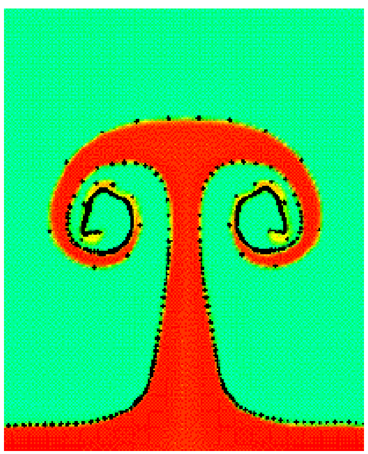
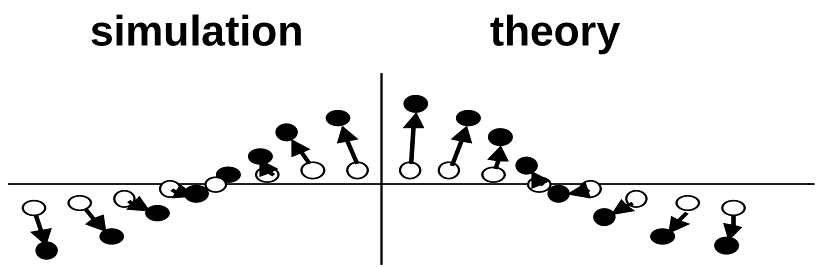
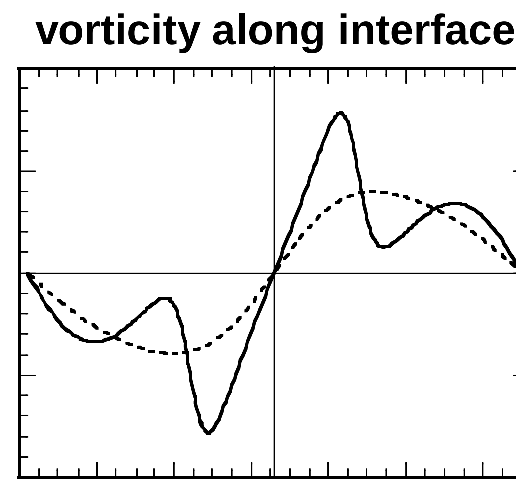
Nonlinear Evolution of an Interface in the Richtmyer-Meshkov Instability







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Introduction

Wouchuk-Nishihara formula(1997) of RMI growth rate indicates that RM instability is driven by velocity shear left by the shocks at the interface without the impulsive gravitational acceleration.

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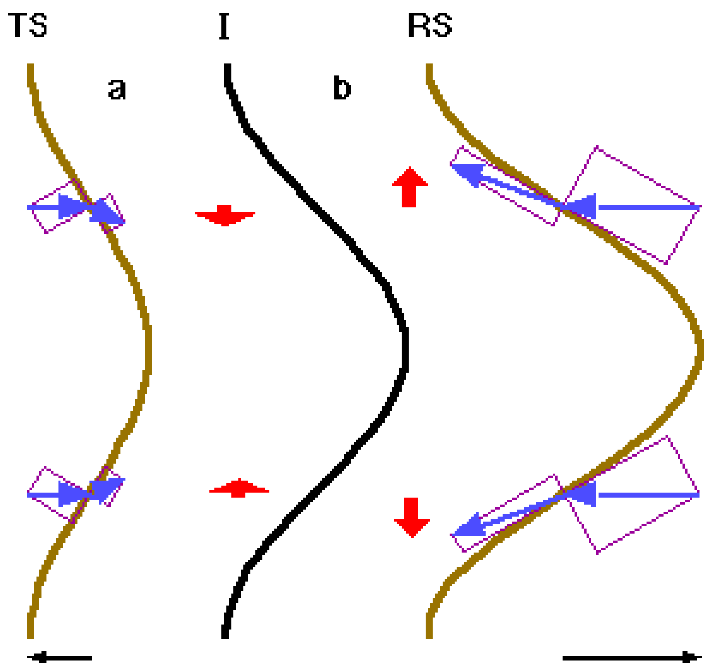

