Vortex-Merger Statistical Model for the Late Time Self-Similar Evolution of the Kelvin-Helmholtz Instability

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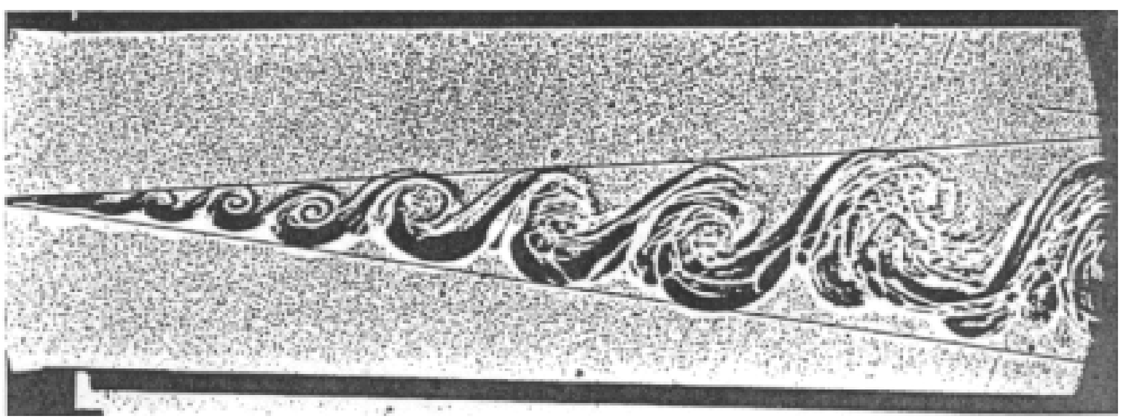

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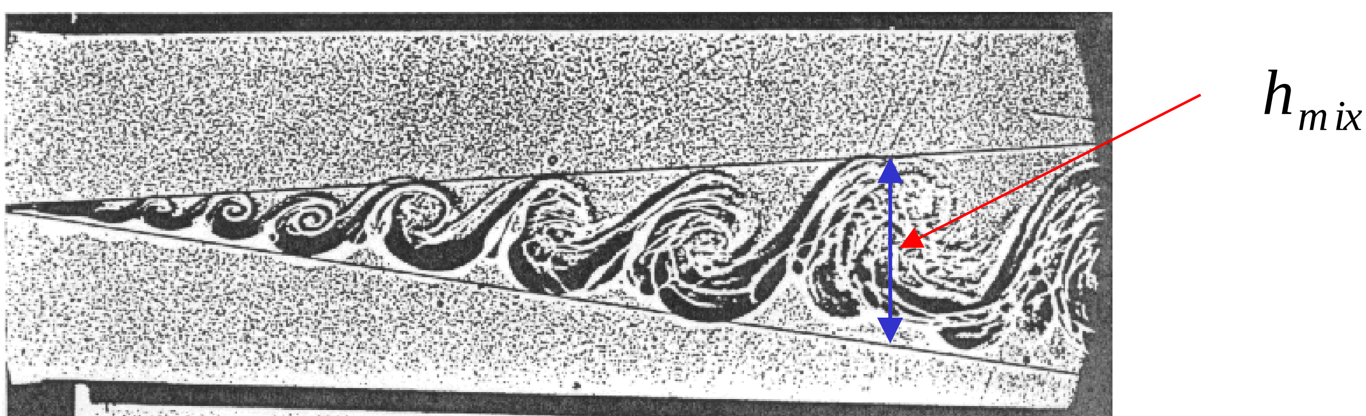
Abstract

The nonlinear growth of the multi-mode incompressible Kelvin-Helmholtz (KH) shear flow instability at all density ratios is treated by a non empirical large scale statistical-mechanics eddy-pairing model, based on the single eddy behavior and the process of two eddy-pairing. From the model, a linear time growth of the mixing zone is obtained, resulting in the linear time growth coefficient for several density ratios as well as an asymptotic lognormal eddy size distribution and the average eddy life time probability. Very good agreement with previous works, full numerical simulations and experiments is achieved.

