

High Mach Number and High Initial Amplitude Effects on
the Evolution of the Single-Mode Richtmyer-Meshkov
Instability – Theoretical Study

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Impulsive Models for the Small Amplitude Single-Mode RM Instability

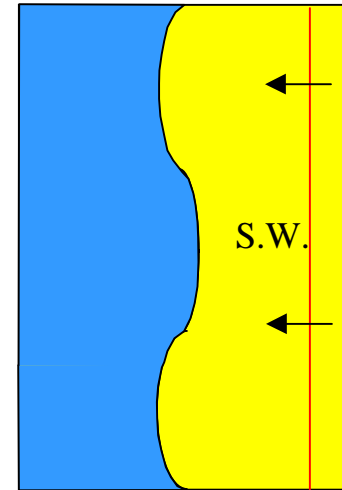
Assuming low mach (SW effects as a -
delta function acceleration)
and small amplitudes ($ak \ll 1$):

$$u_{bubble} = \Delta u_{1d} \cdot k \cdot \frac{\rho_1 - \rho_2}{\rho_1 + \rho_2} a_0$$

k - wavenumber

Δu_{1d} - velocity of unperturbed contact surface induced by shock wave

ρ_1, ρ_2 - shocked densities before and after contact surface



Fast - Slow interaction

Richtmyer Formula :

$$a_0 = a_0^+$$

a_0^+ - post shock amplitude

Slow - Fast interaction (phase inversion)

Meyer-Blewett correction :

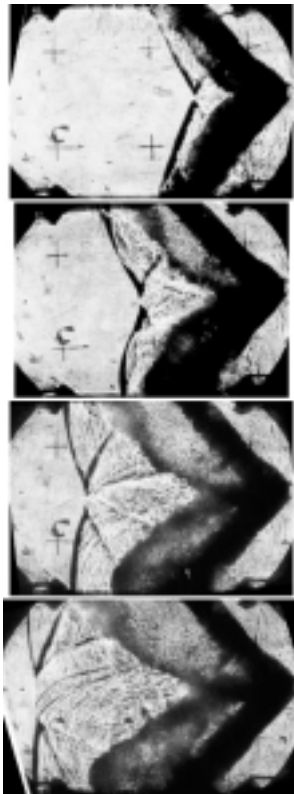
$$a_0 = \frac{a_0^+ + a_0^-}{2}$$

a_0^+ - post shock amplitude

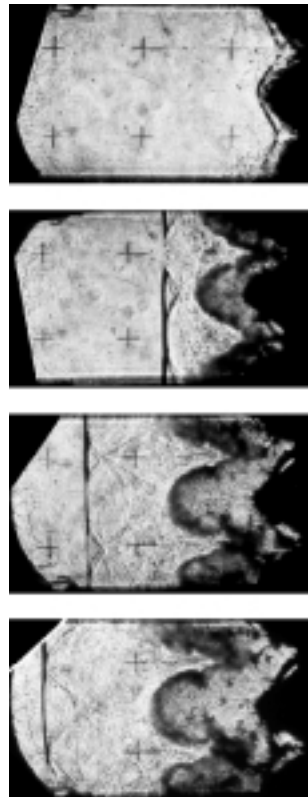
a_0^- - pre shock amplitude

Results from New Shock Tube Experimental by Sadot et. al. $M=1.2$ (E36)

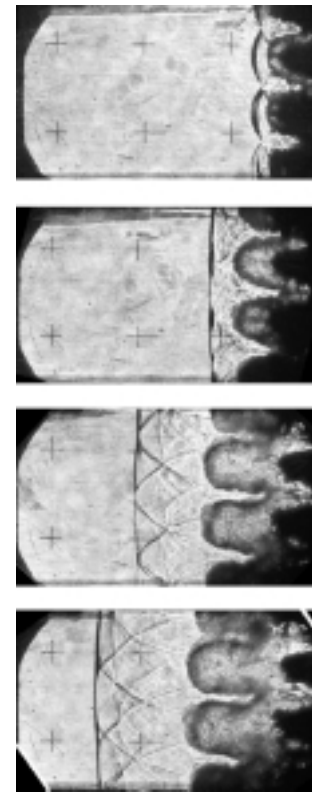
$\lambda=80\text{mm}$
 $a_{\infty}=20\text{mm}$



$\lambda=40\text{mm}$
 $a_{\infty}=12\text{mm}$



$\lambda=26\text{mm}$
 $a_{\infty}=10\text{mm}$



Experimental Velocity Reduction

Class A Experiments:

