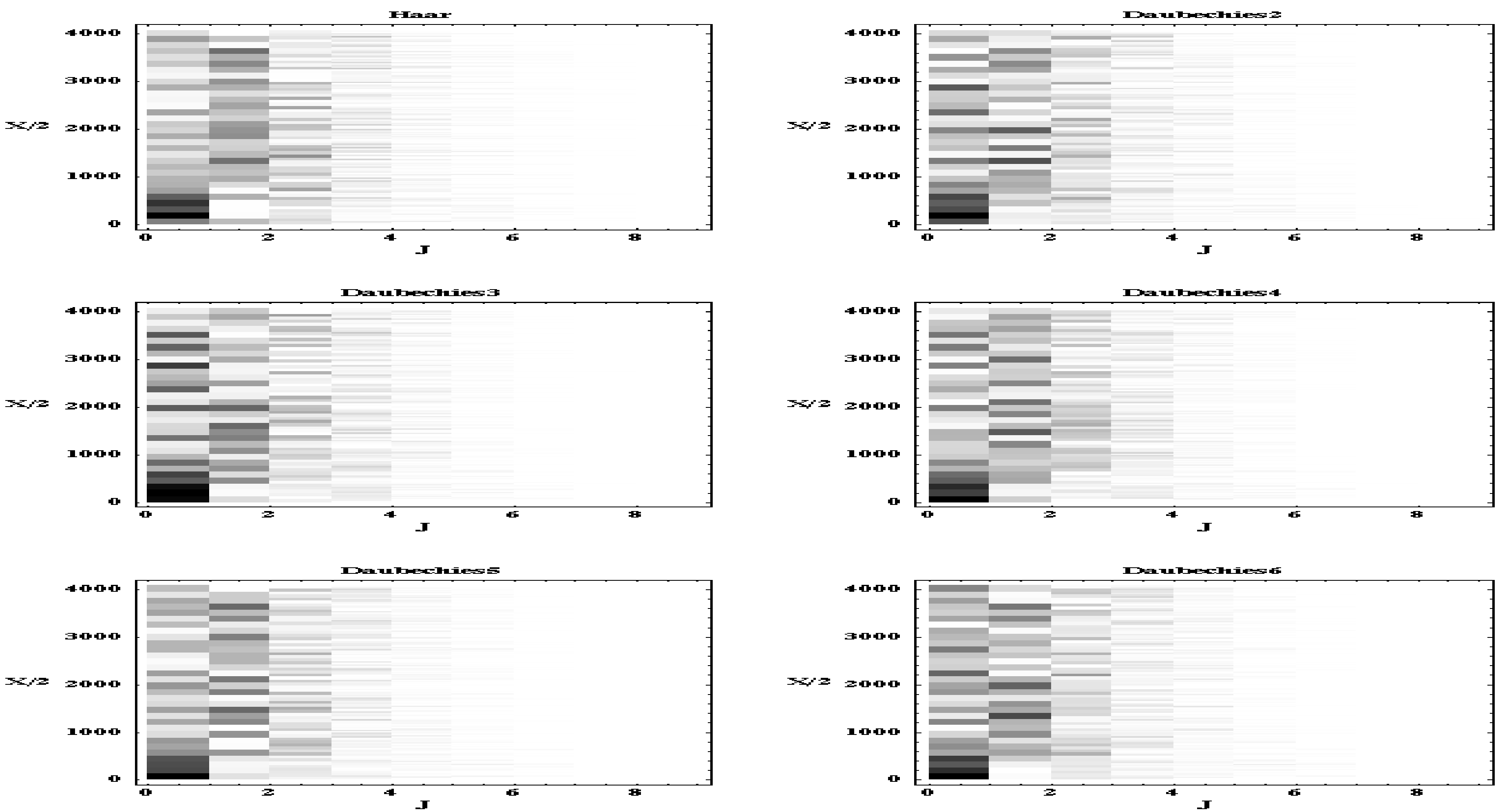






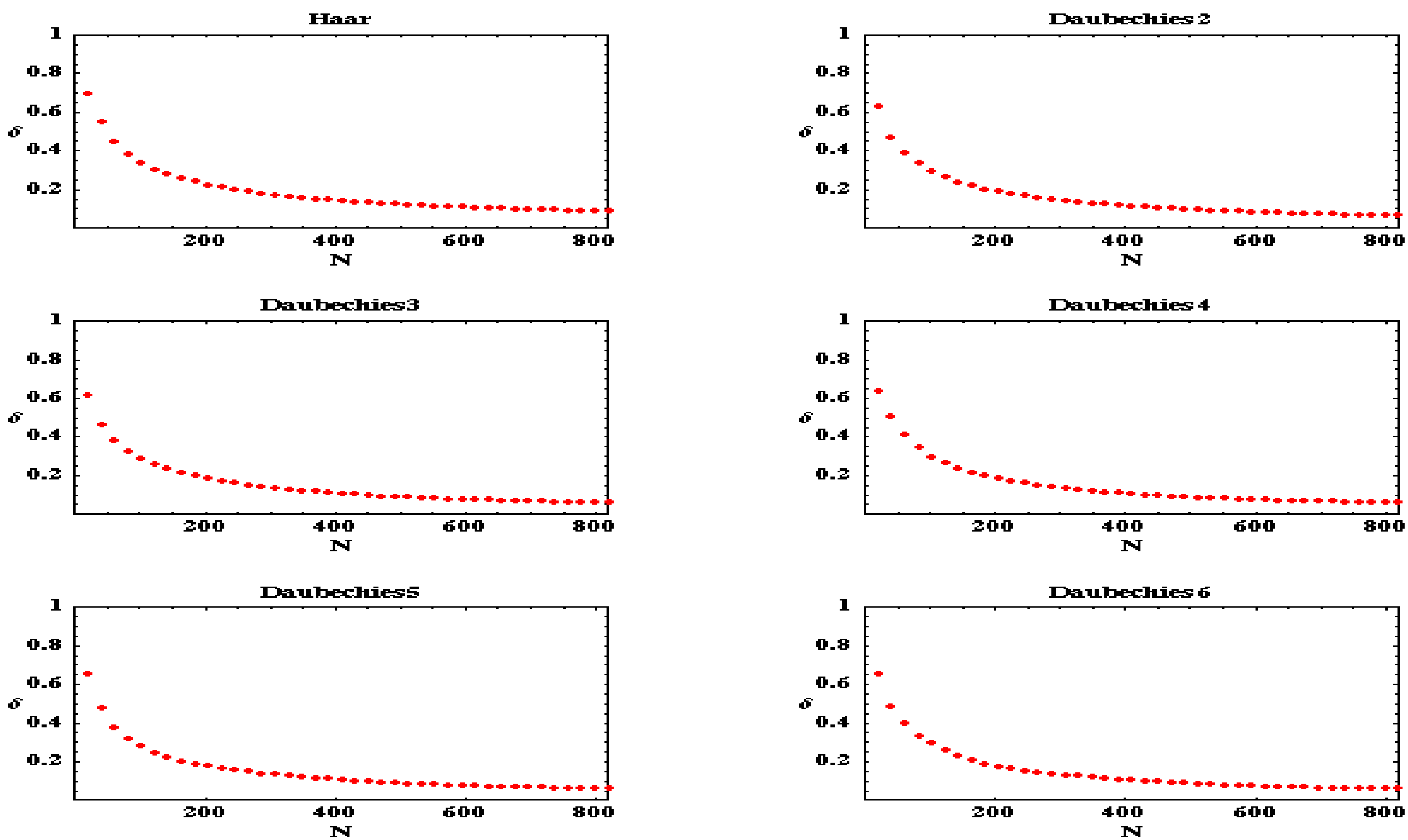


Scaleograms: Waveleters Preferred Way of Judging Tiling in Scale-Translation Space



Least Square Error Incurred By Truncating the WLT Series at N of its Largest Coefficients

Polymath Research Inc.



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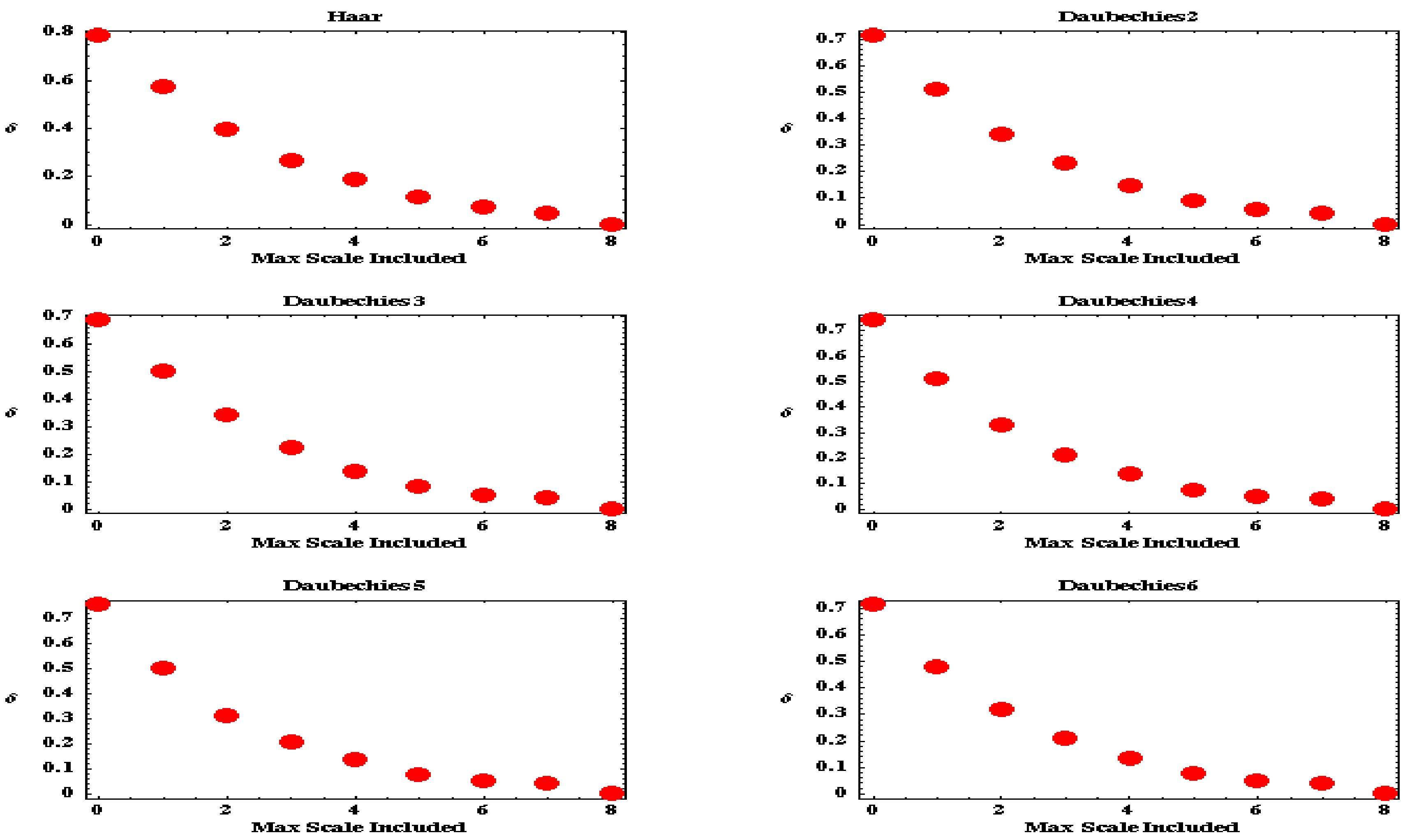
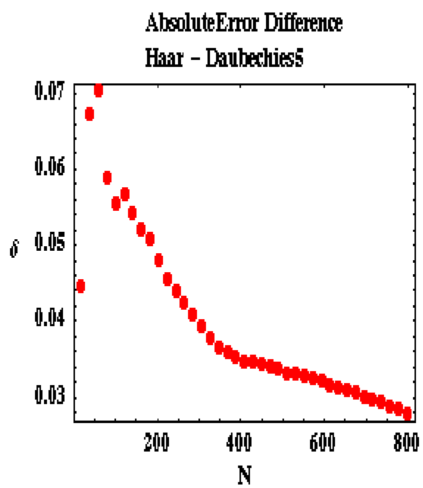
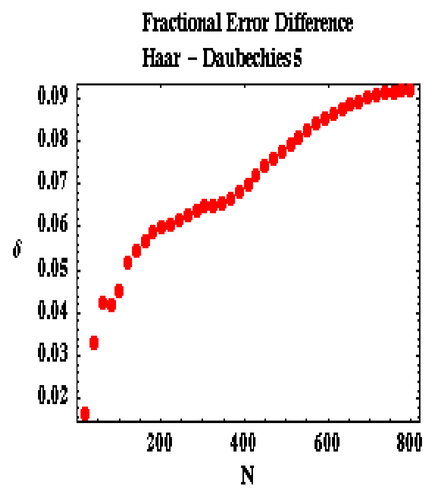
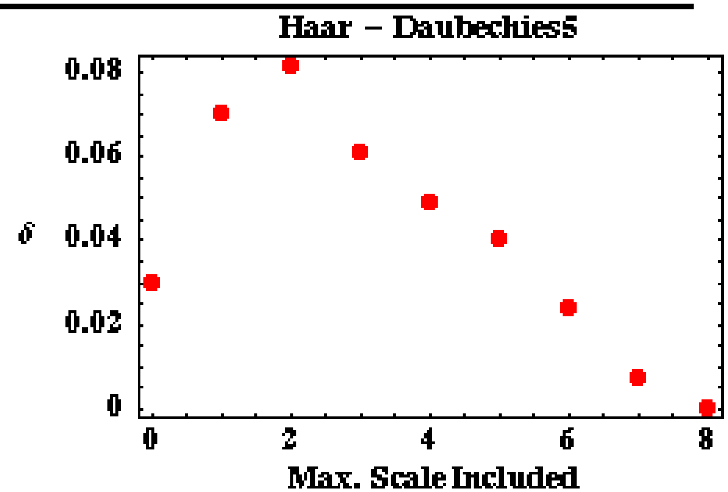
Least Square Error Incurred by Level Thresholding the DWT

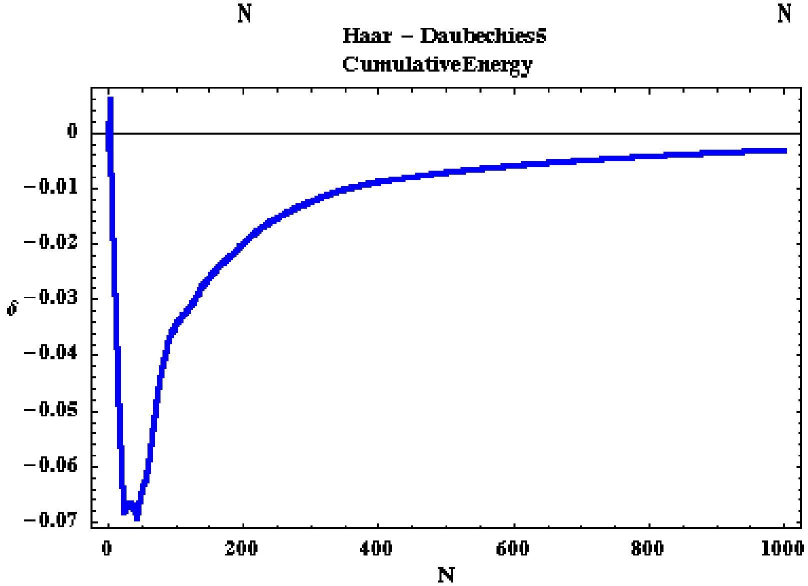


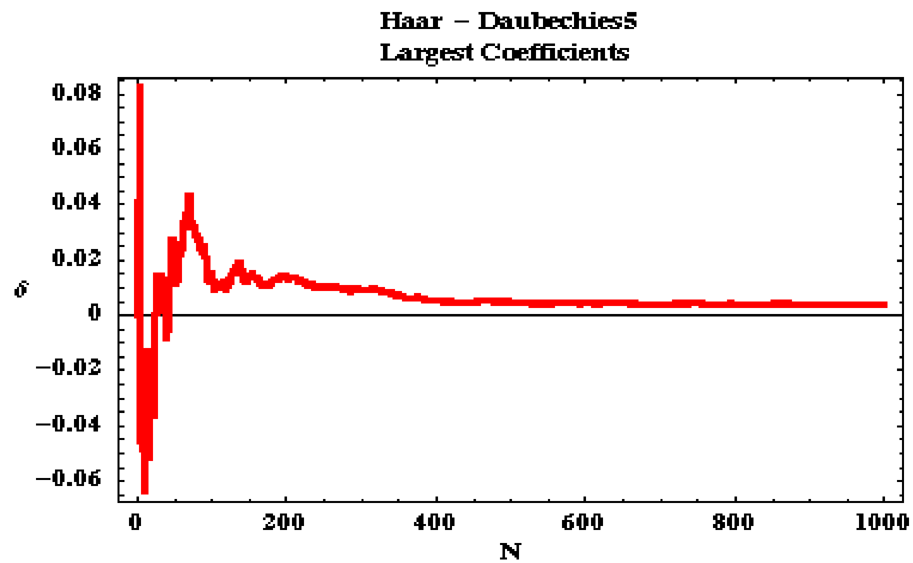
Daubechies 5 Does Much Better than Haar: 5 Quantitative Measures





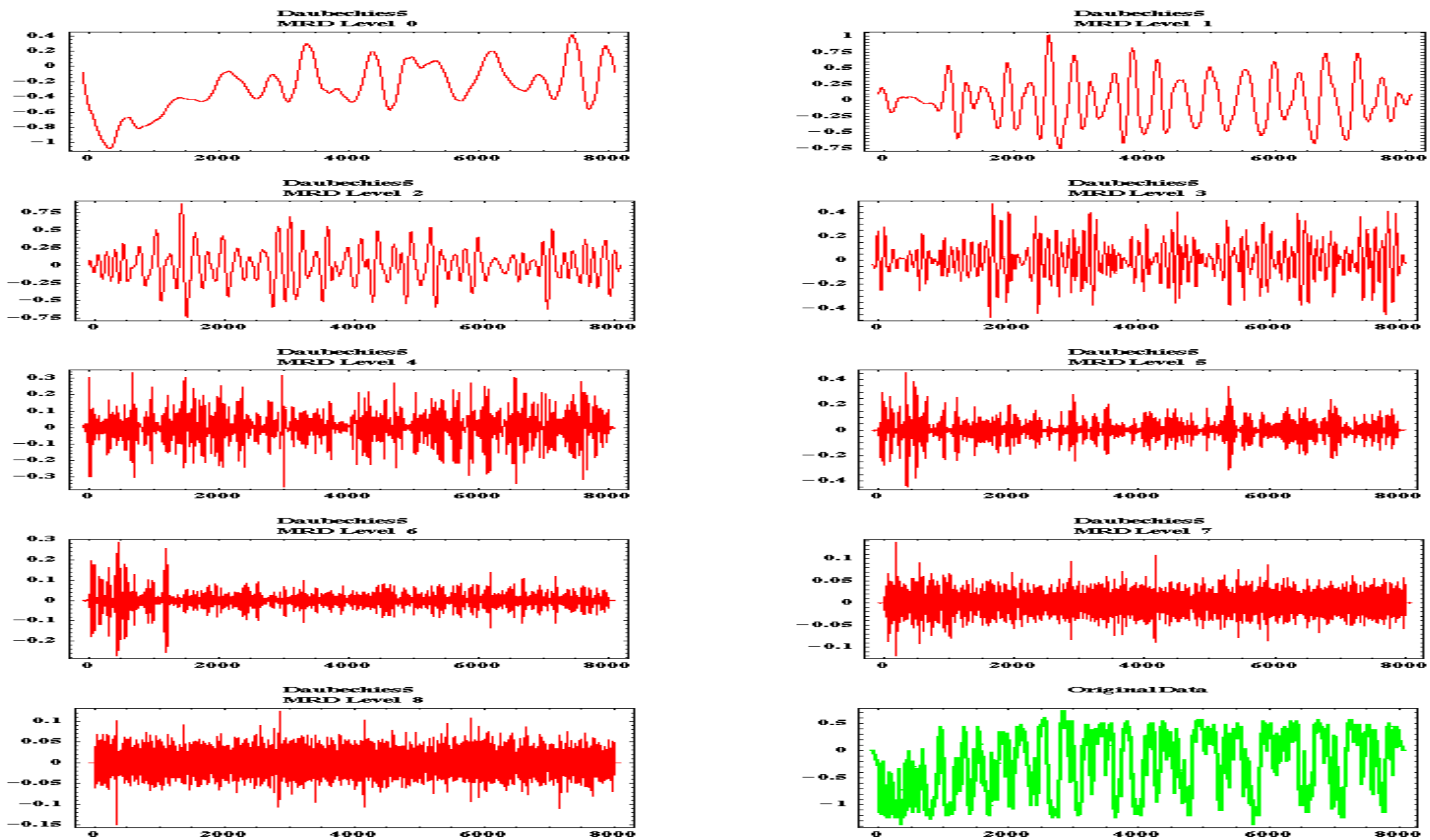






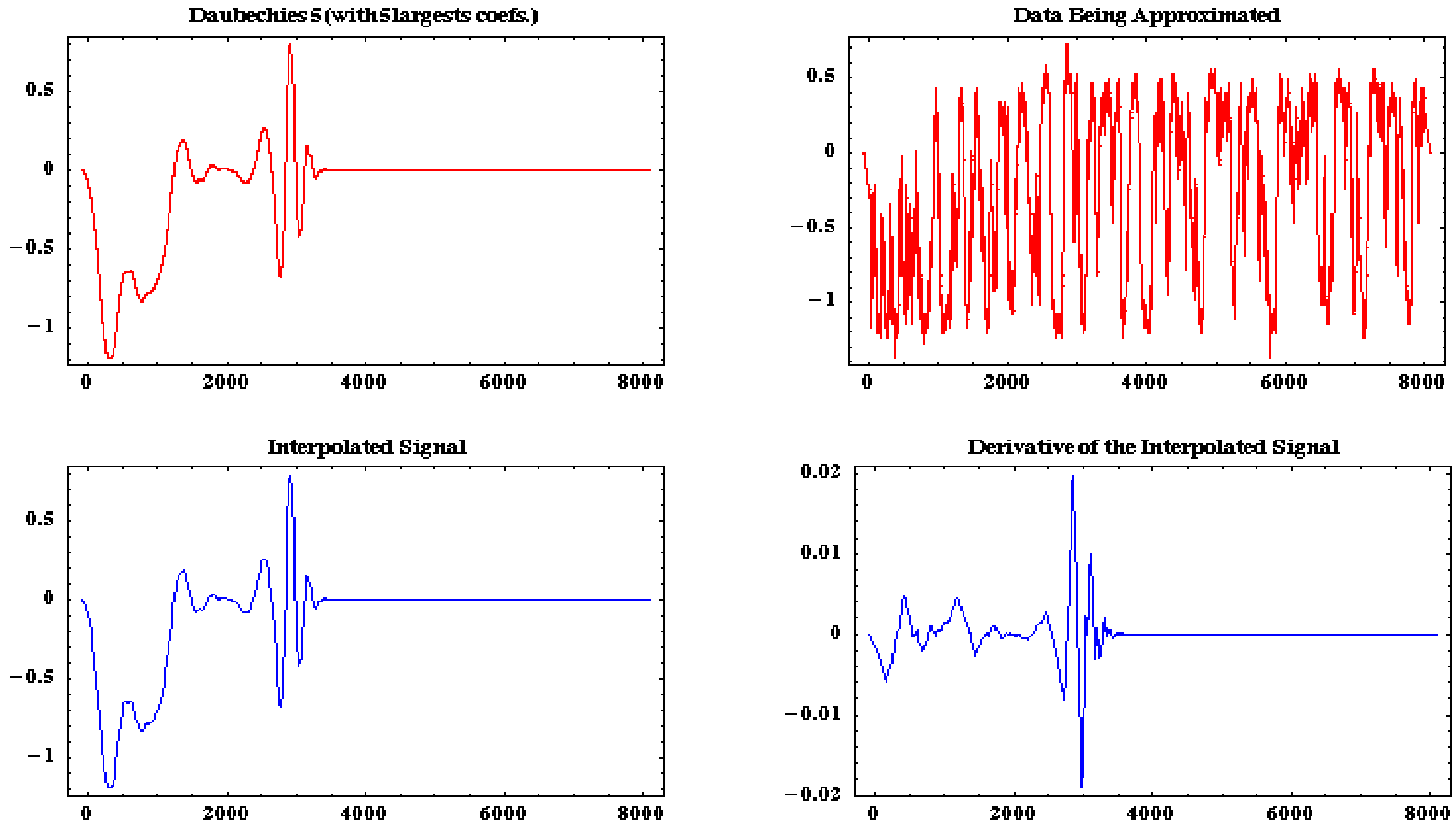
Level by Level Decomposition of the RT Weak Mix Data Using Daub5 WLTs

Polymath Research Inc.



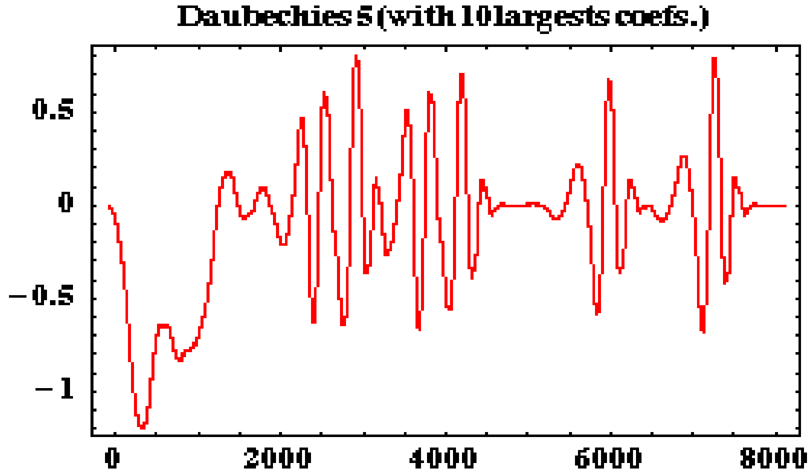
Reconstruction of the Data Using the 5 Largest WLT Coefficients

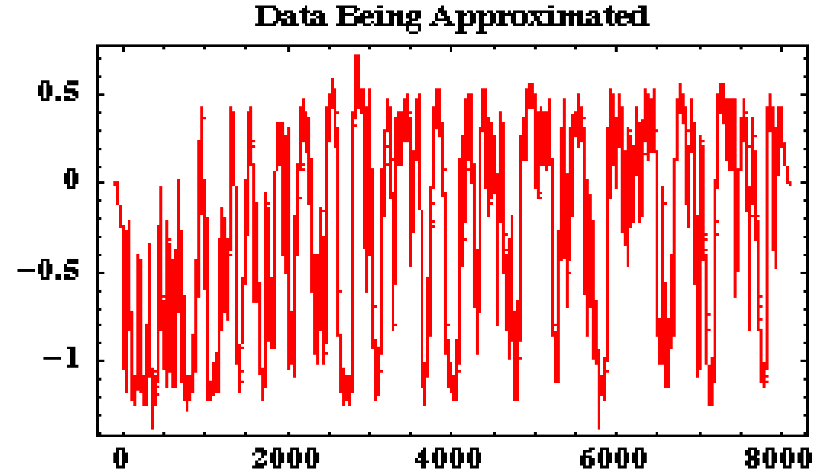
Polymath Research Inc.