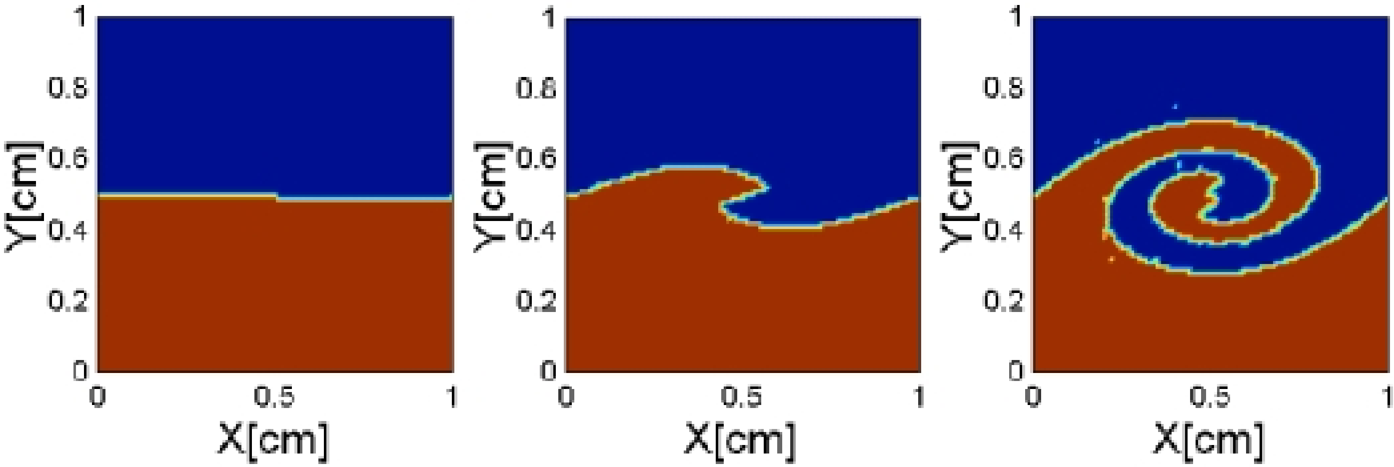
Example of Shadowgraph Photography*



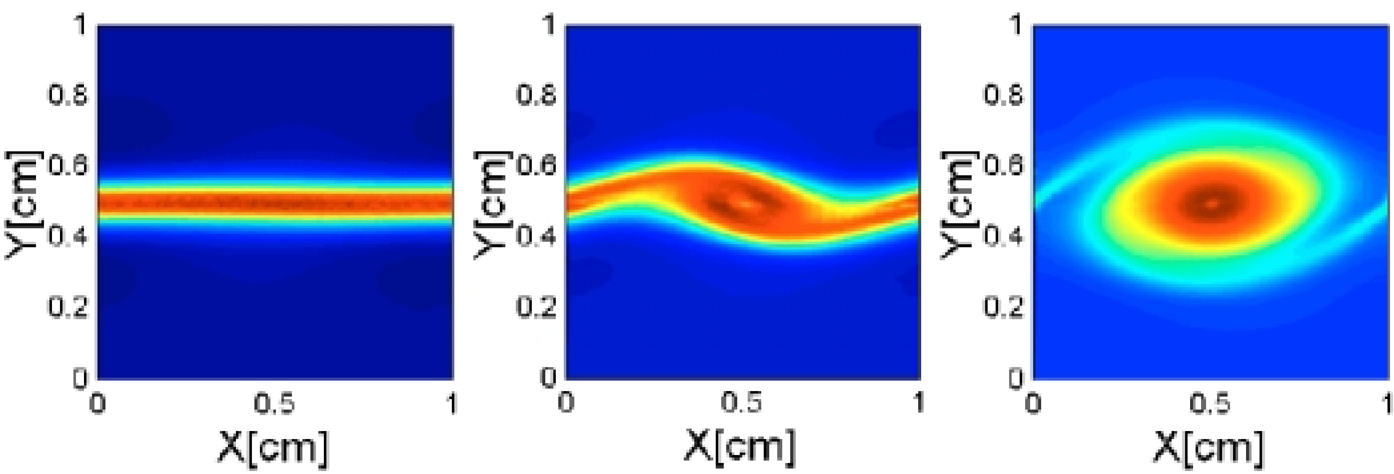
*Roshko & Brown, J. Fluid Mech, 1974.

Numerical Simulations for the Single Mode case

Material interface

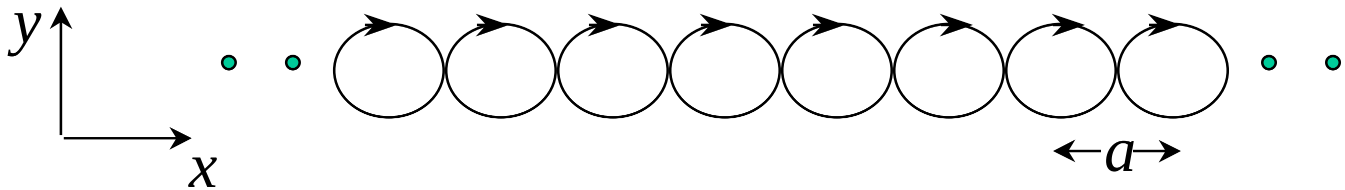


Vorticity Colormaps



Fast
Vortex
Formation

Vortex Line - Analytical Solution



- Vorticity Definition: $\vec{\omega} = \nabla \times V$
- Vortex Strength set by Kelvin Theorem of Circulation:

$$\Gamma = \int_{Surface} \vec{\omega} \cdot ds = \oint_{Contour} v \cdot dl = Const$$

- Resulting Complex Potential of a Vortex Line :

$$W(z) = \sum_{n=-\infty}^{\infty} \left(i \cdot \frac{\Gamma}{2\pi} \right) \ln(z - na) = \frac{i \cdot \Gamma}{2\pi} \ln\left(Sin(\pi \cdot z / a)\right)$$

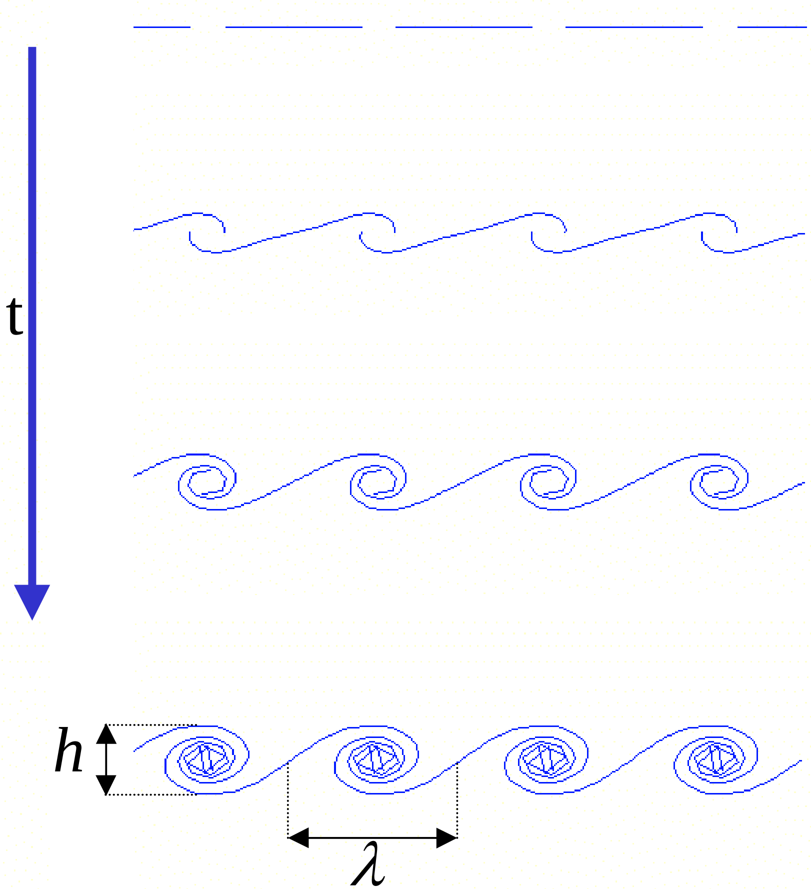
$$(z = x + i \cdot y)$$

Vortex Model for a Single Mode Initial Perturbation

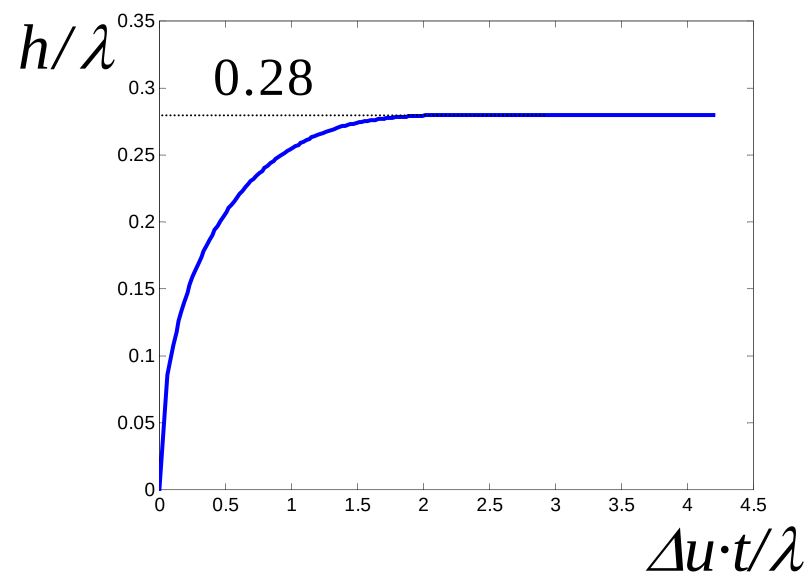
Velocity field calculated as a derivative of the complex potential