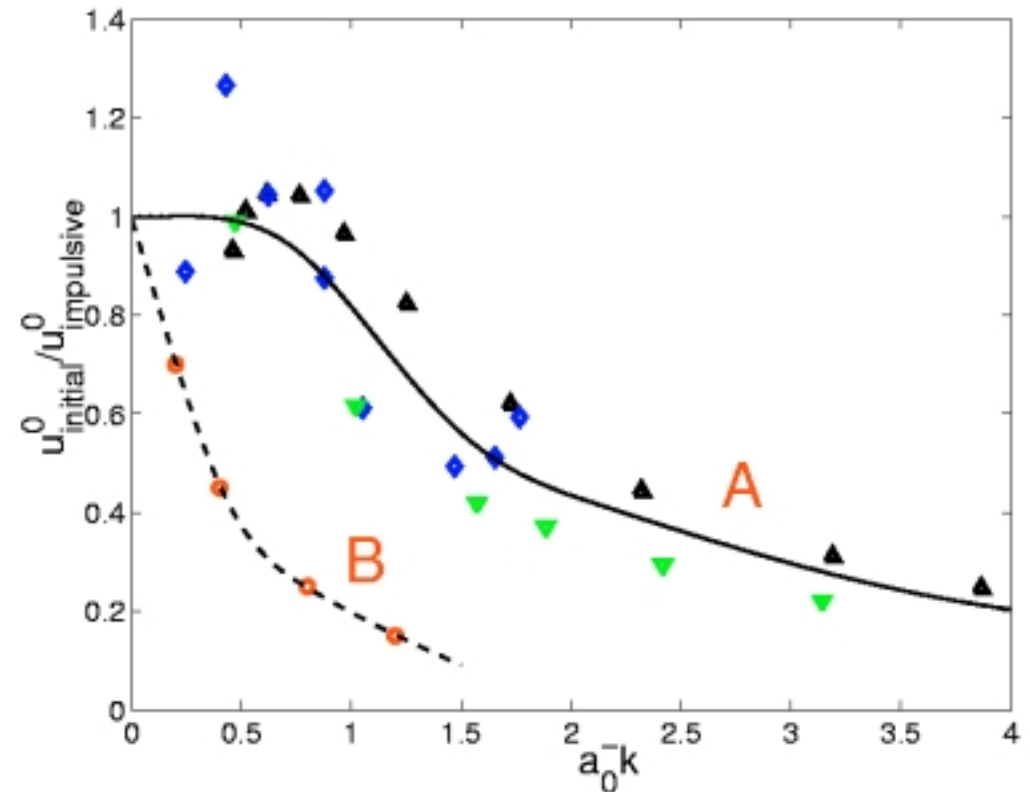
Similar reduction at a range of Mach numbers (1.2-15.3)



Apparent High Amplitude Effect

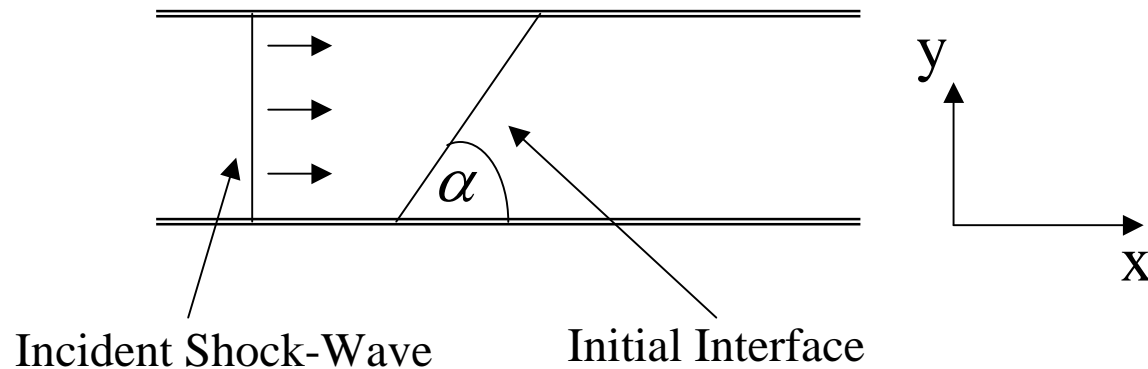
Class B Experiments:

Can be High Mach Effect



- ◆ Dimonte Be → Foam (M=15.3)
- ▲ Aleshin Ar → Xe (M=2.5)
- ▼ Sadot Air → SF₆ (M=1.2)
- Aleshin He → Xe (M=2.5)

