

# **The Dependence of the Shock Induced Richtmyer-Meshkov Instability on Dimensionality and Density Ratio**

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## Buoyancy - Drag Consideration for the Single Mode Case

Newton's second law (for the bubble):

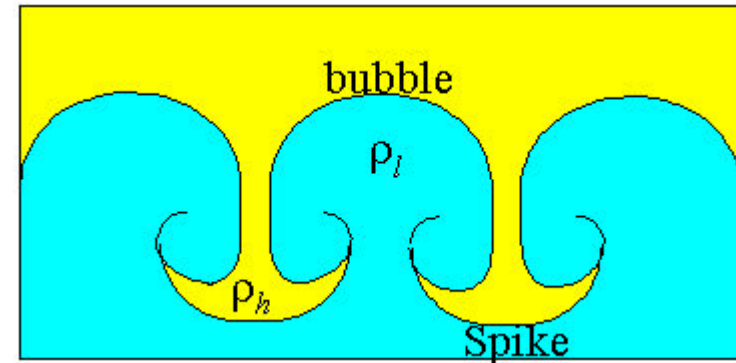
$$(\rho_l \cdot V + C_a \cdot V \cdot \rho_h) \cdot \frac{dU}{dt} = (\rho_h - \rho_l) V \cdot g - C_d \cdot \rho_h \cdot S \cdot U^2$$

For the bubble:

$$(\rho_l + C_a \rho_h) \dot{U} = (\rho_h - \rho_l) \cdot g - \frac{C_d}{\lambda} \rho_h \cdot U^2$$

For the spike: ( $\rho_h \Leftrightarrow \rho_l$ )

$$(\rho_h + C_a \rho_l) \dot{U} = (\rho_l - \rho_h) \cdot g - \frac{C_d}{\lambda} \rho_l \cdot U^2$$



## Linear Stage Single Mode Velocity

Atwood Number:  $A = \frac{\mathbf{r}_H - \mathbf{r}_L}{\mathbf{r}_H + \mathbf{r}_L}$

Using:  $g(t) = U_0 \delta(t)$  and  $U^2 \ll \frac{(\rho_h - \rho_l) \lambda \cdot g}{\rho_h \cdot C_d}$

$U_{\text{linear}} = U_0 \cdot k \cdot h_0 \cdot A$  Constant velocity

$$\left( k = \frac{2\pi}{\lambda} \right)$$

# Asymptotic Single Mode Velocities:

$$(\rho_1 + C_a \rho_h) \dot{U} = (\rho_h - \rho_1) \cdot g - \frac{C_d}{\lambda} \rho_h \cdot U^2$$

$g=0$  ↓



Using \*:

2D:	$C_d=6\pi$	$C_a=2$
3D:	$C_d=2\pi$	$C_a=1$

For the bubble:

$$U_B = \left( \frac{1-A}{1+A} + C_a \right) \cdot \frac{1}{C_d} \cdot \frac{l}{t}$$

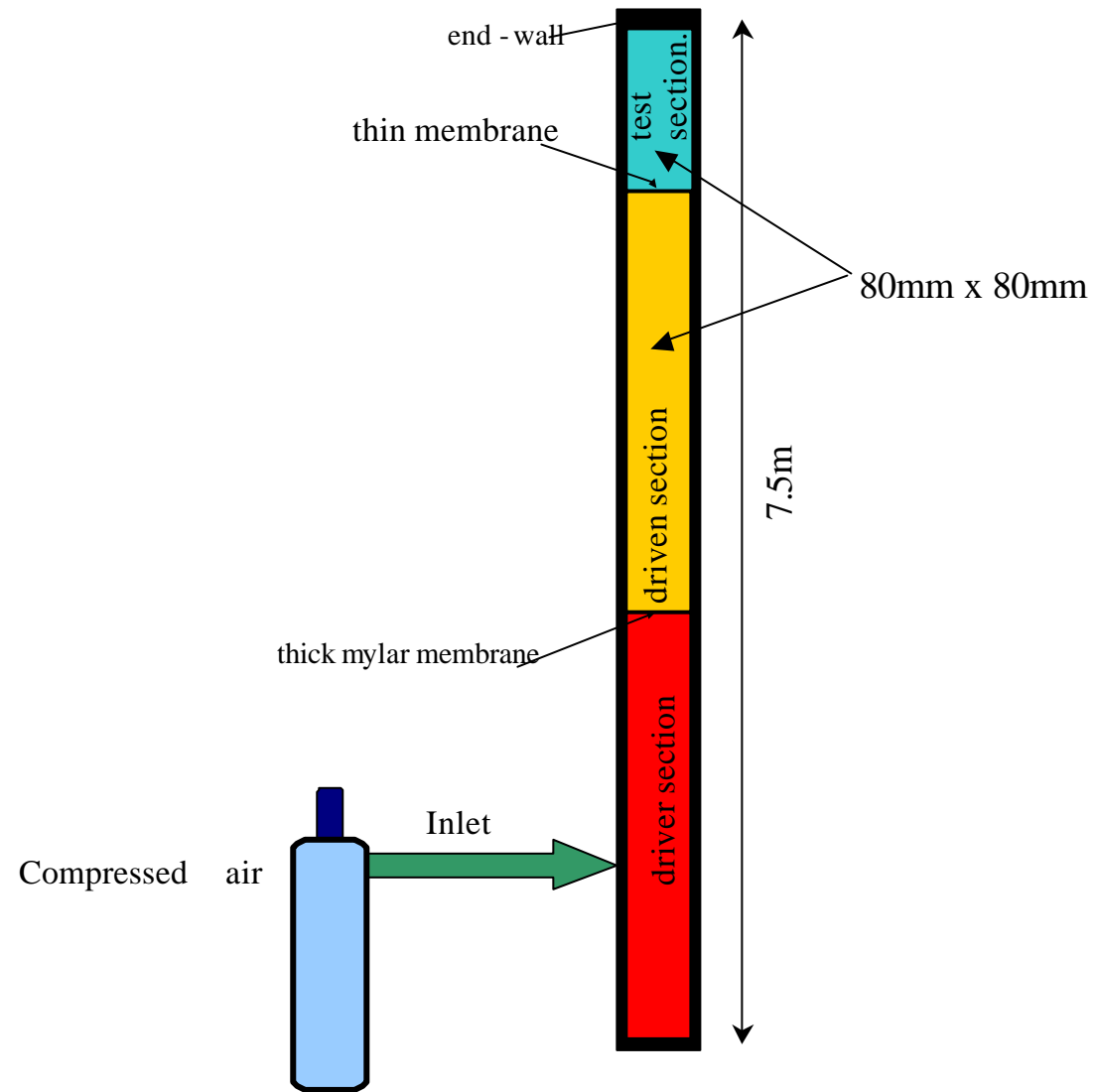
For the spike:  $\rho_h \Leftrightarrow \rho_1$

	$A \approx 0$	$A \approx 1$
2D: 	$\frac{1}{2p} \cdot \frac{l}{t}$	$\frac{1}{3p} \cdot \frac{l}{t}$
3D: 	$\frac{1}{p} \cdot \frac{l}{t}$	$\frac{1}{2p} \cdot \frac{l}{t}$