$$u(z) - i \cdot v(z) = {dW(z)}\Big/{dZ}$$

Interface Evolution



Eddy Height Vs. Time*



$$h(\lambda, t \to \infty) = 0.28\lambda$$

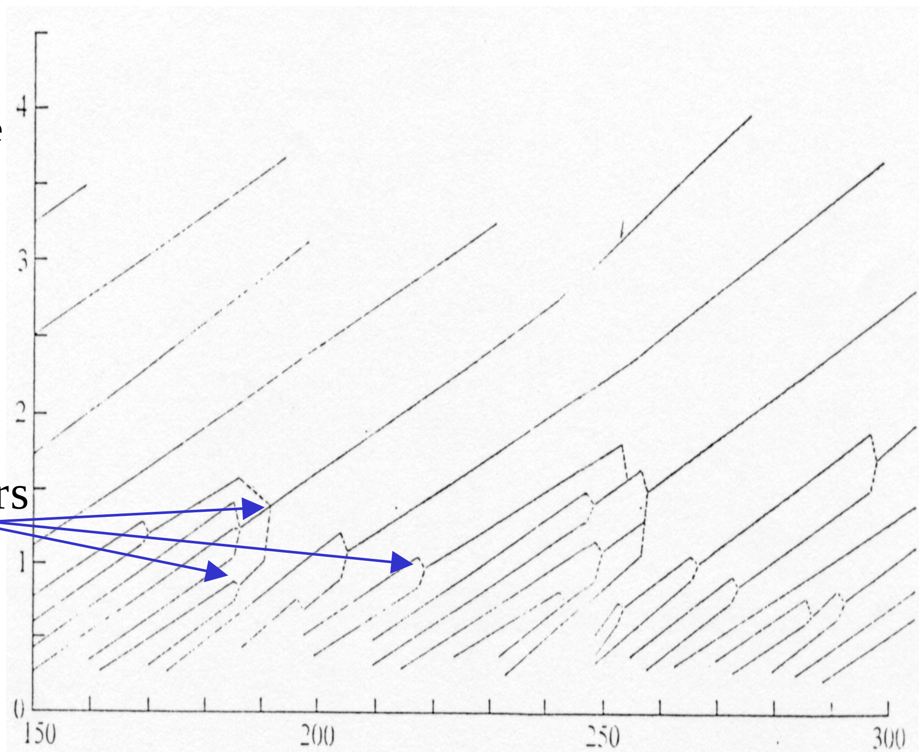
*Analytical Solution by Y. Elbaz

Experimental eddies trajectories show the vortex merger dominance

Vortex Trajectories*

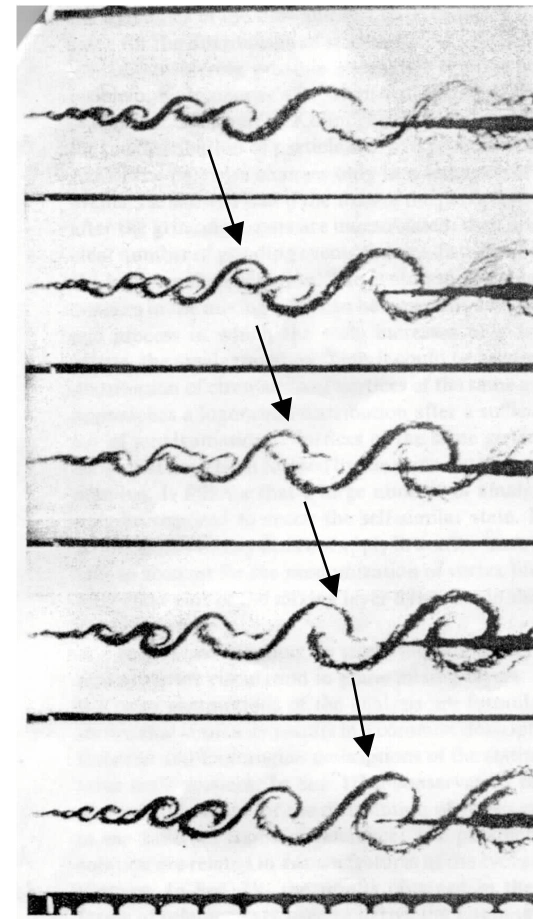
Vortex Travel Distance

mergers



Frame Number (time)

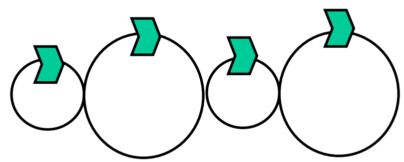
Single Vortex Merger**

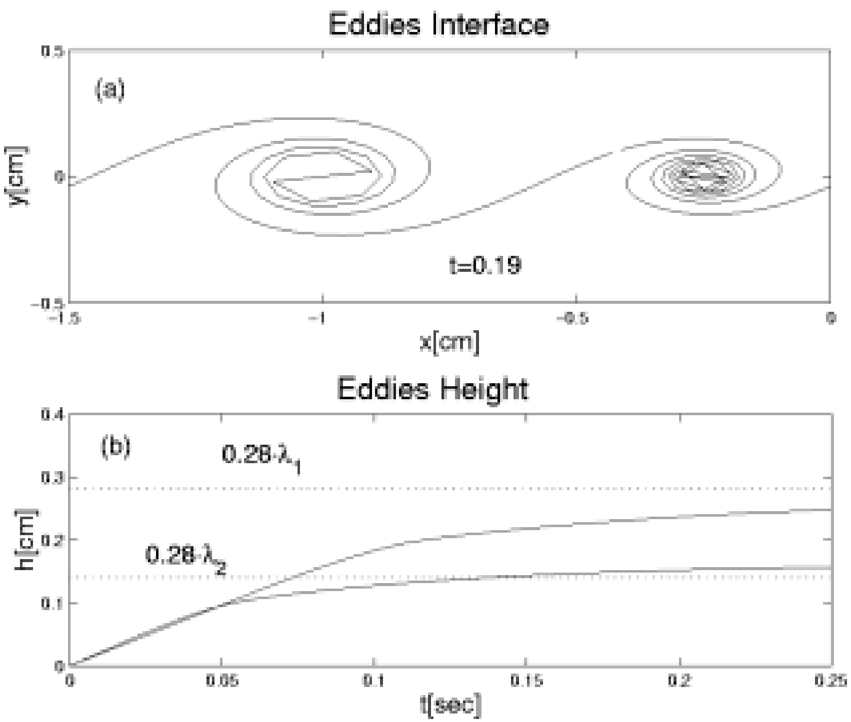


* Roshko & Brown (1974)
** Bernal (1988)