Vorticity Deposition Model



- Local vorticity deposition per unit length*:

$$\Gamma ds = \Gamma_0(M, \gamma_1, \gamma_2, \rho_1, \rho_2) \sin(\alpha) ds$$

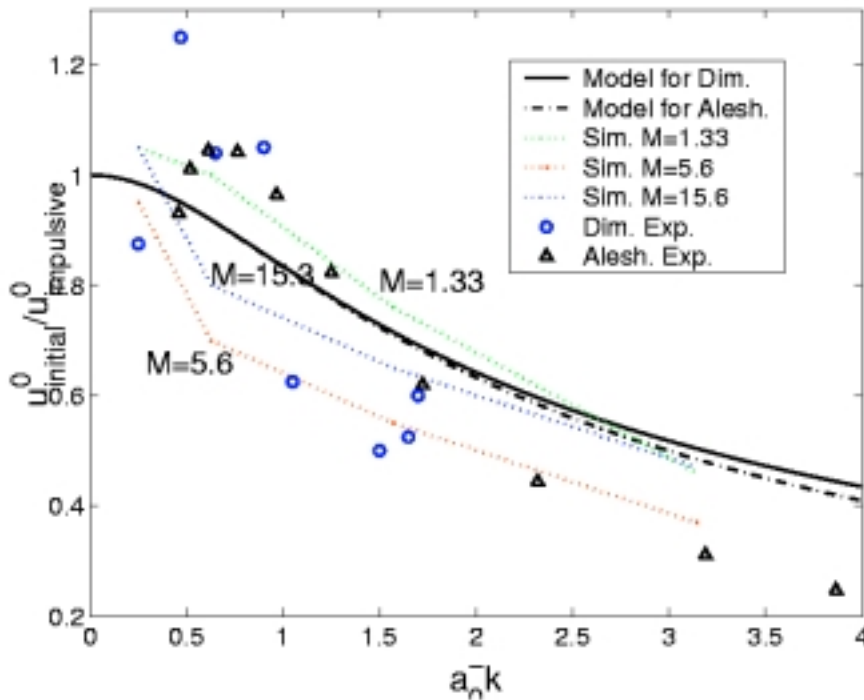
- Bubble tip velocity:

$$u(z) - i \cdot v(z) = 2\pi \int_{\text{interface}} \Gamma(z) \cot((z - z') \cdot \pi / d) dz'$$

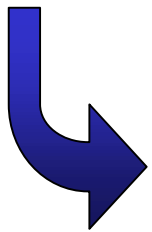
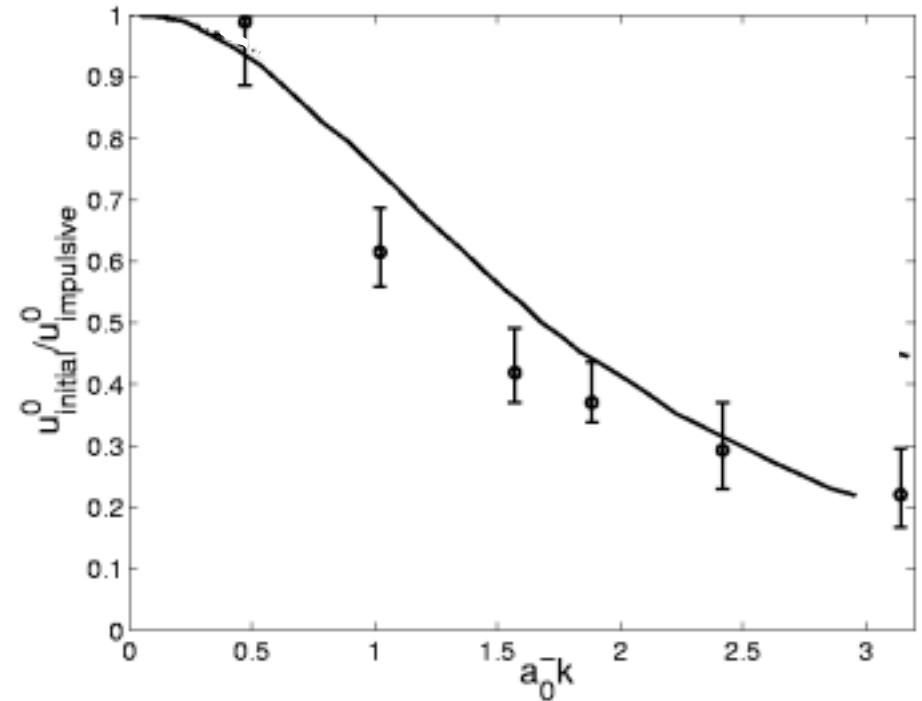
$$z = x + i \cdot y$$