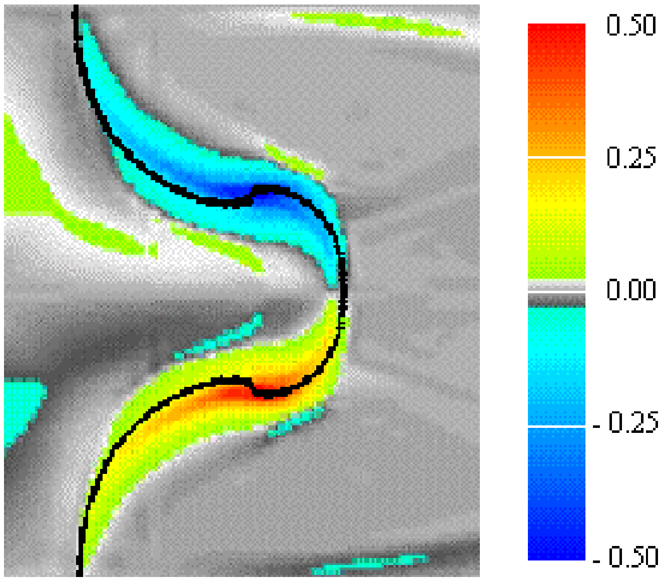

vorticity

$$v_{liu} = \frac{\rho^b \delta v_o^b - \rho^a \delta v_o^a}{\rho^a + \rho^b}$$

$$\delta v_o^a = k\xi_o \left(1 - \frac{u_{st}}{u_{si}}\right) v_i$$

$$\delta v_o^b = k\xi_o \left(1 + \frac{u_{sr}}{u_{si}}\right) (v_l - v_i)$$

Executive summary

- Nonlinear evolution of Richtmyer-Meshkov instability is described with nonlinear self-interaction of a vortex sheet with a density jump across the sheet.
- Lagrangian description of the vortex sheet reveals nonlinear dynamics, such as local stretching and shrinking of the sheet.
- Theory shows local increase and decrease of vorticity along the sheet. They results in spiral formation of spike and appearance of opposite vorticity sign at joint of mushroom umbrella, respectively.
- We have investigated dependence of nonlinear growth and nonlinear evolution of circulation on the sheet on the Atwood number and initial corrugation amplitude of the sheet.

Outline of Talk

- Shocked interface vs vortex sheet

- Analytical model for nonlinear dynamics of vortex sheet

  Lagrangian description, finite Atwood number

- Result and comparison with simulations

  nonlinear growth of spike and bubble

  local strethcing and shrinking of interface

  local increase and decrease of vorticity along interface

  dependence of nonlinear growth and circulation on interface
  on Atwood number and initial corrugation amplitude

By treating the interface as a vortex sheet, fully nonlinear evolution of RM instability is described.