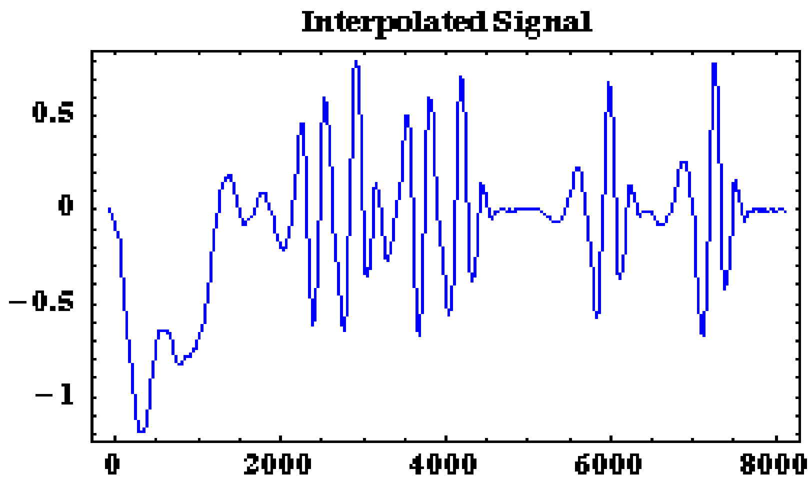


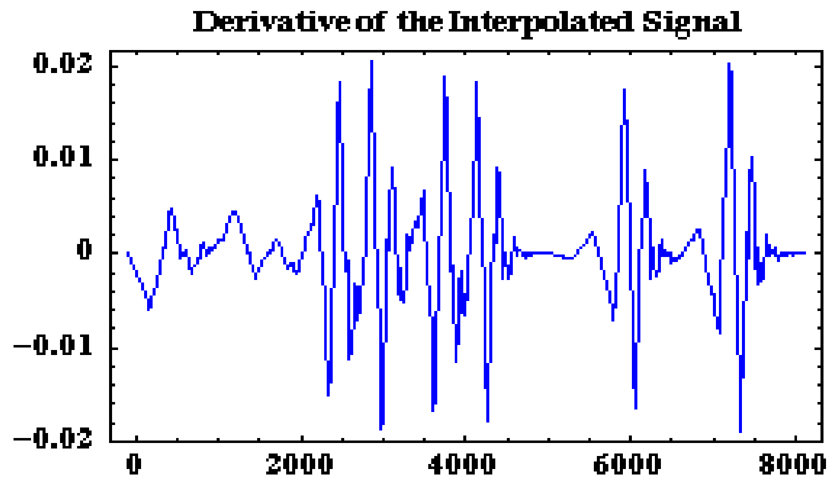
Reconstruction of the Data Using the 10 Largest WLT Coefficients

Polymath Research Inc.

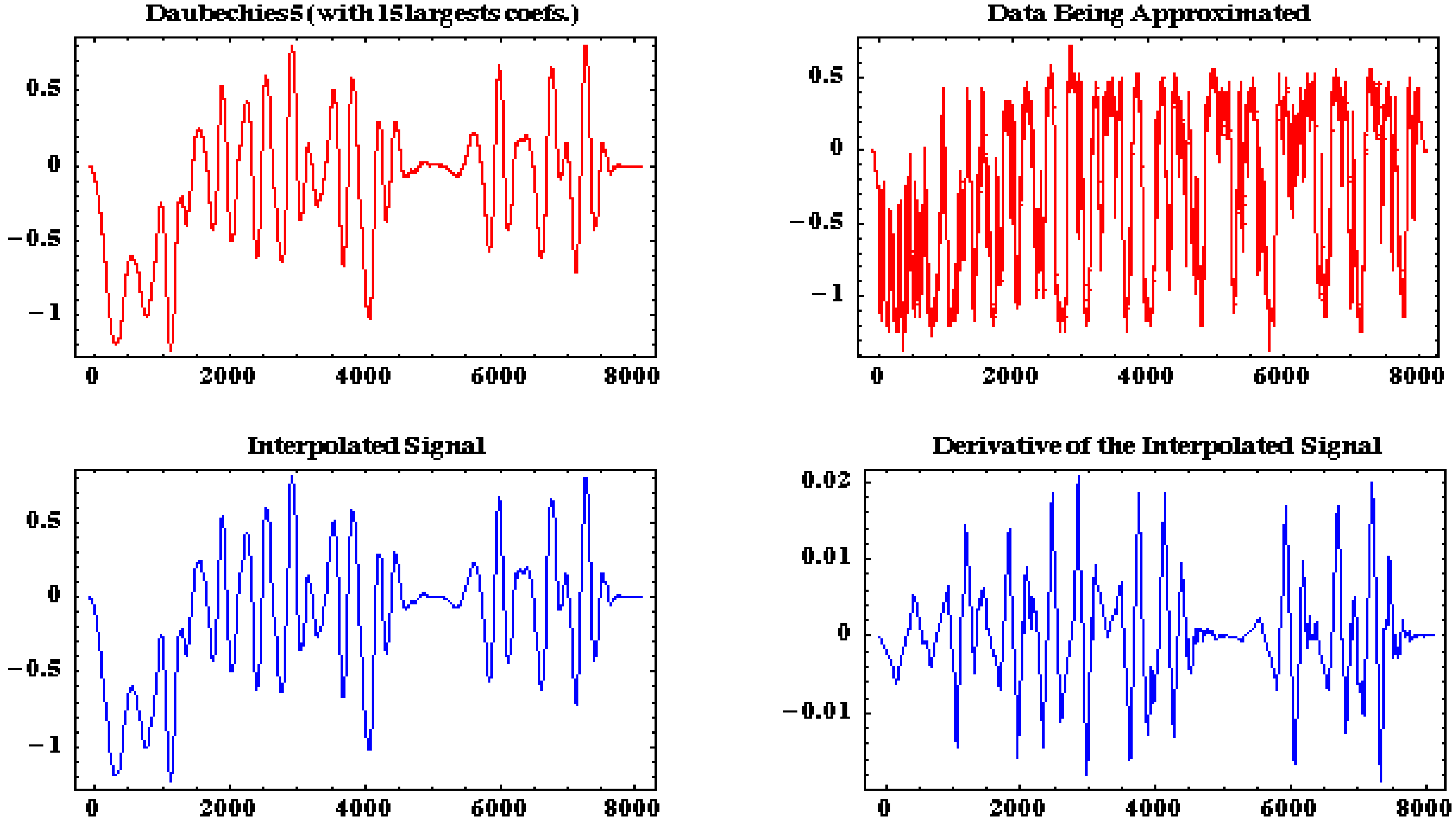






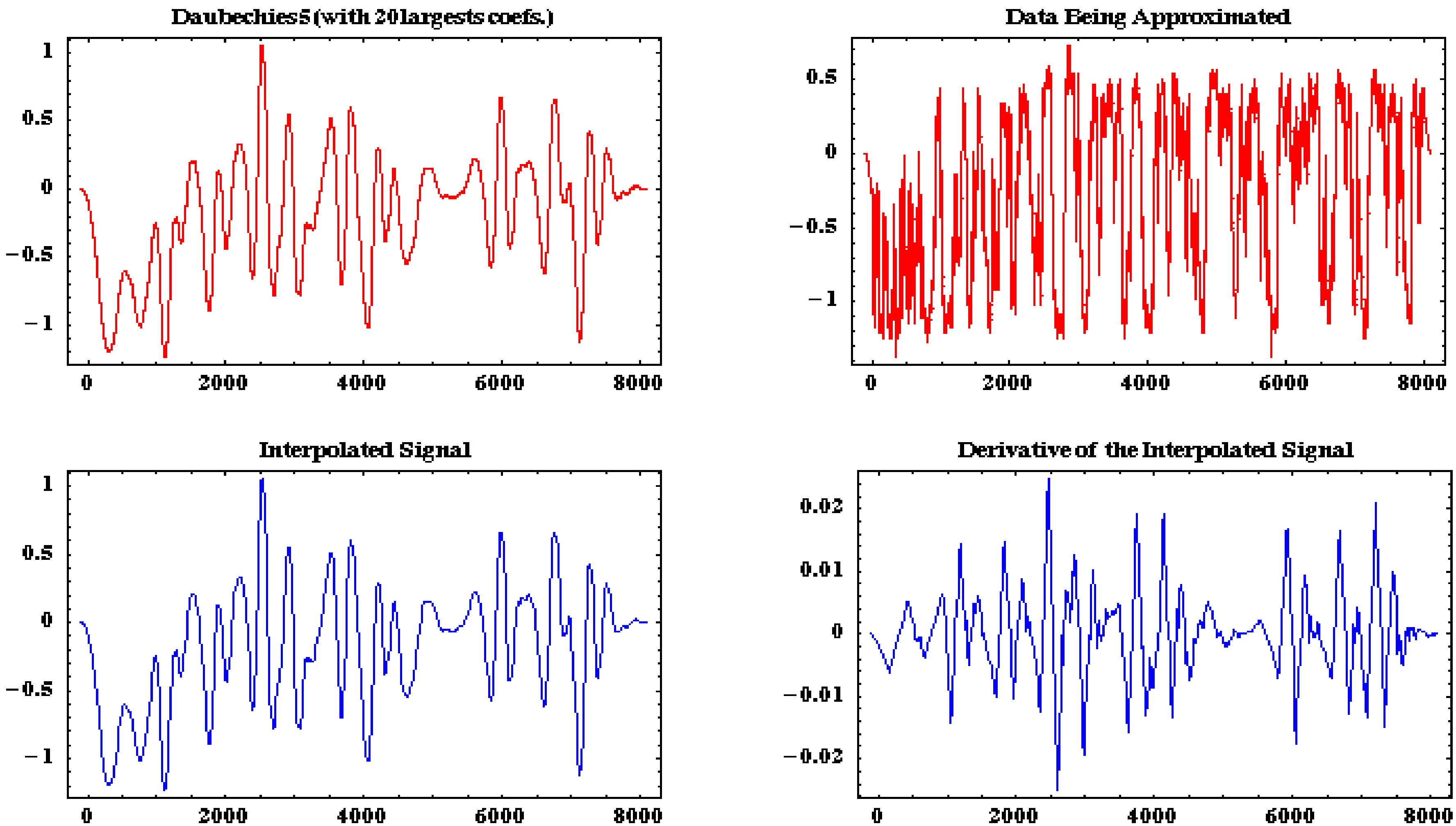


Reconstruction of the Data Using the 15 Largest WLT Coefficients



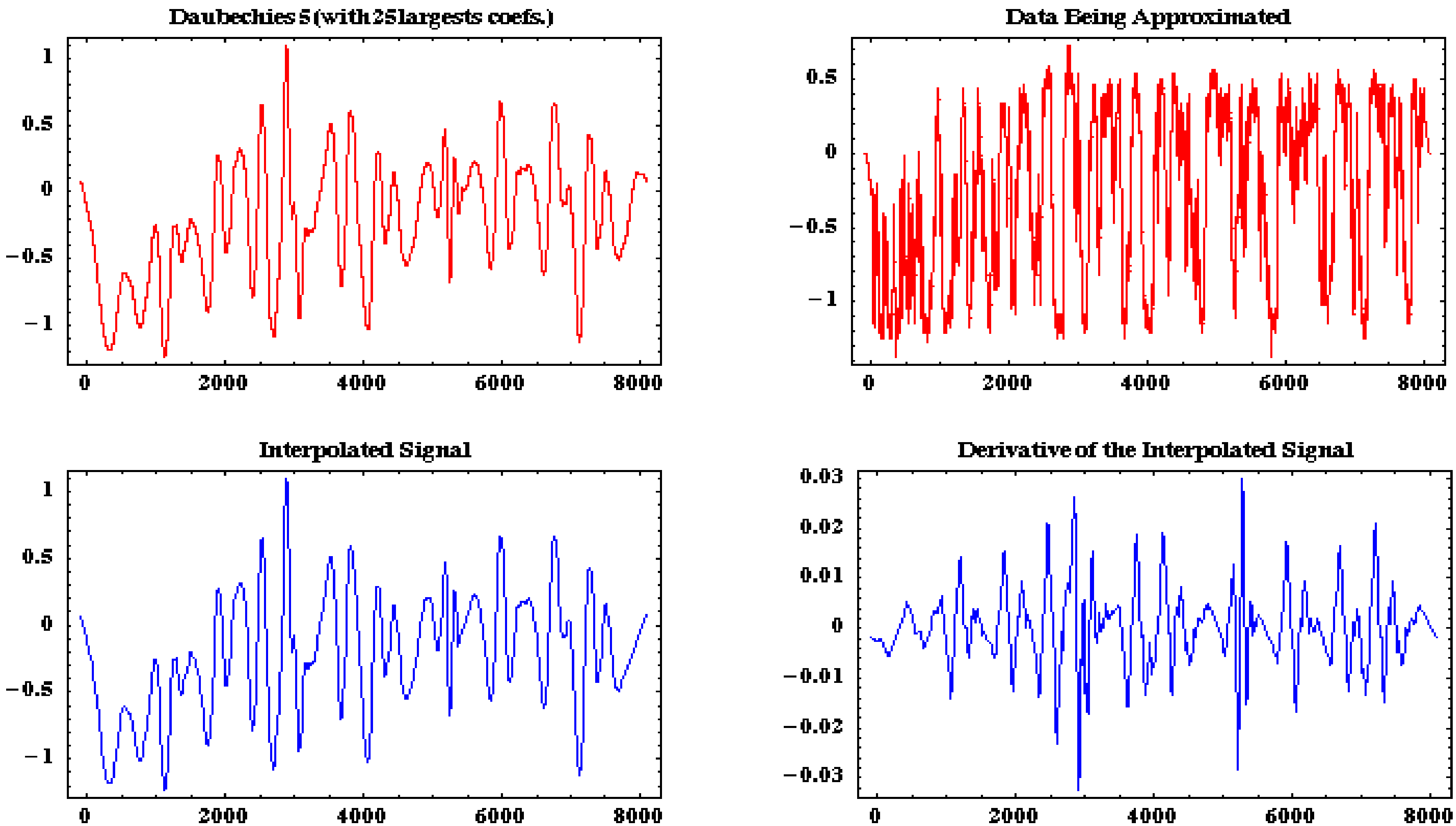
Reconstruction of the Data Using the 20 Largest WLT Coefficients

Polymath Research Inc.

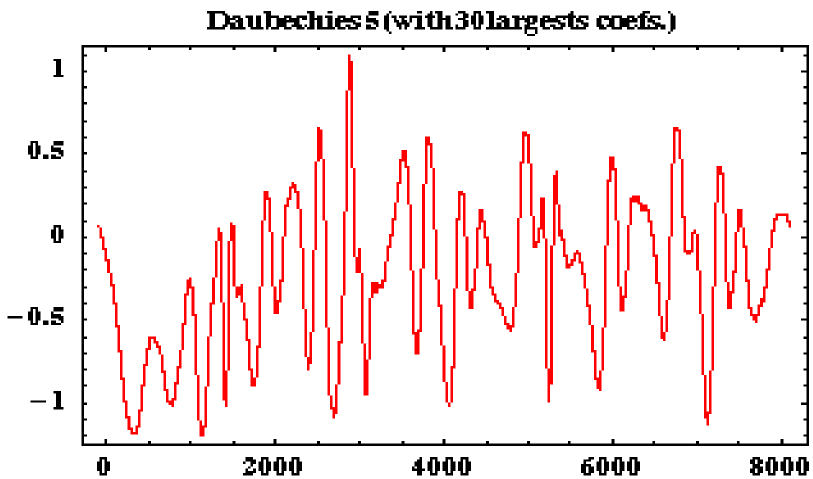


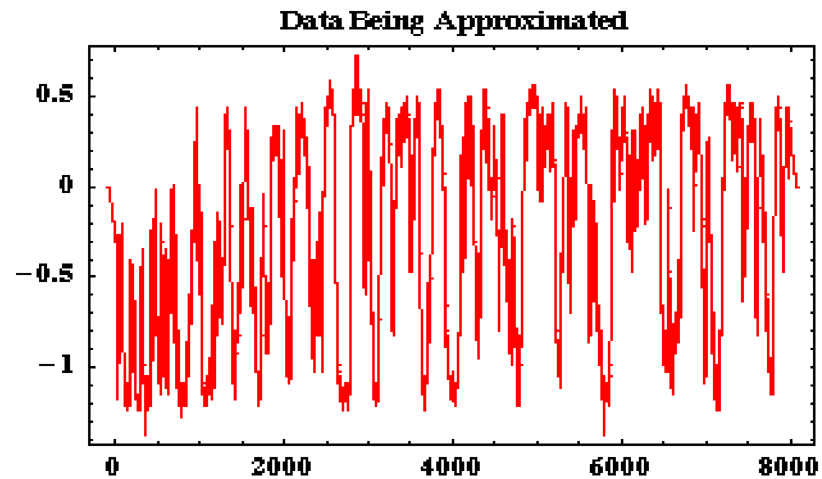
Reconstruction of the Data Using the 25 Largest WLT Coefficients

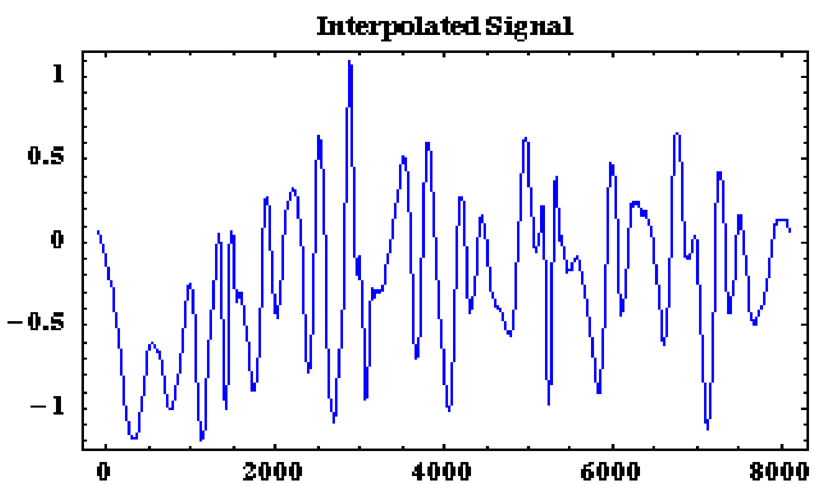
Polymath Research Inc.

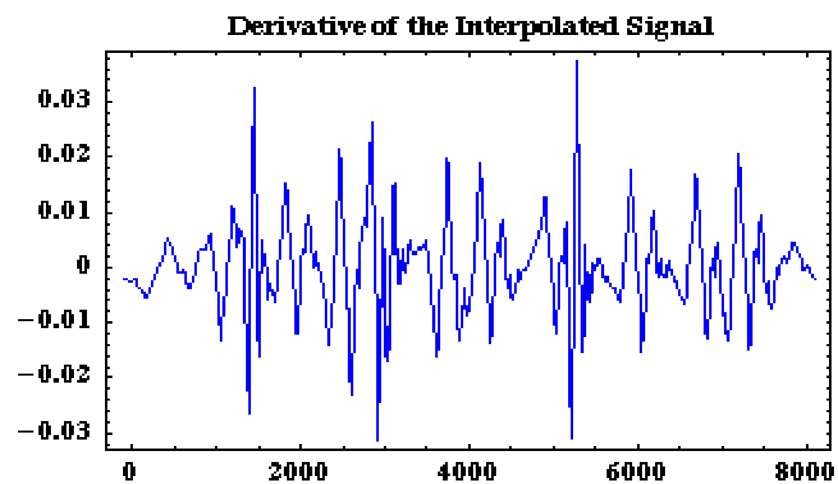


Reconstruction of the Data Using the 30 Largest WLT Coefficients



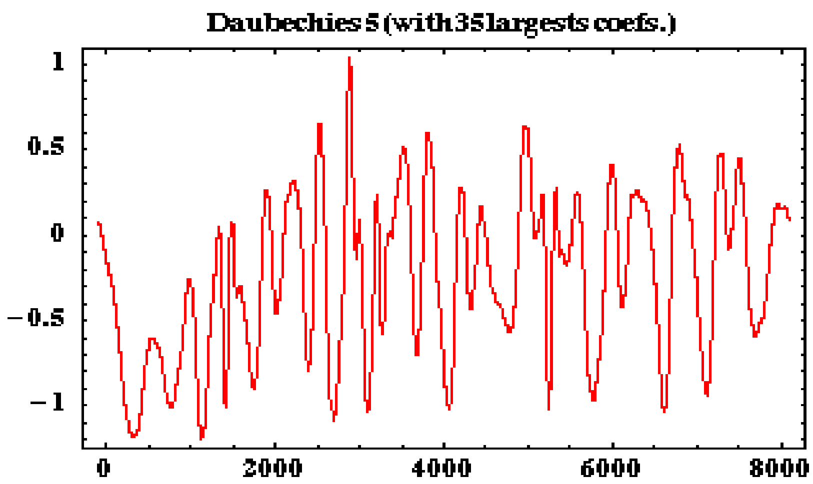


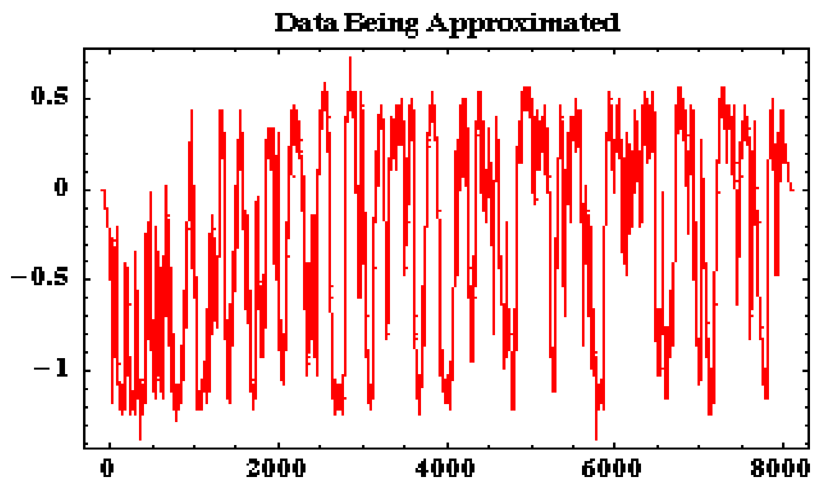


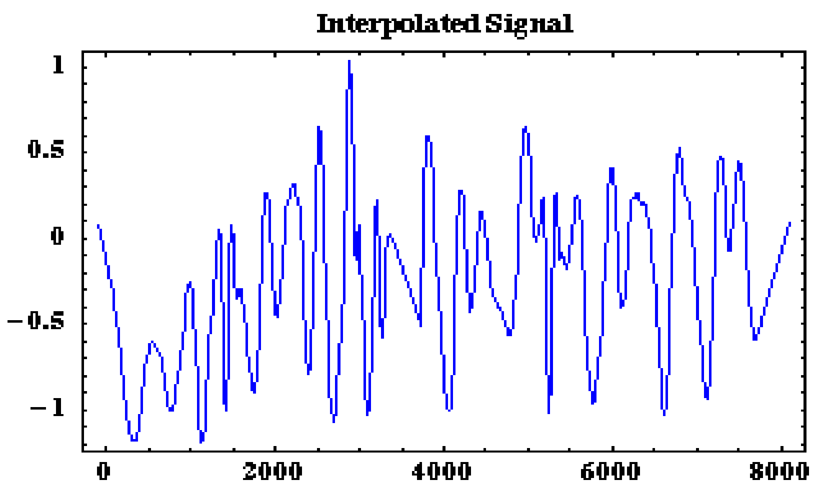


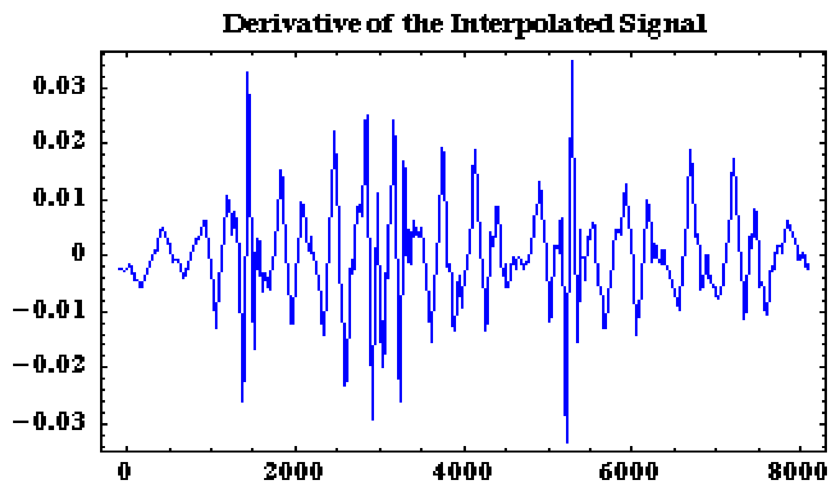
Reconstruction of the Data Using the 35 Largest WLT Coefficients

Polymath Research Inc.



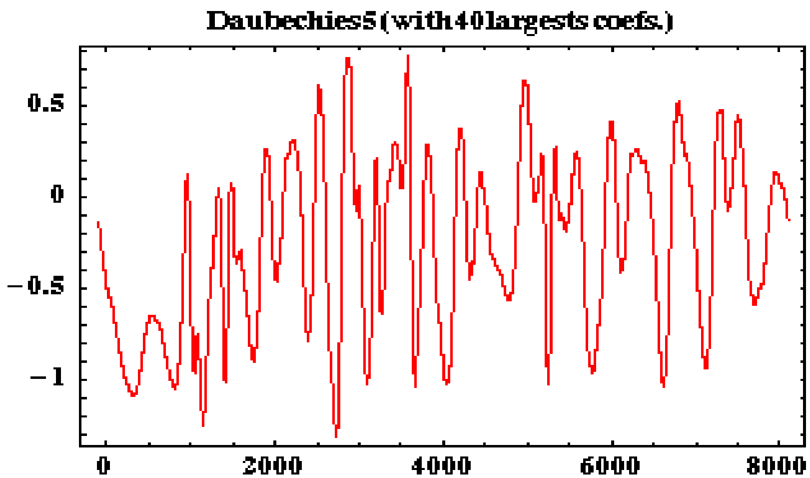


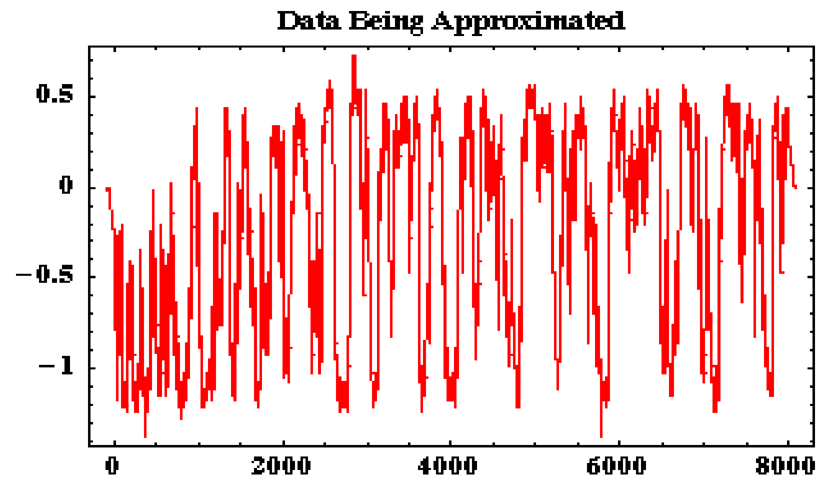


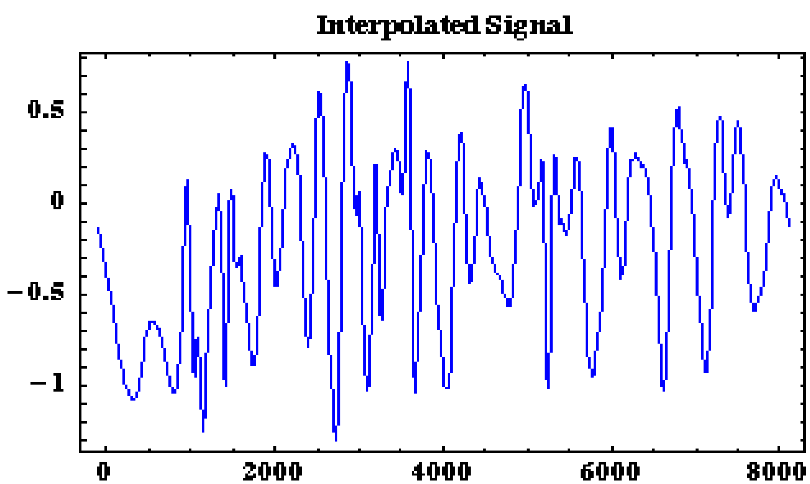


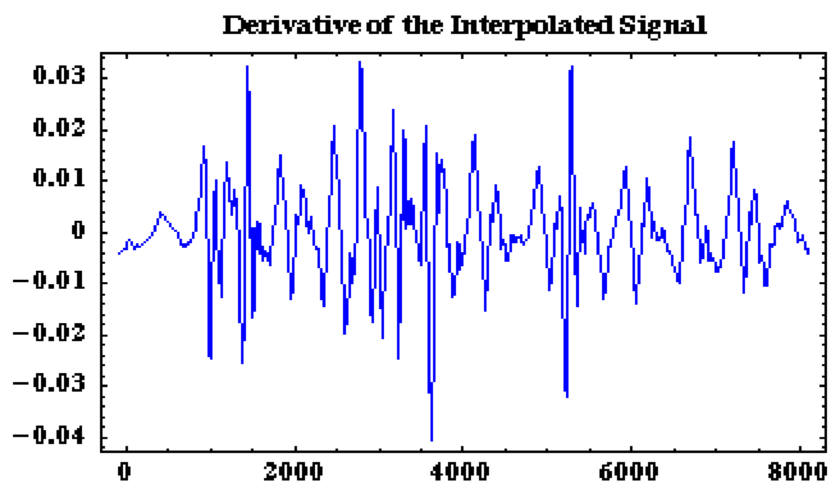
Reconstruction of the Data Using the 40 Largest WLT Coefficients

Polymath Research Inc.