Two Vortex Pairing - Model Results

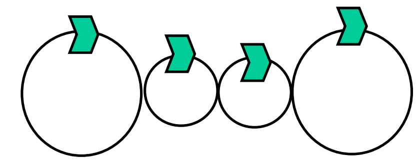
Two Vortex Setup

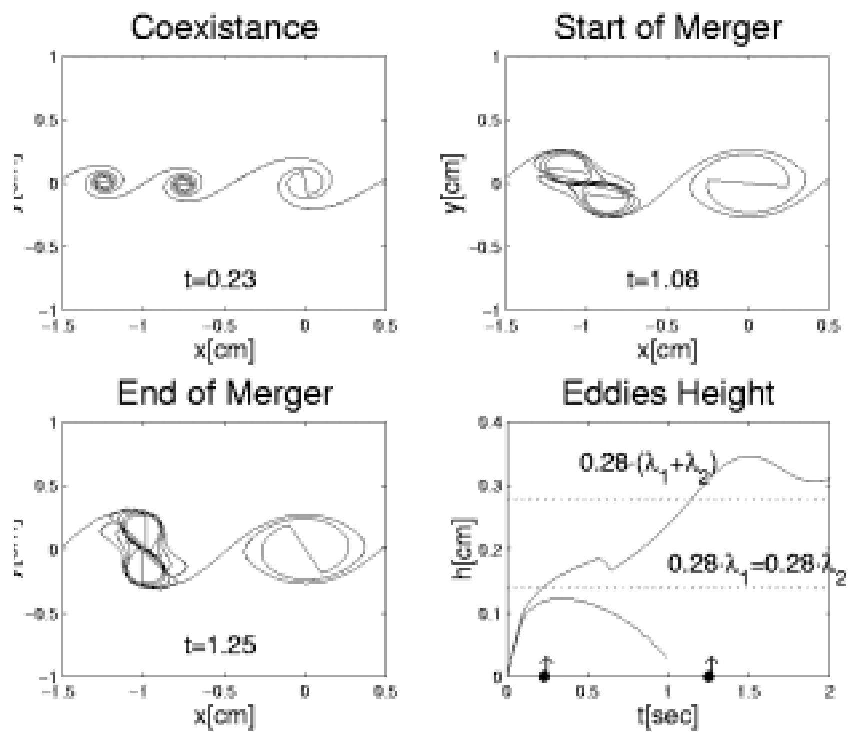




NO MERGER!
3 eddies must be introduced

Three Vortex Setup





New vortex at predicted height of $\lambda_1+\lambda_2$

Statistical Mechanics Model for the Multi Mode Kelvin-Helmholtz Instability

A large ensemble of eddies is set along a line.

Each eddy is assumed to possess its asymptotic height according to the vortex model.

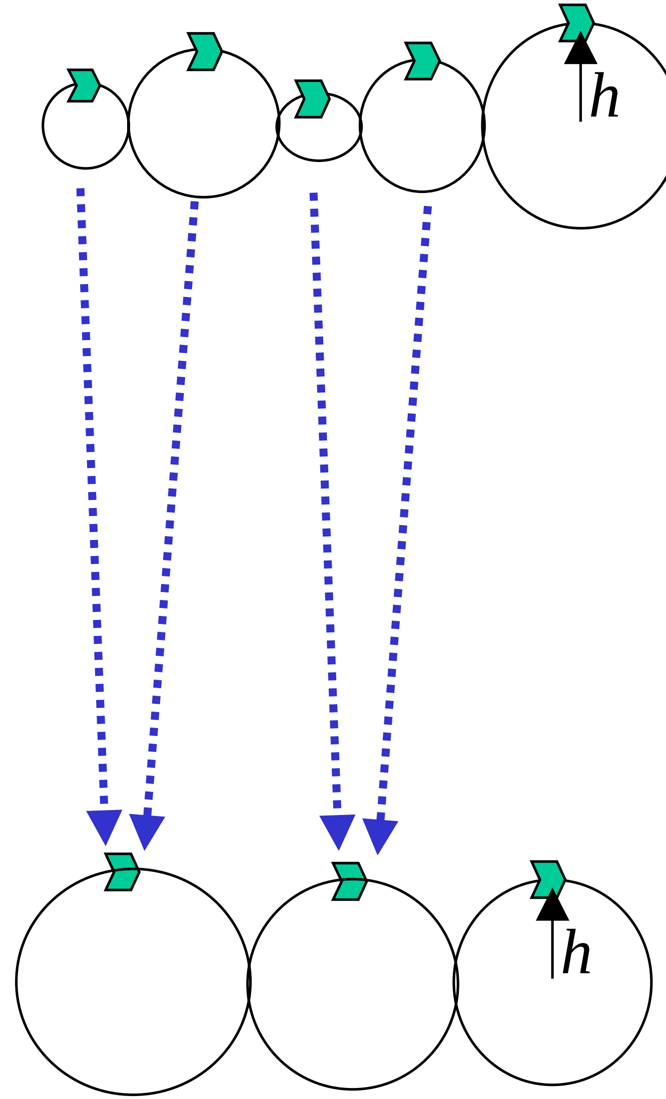
$$h(\lambda, t) = 0.28\lambda$$

Eddies increase in size through occasional mergers, according to the previously calculated merger rate.

$$\lambda_i\,,\ \lambda_{i+1}$$

$$\omega(\lambda_i\,, \lambda_{i+1})$$

$$\lambda_i + \lambda_{i+1}$$



Self-Similar Analyses of the Multi-Mode KH Instability

- Average width: $<h(t)>=0.56<\lambda>=0.56L/N(t)$
- Number of eddies change through mergers: $d(N(t))/dt=-<\omega>N(t)/2$
- Merger rate scales as: $<\omega>=\Delta u<\omega_0>/<\lambda>$
- Resulting number of eddies: $N(t)=0.56L/(\Delta u\cdot t)/<\omega_0>$

Integrating the above, linearity is achieved through simple arguments.