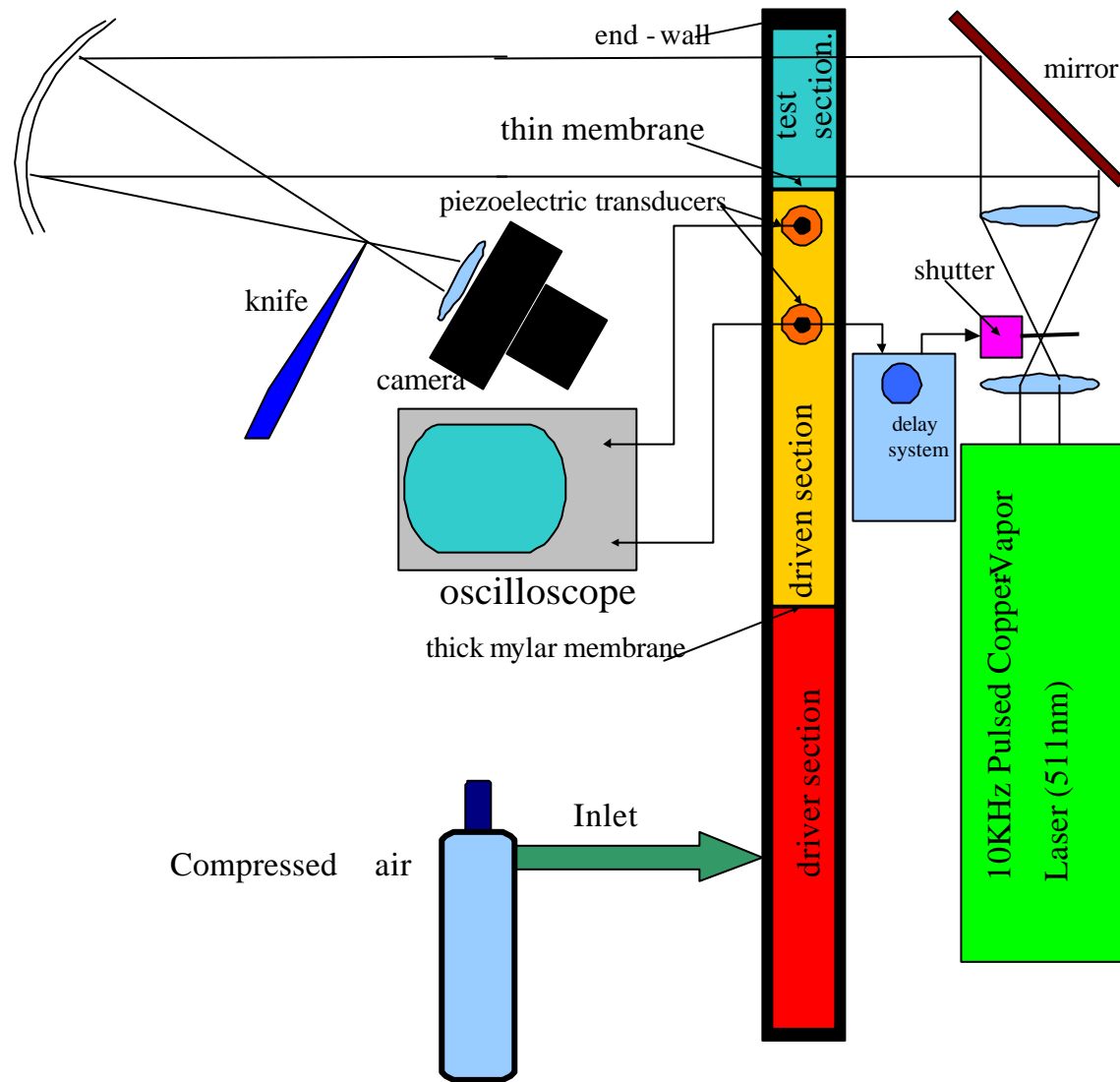
$$U_s = \begin{cases} (A+1)/(A-1) \cdot (3-A)/(3+A) \cdot U_B & \text{for 2D} \\ (A+1)/(A-1) \cdot U_B & \text{for 3D} \end{cases}$$

\* Layzer (1955);  
Hecht *et al.* (1994).