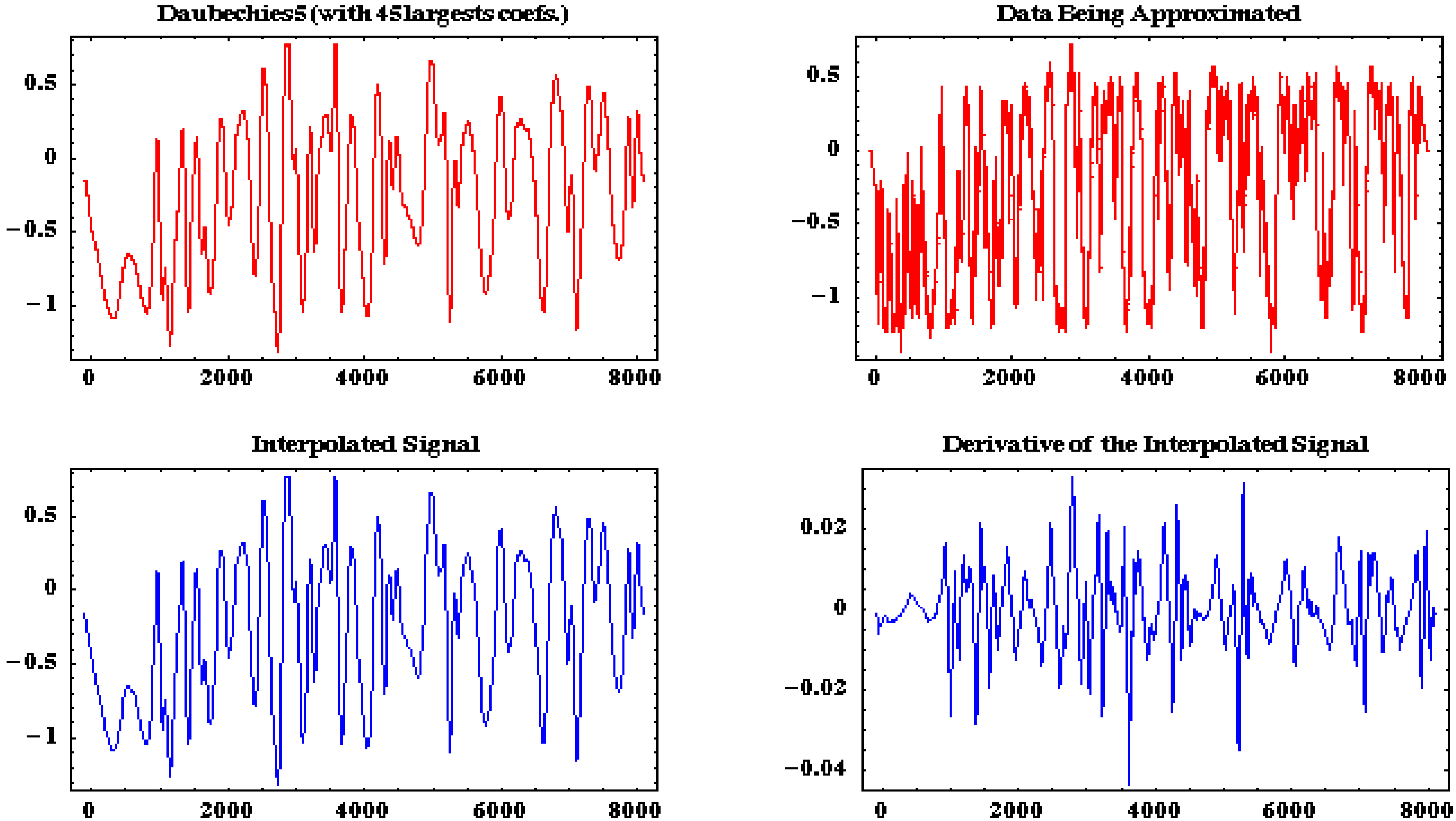




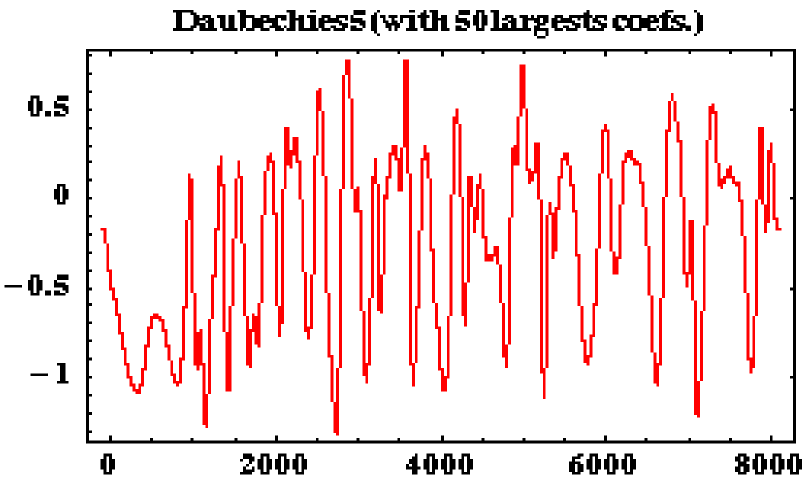


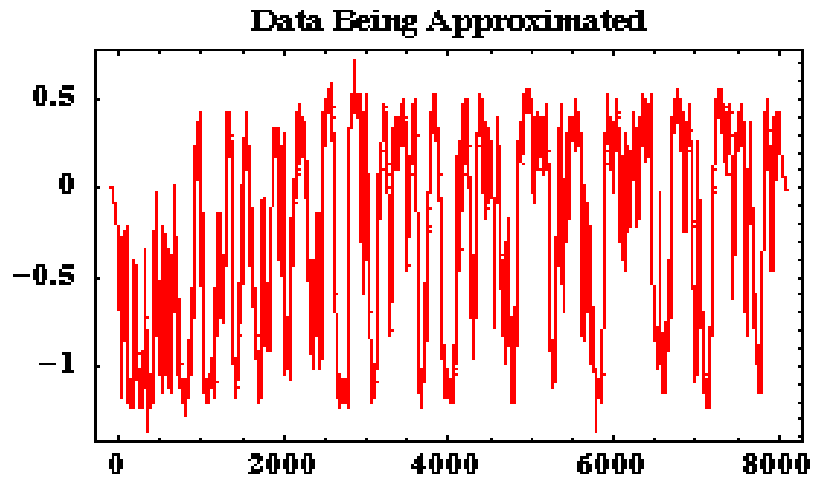


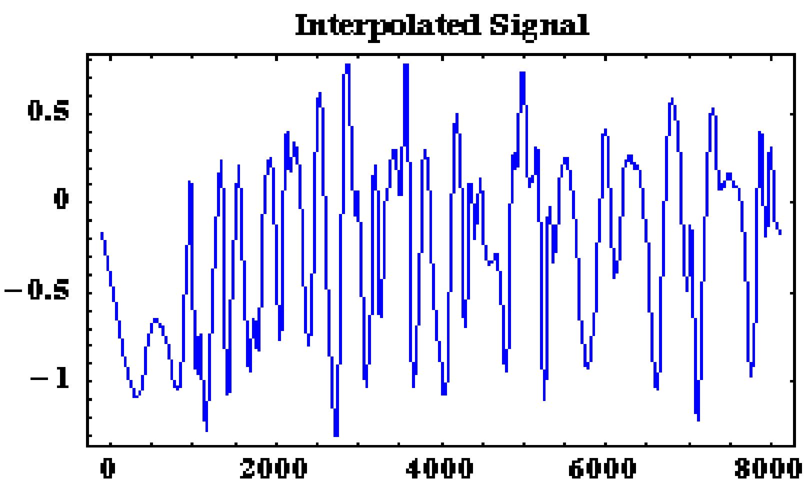
Reconstruction of the Data Using the 45 Largest WLT Coefficients

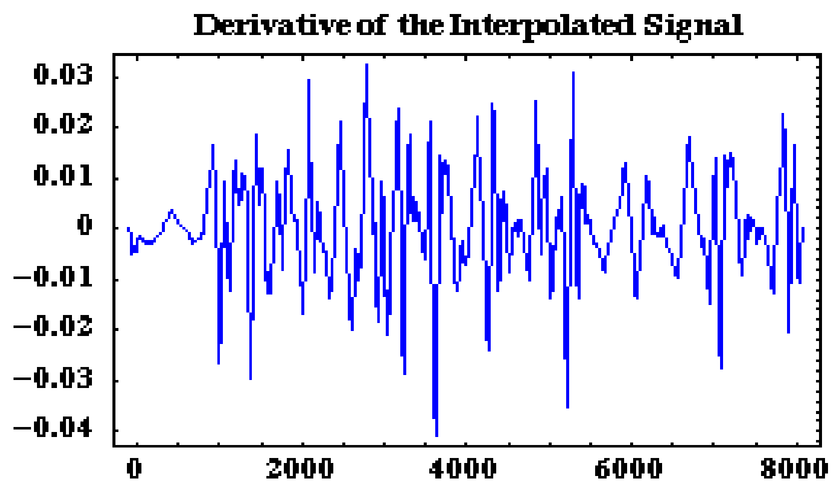


Reconstruction of the Data Using the 50 Largest WLT Coefficients



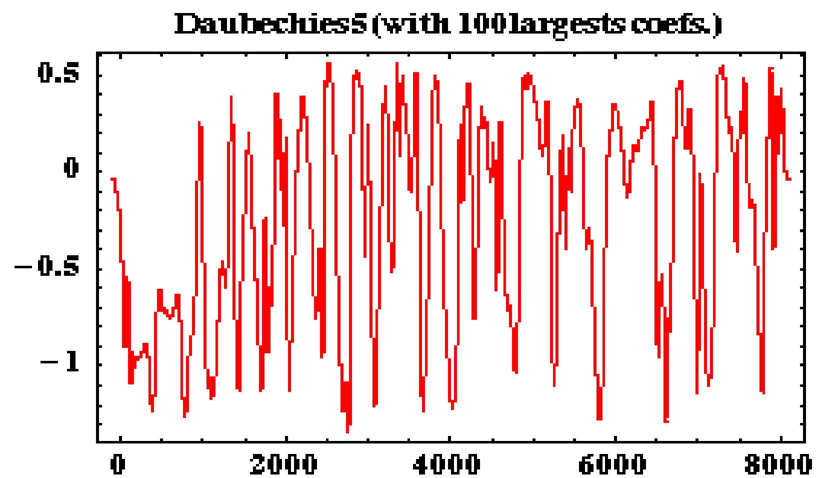


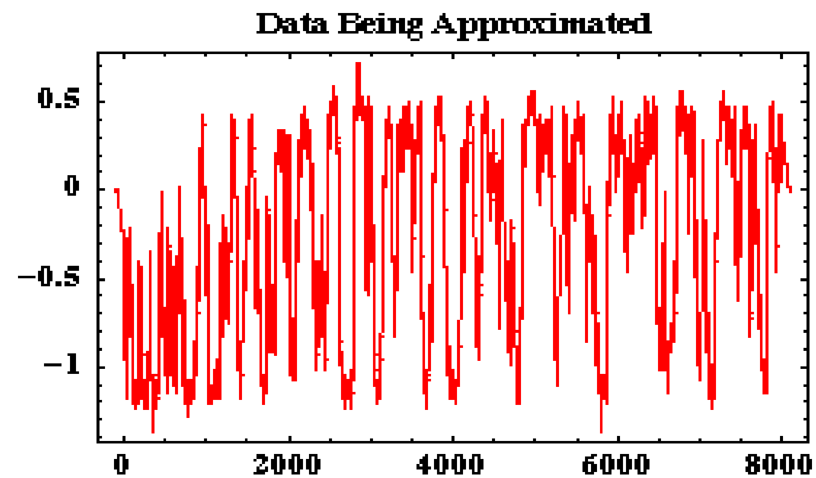


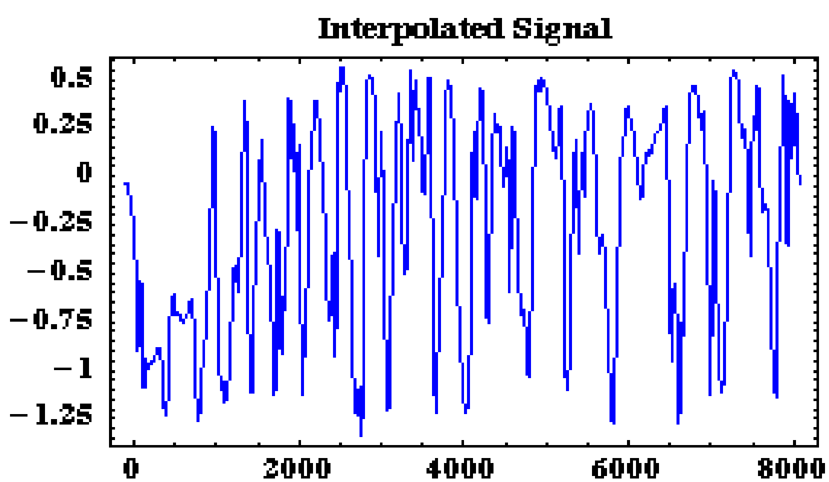


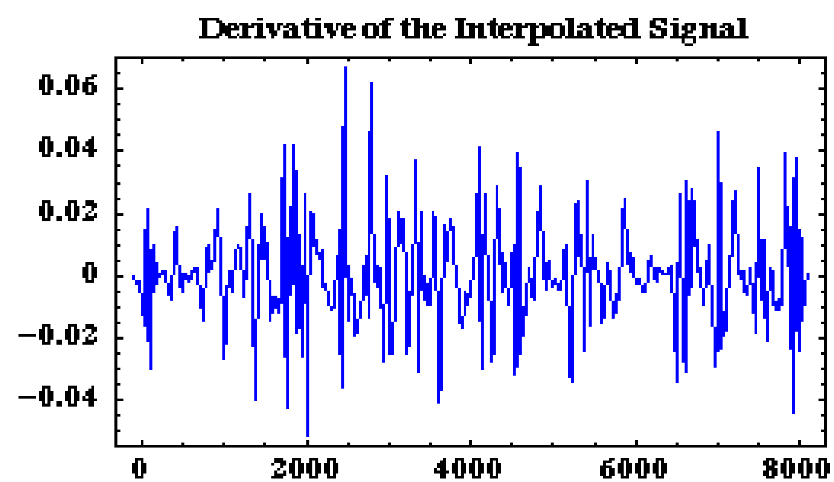
Reconstruction of the Data Using the 100 Largest WLT Coefficients

Polymath Research Inc.



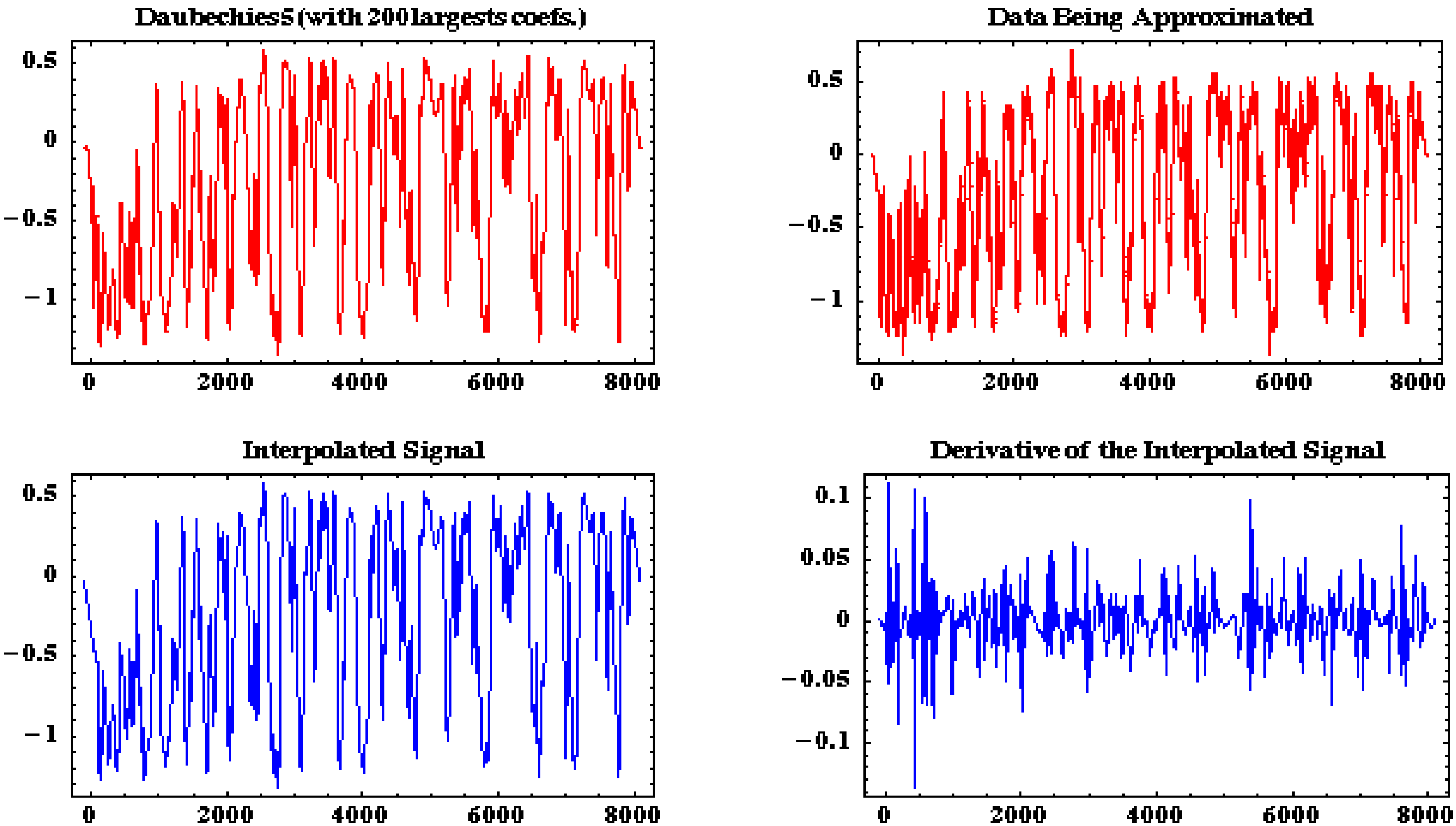




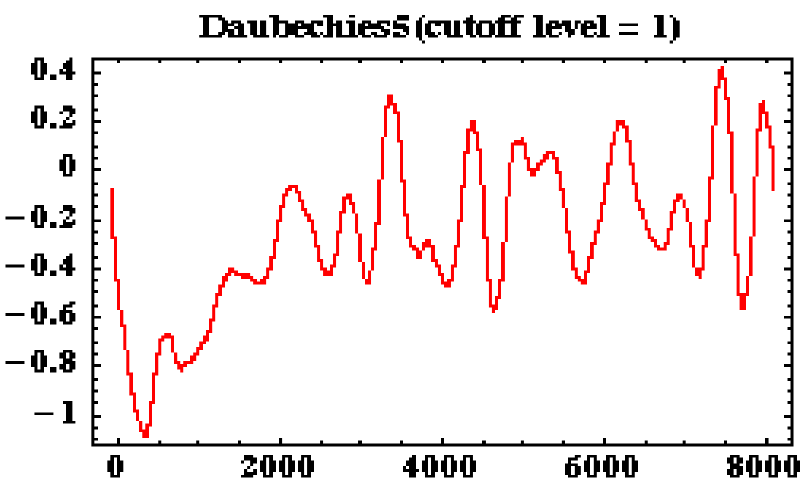


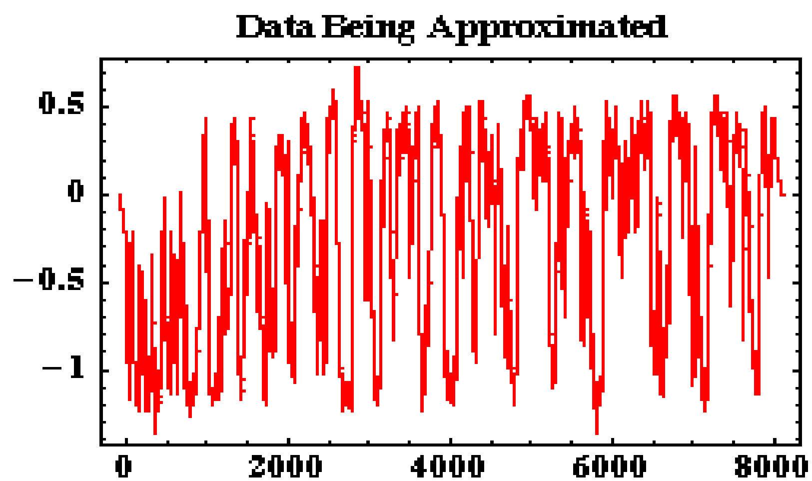
Reconstruction of the Data Using the 200 Largest WLT Coefficients

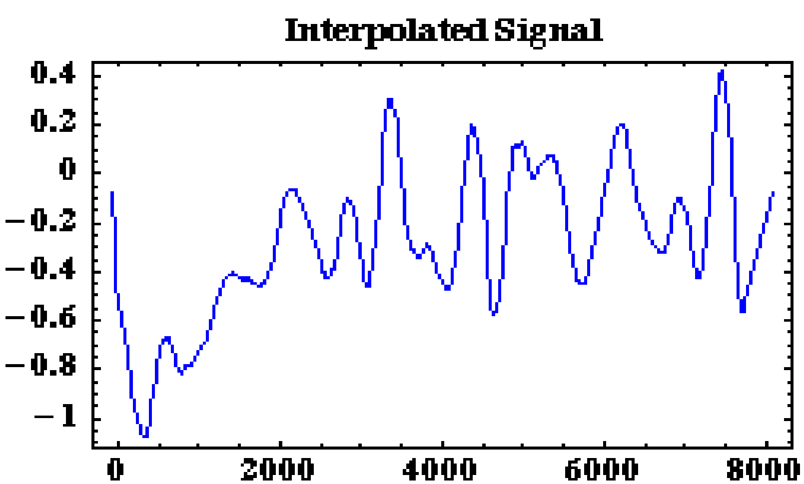
Polymath Research Inc.