Model Velocity Reduction Compared with class 'A' Experiments and Simulations

Experiments by Dimonte and Aleshin



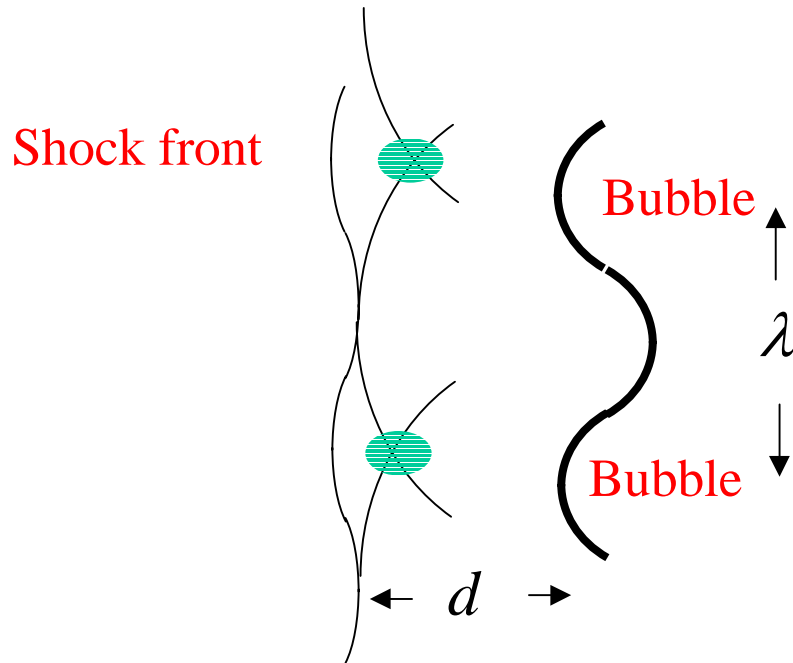
New shock Tube Experiments



In class 'A' experiments the velocity reduction is mainly attributed to high amplitudes effects.

Compressibility dominated regimes

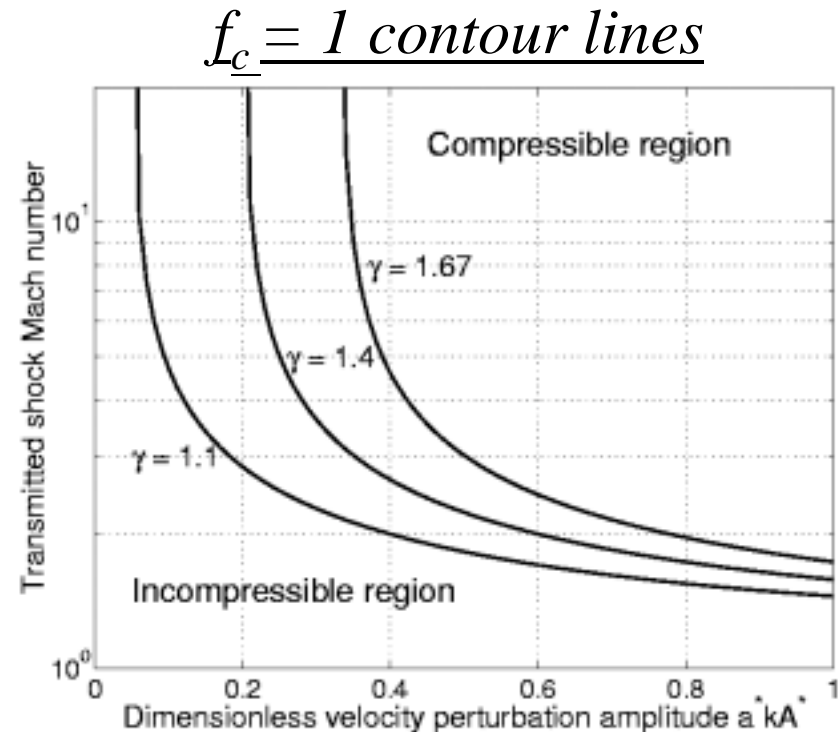
Compressibility effects are expected to dominate the flow when the shock wave is in proximity with the interface.



Proximity criterion:

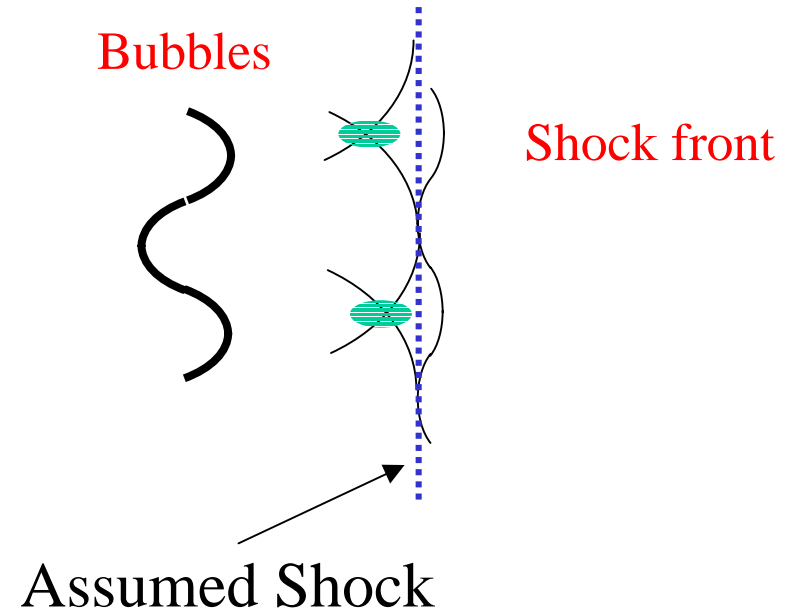
$$f_c = \frac{u_{bubble}}{u_{shock} - u_{1d}}$$

Conjecture: f_c characterizes the flow at moderate Mach numbers



“Wall” model for moderate Mach RM instability

- Shock wave is treated as a rigid straight wall moving in the $1d$ shock velocity.
- Secondary high pressure points are not considered.
- Model reduction depends only on f_c .
- Model is solved by using previous models* while inhibiting the shock as a moving boundary condition.