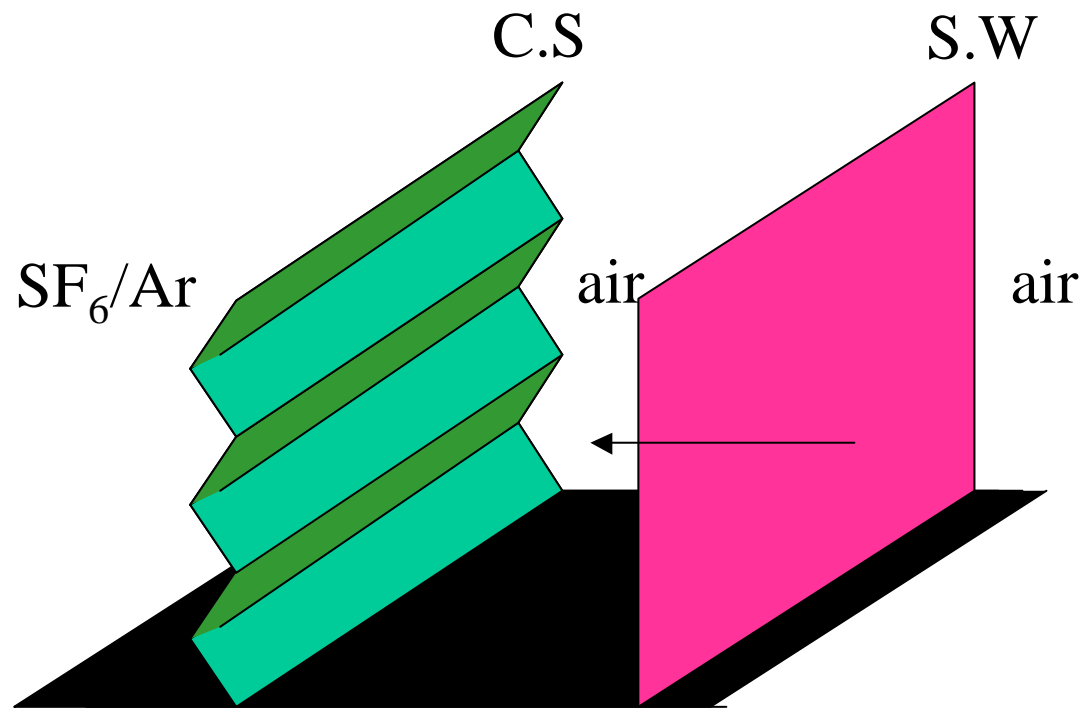
# Experimental Apparatus



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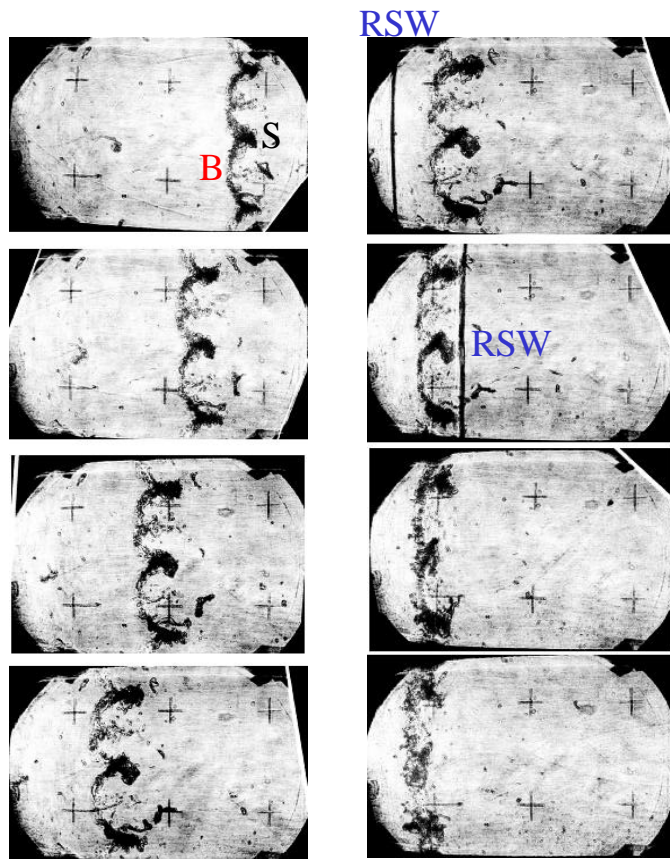