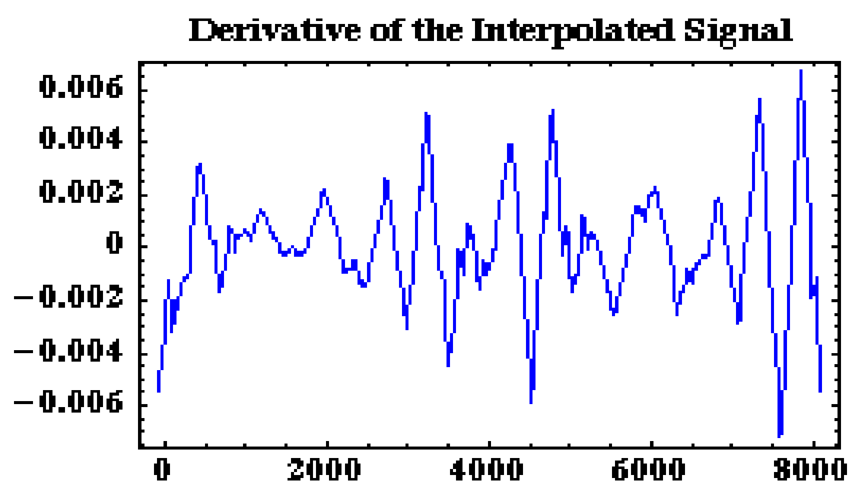


Reconstruction of the LPF Data Using the First MRD Level

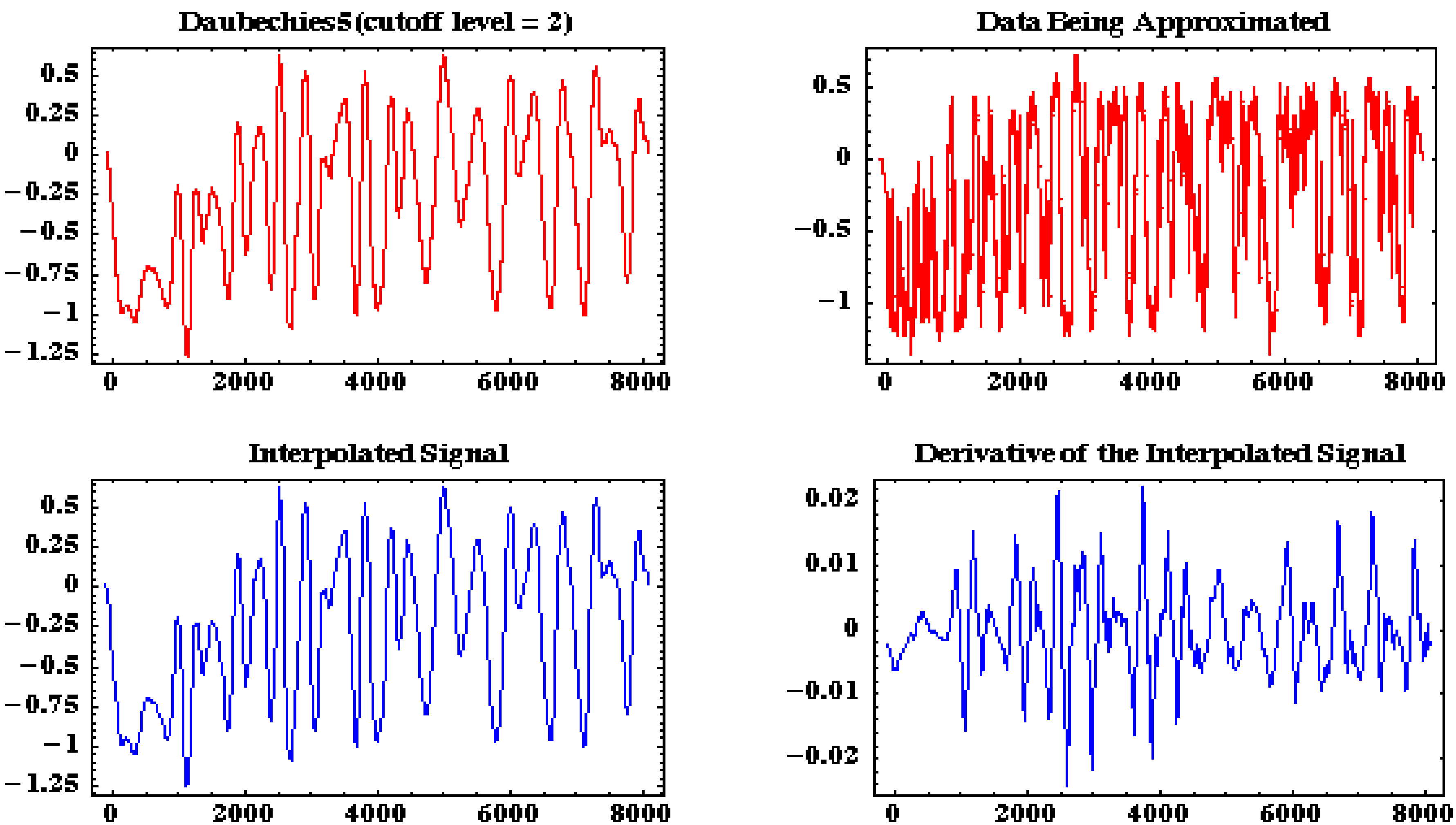




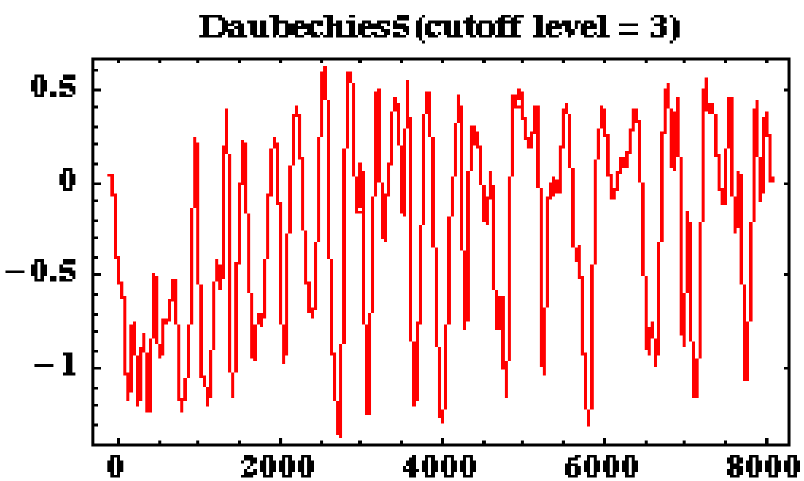


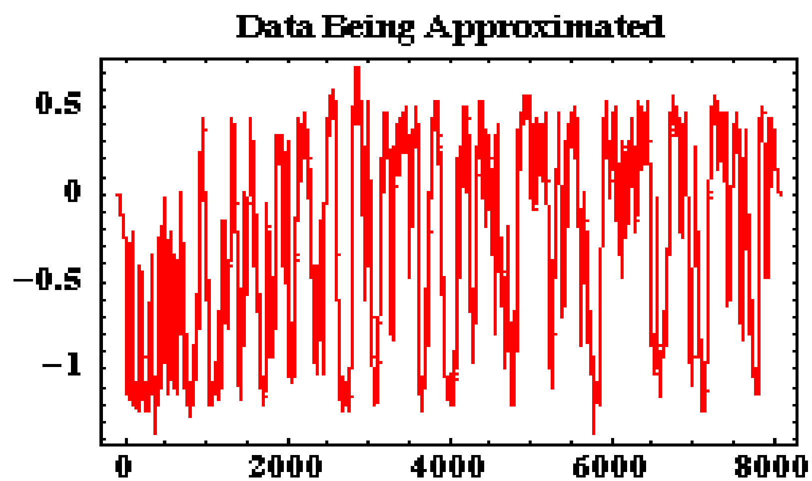


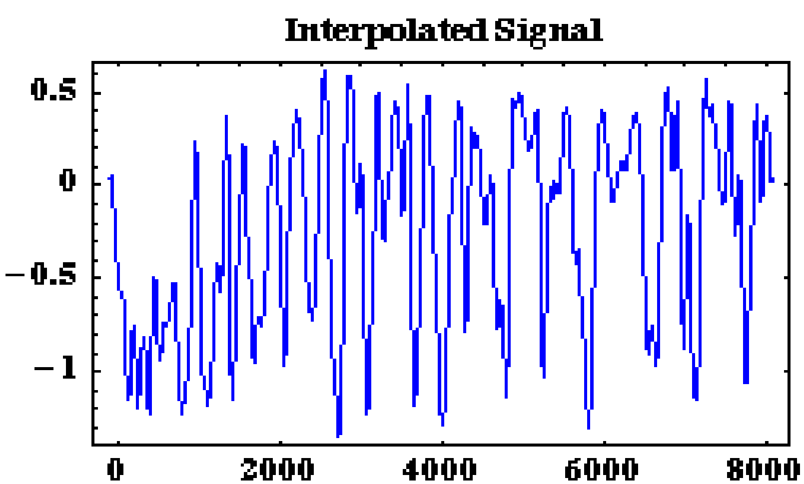
Reconstruction of the Weak Mix Data Using First 2 MRD Levels

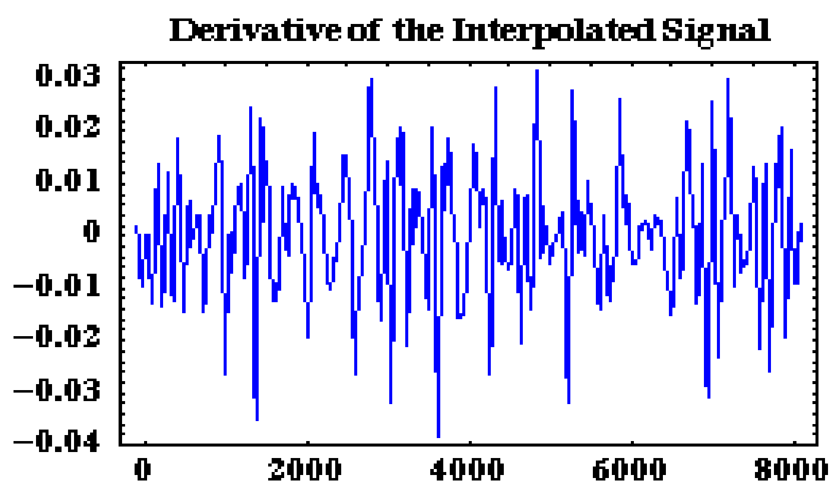


Reconstruction of the Weak Mix Data Using First 3 MRD Levels









Reconstruction of the Weak Mix Data Using First 4 MRD Levels

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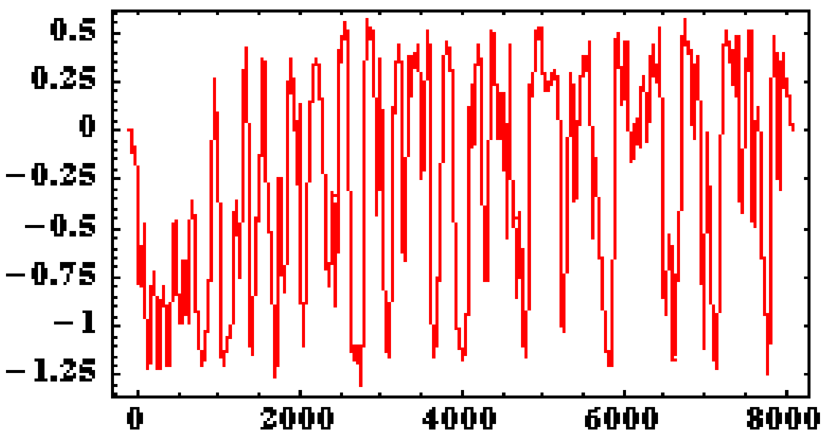
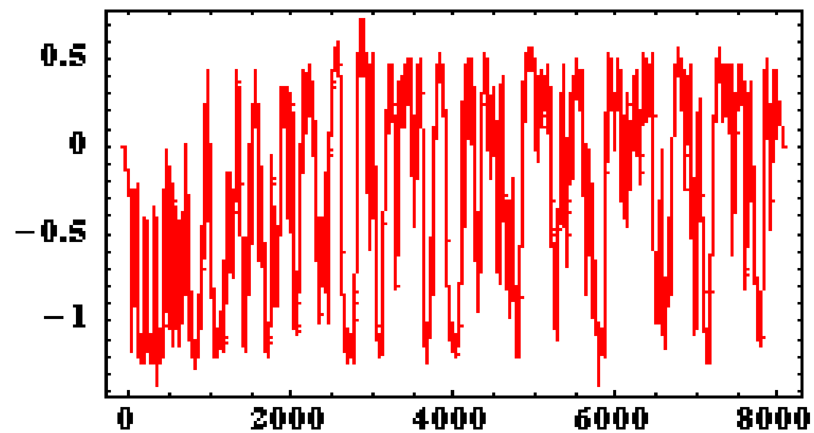
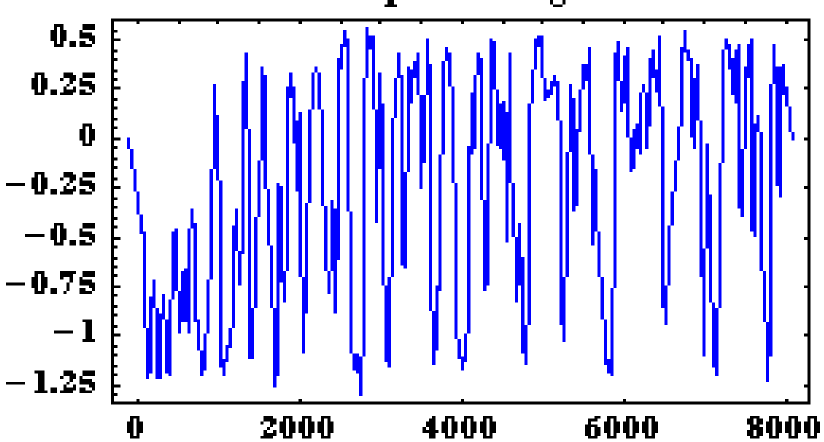
Daubechies5 (cutoff level = 4)



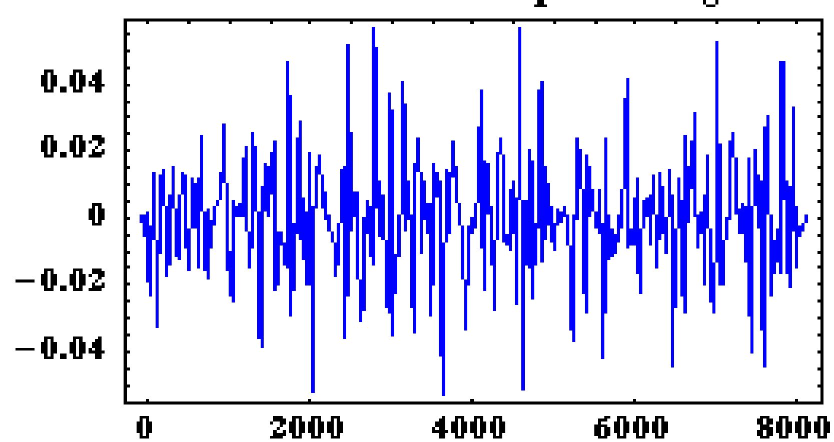
Data Being Approximated



Interpolated Signal

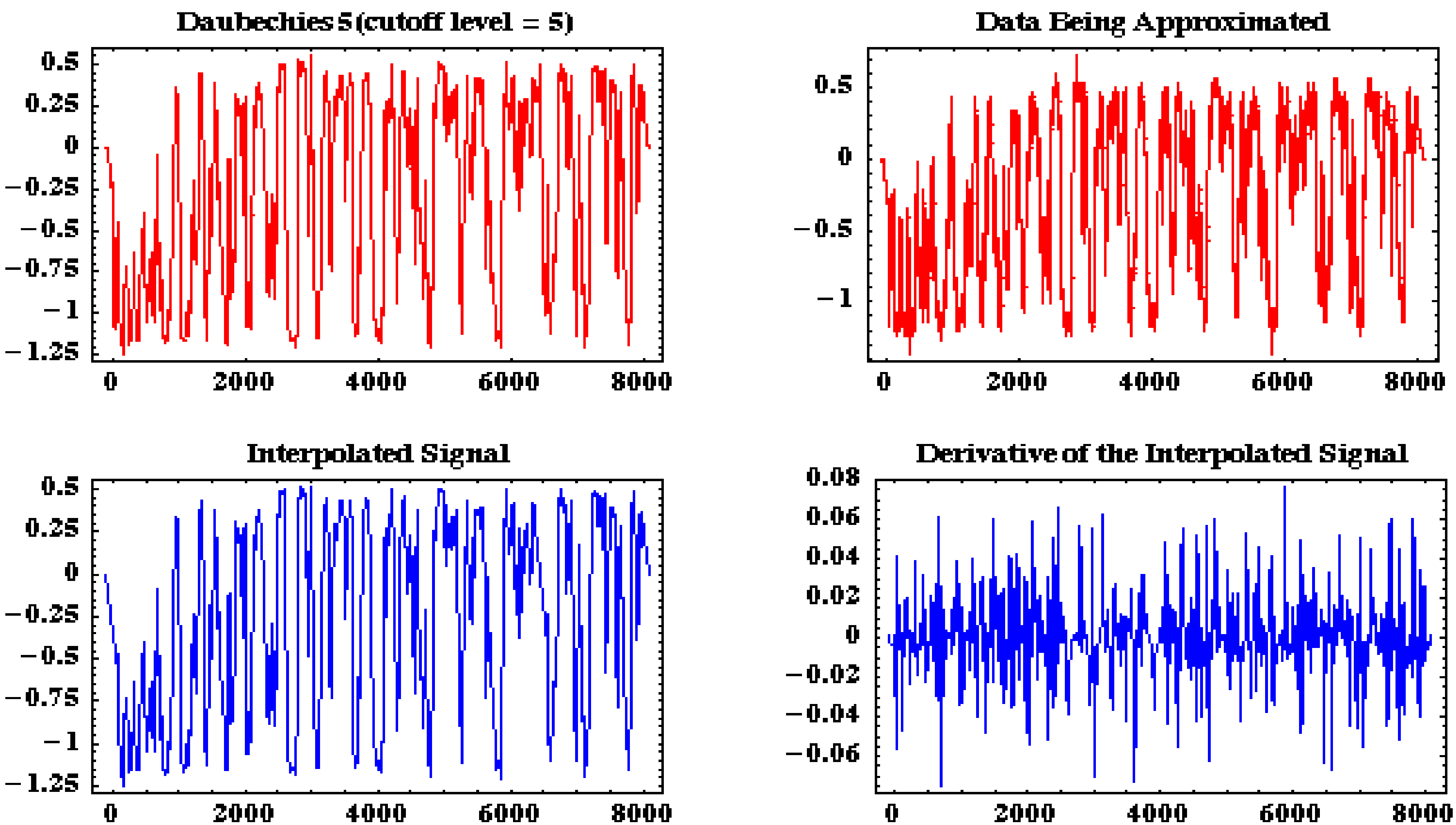


Derivative of the Interpolated Signal



Reconstruction of the Weak Mix Data Using First 5 MRD Levels

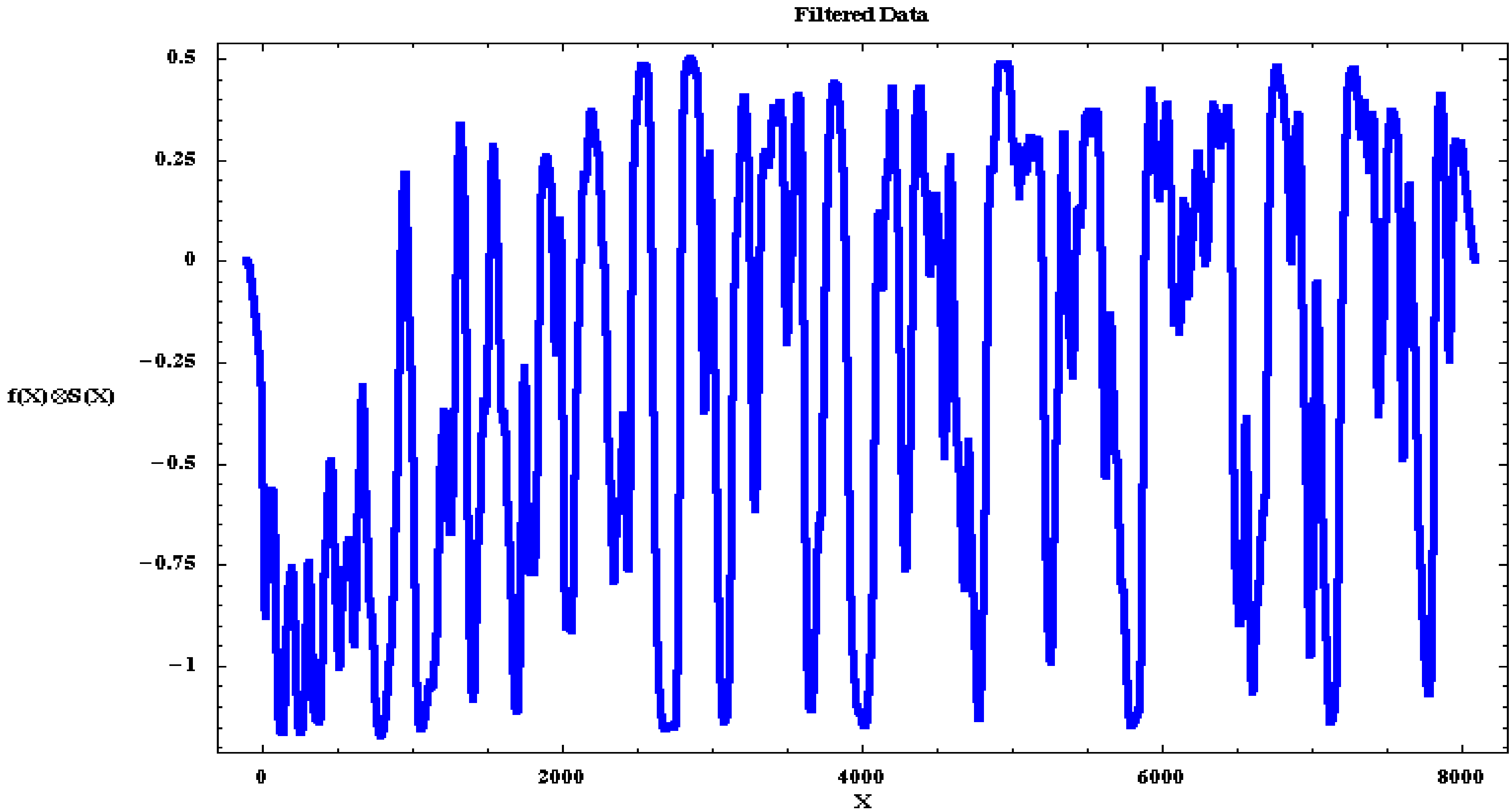
Polymath Research Inc.



Conclusions Regarding the WLT Analysis of the RT Weak Mix Data

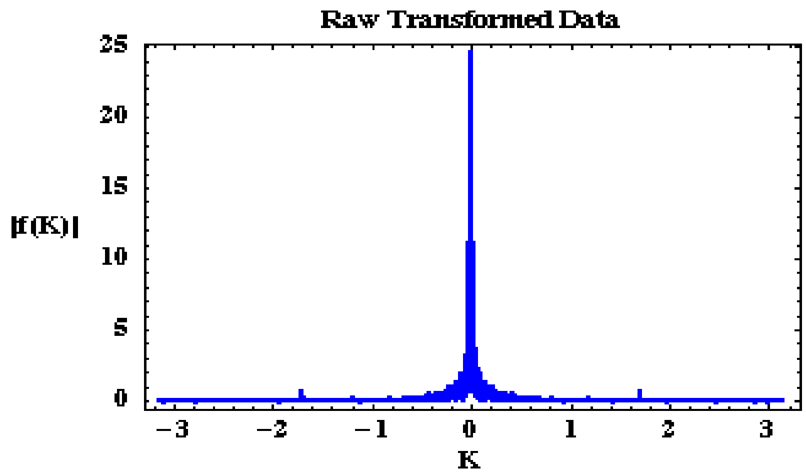
Low Pass Filtered (LPF) Padded and Faded RT Weak Mix Data

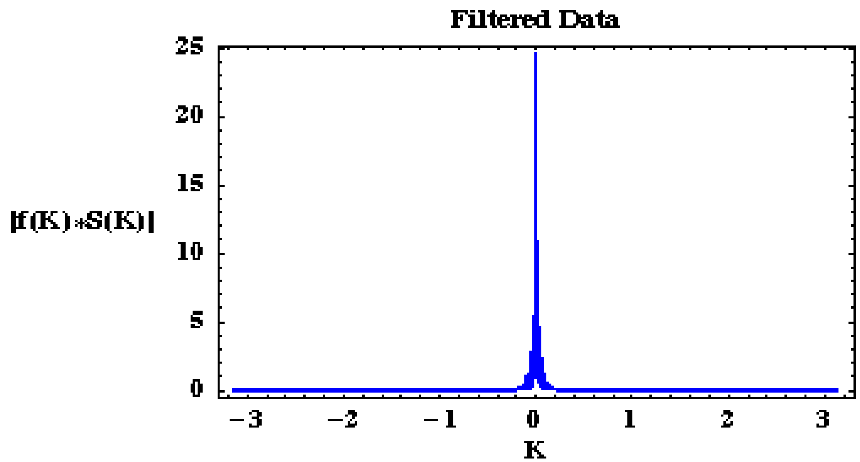
Polymath Research Inc.

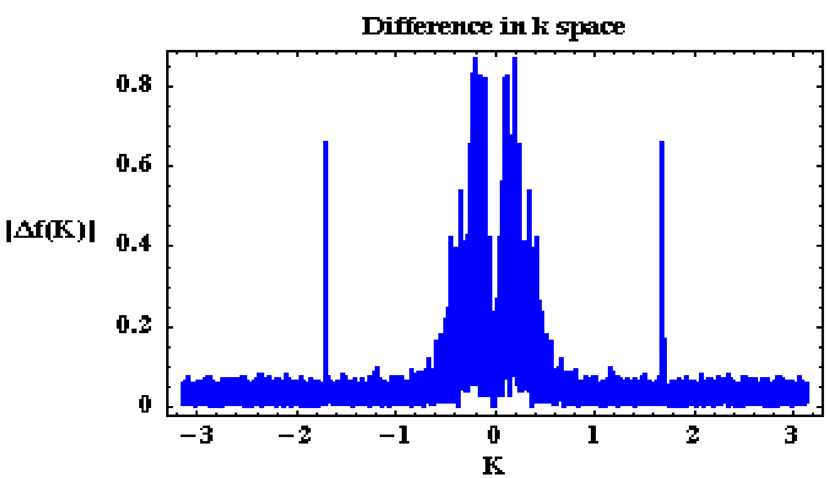


The Filtering Has This Form and Effect on the Data in k-Space

Polymath Research Inc.