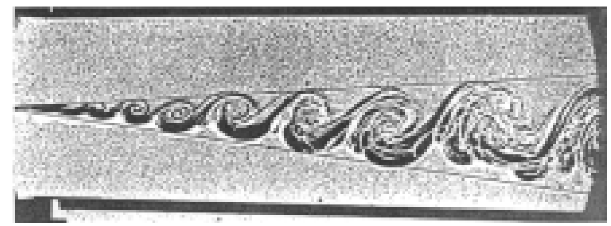
$$<h(t)>=<\omega_0>\Delta u\cdot t$$

Results from the Large-Scale Statistical Model

Statistical Model

$$h_{mix}(t) = a_0 \cdot t \quad ; \quad a_0 \approx \underline{0.2}$$

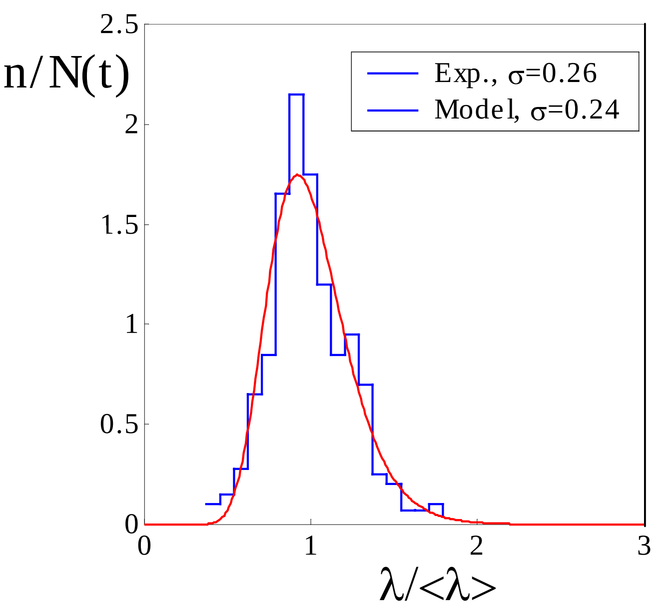
Experiment



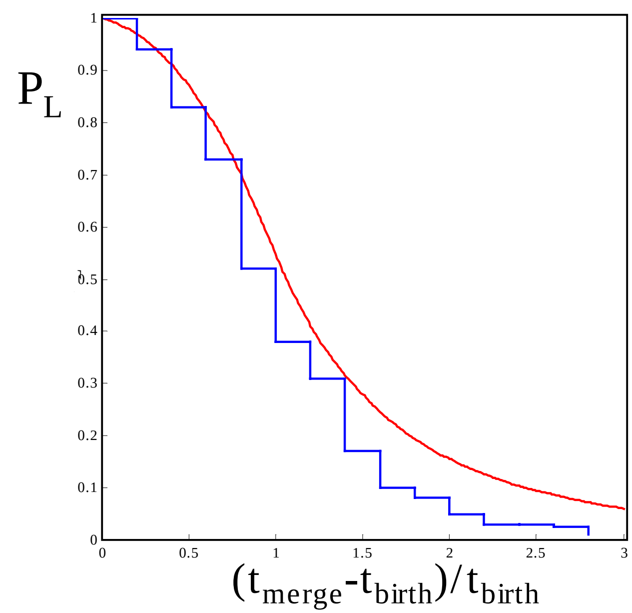
$h_{mix}(t) = a_0 \cdot t$

$a_0(\rho_1 = \rho_2) \approx \underline{0.2}$

Eddy Size Distribution

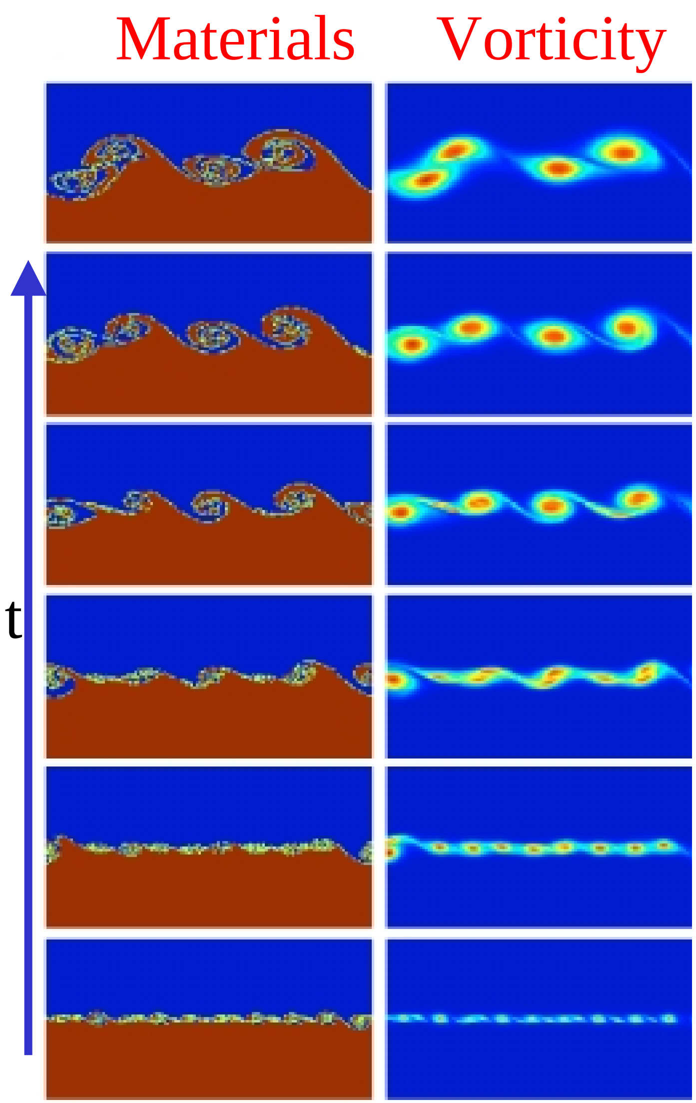


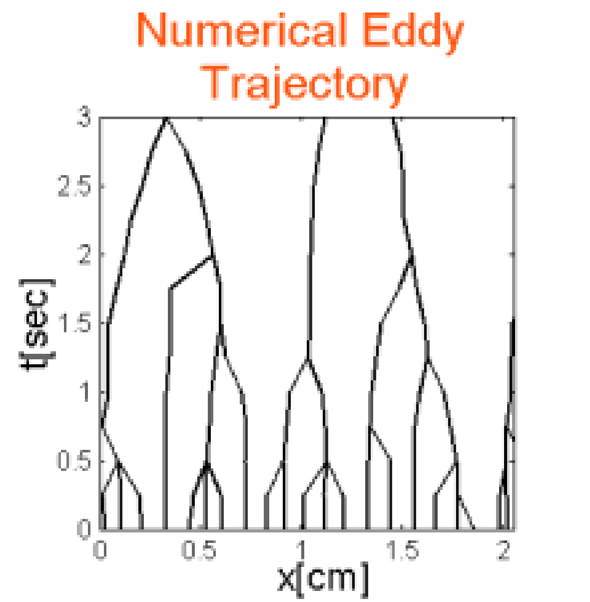
Life-time Probability

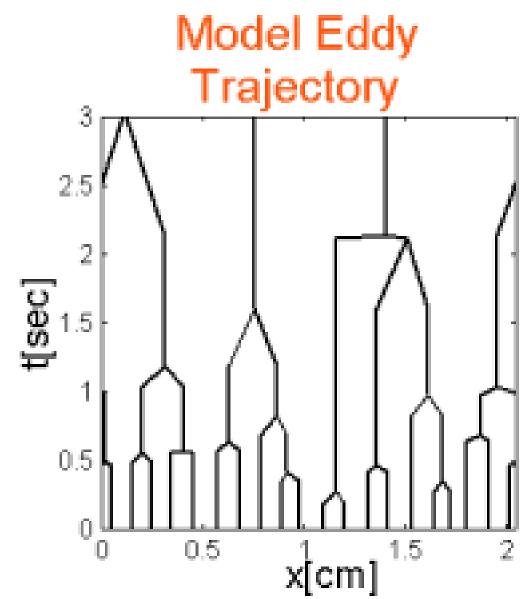


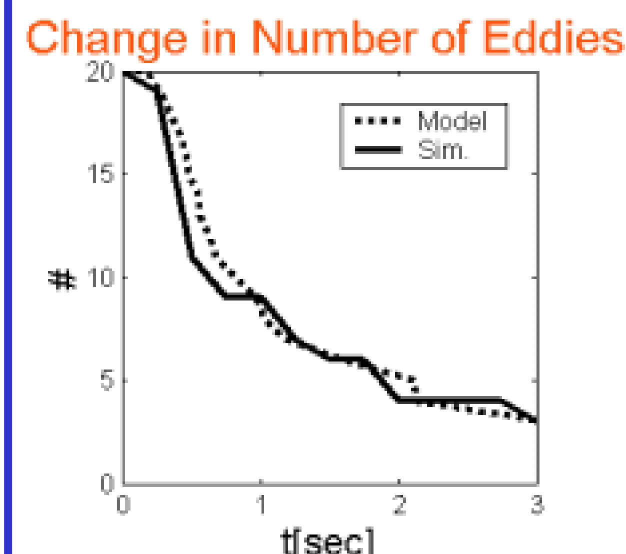
Experimental results taken from Roshko & Brown, J. Fluid Mech, 1974 & L.P. Bernal, Phys Fluids (1988).