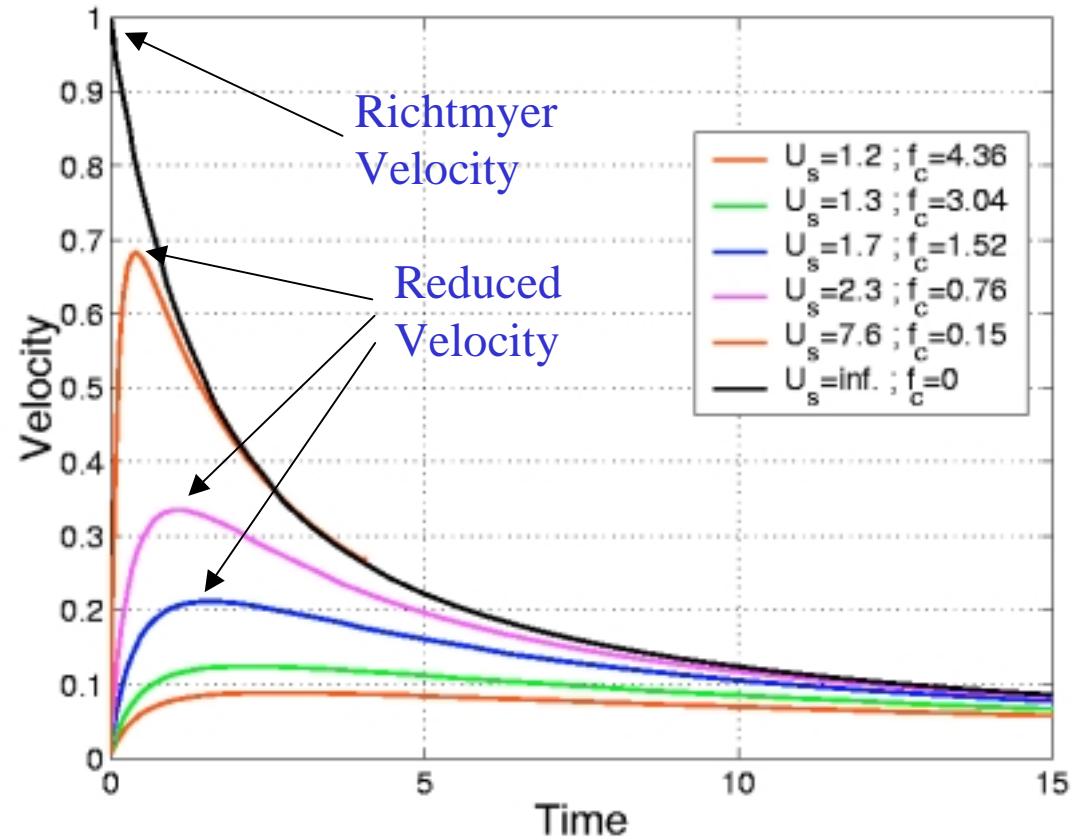
* Potential model for A close to 1 and Vortex model for A close to 0.

Example of results from the Wall model

- As the shock velocity increases (f_c decreases) the velocity profile is closer to the incompressible case.

- The reduction factor is calculated by:

$$\frac{\text{Reduced Velocity}}{\text{Richtmyer Velocity}}$$



Comparison with Aleshin He \rightarrow Xe experiments

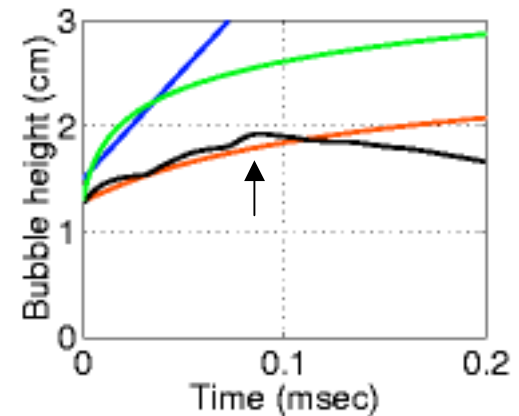
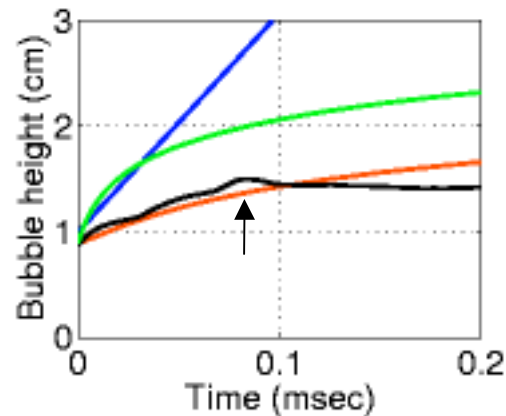
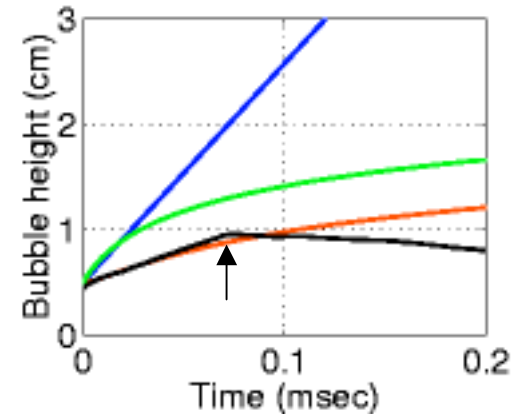
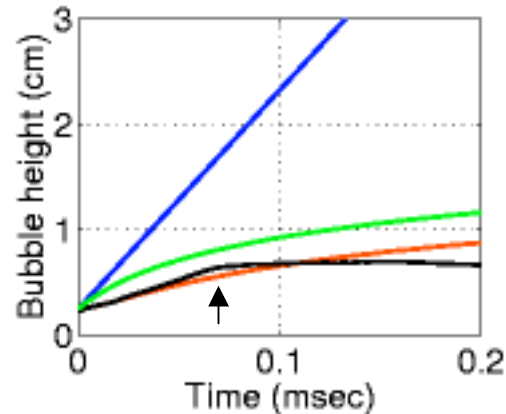
— Rictmyer Velocity

— Classical Model

— Wall Model

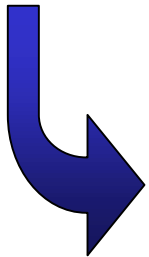
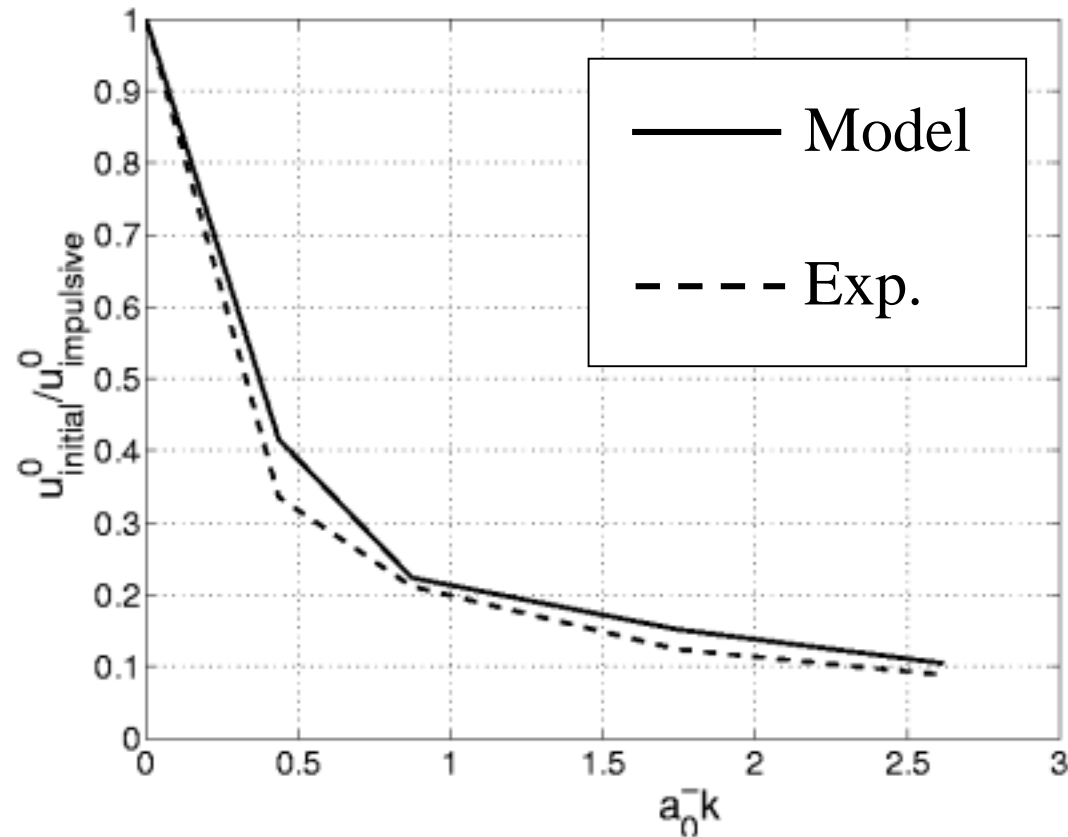
— Sim./Exp.