# Experimental Setup - The Membrane

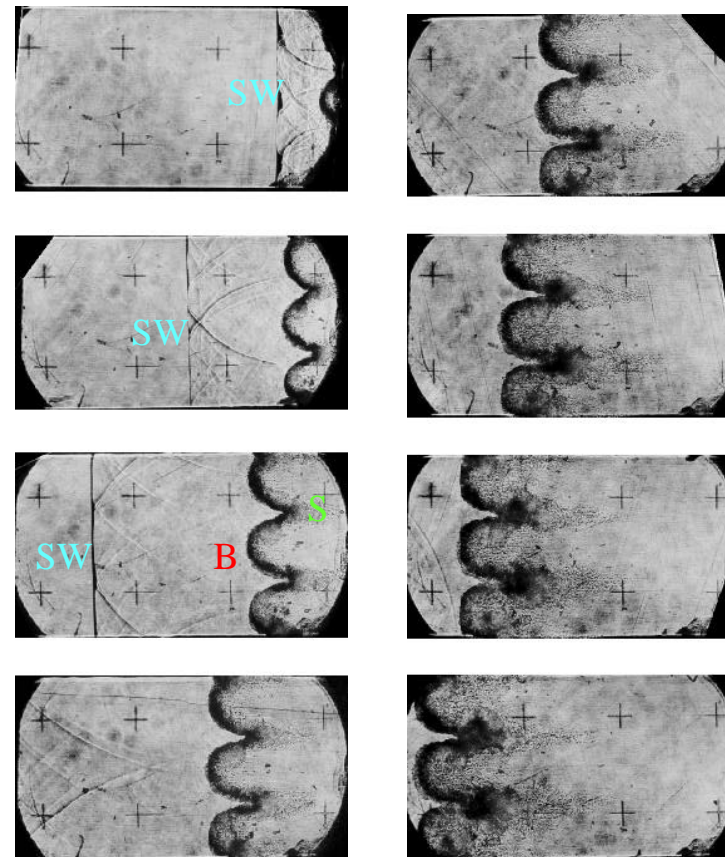


# 2D - Low and High Atwood number Experiments

A=0.2 (Air to Ar)  $\lambda=26\text{mm}$



A=0.7 (Air to SF<sub>6</sub>)  $\lambda=26\text{mm}$

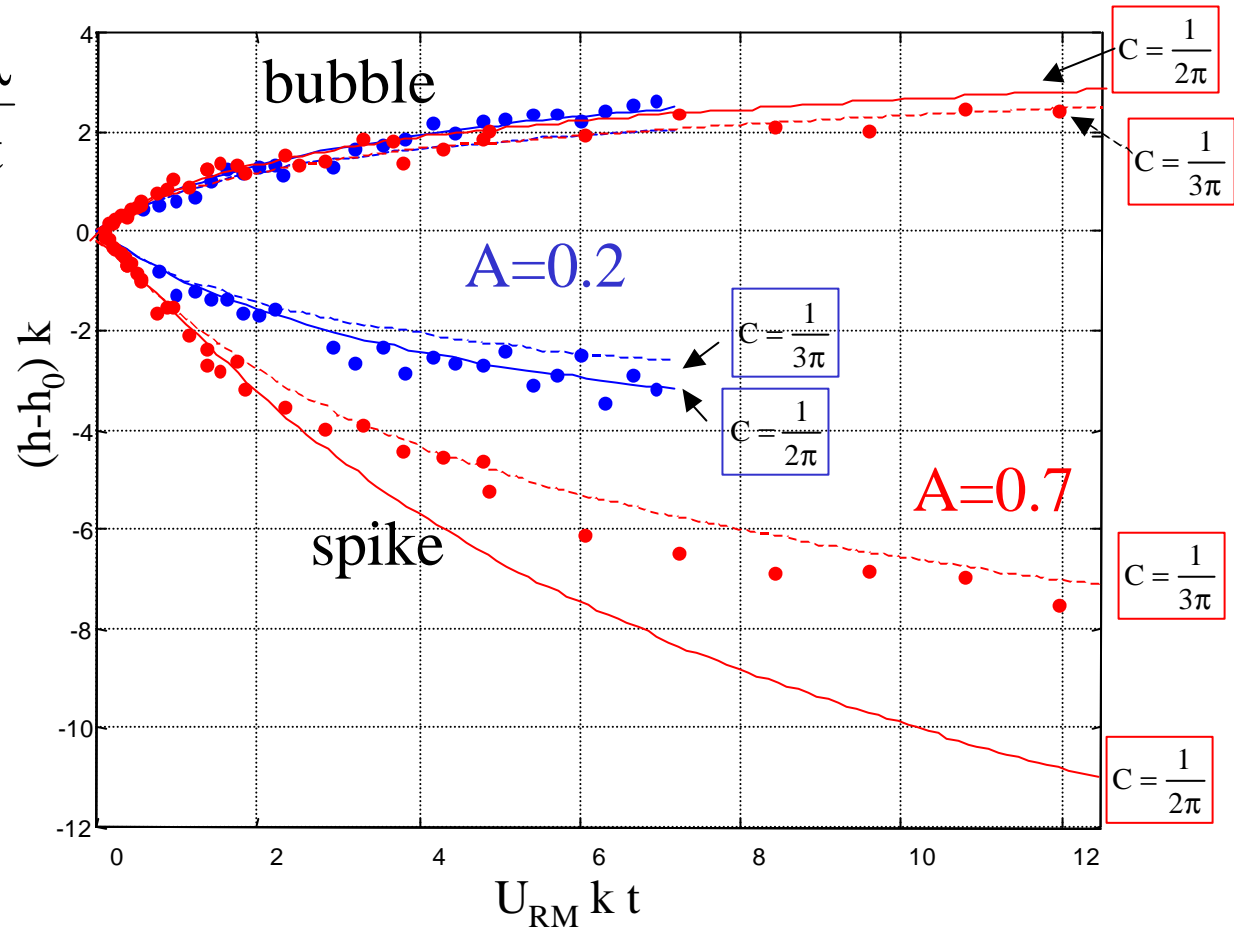




# Experiment vs. Model for the Atwood Number Dependence

$$U_{\text{asy}} = C \cdot \frac{\lambda}{t}$$

	A=0.2	A=0.7
1/C	$2\pi$	$3\pi$
$U_b/U_s$	0.77	0.28

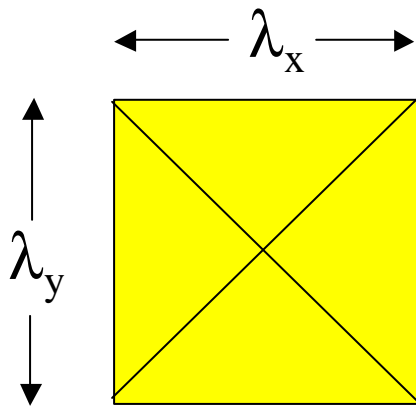


## Dimensionality Investigation:

$$U_{B_{A \approx 1}}^{(\text{asy.})} = \begin{array}{|c|c|} \hline 2\text{D:} & \frac{1}{3p} \cdot \frac{I}{t} \\ \hline 3\text{D:} & \frac{1}{2p} \cdot \frac{I}{t} \\ \hline \end{array}$$

## The Effective Wavelength

3D



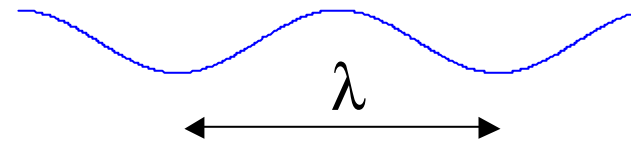
$$k_i = \frac{2\pi}{\lambda_i}$$

$$i = x, y$$

$$|\mathbf{k}| = \sqrt{k_x^2 + k_y^2}$$

$$\bar{\lambda} = \frac{2\pi}{|\mathbf{k}|}$$

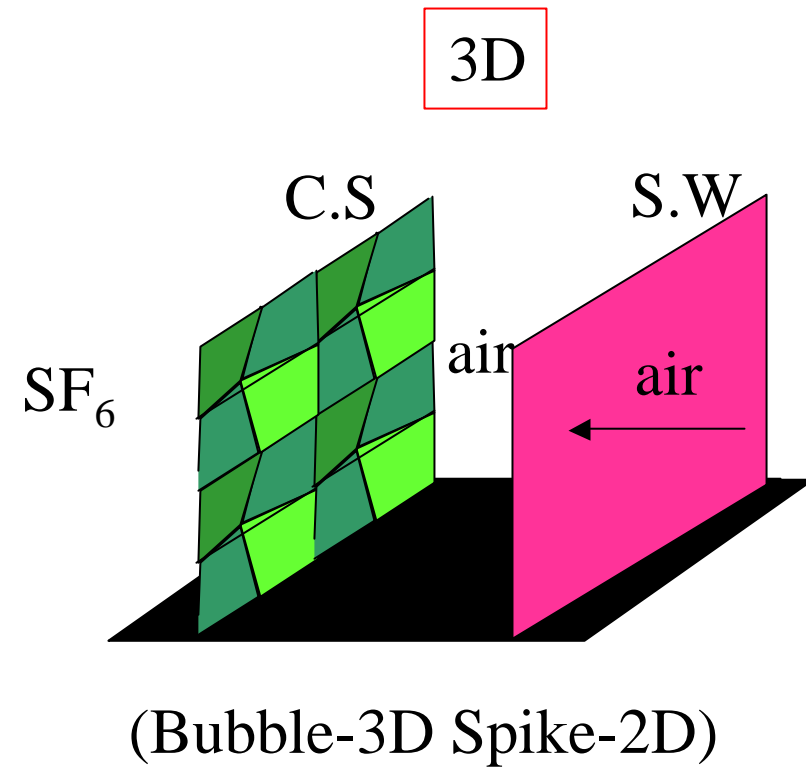
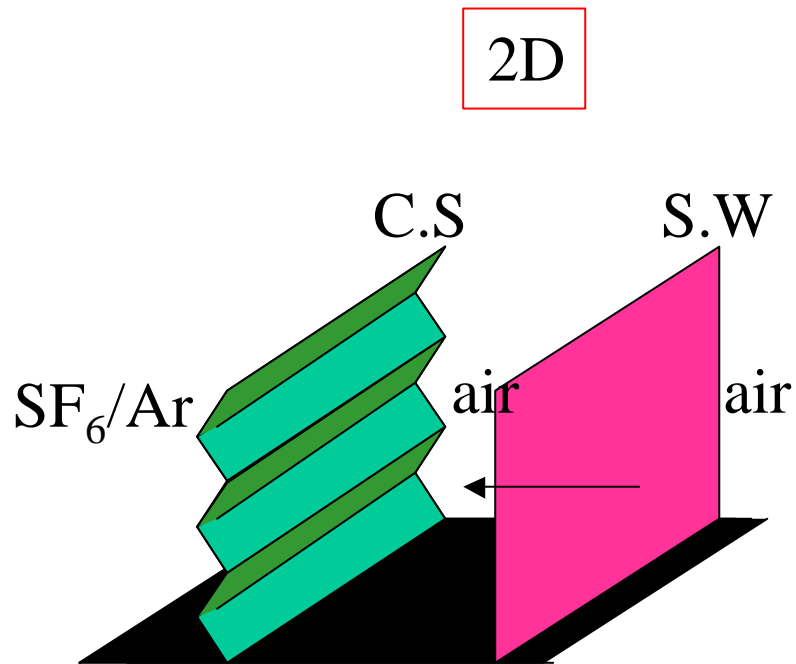
2D