Filter was of the form:

$$S(k) = \exp\left[-\left(\frac{k}{k_{width}} \right)^{2\alpha} \right]$$

Where α=5 and k_{width} = 400

MRD Coefficients of the LPF RT Weak Mix Data in 6 Different Daubechies WLT Bases

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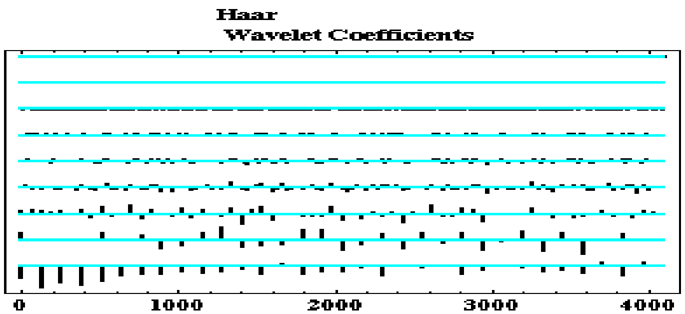
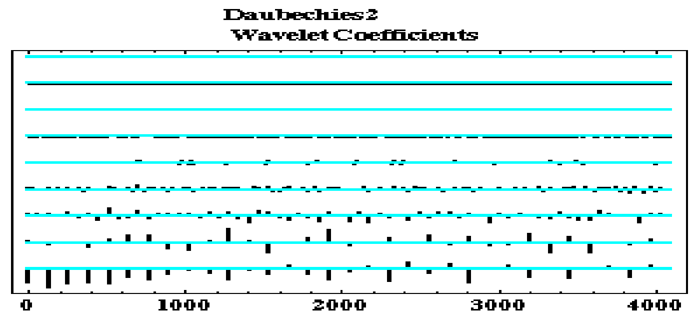
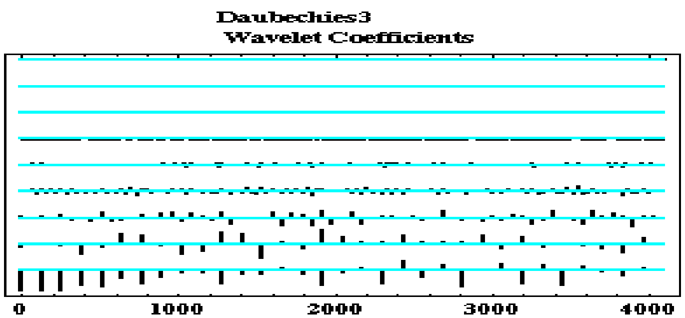
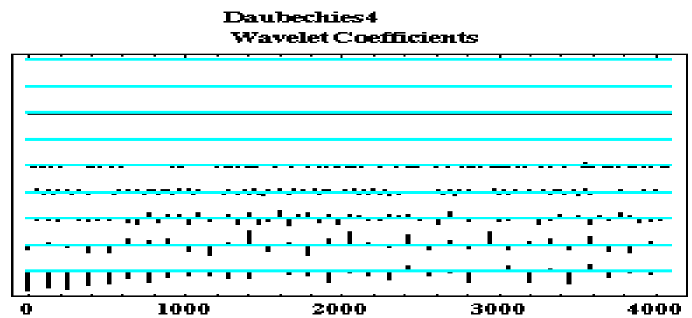
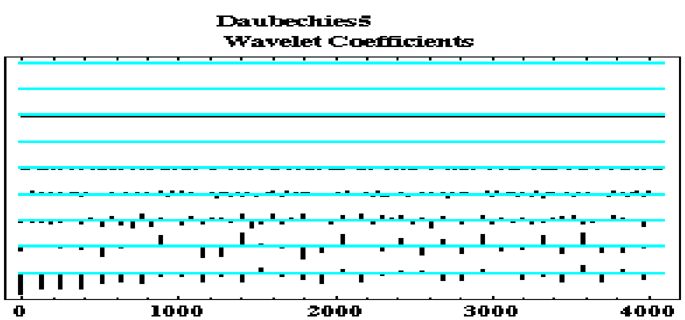
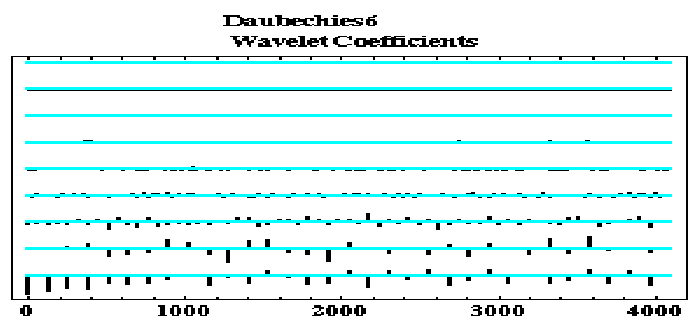






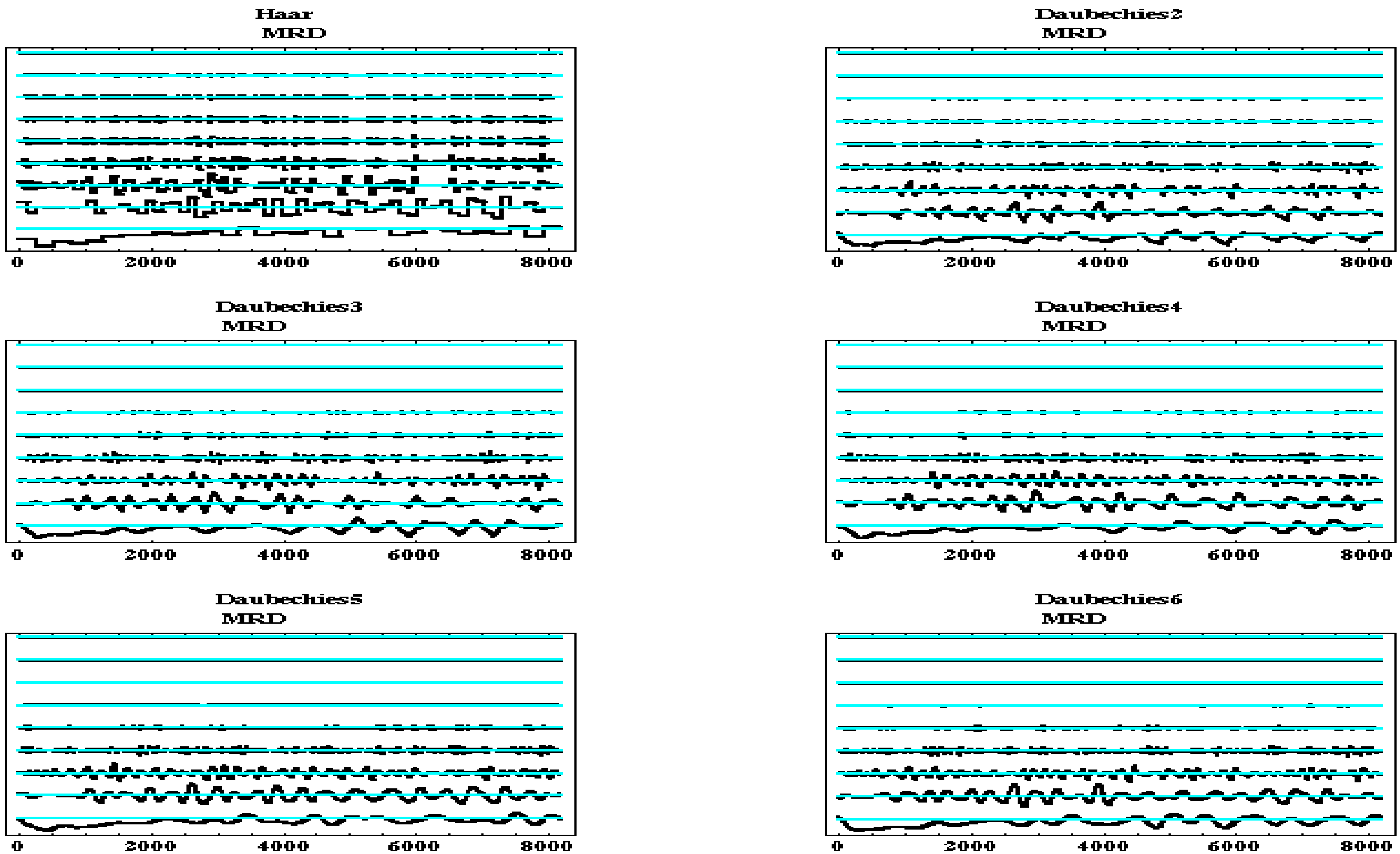




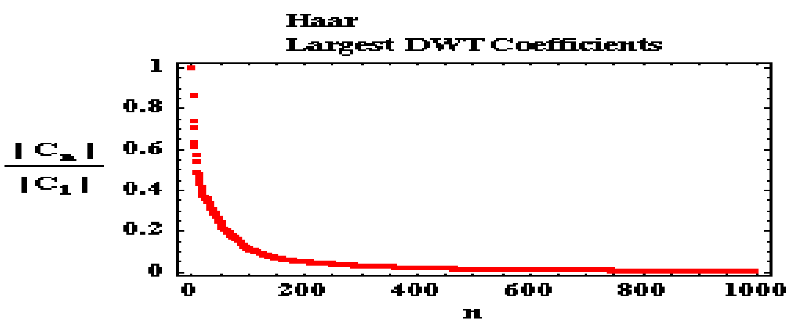


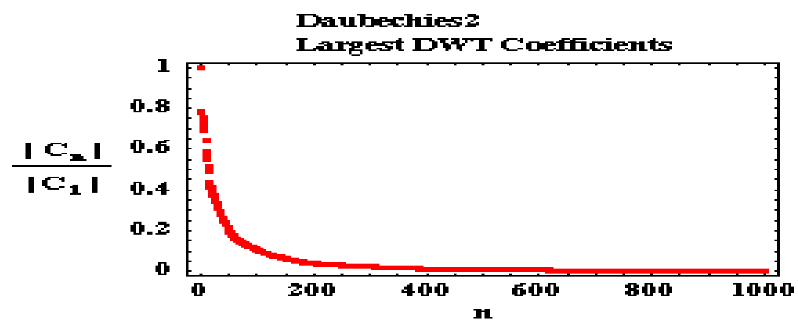
Actual MRDs of the LPF RT Weak Mix Data in 6 Different Daubechies WLT Bases

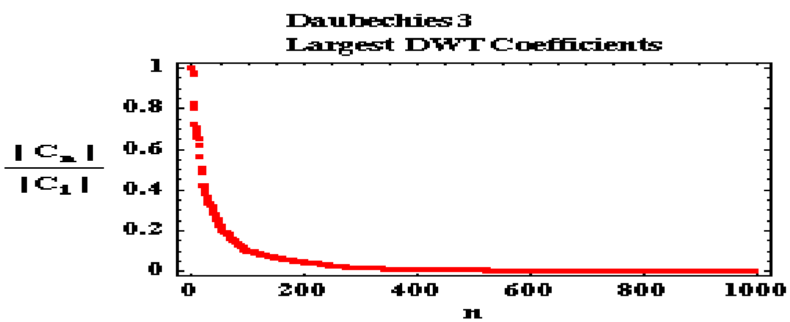
Polymath Research Inc.

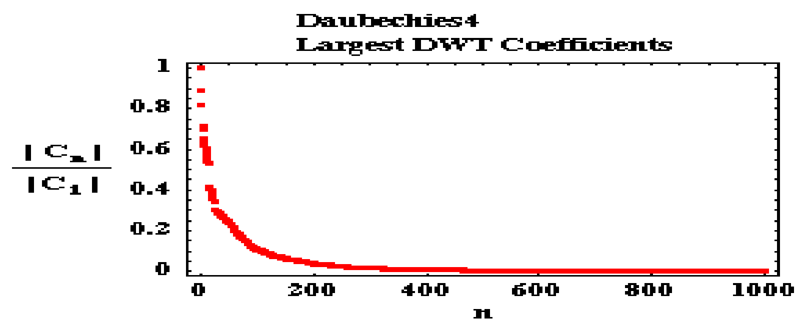


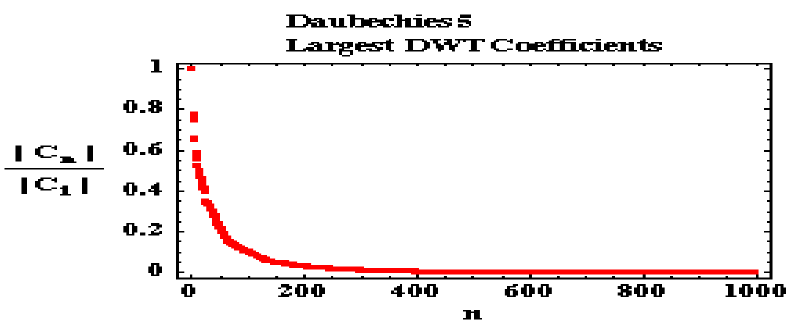
Decay Rate of Largest Coefficient vs Number of Coefficients Kept in 6 Different Daub WLT Decomps

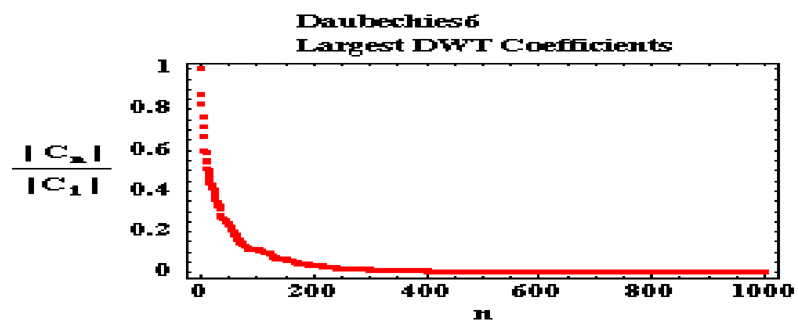




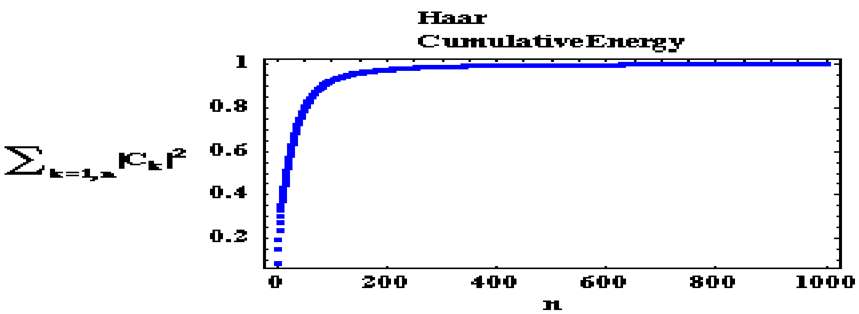


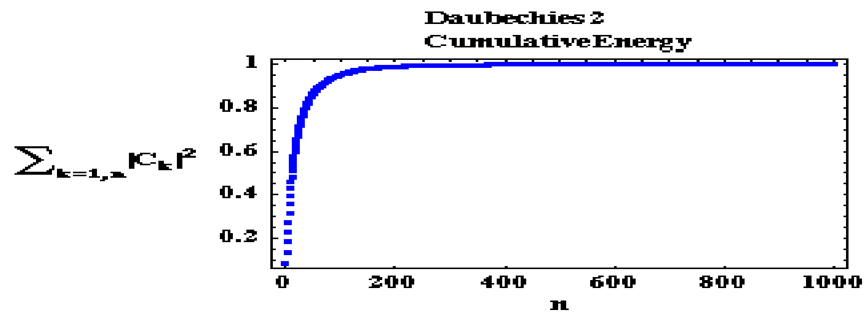


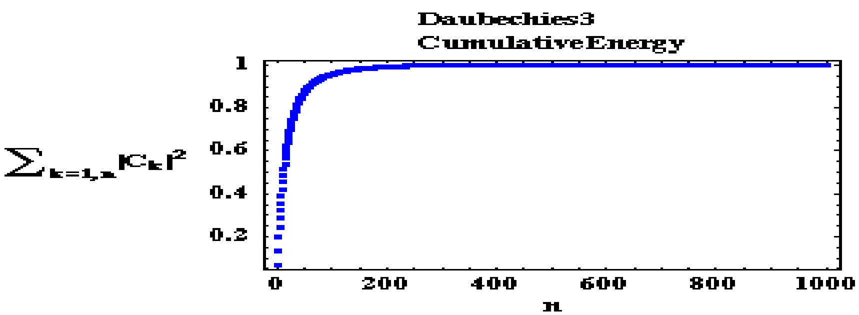


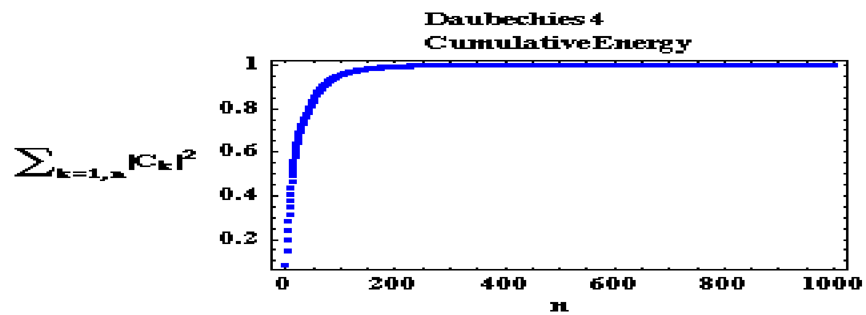


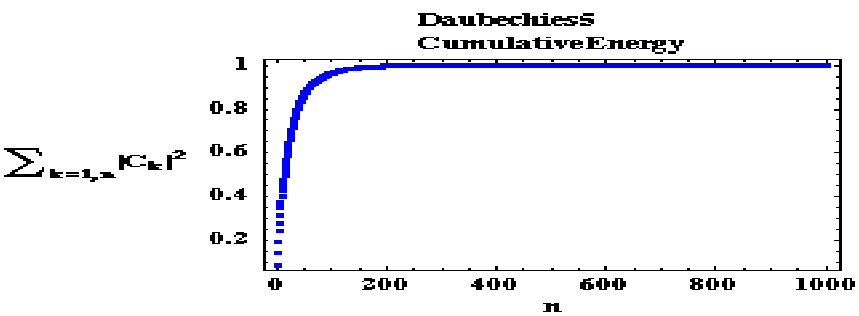
Energy Accumulation Rate in Coefficient Space vs # of WLTs Kept in 6 Different Daub Decomps.

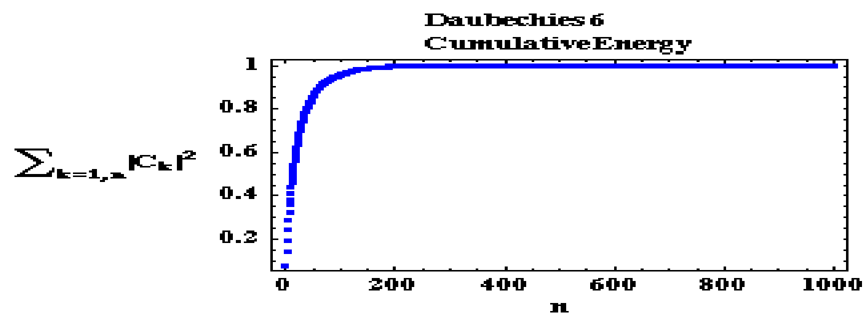






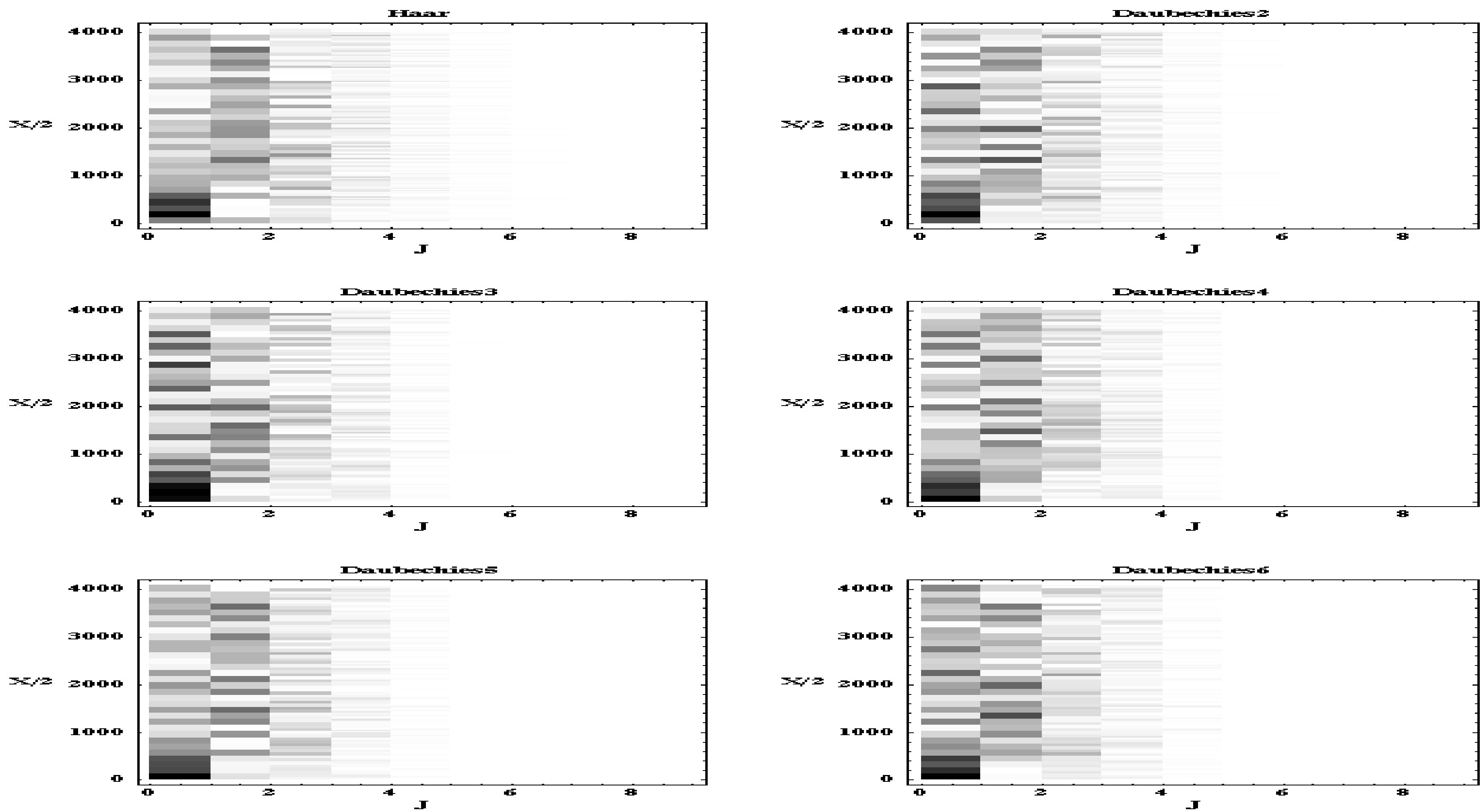






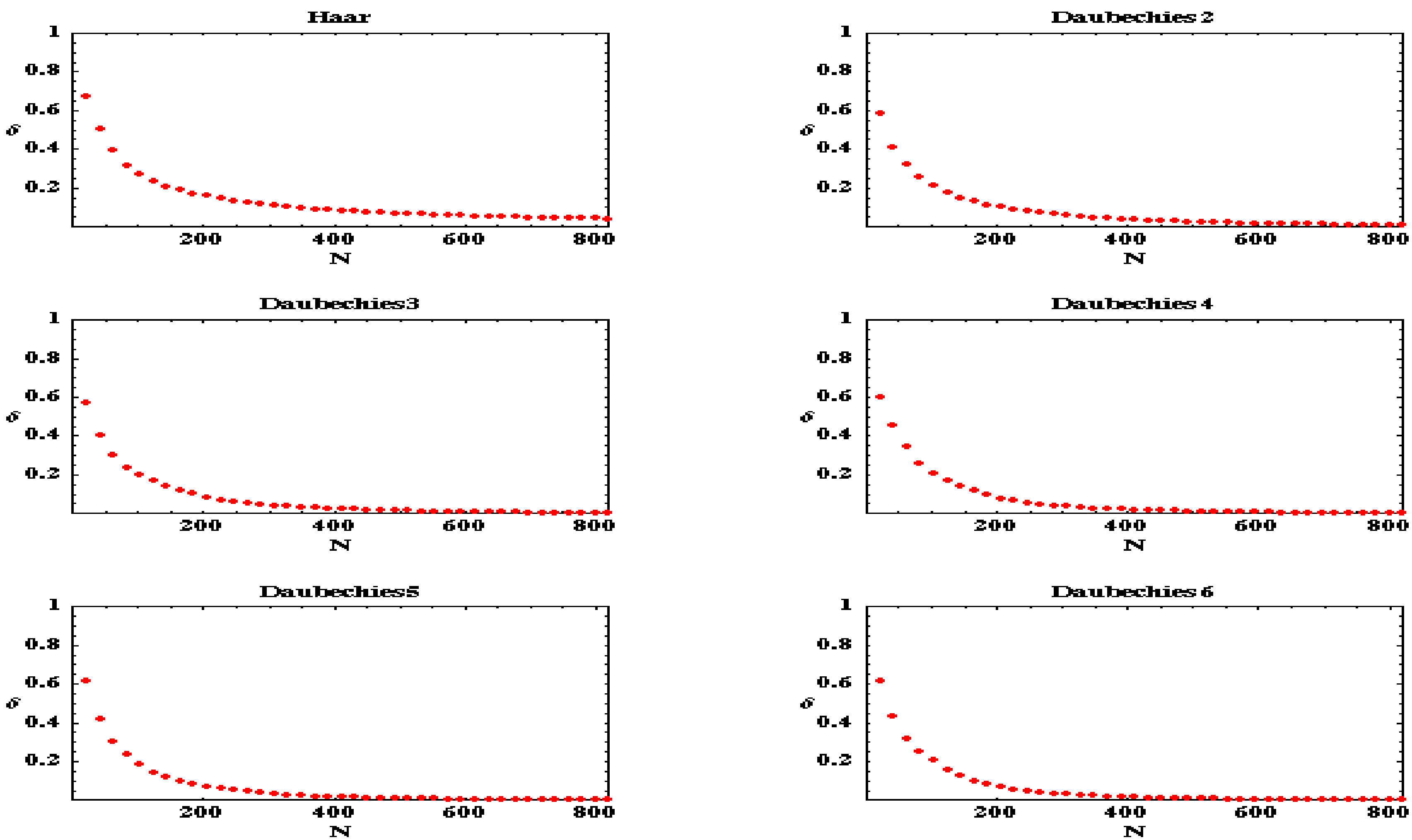
Scaleograms: Waveleters Preferred Way of Judging Tiling in Scale-Translation Space

Polymath Research Inc.



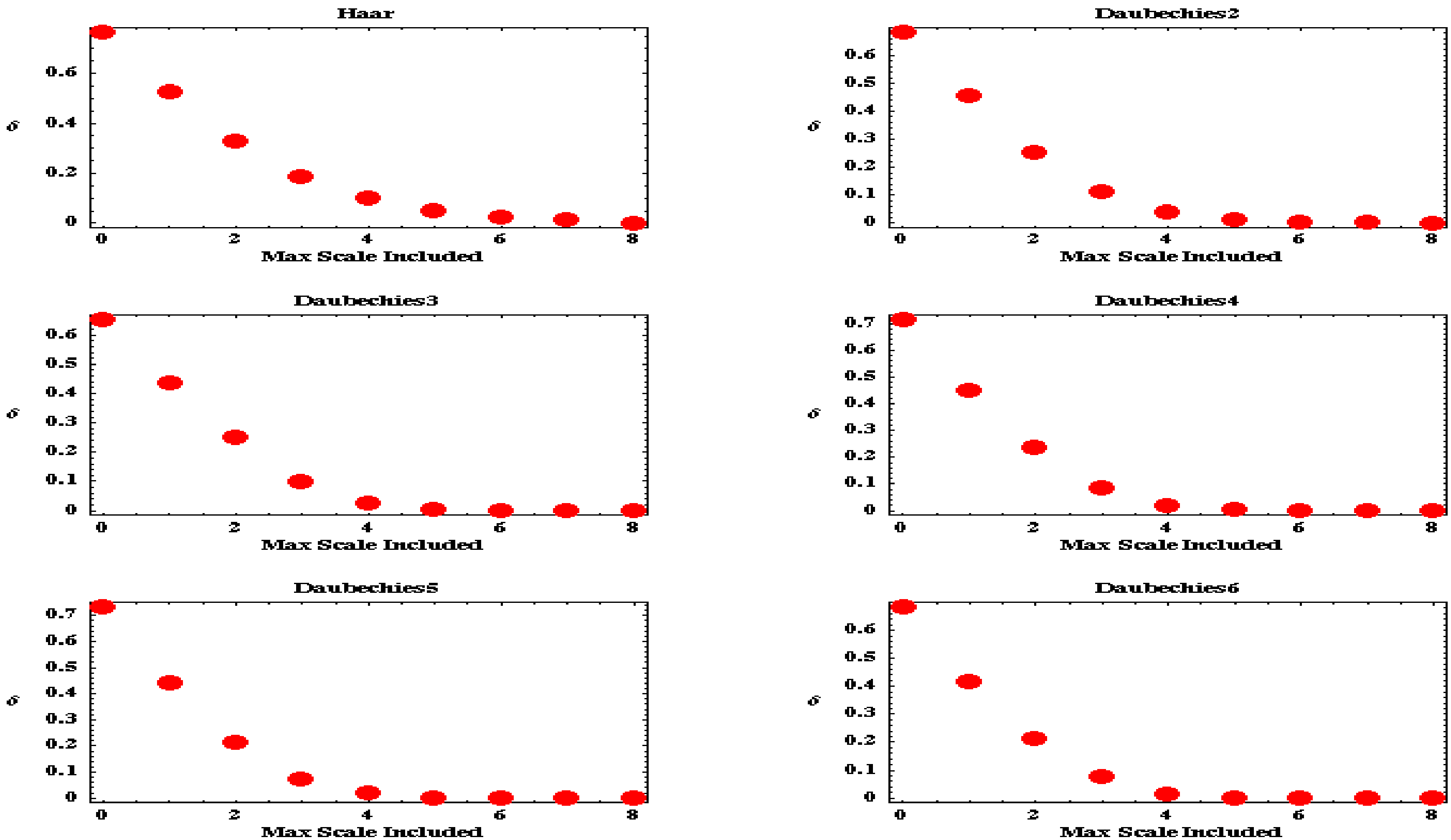
Least Square Error Incurred By Truncating the WLT Series at N of its Largest Coefficients

Polymath Research Inc.