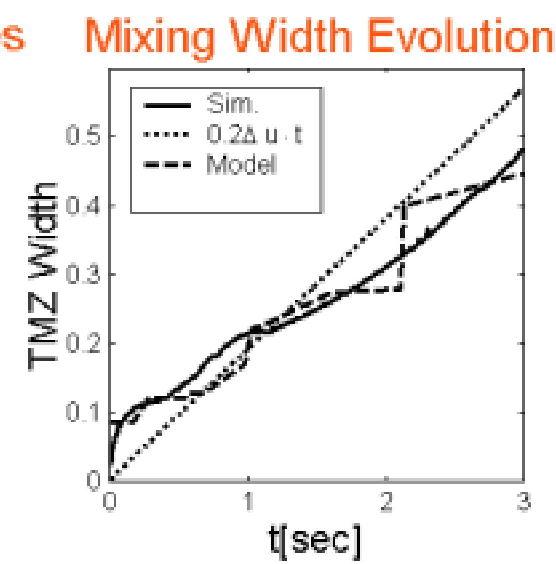
Multi Mode Numerical Simulation





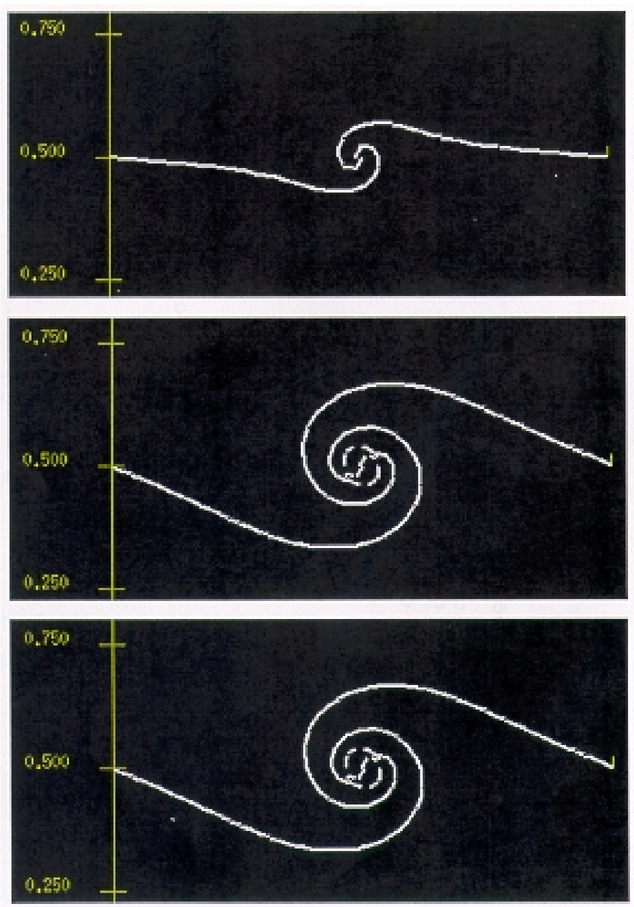




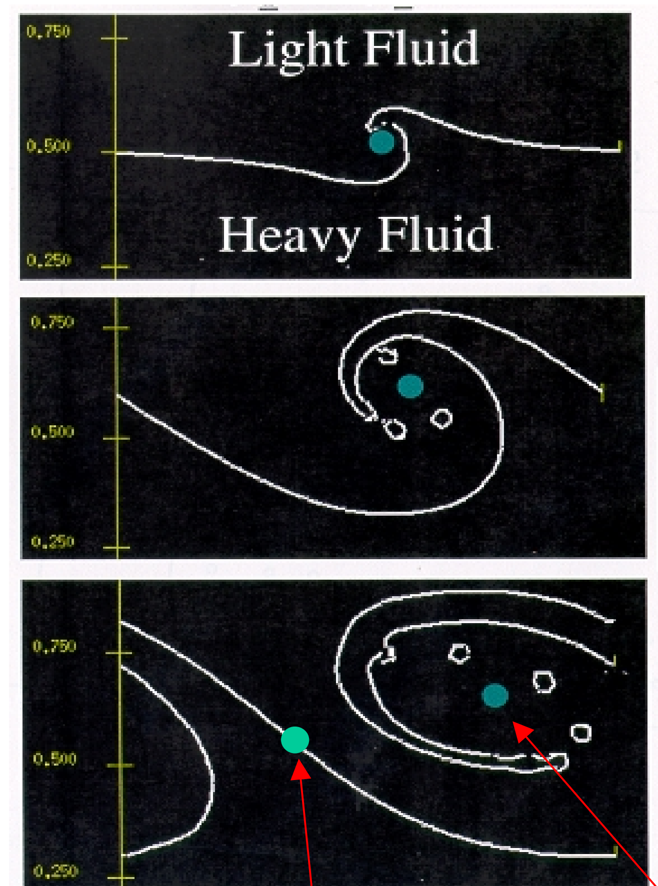


Density Ratio Effects on the KH Instability: Single Mode - Numerical Simulations

$\rho_1=\rho_2$; $v_1=v_2$



$\rho_1=7\rho_2$; $v_1=v_2$



Vortex drifts at

$$v_c \neq \frac{v_1 + v_2}{2}$$