$$k = \frac{2\pi}{\lambda}$$

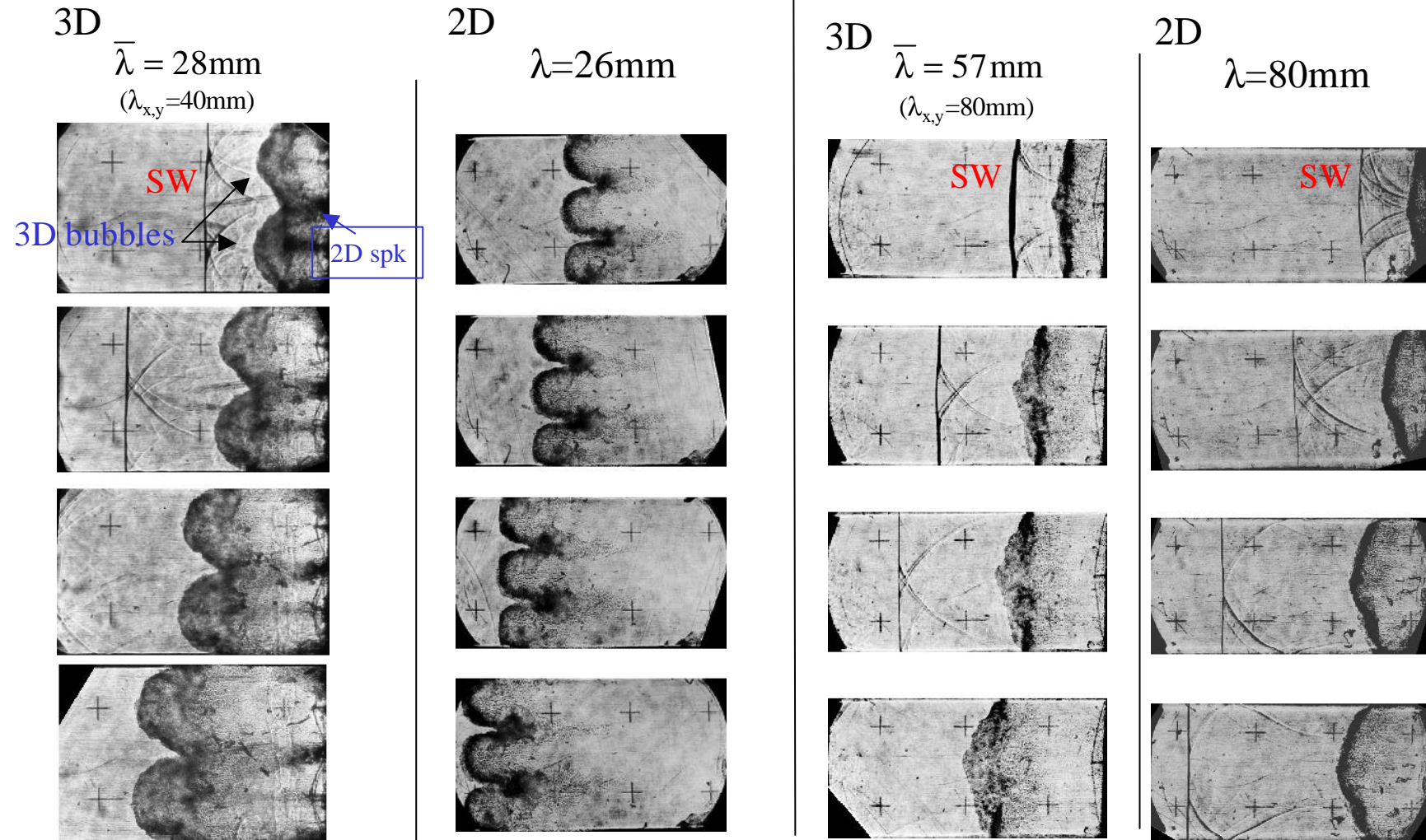
# Experimental Setup - The Membrane - 2D/3D case

Periodic Initial Conditions:



# Results of 2D vs. 3D Experiments

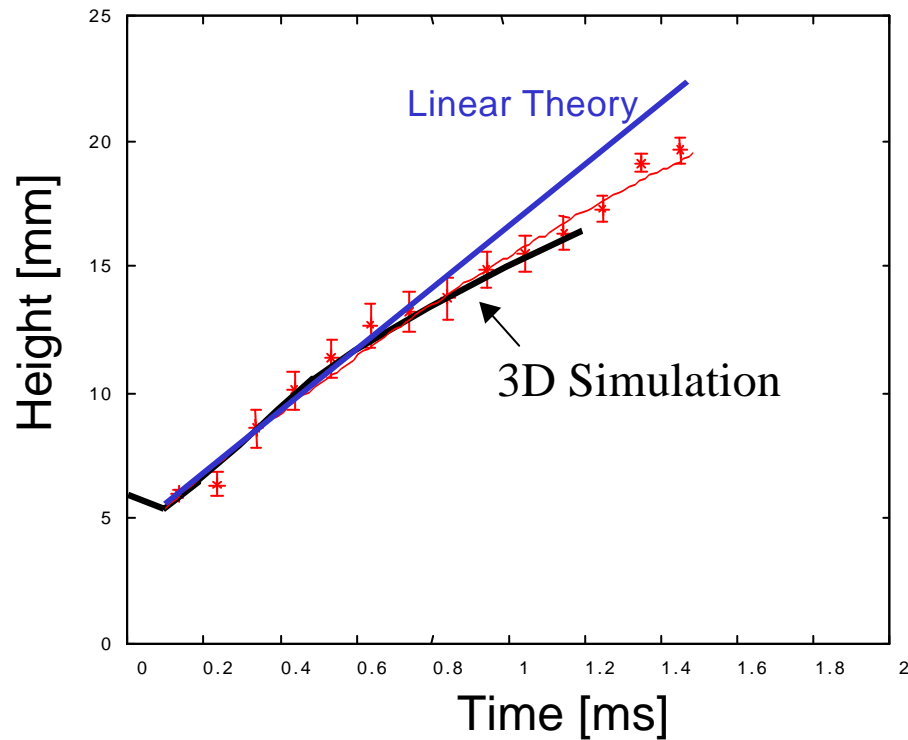
(M=1.20 Air to SF<sub>6</sub>)