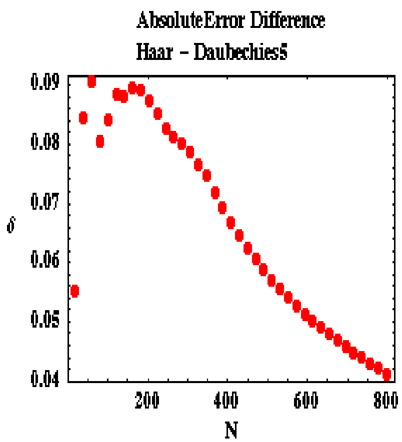


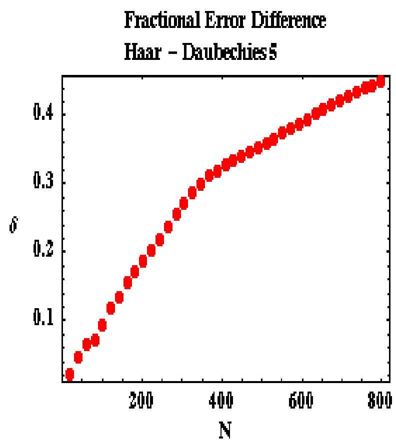
Least Square Error Incurred by Level Thresholding the DWT

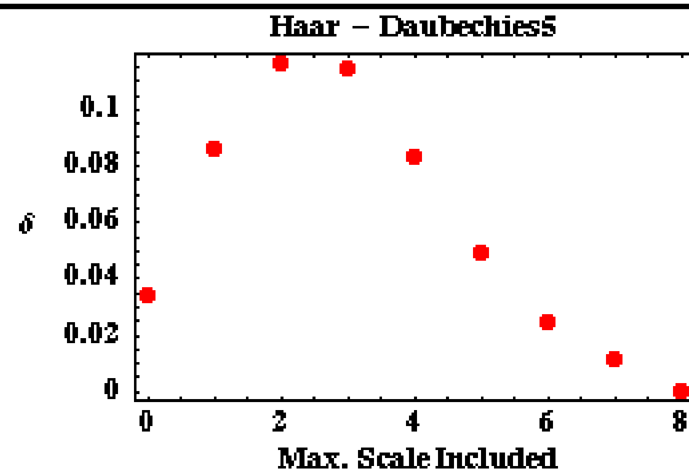
Polymath Research Inc.

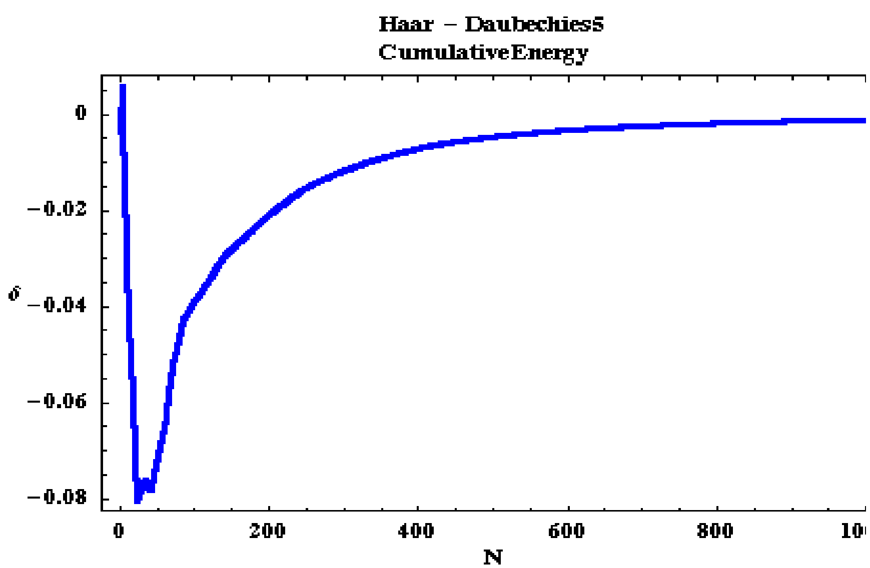


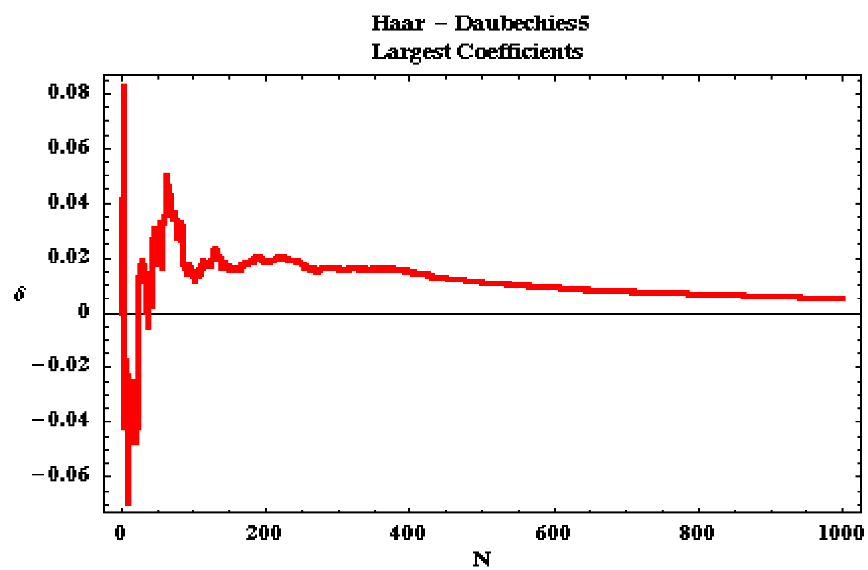
Daubechies 5 Does Much Better than Haar: 5 Quantitative Measures





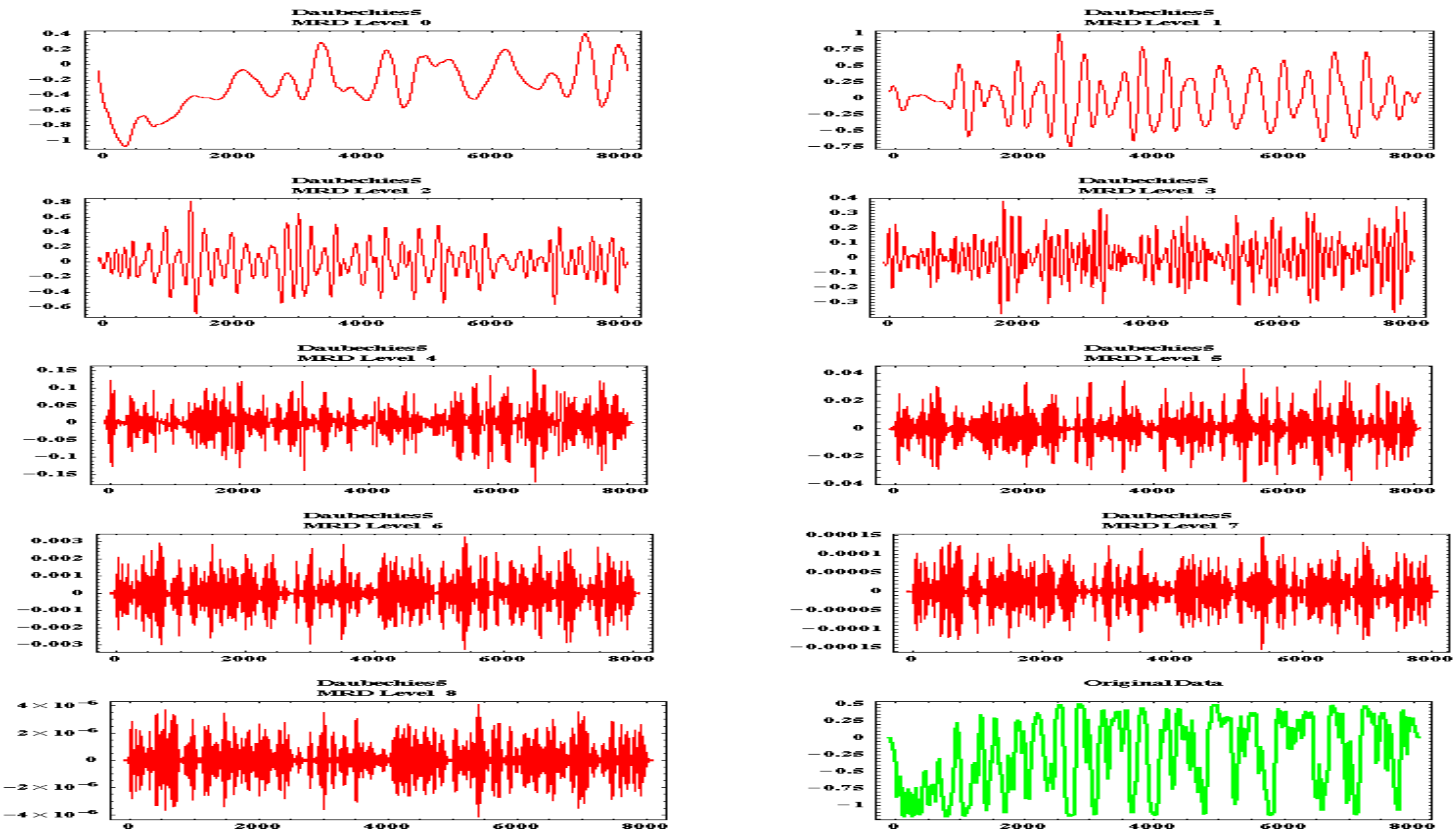




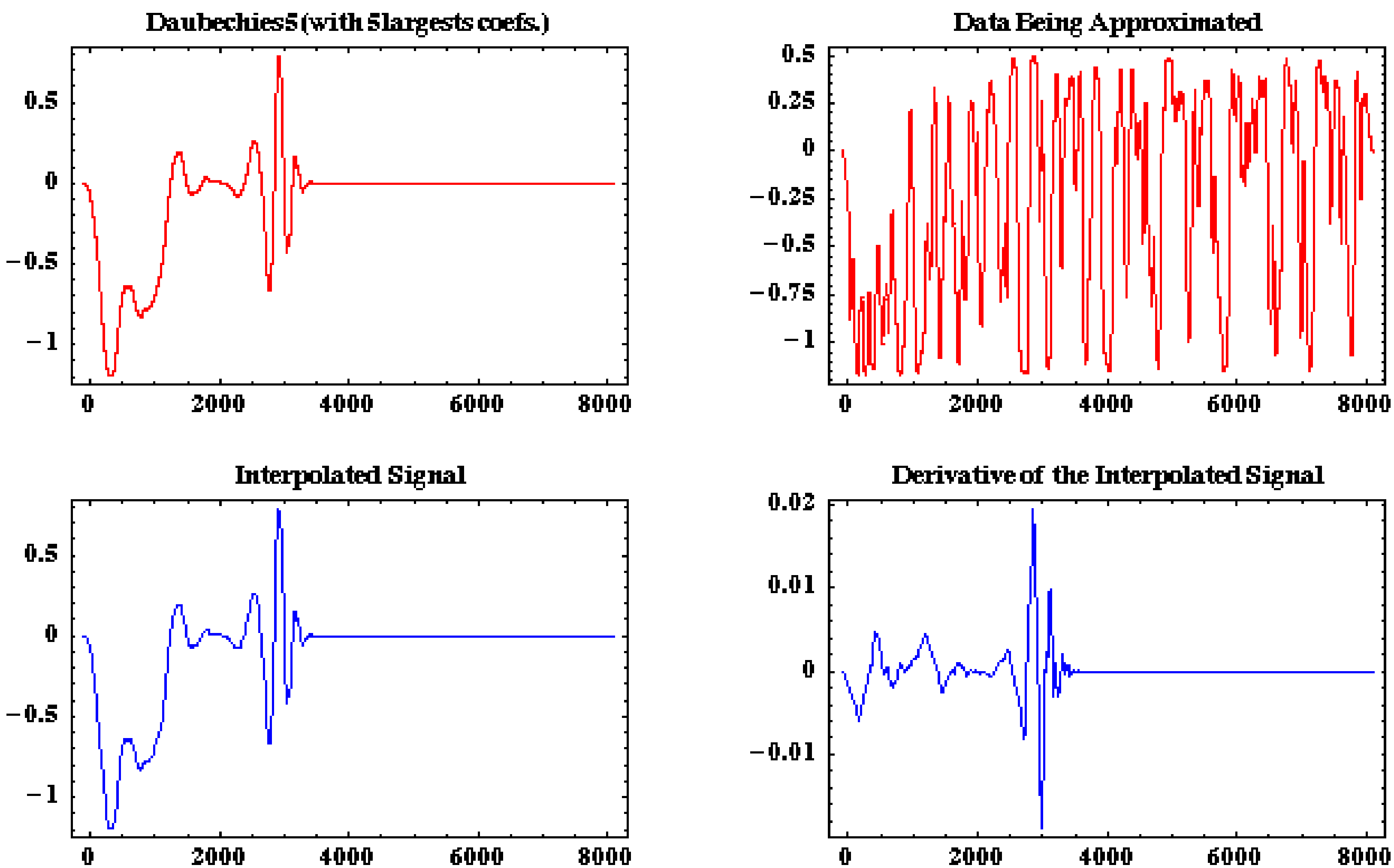


Level by Level Decomposition of the LPF RT Weak Mix Data Using Daub5 WLTs

Polymath Research Inc.

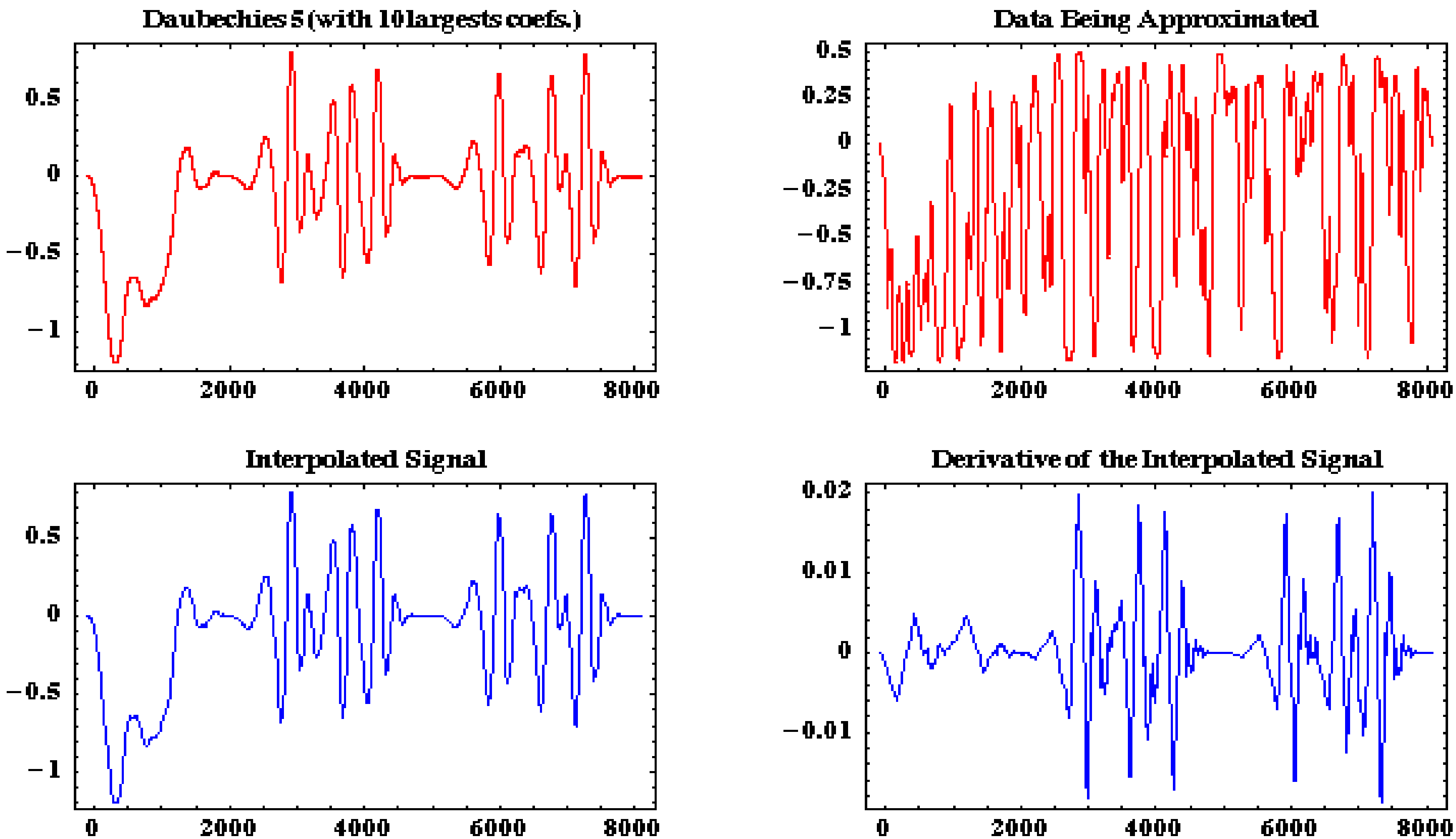


Reconstruction of the LPF RT Weak Mix Data Using the 5 Largest WLT Coefficients



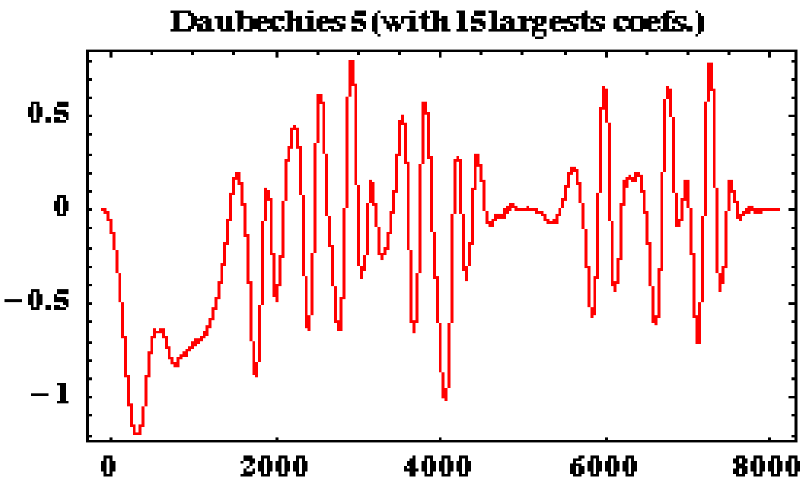
Reconstruction of the LPF RT Weak Mix Data Using the 10 Largest WLT Coefficients

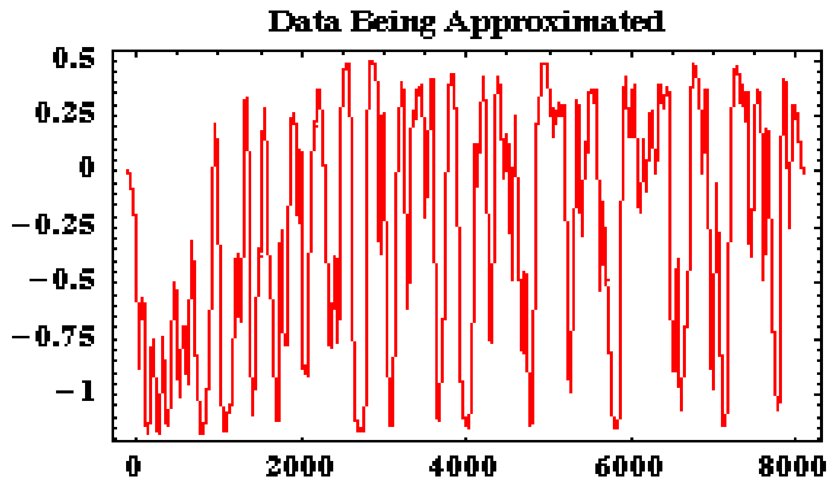
Polymath Research Inc.

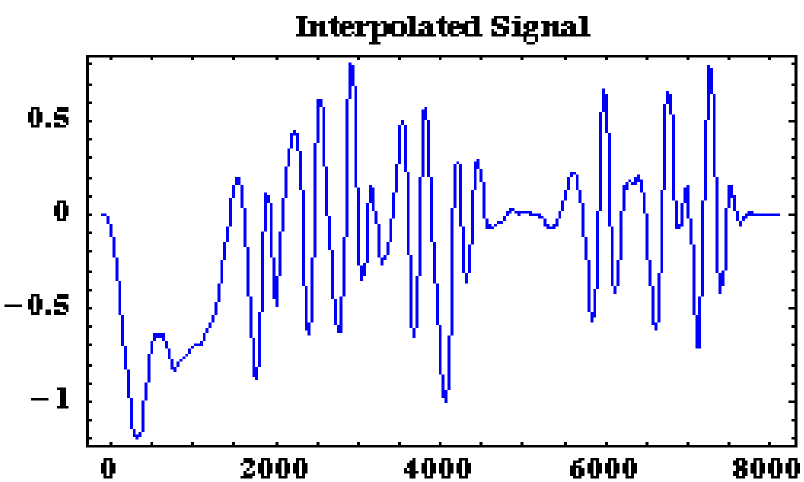


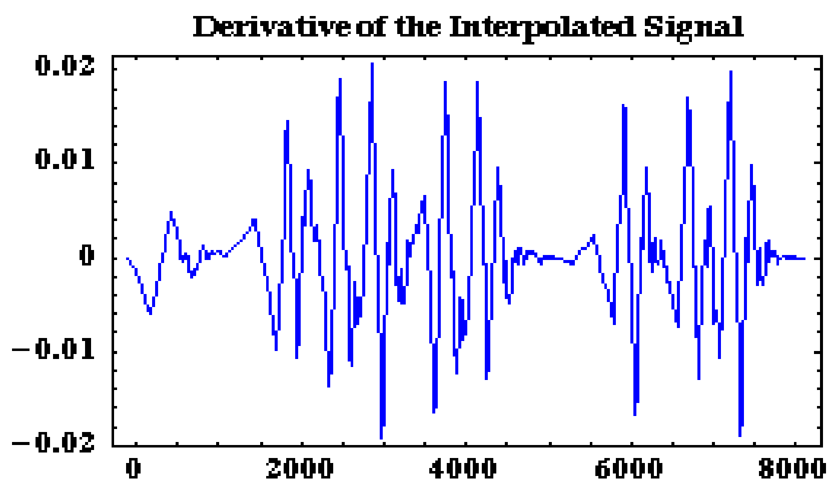
Reconstruction of the LPF RT Weak Mix Data Using the 15 Largest WLT Coefficients

Polymath Research Inc.



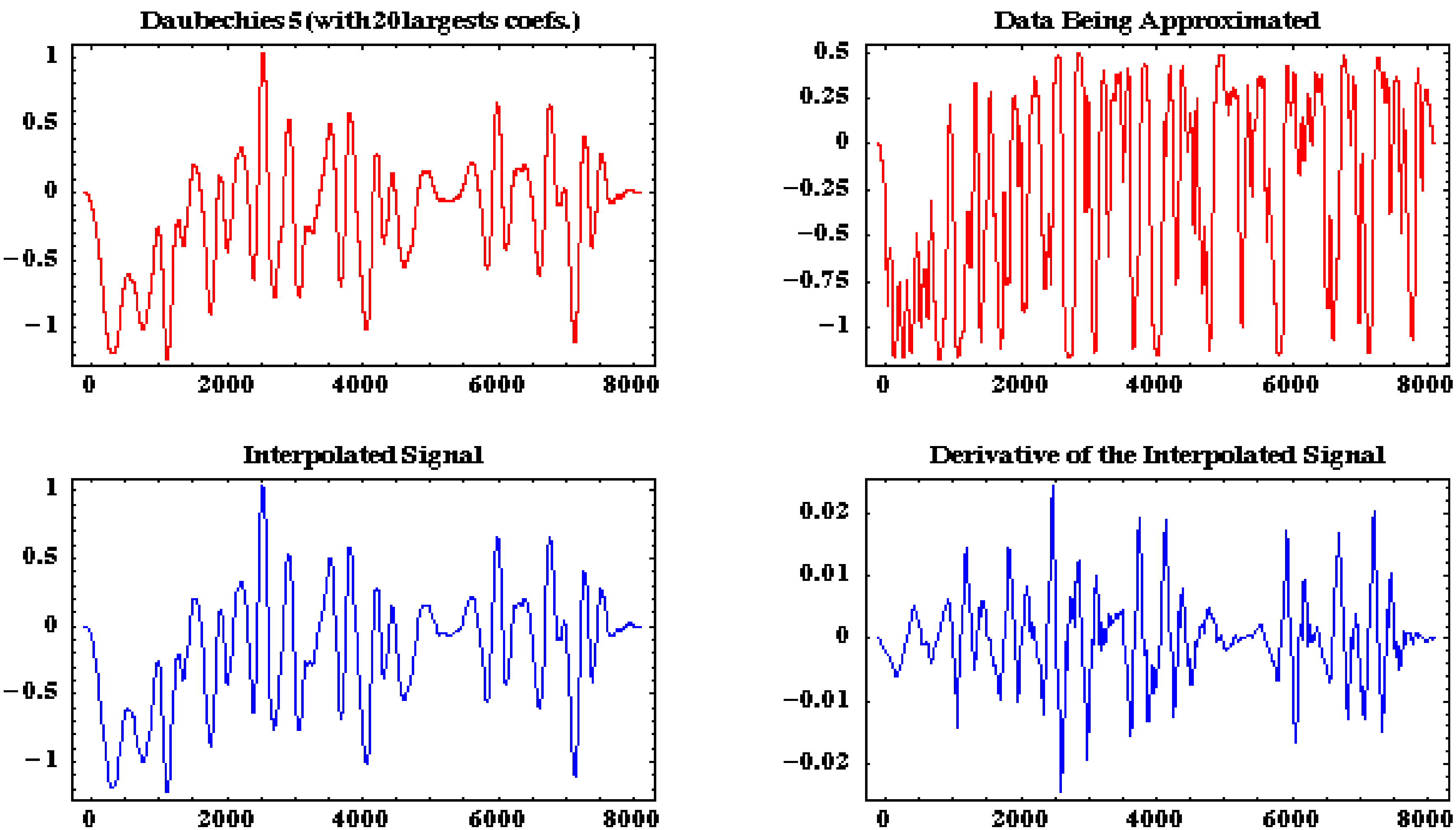




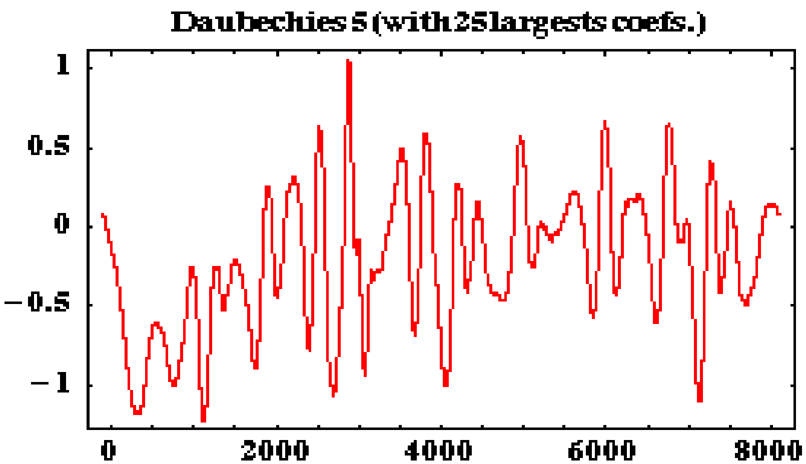


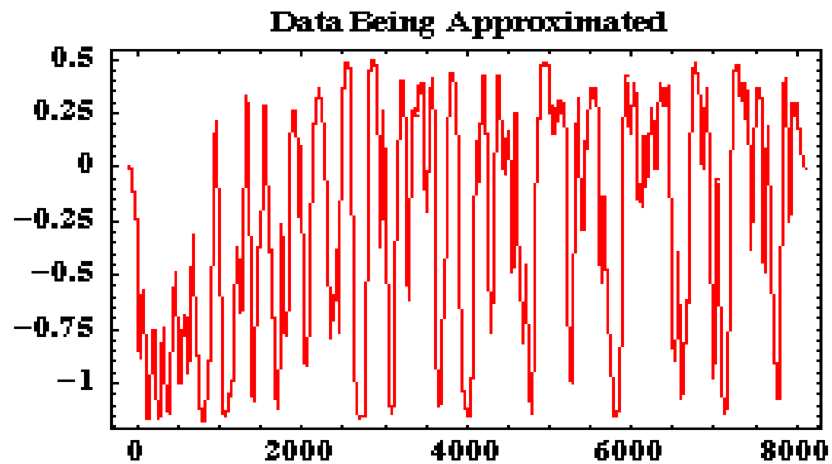
Reconstruction of the LPF RT Weak Mix Data Using the 20 Largest WLT Coefficients

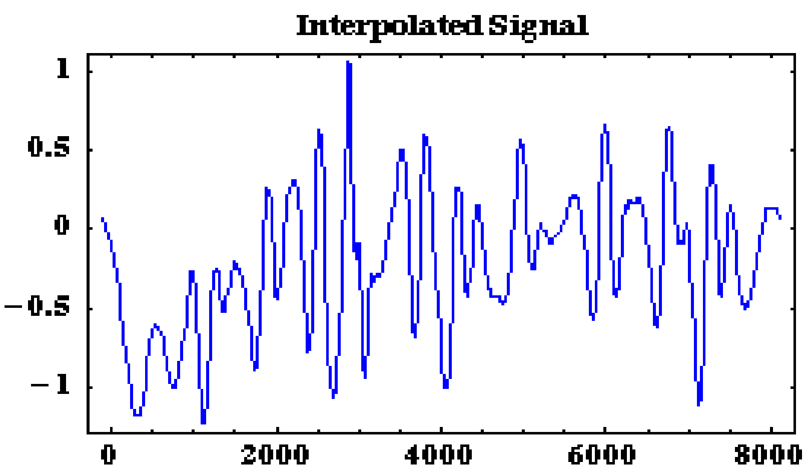
Polymath Research Inc.