according to pressure equilibrium on stagnation points between neighboring vortices*:

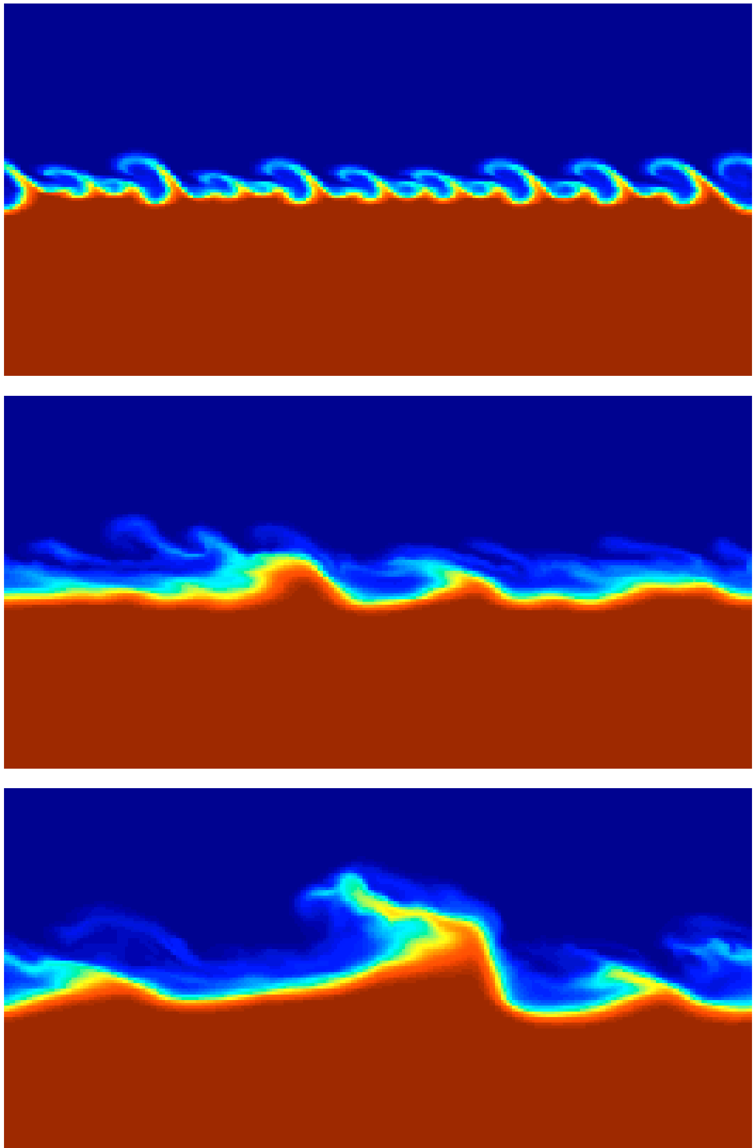
$$\rho_1(v_1 - v_c)^2 = \rho_2(v_c - v_2)^2$$

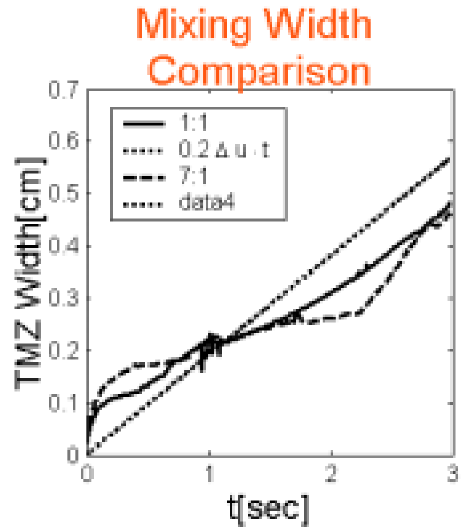
Stagnation point

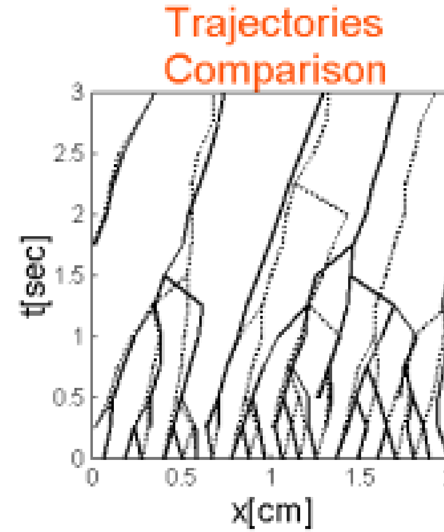
Vortex

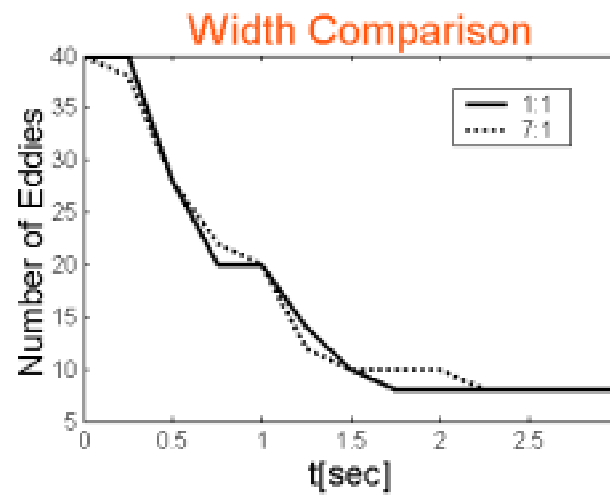
*Dimotakis et. Al., AIAA J. **24**, 1791 (1986)