# Dimensionality Dependence Results - Bubbles

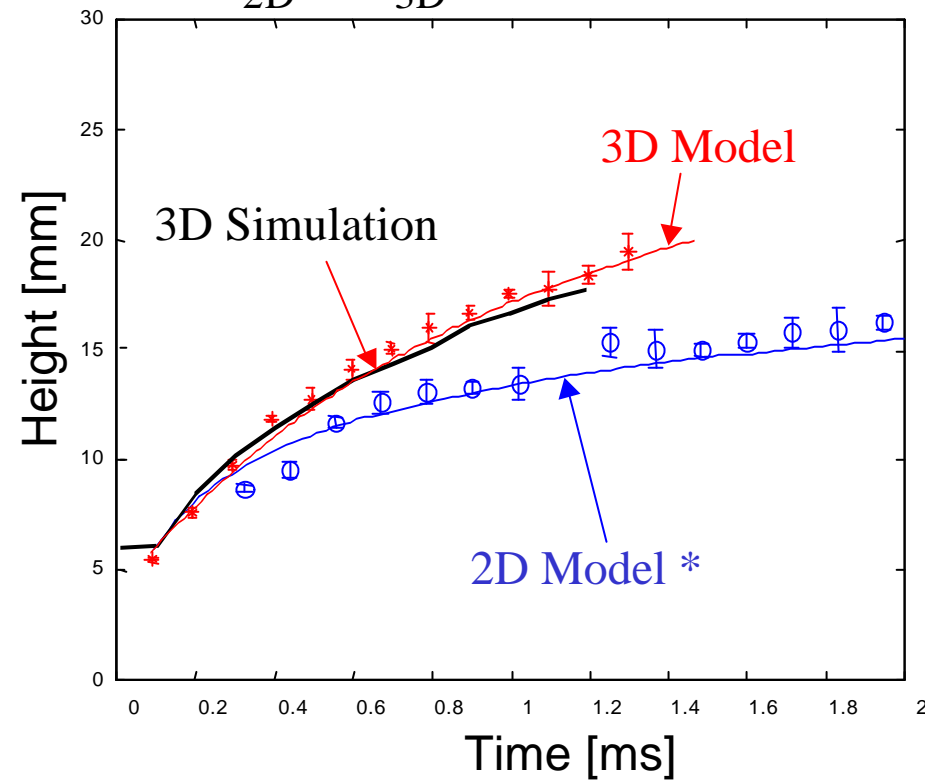
Linear stage

$$k_{3D} \approx 0.1 \text{ mm}^{-1}$$



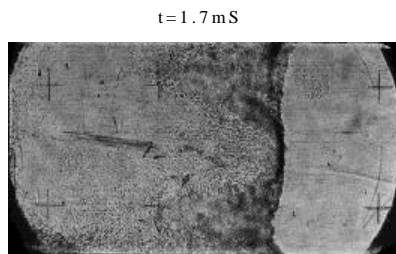
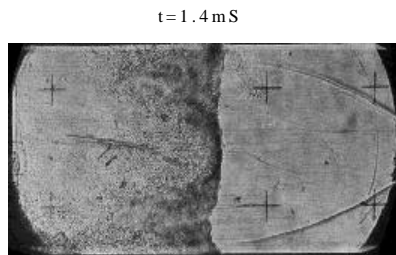
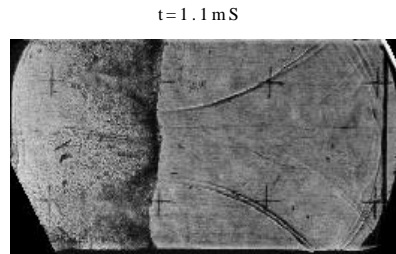
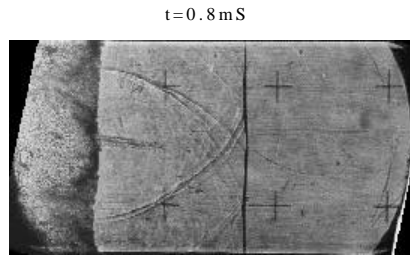
Nonlinear stage

$$k_{2D} \approx k_{3D} \approx 0.2 \text{ mm}^{-1}$$

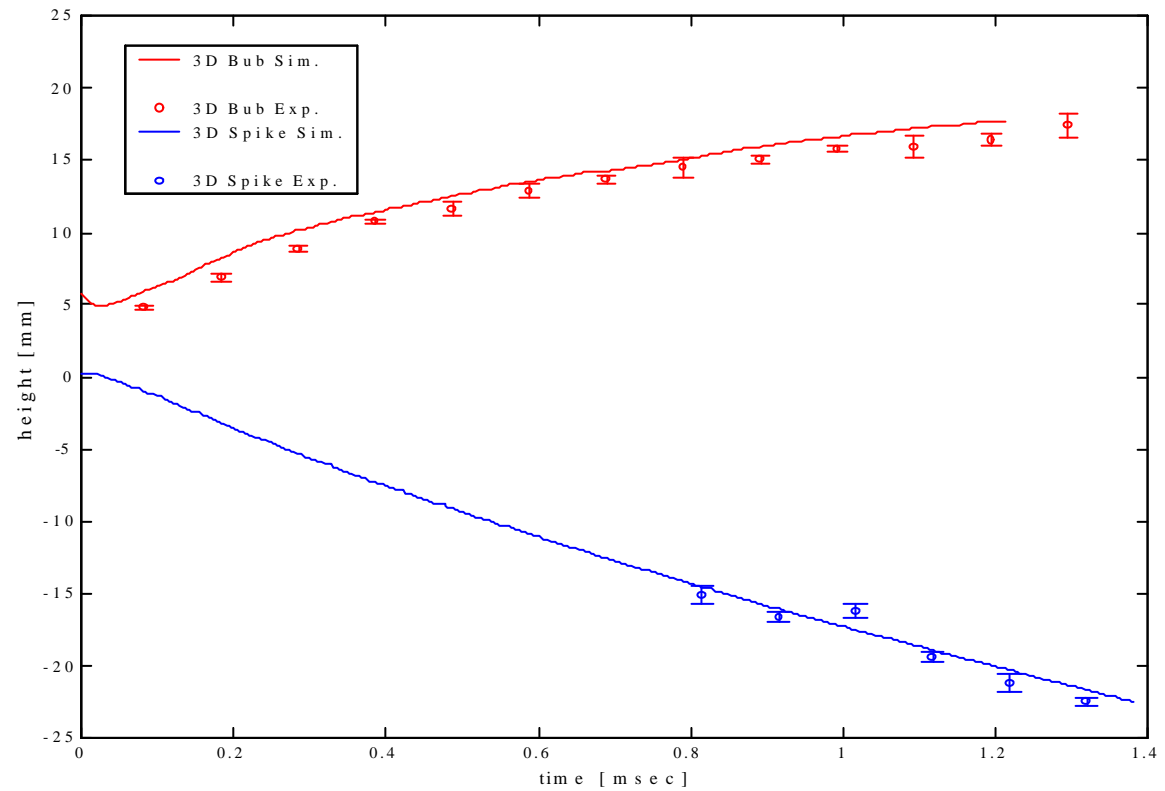
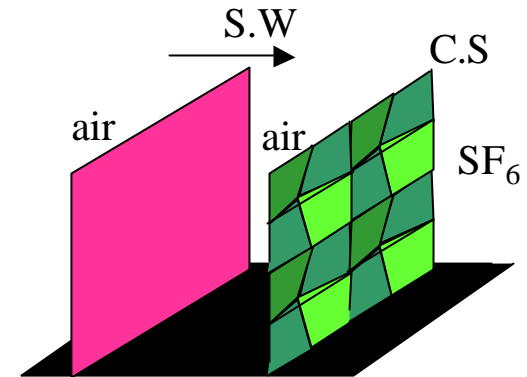


\* [Sadot *et al.* (1998)]