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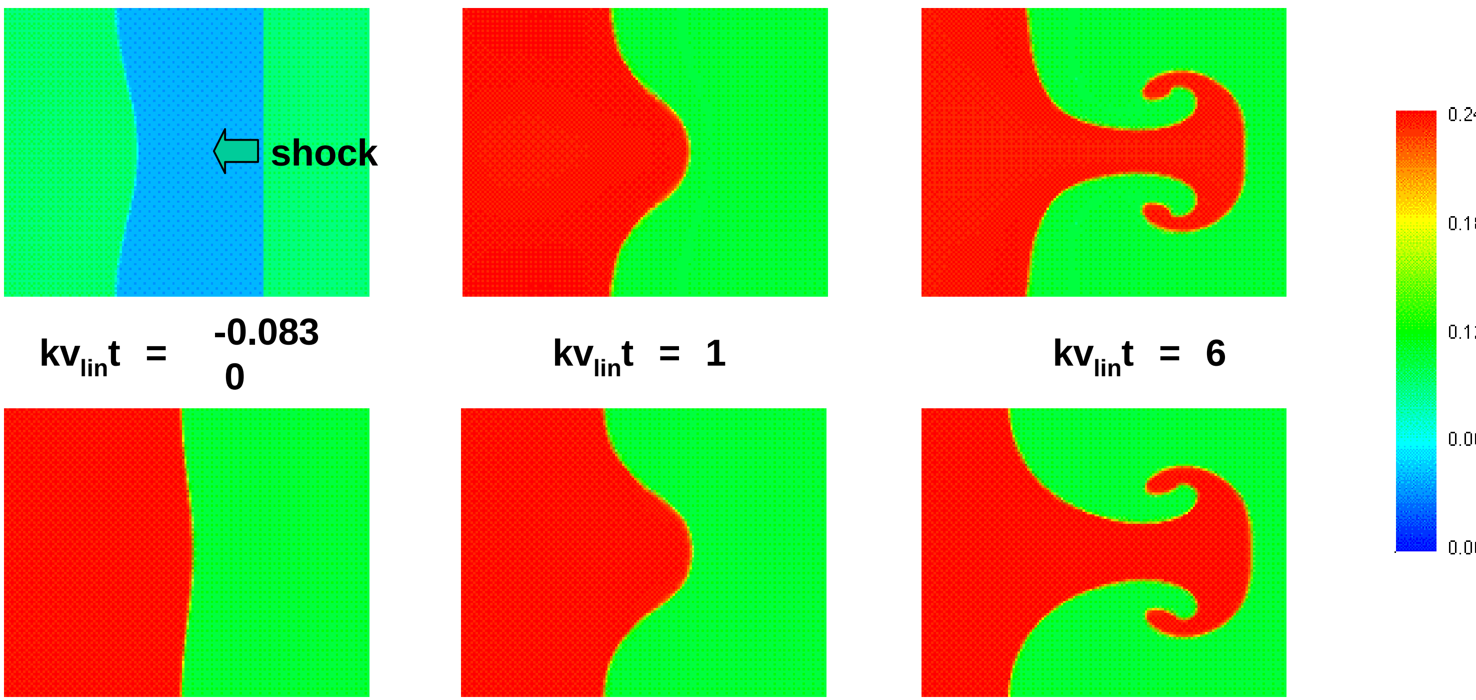
M=2, **before shock** $A_0=0.4$ $\xi_0/\lambda=0.0362$

after shock $A'_0=0.376$ $\xi'_0/\lambda=0.02$ $\xi'_0=\xi_0\,(1-u_{st}/u_{si})$

shock



velocity shear

$$\frac{\mathbf{v}}{\mathbf{v}_{\mathbf{lin}}}=\left(-\frac{\partial}{\partial \mathbf{x}}\psi,\frac{\partial}{\partial \mathbf{y}}\psi,\right) \qquad \psi=\mathbf{e}^{\mp\mathbf{kx}}\sin\mathbf{ky}$$

In case of finite Atwood number, baroclinic term induces vorticity on interface, and streching and shrinking of interface occurs locally in tangential direction

By introducing fluid velocity on the interface

$$\mathbf{u}_+ \equiv \frac{\rho_1 \mathbf{u}_1 + \rho_2 \mathbf{u}_2}{\rho_1 + \rho_2}$$

we obtain from Euler equation

$$\frac{\mathbf{D}\Gamma}{\mathbf{Dt}} = \left(\frac{\partial}{\partial \mathbf{t}} + \mathbf{u}_+ \cdot \nabla\right)\Gamma = 2\mathbf{A}\left[\frac{\mathbf{D}\Phi}{\mathbf{Dt}} - \frac{1}{2}\mathbf{q}\cdot\mathbf{q} - \frac{1}{8}\boldsymbol{\kappa}\cdot\boldsymbol{\kappa} + \frac{\mathbf{A}}{2}\boldsymbol{\kappa}\cdot\mathbf{q}\right]$$

where

$\mathbf{A} = \dfrac{\rho_2 - \rho_1}{\rho_2 + \rho_1}$ **(Atwood number) ,** $\Gamma = \phi_1 - \phi_2$ **(circulation),** $\Phi = \phi_1 + \phi_2$

