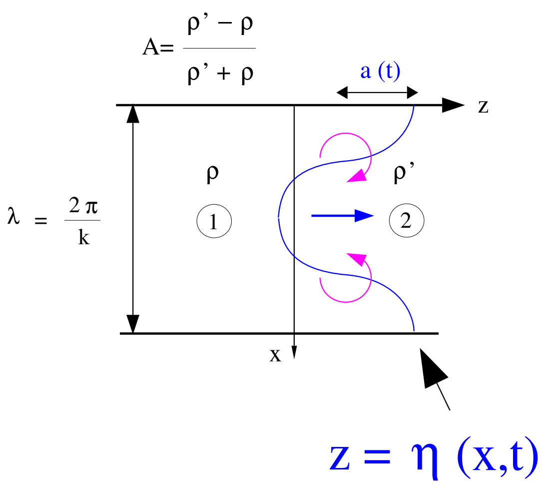
ANALYTICAL GROWTH RATE OF A NON-

M. Vandenboomgaerde - Commissariat à l'Energie Atomique -

ACCURATE APPROXIMATE PERTURBATION



$$k\eta = \sum_{n=1}^{\infty} (a_0 k \sigma t)^n \sum_{j=1}^{n} a_j^{(n)} \cos(jkx) \quad \Rightarrow$$

$$P1[A] = 1$$

$$P3[A] = \frac{-1 + 2 * A^2}{6}$$

$$P5[A] = \frac{19 - 125 * A^2 + 92 * A^4}{240}$$

$$P7[A] = \frac{-264 + 3686 * A^2 - 6997 * A^4 + 3234 * A^6}{5040}$$

* See T32 Oral : Efficient perturbation method for the RM and the RT instabilities: Weakly nonlinear stage and beyond

LINEAR SINGLE-MODE R-M INSTABILITY

BP 12 - 91680 Bruyeres-Le-Chatel- FRANCE

E-mail: marc.vandenboomgaerde@cea.fr

THEORY* GIVES SIMPLE FORMULA FOR a(T)

$$k\,a(T) = \sum_{p=0}^{\infty} P_{2p+1}[A]\,T^{2p+1} \quad \text{with} \quad T = a_0 k \sigma t$$

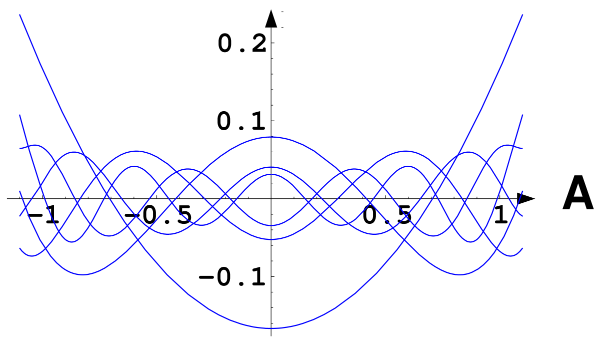
$$P9[A] = \frac{117663 - 2855274 * A^2 + 10086083 * A^4 - 11093856 * A^6 + 3805728 * A^8}{2903040}$$

$$P11[A] = (-5507319 + 206796915 * A^2 - 1168865775 * A^4 + 2250383605 * A^6 - 1755444326 * A^8 + 483163144 * A^{10})/159667200$$

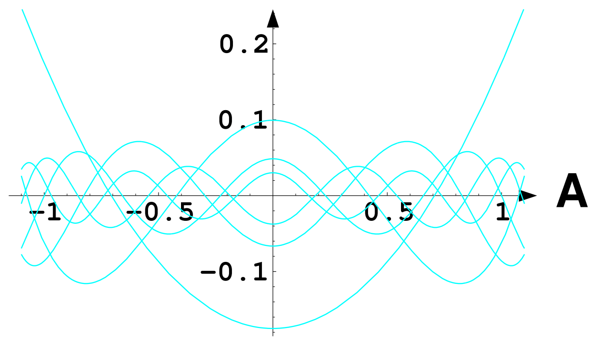
$$P13[A] = (31171393337 - 167864265395 * A^2 + 1387729381613 * A^4 - 4085608349133 * A^6 + 5380115327442 * A^8 - 3272269554968 * A^{10} + 755653587648 * A^{12})/99632332800$$

$P_{2\,P+1}$[A] POLYNOMIALS HAVE PECULIAR SHAPES

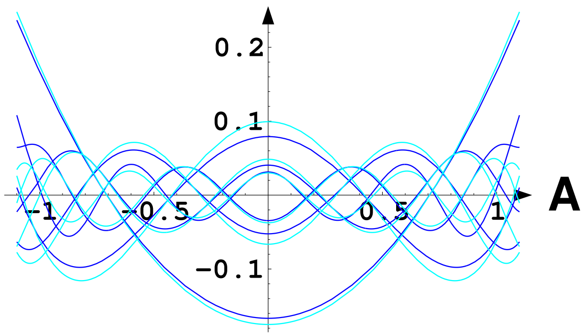
P_{2P+1}[A]