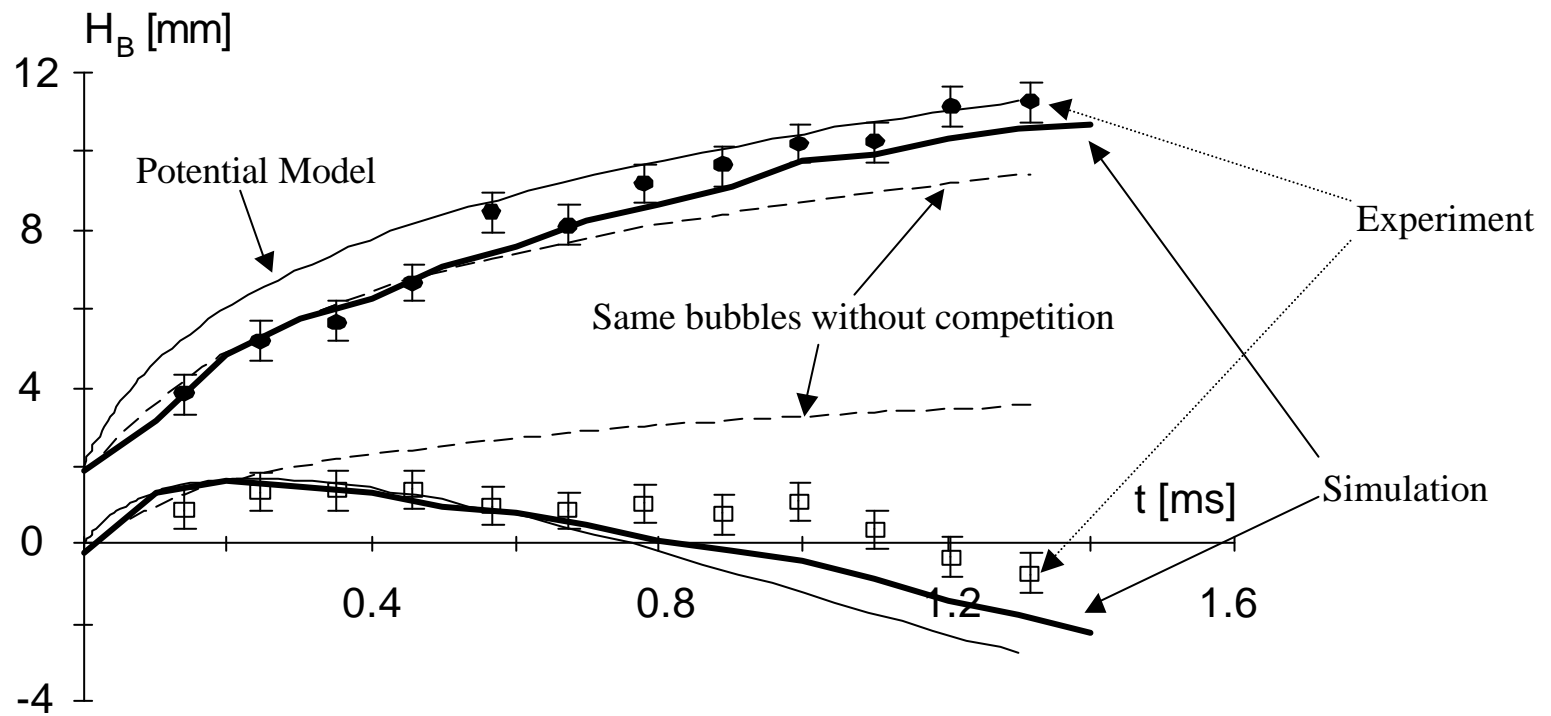
# Completing the picture 3D Spikes



3D Spike , 2D Bubble



# Bubble Competition 2D

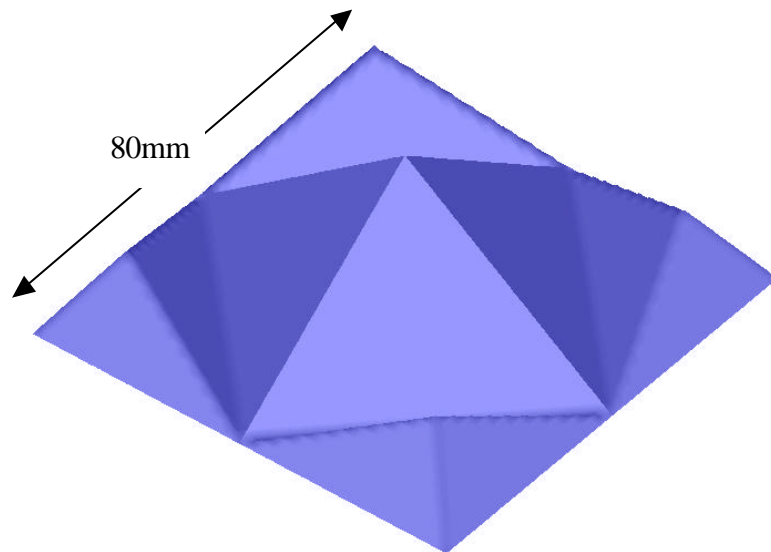


$M=1.2$   $A=0.67$   $\lambda_1=10\text{mm}$   $\lambda_2=25\text{mm}$

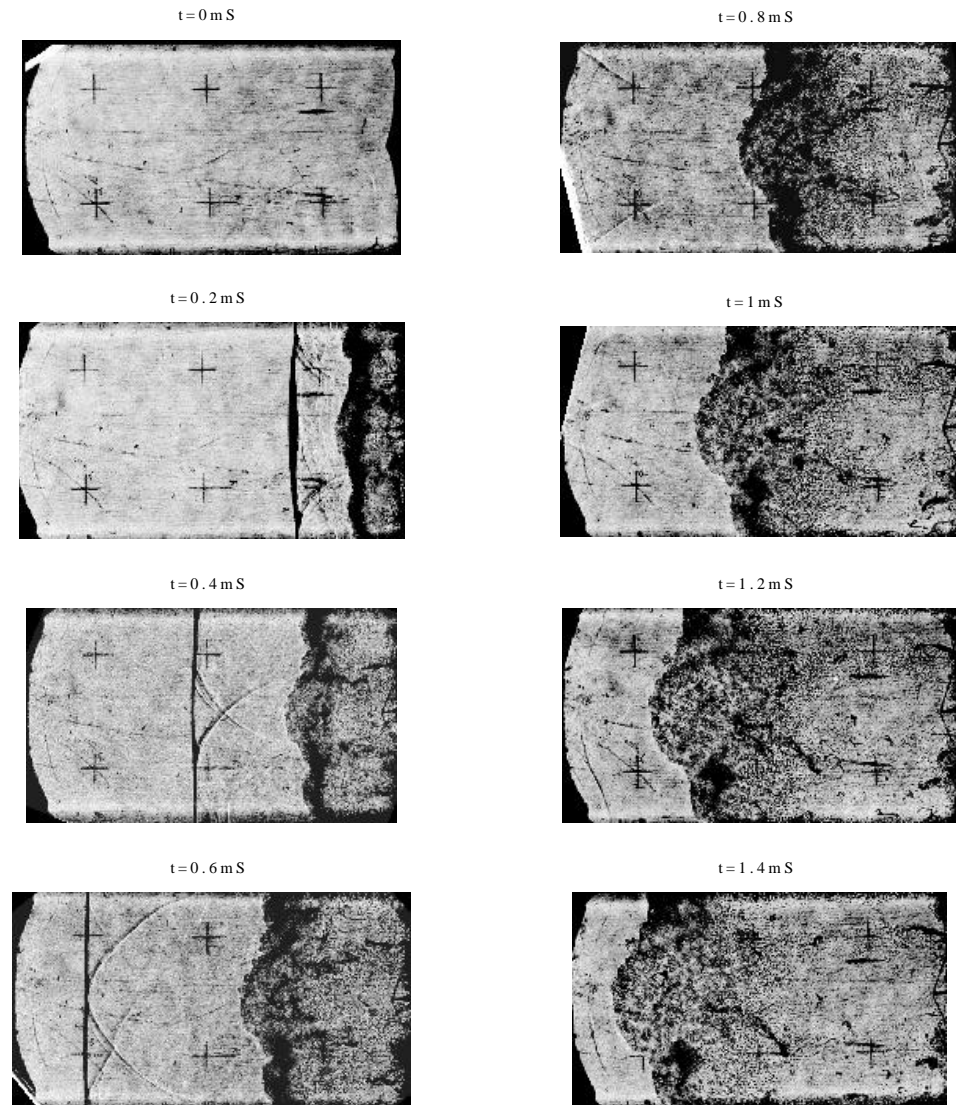
[Sadot *et al.* (1998)]