$\boldsymbol{\kappa} = \nabla\Gamma = \mathbf{u}_1 - \mathbf{u}_2$ **(vorticity),** $\mathbf{q} = \dfrac{1}{2}(\mathbf{u}_1 + \mathbf{u}_2)$**,** $\mathbf{u}_\mathbf{i} = \nabla\phi_\mathbf{i}\,(\mathbf{i} = 1,2)$

Modified Birkhoff-Rott equation becomes

$$\frac{\partial}{\partial \mathbf{t}}\mathbf{Z}^* = \mathbf{u}_+ = \mathbf{q} - \frac{\mathbf{A}}{2}\boldsymbol{\kappa} = \frac{1}{2\pi\mathbf{i}}\int\frac{\boldsymbol{\kappa}\mathbf{ds}'}{\mathbf{Z}-\mathbf{Z}'} - \frac{\mathbf{A}}{2}\boldsymbol{\kappa},\quad \mathbf{Z}(\mathbf{t}) = \mathbf{x} + \mathbf{iy}$$ **(position of the interface)**

Nonlinear Theory of a Vortex Sheet(1): Basic Equations

Circulation on vortex sheet

$$\frac{\mathbf{D}}{\mathbf{Dt}}\Gamma = \left(\frac{\partial}{\partial \mathbf{t}} + \mathbf{u}_+ \cdot \nabla\right)\Gamma = 2\mathbf{A}\left[\frac{\mathbf{D}\Phi}{\mathbf{Dt}} - \frac{1}{2}\mathbf{q}\cdot\mathbf{q} - \frac{1}{8}\boldsymbol{\kappa}\cdot\boldsymbol{\kappa} + \frac{\mathbf{A}}{2}\boldsymbol{\kappa}\cdot\mathbf{q}\right]$$

where

$$\mathbf{A} = \frac{\rho_2 - \rho_1}{\rho_2 + \rho_1} \quad , \quad \Gamma = \phi_1 - \phi_2 \quad , \quad \Phi \equiv \phi_1 - \phi_2 \quad ,$$

$$\mathbf{u}_+ = \frac{\rho_1 \mathbf{u}_1 + \rho_2 \mathbf{u}_2}{\rho_1 + \rho_2} \quad , \quad \boldsymbol{\kappa} = \mathbf{u}_1 - \mathbf{u}_2 \quad , \quad \mathbf{q} = \frac{1}{2}\left(\mathbf{u}_1 + \mathbf{u}_2\right)$$

incompressible and irrotational fluid

$$\nabla^2 \phi_\mathbf{i} = 0 \quad , \quad \mathbf{u}_\mathbf{i} = \nabla\phi_\mathbf{i} \, (\mathbf{i} = 1,2)$$

kinematic boundary conditions

$$\mathbf{r}_\mathbf{t} \cdot \mathbf{n} = \mathbf{u}_+ \cdot \mathbf{n} \quad , \quad \mathbf{r}_\mathbf{t} \cdot \mathbf{t} = \mathbf{u}_+ \cdot \mathbf{t} \quad , \quad \mathbf{u}_1 \cdot \mathbf{n} = \mathbf{u}_2 \cdot \mathbf{n}$$

Nonlinear Theory of a Vortex Sheet(2) : Expantion

Lagrangian marker of vortex sheet

$$x(t) = \theta + X(\theta, t)$$
$$y(t) = Y(\theta, t)$$

expansion

$$X = \sum_n \varepsilon^n X^{(n)} \quad , \quad Y = \sum_n \varepsilon^n Y^{(n)}$$

$$\phi_1 = \sum_n \varepsilon^n \phi_i^{(n)} e^{\mp nky} \cos nkx \quad ,$$

By introducing normalization, kx , $kv_{lin}t$, and $k\phi_i / v_{lin}$, nonlinear evolution of a vortex sheet is determined from two parameters

A (Atwood number) and $k\xi_0$(initial corrugation of sheet),

where $v_{lin} = \dfrac{\rho_1 \delta v_{10} - \rho_2 \delta v_{20}}{\rho_1 + \rho_2}$, (Wouchuk – Nishihara formula)

Comparison of Nonlinear Growth Rates (Spike grows faster than bubble)