↑
Reverberation
arrival
time



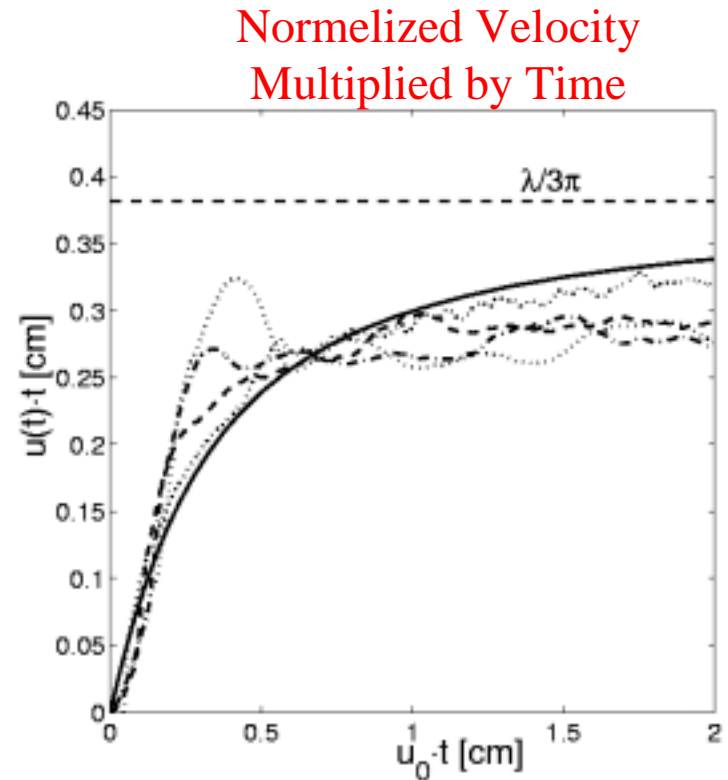
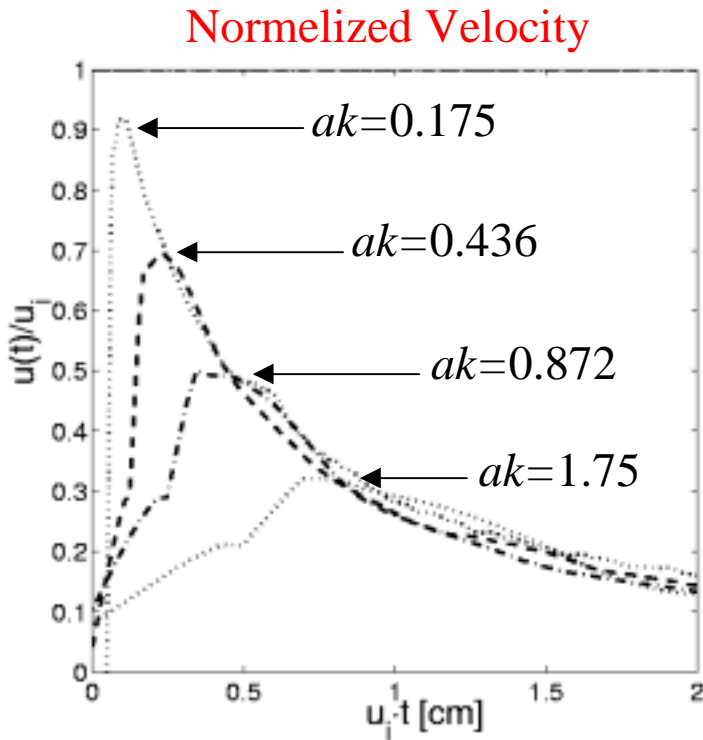
\Rightarrow By Introducing f_c to the potential model, good agreement is achieved with experiments.