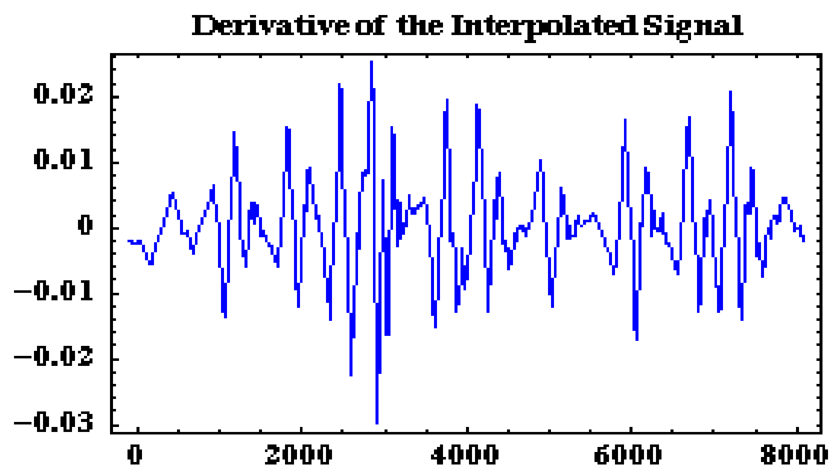


Reconstruction of the LPF RT Weak Mix Data Using the 25 Largest WLT Coefficients



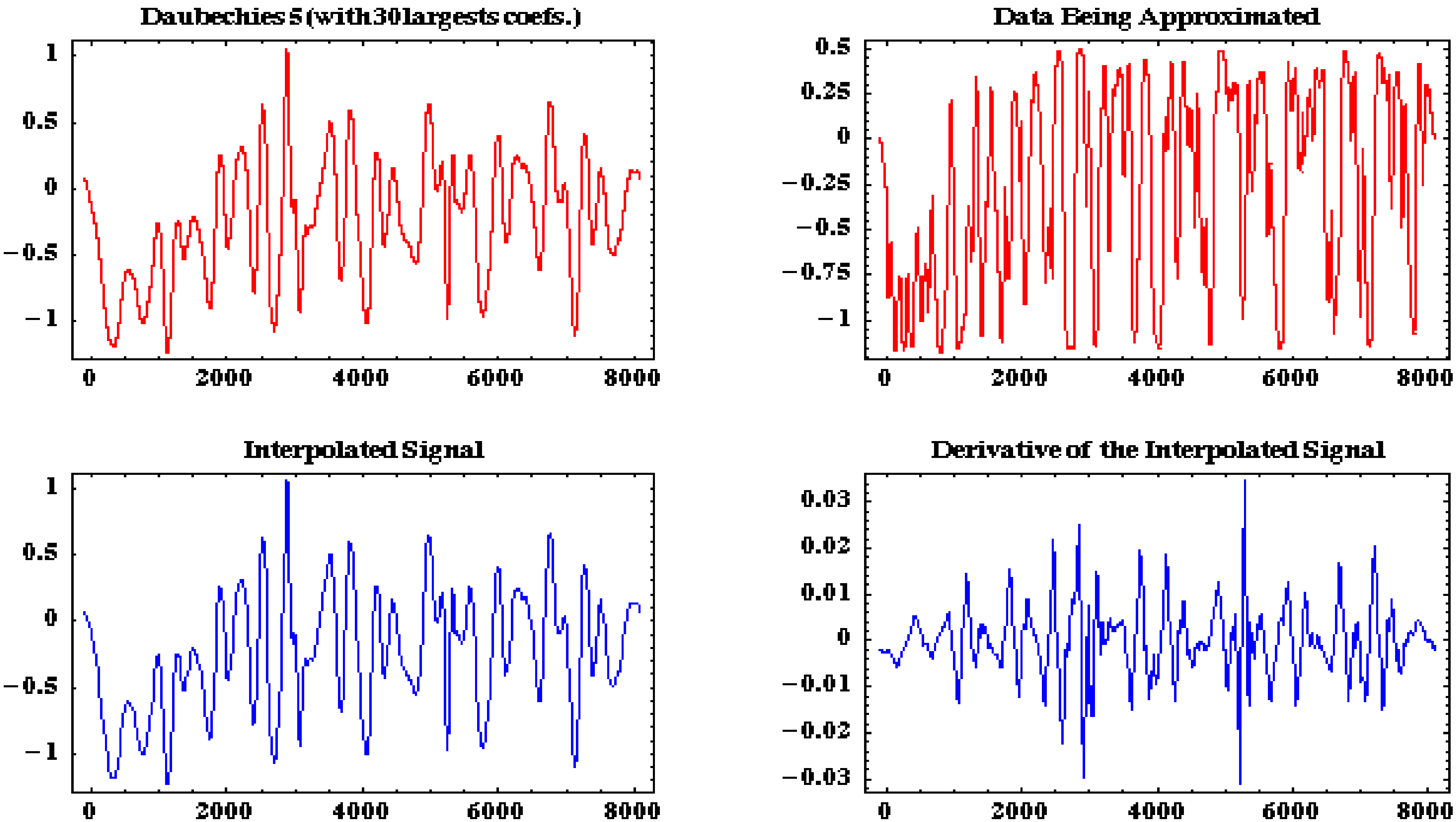




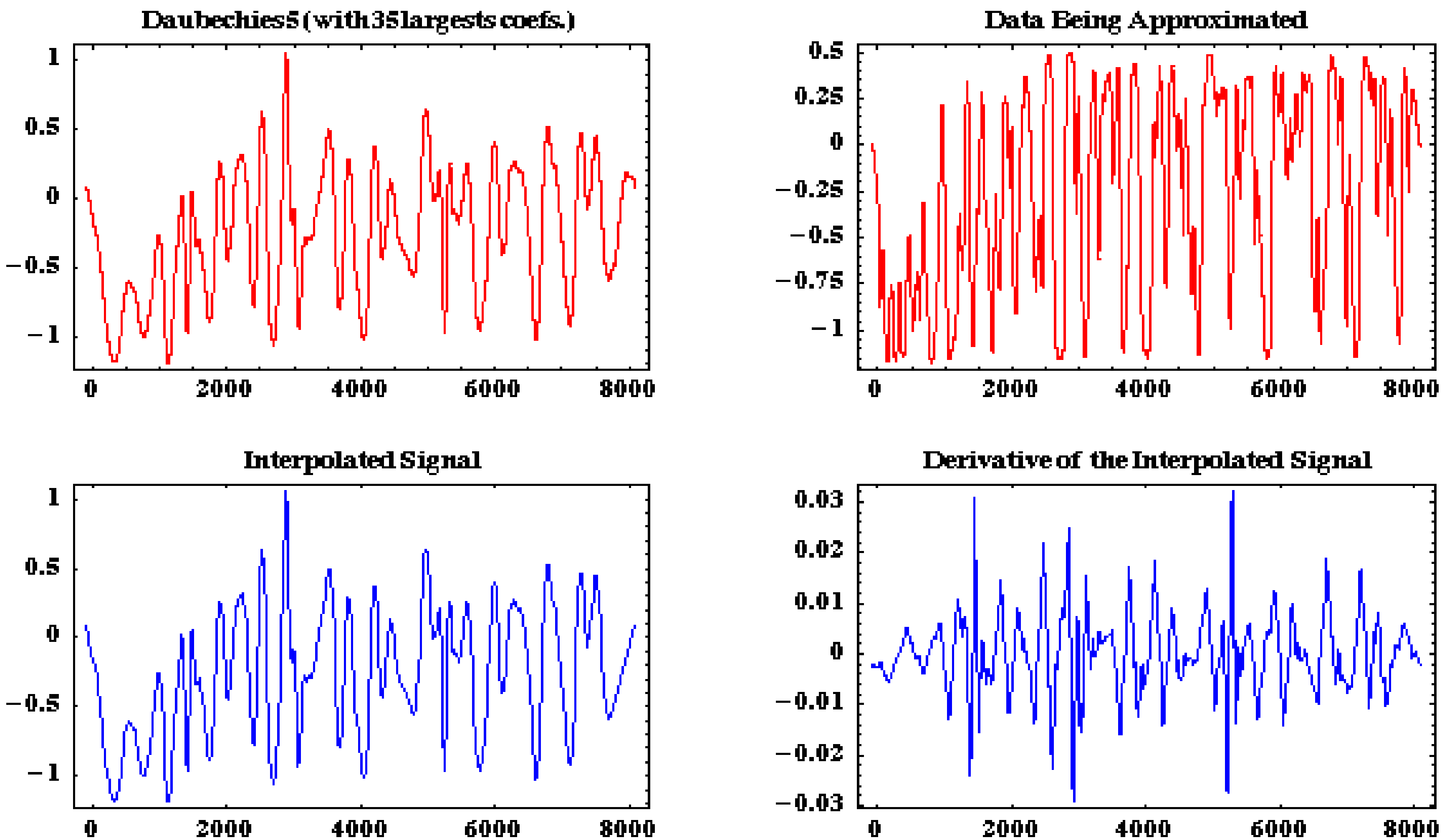


Reconstruction of the LPF RT Weak Mix Data Using the 30 Largest WLT Coefficients

Polymath Research Inc.

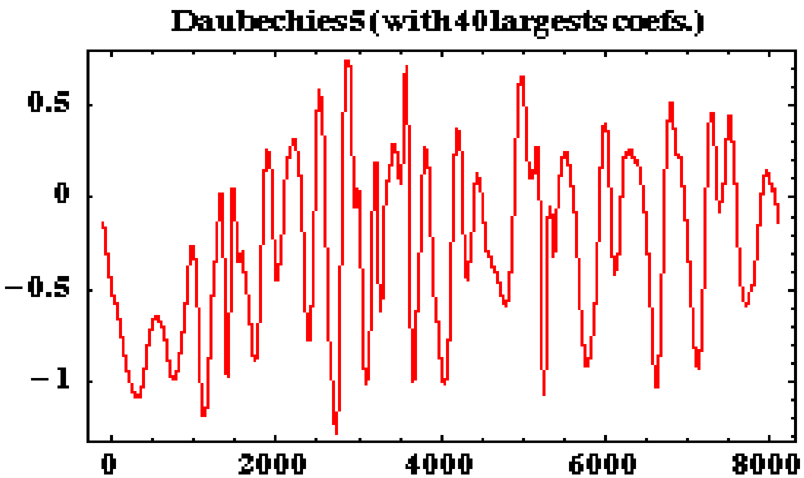


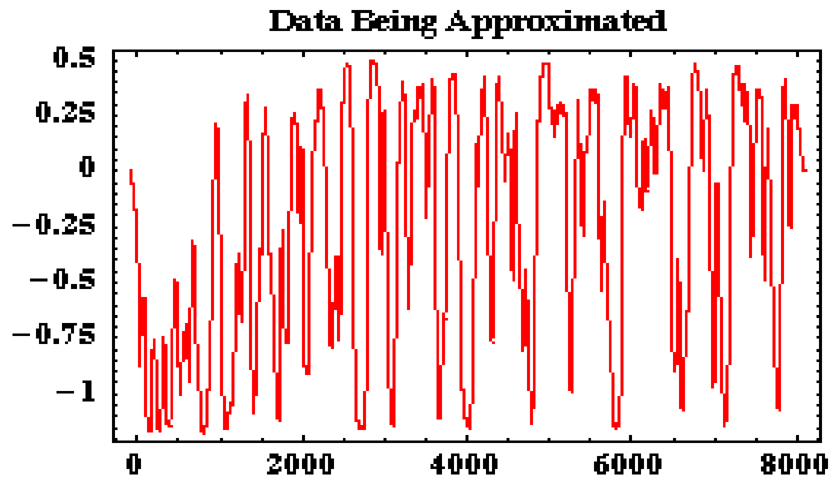
Reconstruction of the LPF RT Weak Mix Data Using the 35 Largest WLT Coefficients

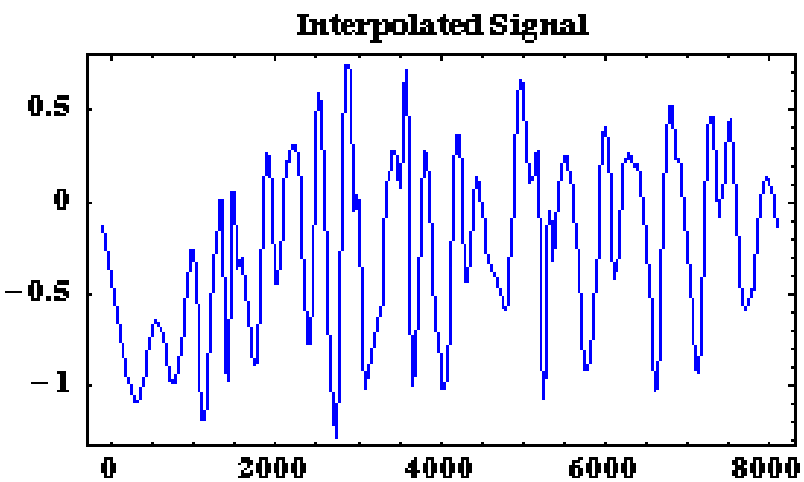


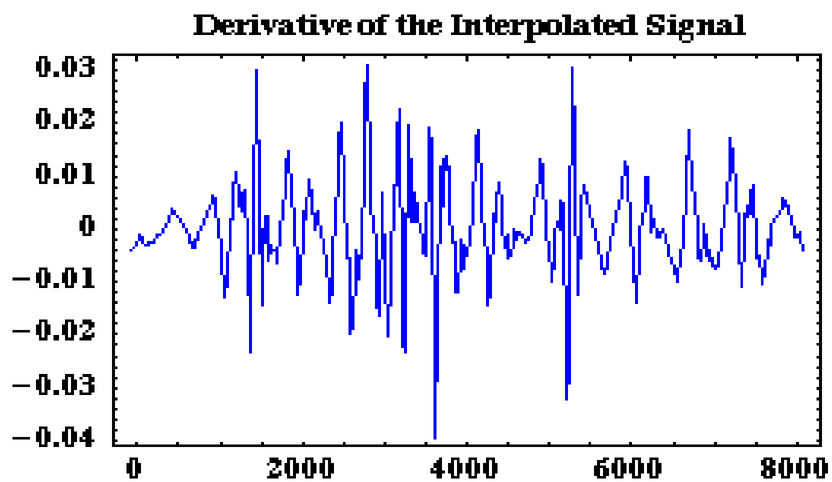
Reconstruction of the LPF RT Weak Mix Data Using the 40 Largest WLT Coefficients

Polymath Research Inc.





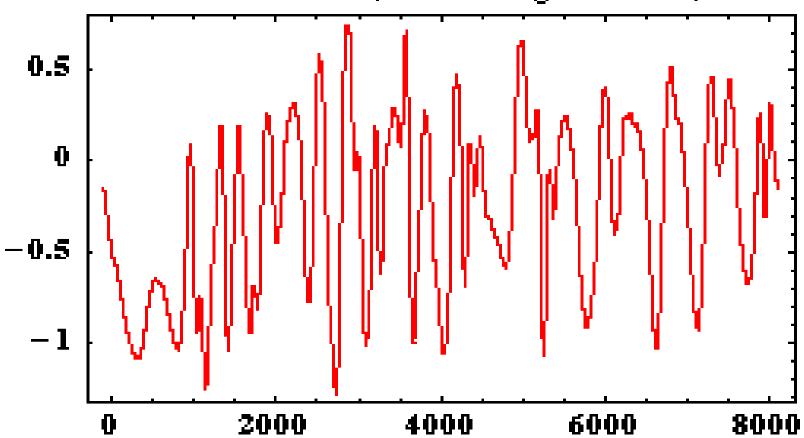




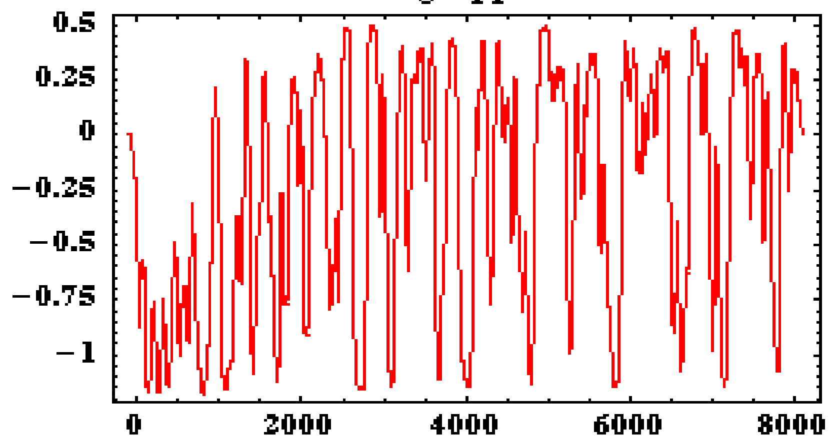
Reconstruction of the LPF RT Weak Mix Data Using the 45 Largest WLT Coefficients

Polymath Research Inc.

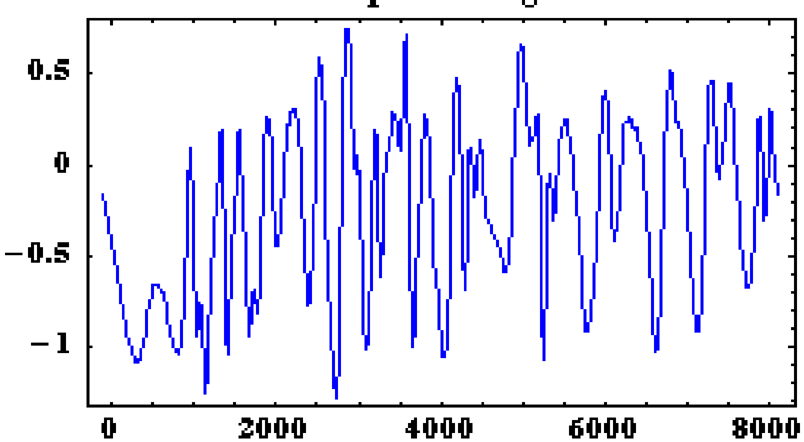
Daubechies5 (with 45 largests coefs.)



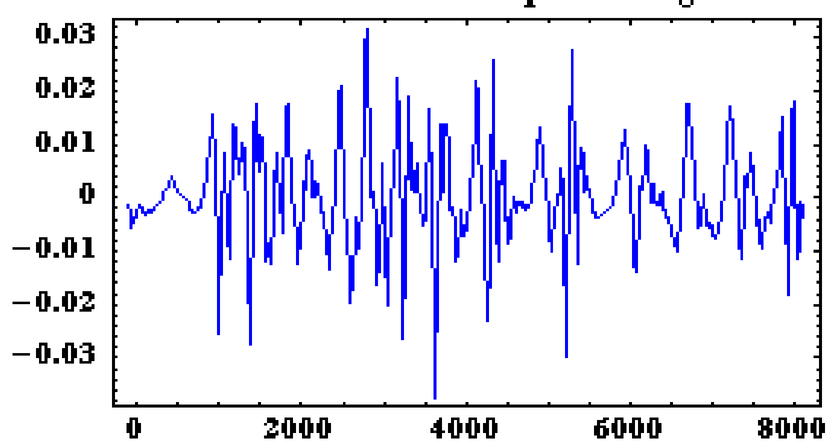
Data Being Approximated



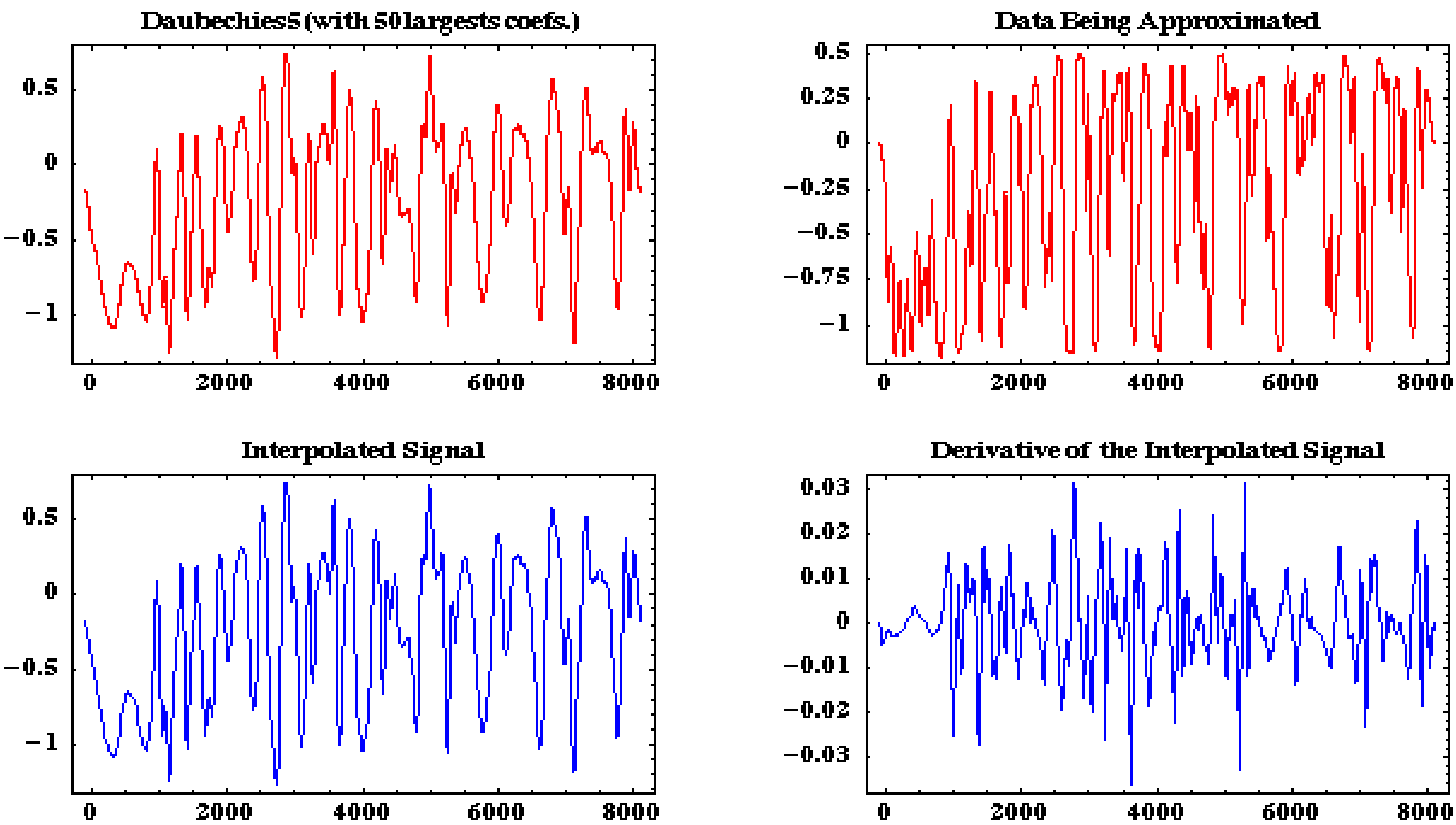
Interpolated Signal



Derivative of the Interpolated Signal

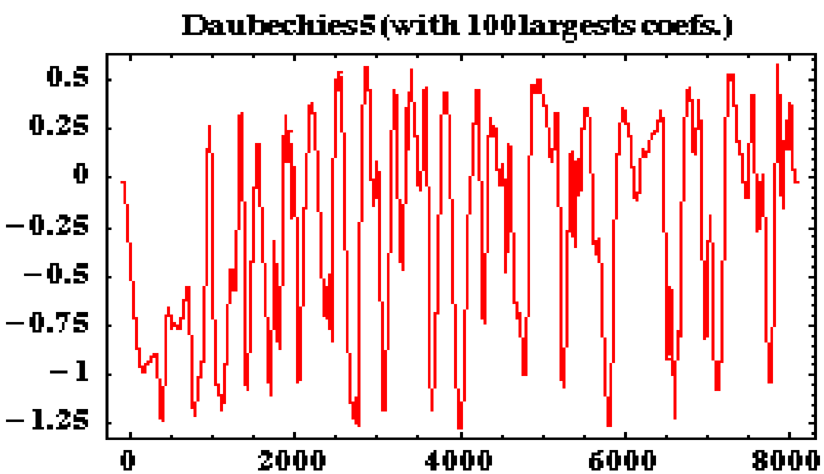


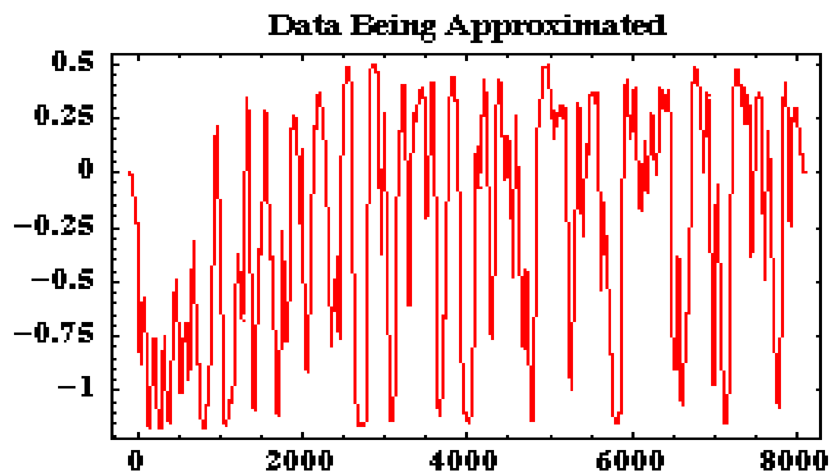
Reconstruction of the LPF RT Weak Mix Data Using the 50 Largest WLT Coefficients

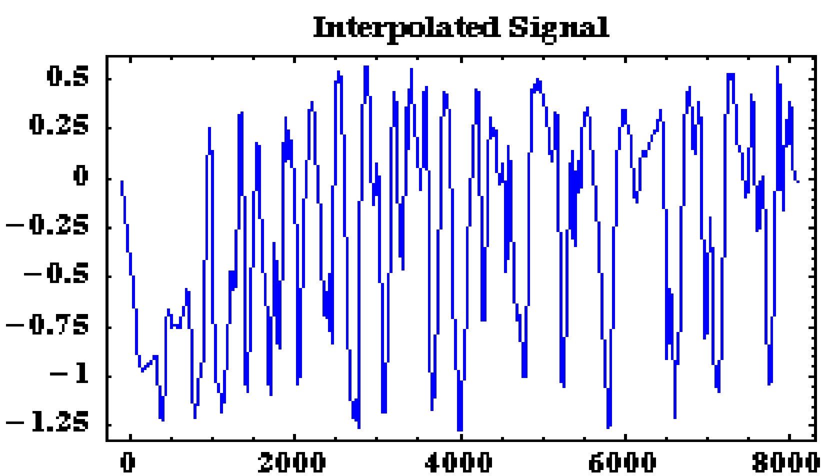


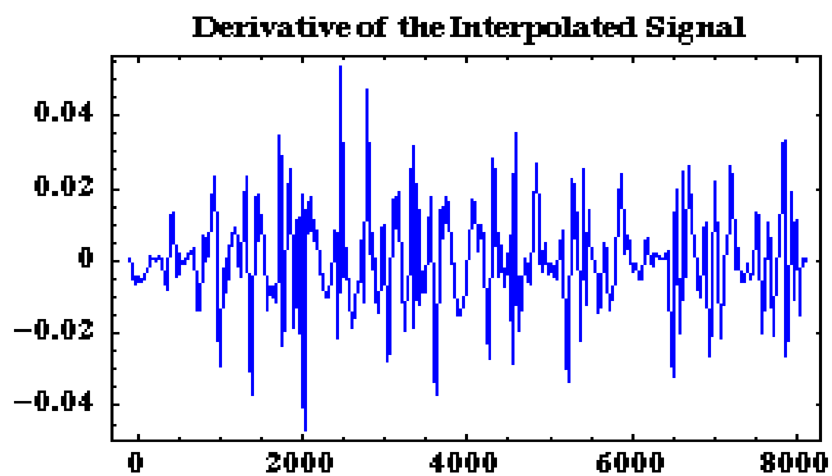
Reconstruction of the LPF RT Weak Mix Data Using the 100 Largest WLT Coefficients

Polymath Research Inc.