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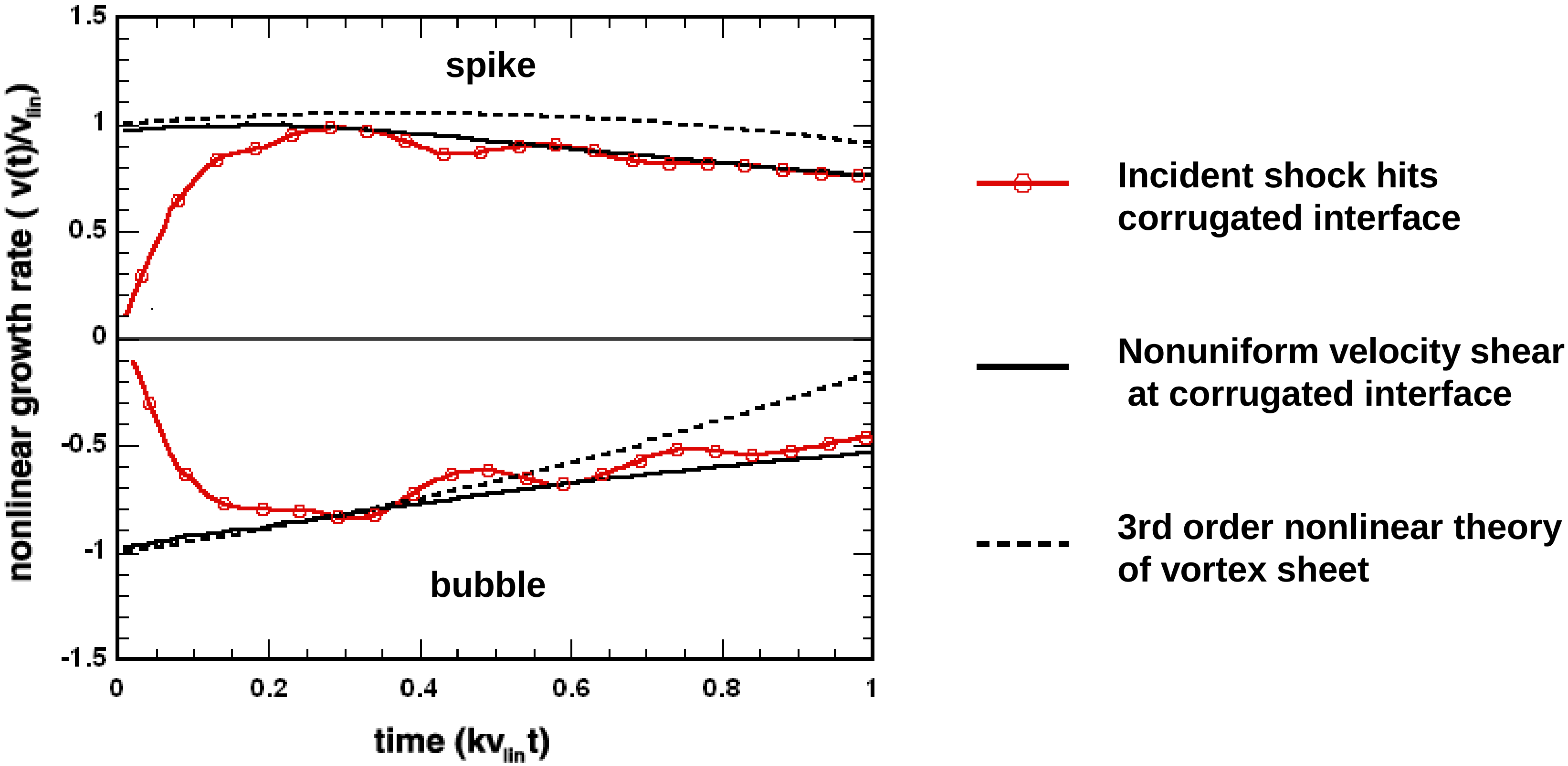
M=2 A_0=0.4 ξ'_0/λ=0.02



Theoretical dependence of nonlinear growth and vorticity on Atwood number and corrugation amplitude agrees well simulations