Multi-Mode Case: Numerical Simulation



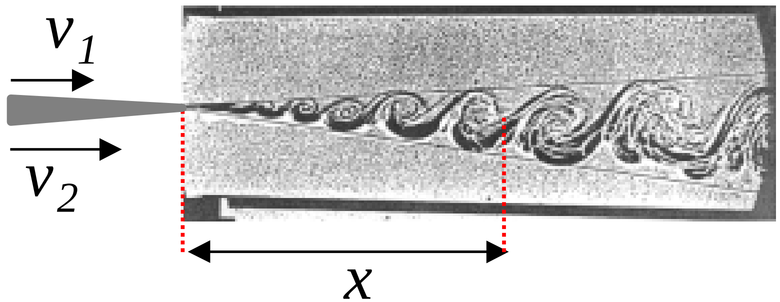






No true density ratio effects on the large scale evolution of the flow

Drift Velocity Effect on the Linear Growth Coefficient



$$\bar{v} = (v_1 + v_2)/2$$

$$v_d = \textit{drift velocity}$$

$t(x) = ?$

$v_d = 0 \ (\rho_1 = \rho_2)$

$$t = \frac{x}{\bar{v}}$$

$$h_{mix} = 0.2 \cdot \Delta u \cdot t = 0.4 \frac{v_1 - v_2}{v_1 + v_2} x$$

$v_d \neq 0 \ (\rho_1 \neq \rho_2)$

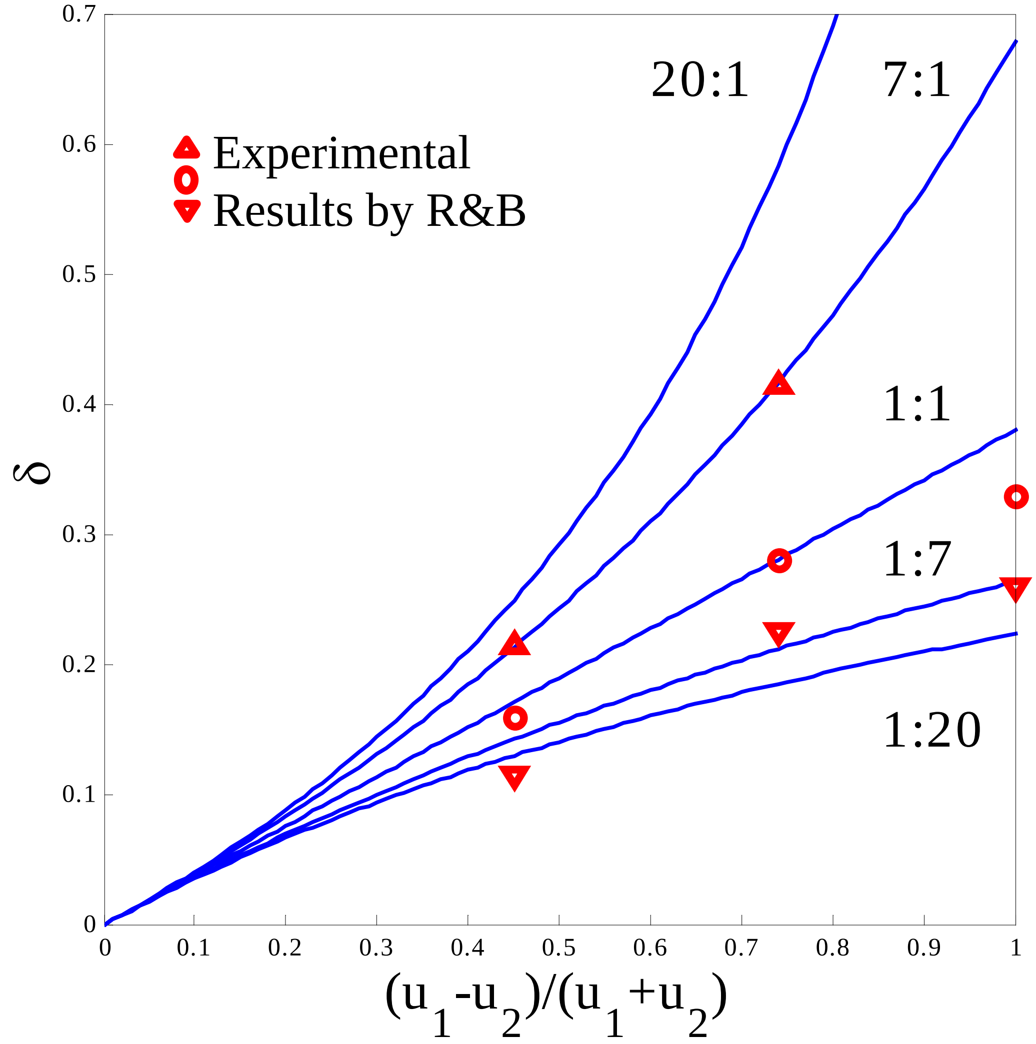
$$t = \frac{x}{v_c} = \frac{x}{\bar{v} + v_d}$$

$$h_{mix} = 0.2 \cdot \Delta u \cdot t = 0.4 \frac{f}{1 \pm c \cdot f} x$$

$$with \quad f = \frac{v_1 - v_2}{v_1 + v_2}$$

Linear Growth Coefficient in Agreement with Experiments

For mixing zone height of: $h(x) = \delta \cdot x$



$h_{mix}(x)$ not linear with Δu for $v_d \neq 0$

Summary

- A large scale statistical model for the KH instability was developed resulting in:
  - Linear growth rate coefficient for all density ratios.
  - Full statistical characterization of the flow.
  - The vortex merger is established as the growth mechanism of the mixing zone.

- Very good agreement with simulations and experiments is achieved.