Class B Reduction Factor - Theory Vs. Experiments



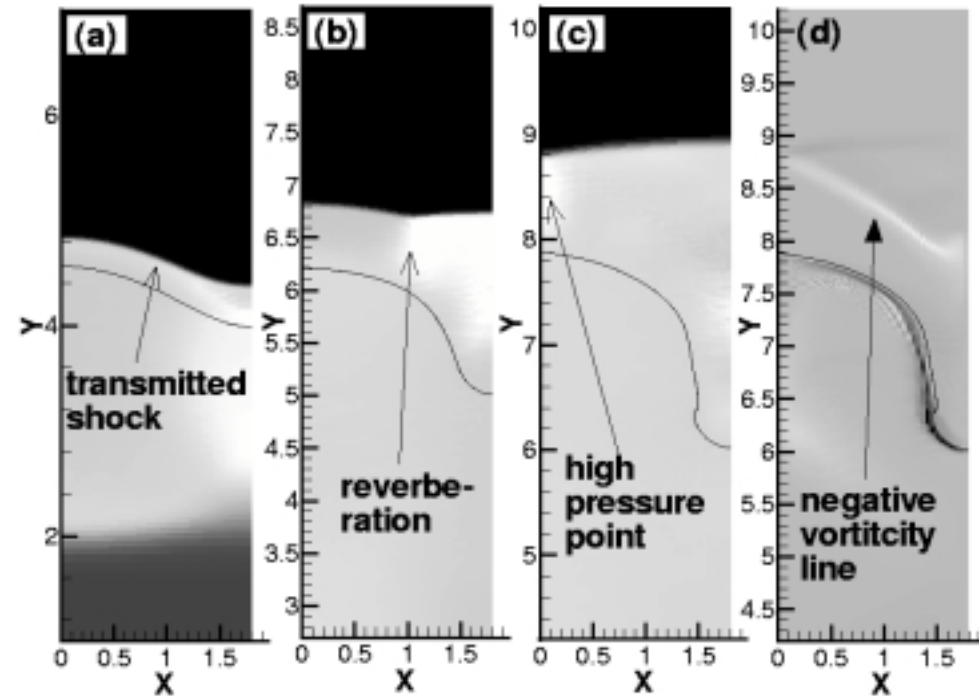
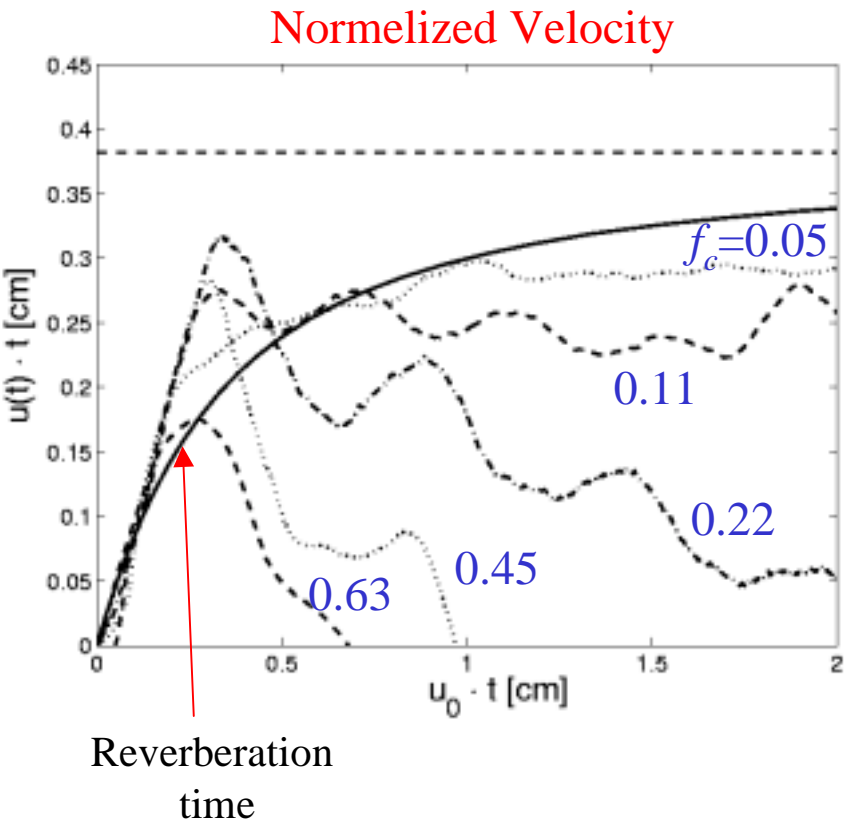
In class 'B' experiments the velocity reduction is mainly attributed to high Mach effects.

Late Nonlinear Stages of the Flow - Numerical Simulations at $f_c=0.05$ and $ak=0.175 - 1.75$.



⇒ Normalizing the late stages of the flow by the initial velocity from the High Amplitudes Model, deduces high amplitudes effects. Hence the classic behavior is regained.

Late Nonlinear Stages of the Flow - Numerical Simulations for $f_c=0.05 - 0.625$ and $ak=0.43$.



\Rightarrow At High values of f_c new phenomena arises due to secondary high pressure points, drastically affecting the flow.

Summary

- Effects of high initial amplitudes and Mach numbers were quantified for the early linear stages of the flow.
- Classes 'A' and 'B' of experiments were recognized, distinguishing between the two effects.
- For the late nonlinear stages of the flow:
 - No true effects were found for high initial amplitudes.
 - New dominant effects were found for high Mach numbers.