JACOBI'S POLYNOMIALS



HYPOTHESIS 1 : $P_{2P+1}[A] = \frac{1}{P+2}$ JACOBIP[2 P, 0.1, 0.1, 0.79 A]



A BASIS OF JACOBI'S POLYNOMIALS IS USED FOR a(T)

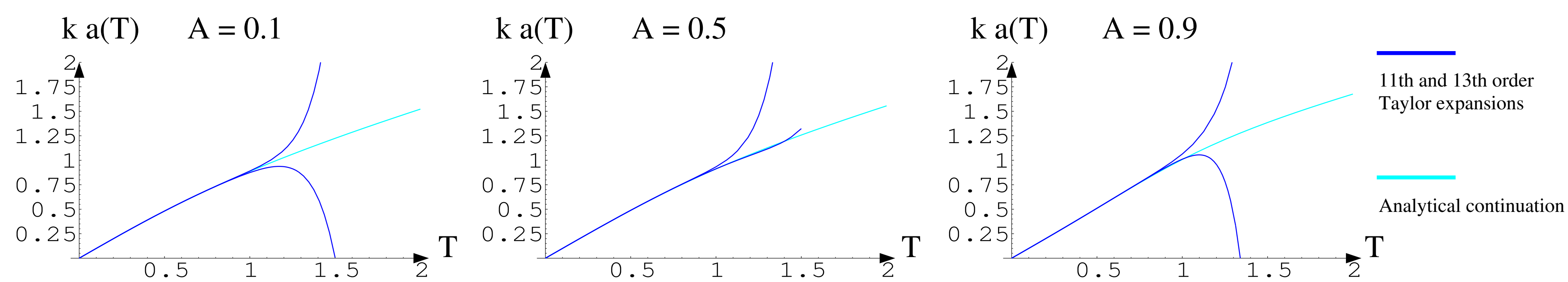
$$k\left(a(T)-a_0\right)=\sum_{p=0}^{\infty} P_{2p+1}[A]\,T^{2p+1}=\sum_{p=0}^{\infty}\frac{1}{p+2}\,\mathbf{JacobiP}\,[2p,0.1,0.1,0.79A]\;T^{2p+1}+remainder_{2p+1}[A]\,T^{2p+1}$$

$$=\frac{1}{2}T+\frac{1}{T^3}\int_0^T\frac{\tau^3}{2}\{F(A,\tau)+F(A,-\tau)\}d\tau \qquad + RemainderFunction[A,T]$$

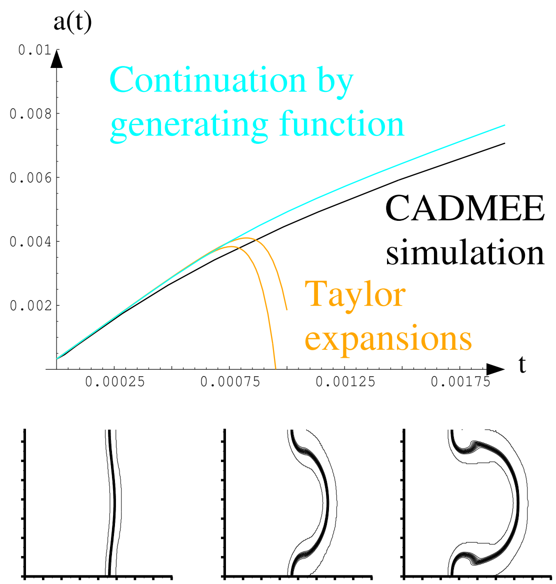
$F(A,\tau)$ = Generating function of Jacobi's polynomials

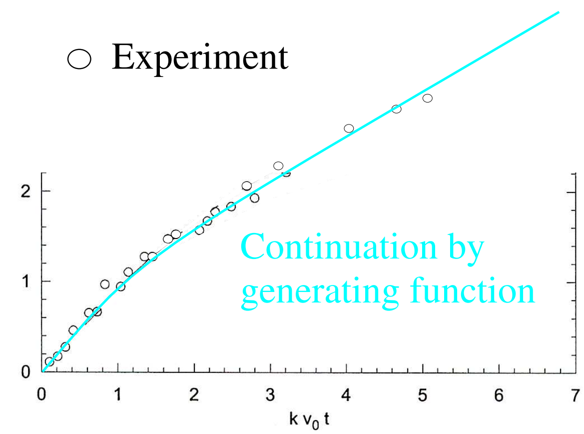
HYPOTHESIS 2 : $k\left(a(T)-a_0\right)\approx\frac{1}{2}T+\frac{1}{T^3}\int_0^T\frac{\tau^3}{2}\{F(A,\tau)+F(A,-\tau)\}d\tau$