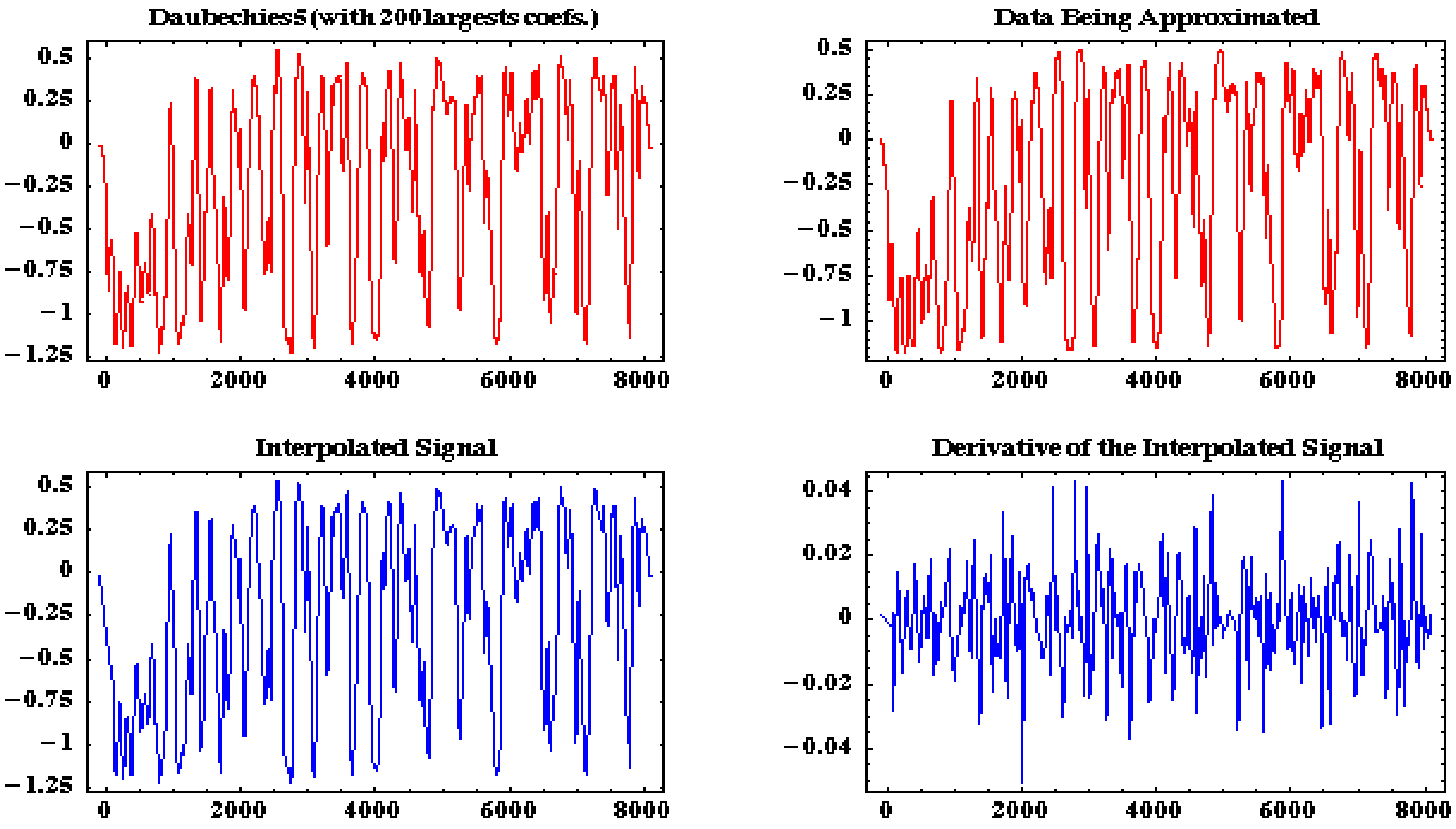








Reconstruction of the LPF RT Weak Mix Data Using the 200 Largest WLT Coefficients



Reconstruction of the LPF RT Weak Mix Data Using the First MRD Level

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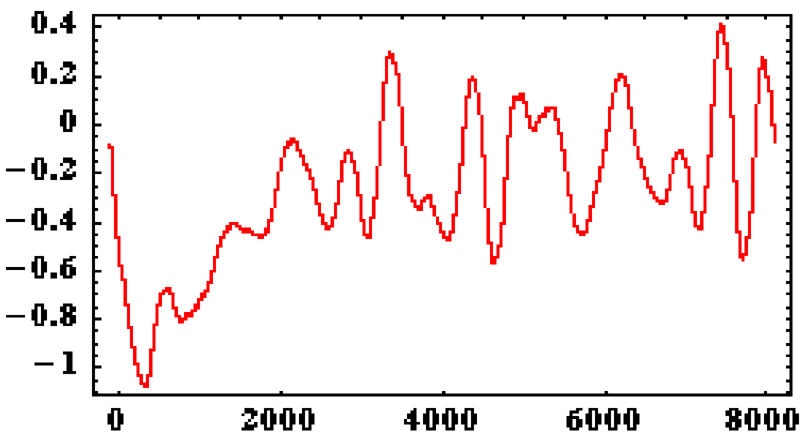
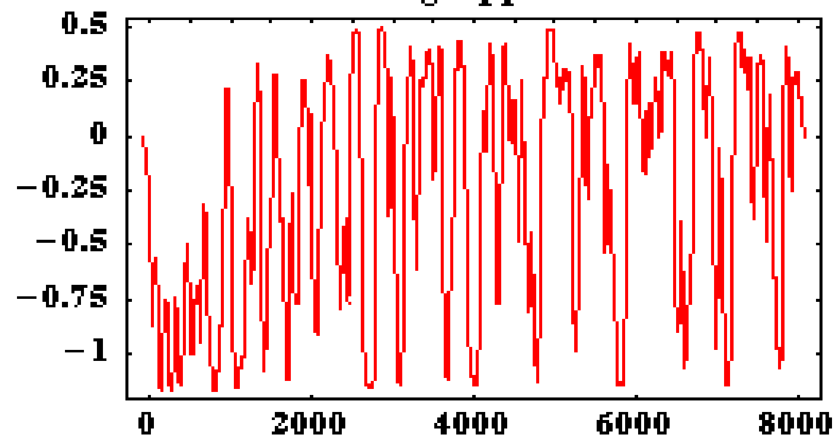
Research Inc.

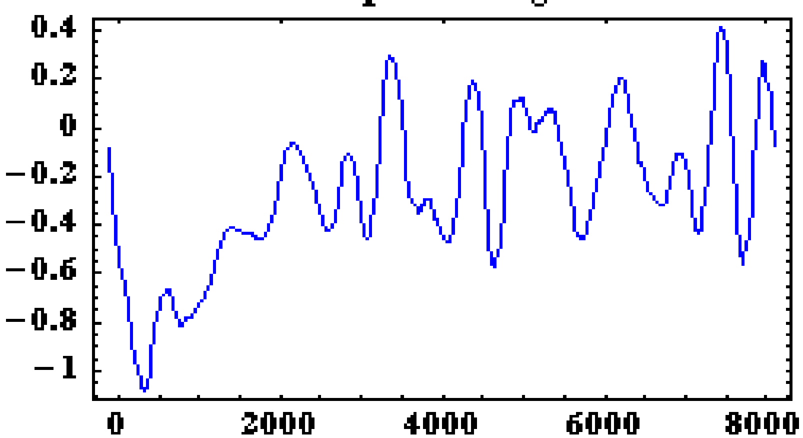
Daubechies 5 (cutoff level = 1)



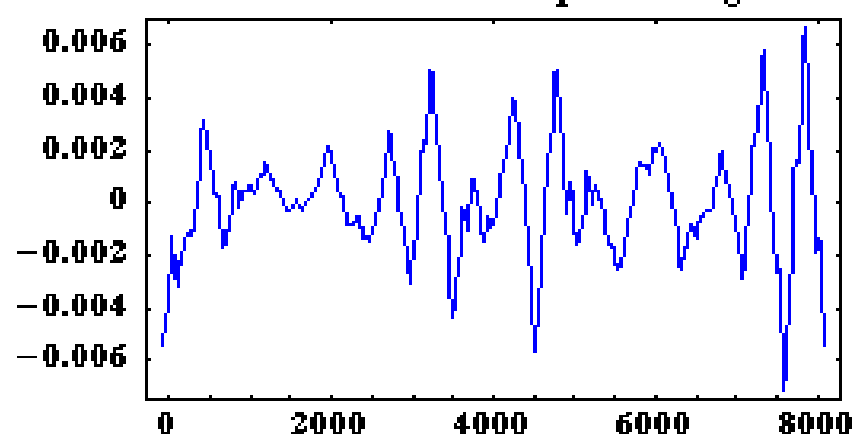
Data Being Approximated



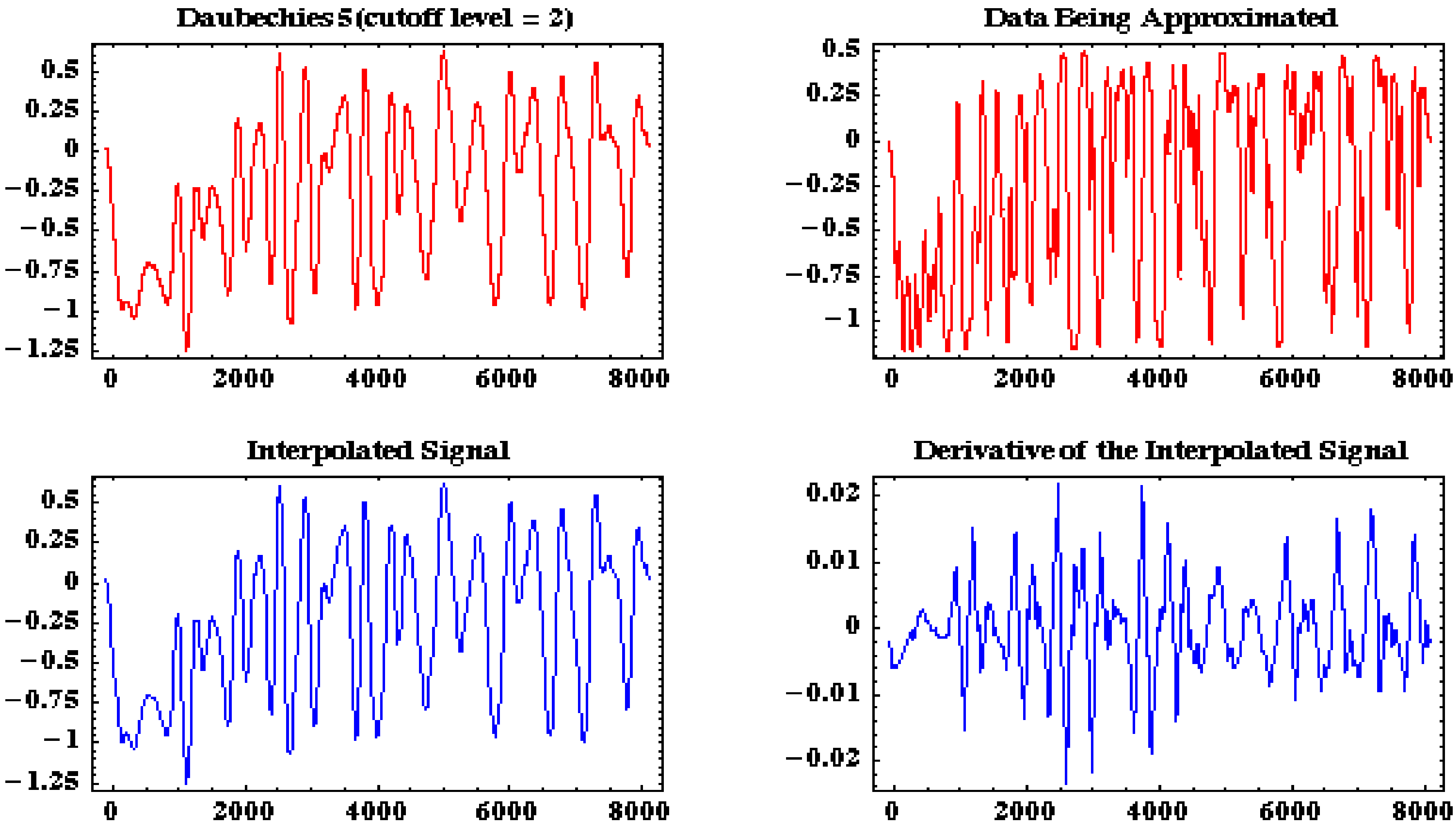
Interpolated Signal



Derivative of the Interpolated Signal

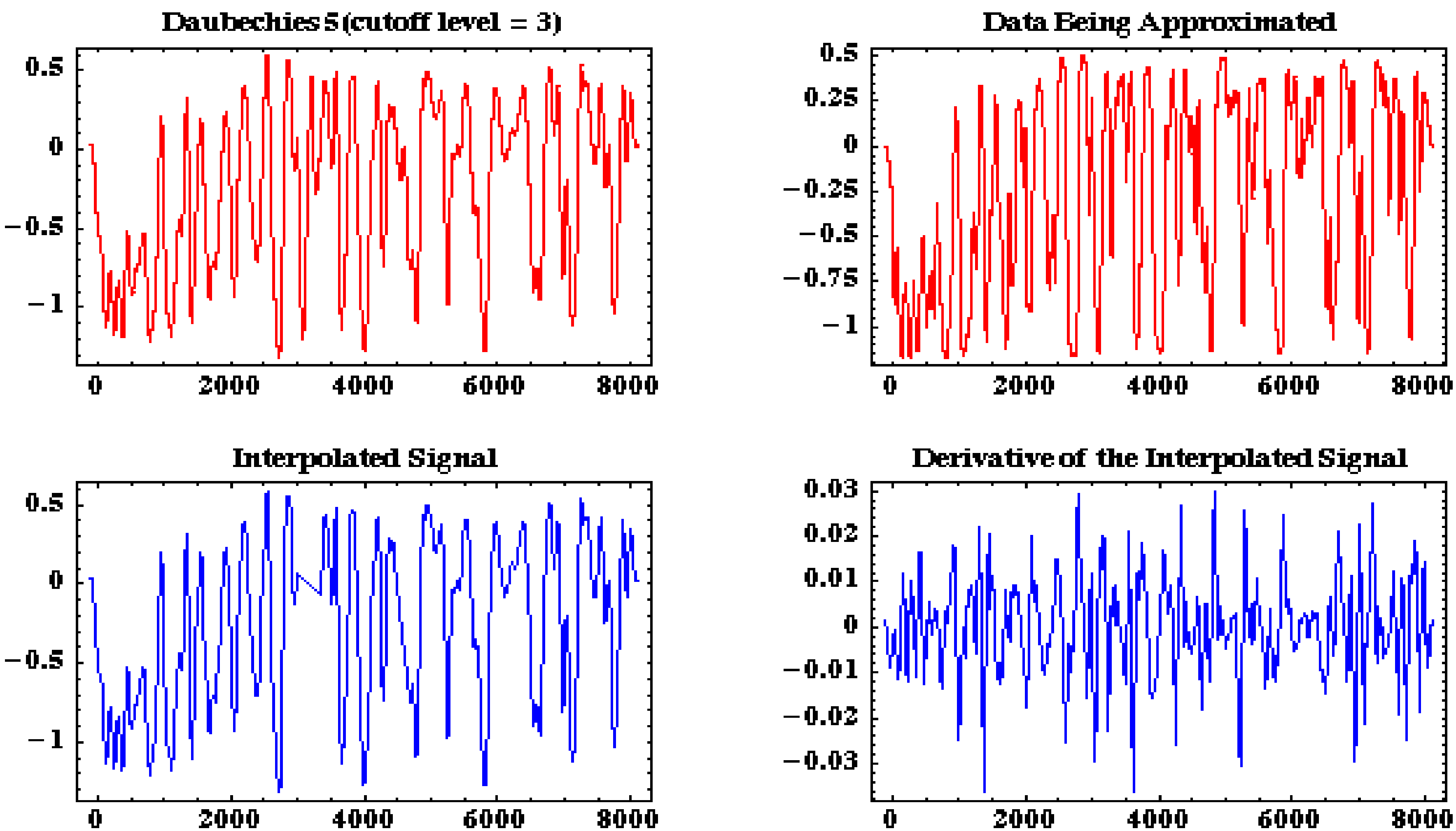


Reconstruction of the LPF RT Weak Mix Data Using the First 2 MRD Levels



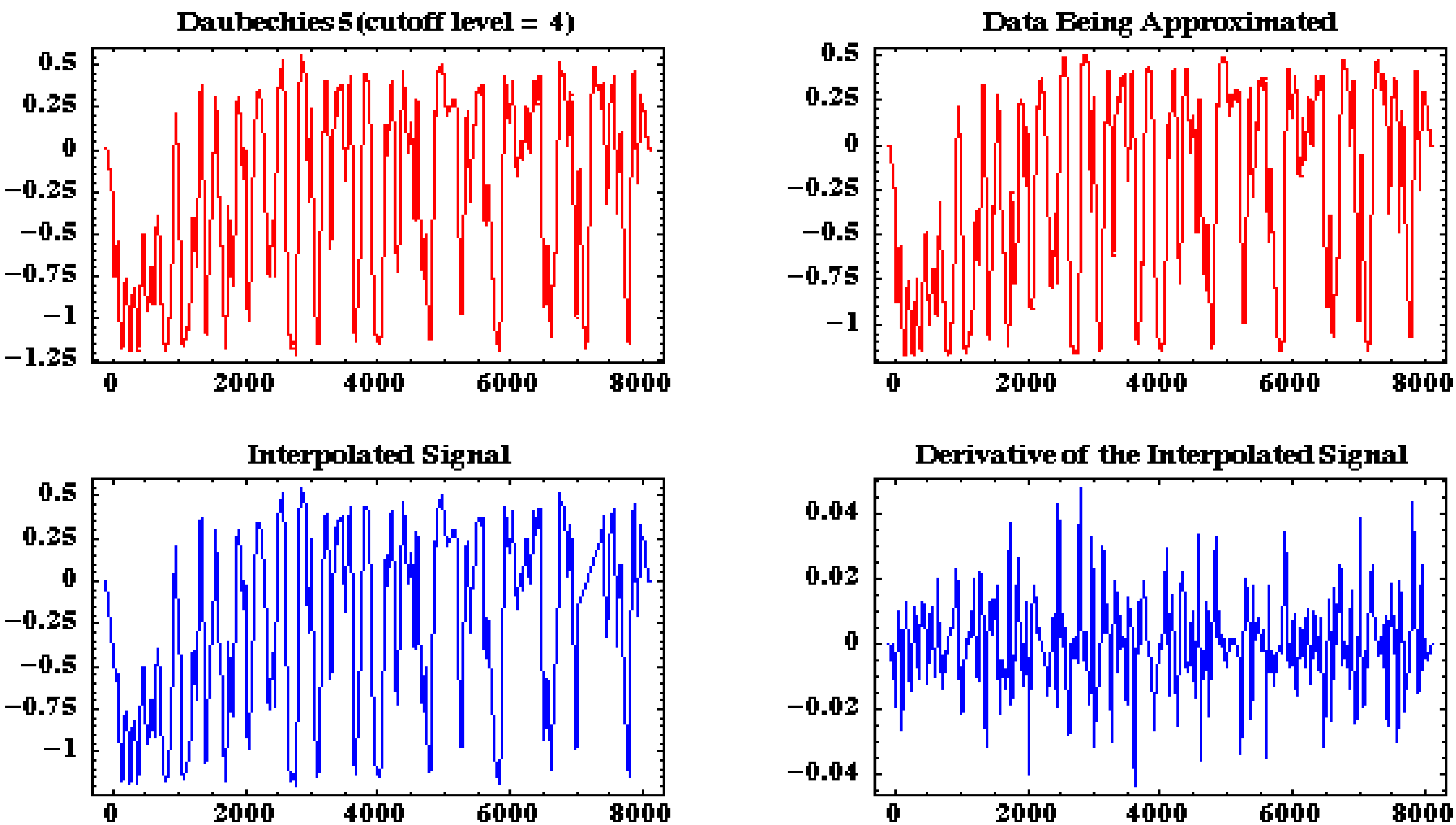
Reconstruction of the LPF RT Weak Mix Data Using the First 3 MRD Levels

Polymath Research Inc.



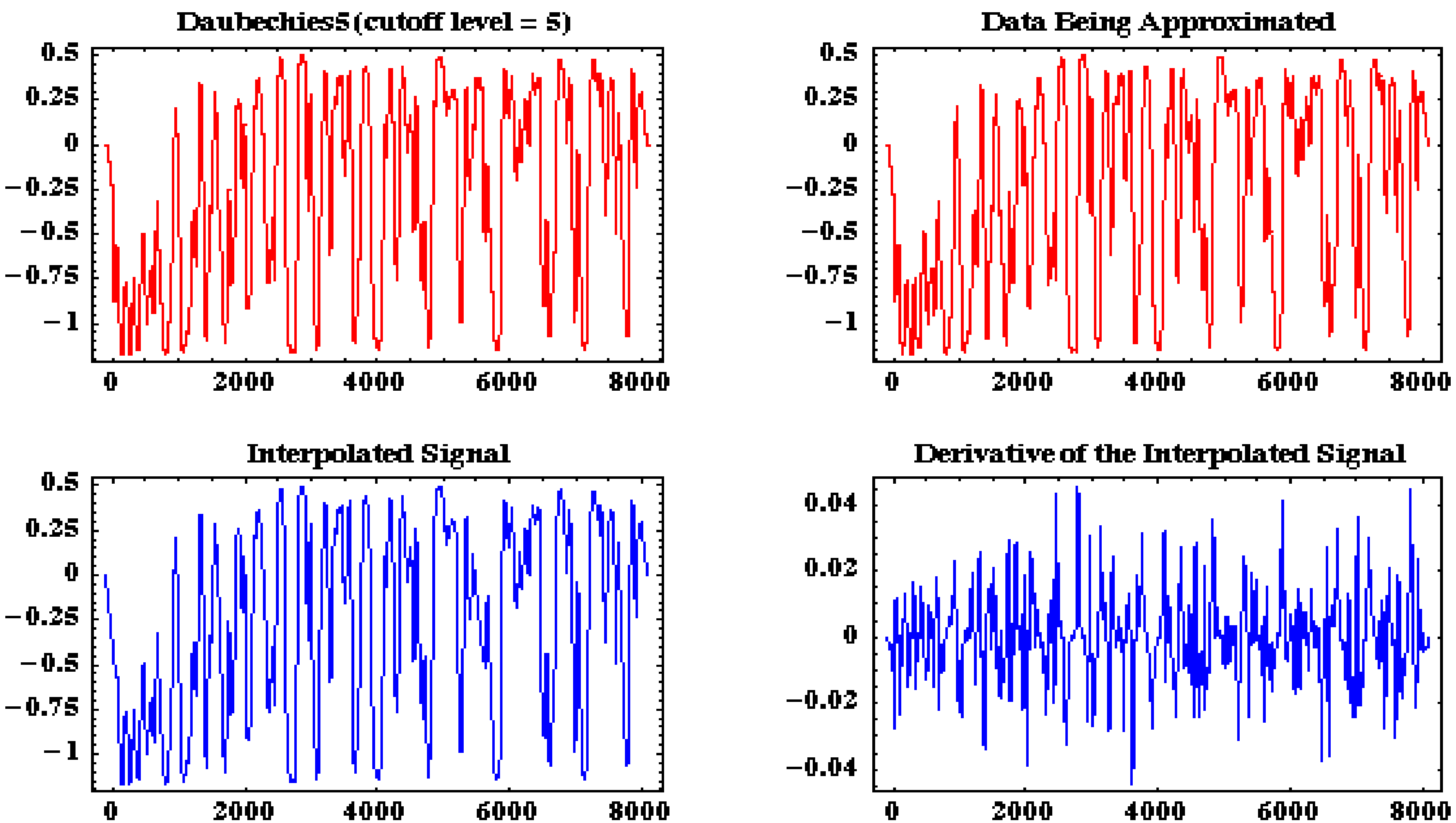
Reconstruction of the LPF RT Weak Mix Data Using the First 4 MRD Levels

Polymath Research Inc.



Reconstruction of the LPF RT Weak Mix Data Using the First 5 MRD Levels

Polymath Research Inc.



Conclusions Based on the LPF RT Weak Mix Data's WLT Analyses