# Bubble Competition in 3D - Experiment

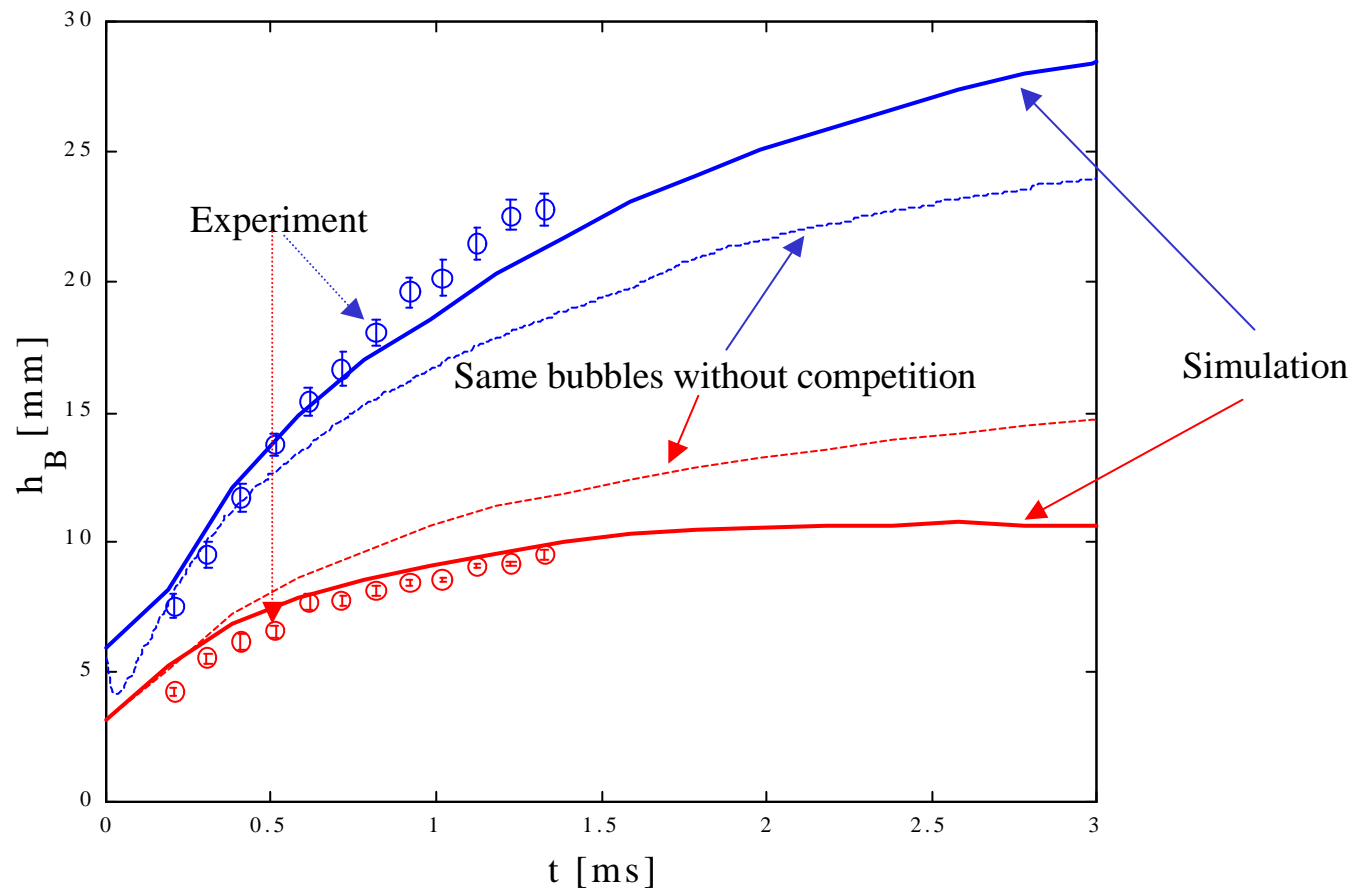


( $M=1.20$  Air to  $SF_6$ )





# Bubble Competition in 3D



## Summary

Experiments were performed to investigate the dependence of the RM instability (Bubbles and Spikes) on the dimensionality and the Atwood number.


Good agreement was found between the results of the experiments and the prediction of the model as well as with the results of full simulations.

Bubble Competition was shown to exist also in the 3D case.

$$U_B =$$

	Inertia of the bubble	
	$A \approx 0$	$A \approx 1$
2D:	$\frac{1}{2p} \cdot \frac{l}{t}$	$\frac{1}{3p} \cdot \frac{l}{t}$
3D:	$\frac{1}{p} \cdot \frac{l}{t}$	$\frac{1}{2p} \cdot \frac{l}{t}$

Dimensional effect



# Summary: Implications for Multimode Theory for the RM Instability

Alon *et. al.* (1994, 1995), D. Oron *et. al.* (1998), Shvarts *et. al.* (2000)

Two key elements govern the multimode bubble front evolution :

- 1) The asymptotic velocity of a single bubble. <= **Shown as depended on Dimensionality and Atwood.**
- 2) The merger rate between two neighboring bubbles. <= **Merging was observed in 3D as well.**

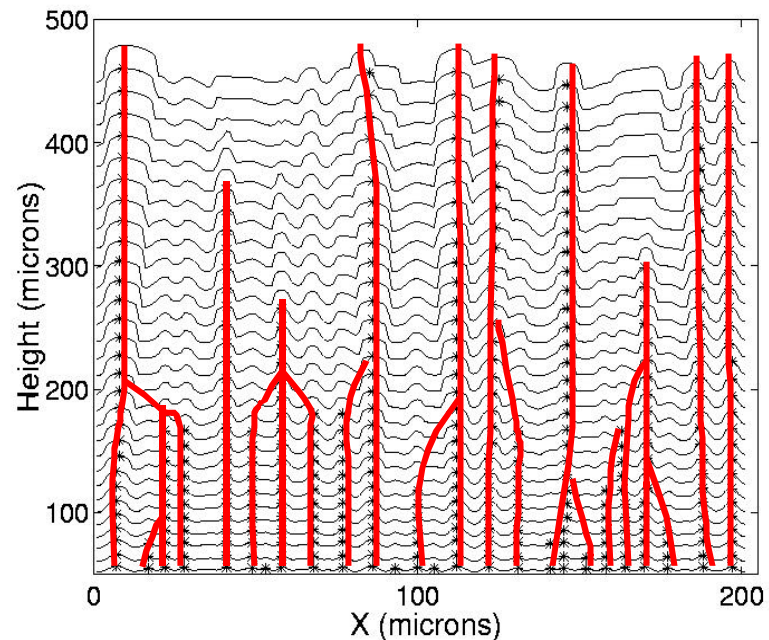


$$h_B = a_0 \cdot t^{\theta_B(2D/3D,A)}$$

$$h_S = b_0 \cdot t^{\theta_s(2D/3D,A)}$$

Present work verifies the:  
2D/3D and Atwood Number  
Dependence of Single-Mode  
Bubble/Spike

## Bubble Envelope (Simulation)



— Rising bubbles



$$q_{2D} \cong 0.4 \neq q_{3D} \cong 0.2$$