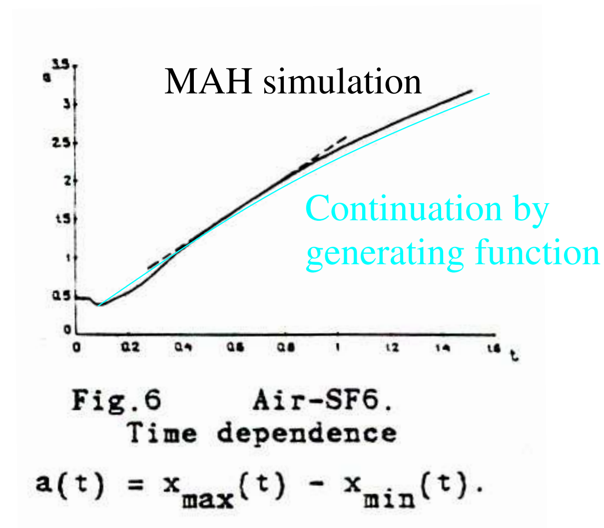
EXTENDED GROWTH RATE BY GENERATING FUNCTION FITS WITH TAYLOR EXPANSION



EXTENTED GROWTH RATE BY GENERATING FUNCTION FITS NONLINEAR R-M INSTABILITIES







CADMEE simulation

He/Air M_0= 1.09
k=224.855
A= 0.764
a_0=0.35 mm

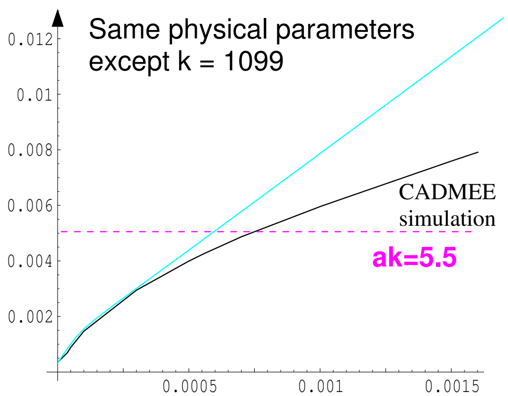
From Jacobs & Krivets 2001
Proceedings of the 23rd ISSW

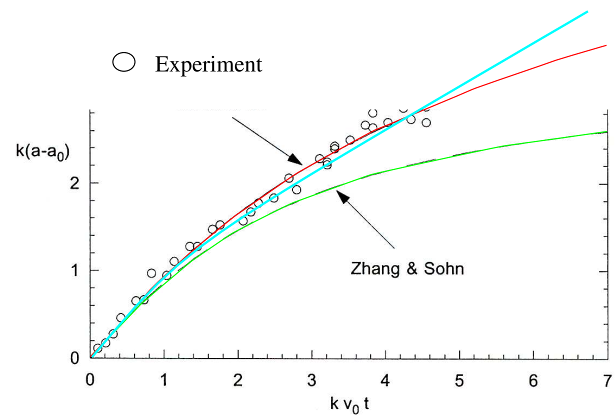
Air/SF6 M_0=1.3
a_0k=0.23
A=0.635

From Anuchina & Volkov
Proceedings IWPCTM 91

Air/SF6 M_0=1.32
k=1.112
A=0.585
a_0=0.183

ANALYTICAL CONTINUATION IS ACCURATE FOR a(T) k < 5.5





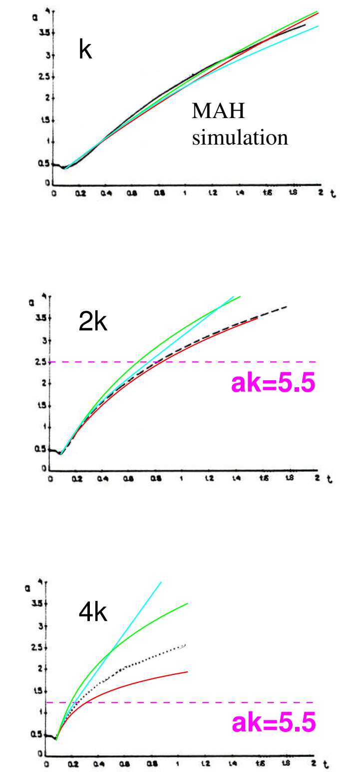


Fig.10 Air-SF6. Time dependences a(t) for some various wavelengths.

Analytical continuation

Sadot's model
$$\frac{da_{b/s}}{dt} = \frac{a_0 k\sigma(1 + a_0 k\sigma t)}{1 + (1 \pm A)a_0 k\sigma t + \frac{1}{2\pi C}\left(\frac{1+A}{1\pm A}\right)(a_0 k\sigma t)^2}$$
$$C = \frac{1}{3}\pi \quad \text{for } A > 0.5$$
$$C = \frac{1}{2}\pi \quad \text{for } A \to 0$$

Zhang's model
$$\frac{da}{dt} = \frac{a_0\sigma}{1 + a_0^2 k^2 \sigma t + \mathrm{Max}\left[0, a_0^2 k^2 - A^2 + \frac{1}{2}\right](a_0 k\sigma t)^2}$$

ANALYTICAL CONTINUATION HAS A RELIABLE RANGE OF VALIDITY

CONCLUSIONS

- Continuation of the growth rate by generating function has a reliable range of validity

  * for $ak < 5.5$ Relative error < 15%

  * for $ak > 5.5$ non-physical linear behavior :
    - remainder part no more negligeable
    - other polynomial basis should be used
    - single-evaluated interface model has reached its limit

- No adjustable parameter in the model
- The growth rate is solution of governing equations