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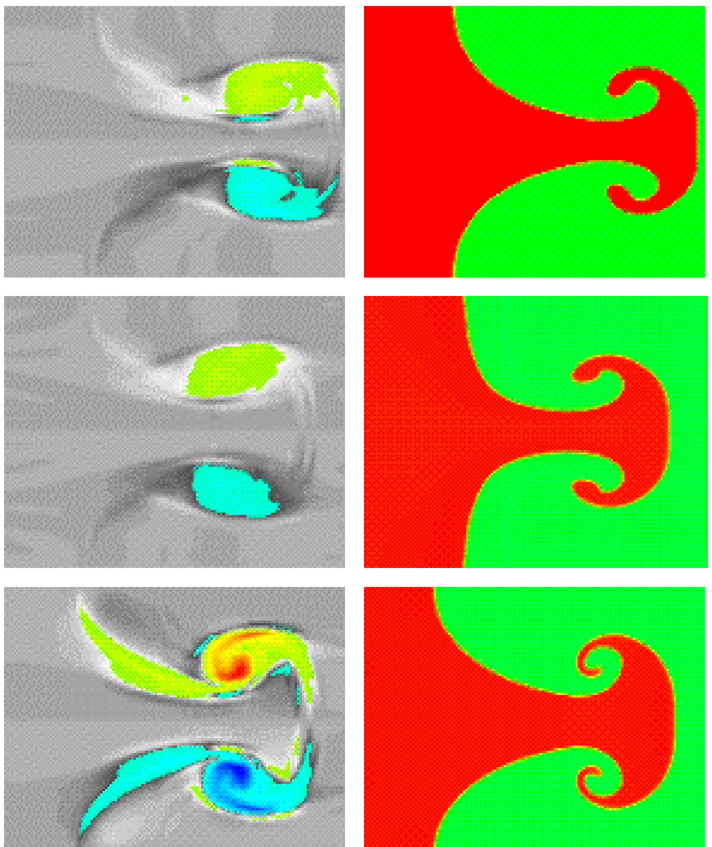
vorticity density



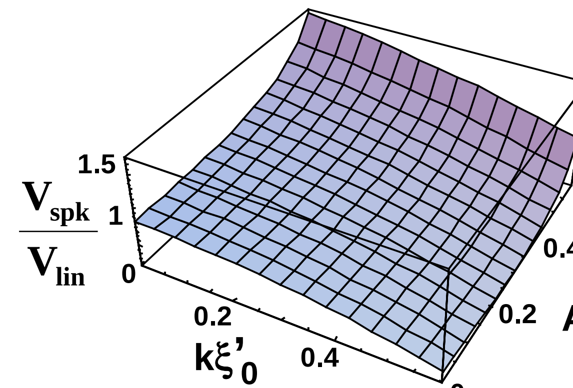
$A_0 = 0.6$, $\xi'_0/\lambda = 0.02$

$A_0 = 0.4$, $\xi'_0/\lambda = 0.02$

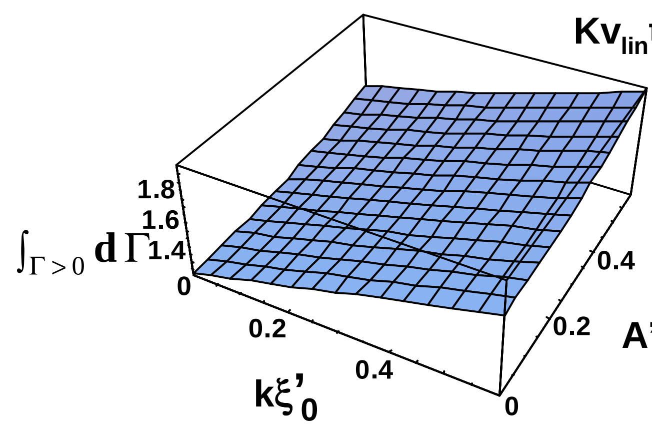
$A_0 = 0.4$, $\xi'_0/\lambda = 0.08$

$kv_{lin}t = 6$

Larger Atwood number leads to larger nonlinear growth



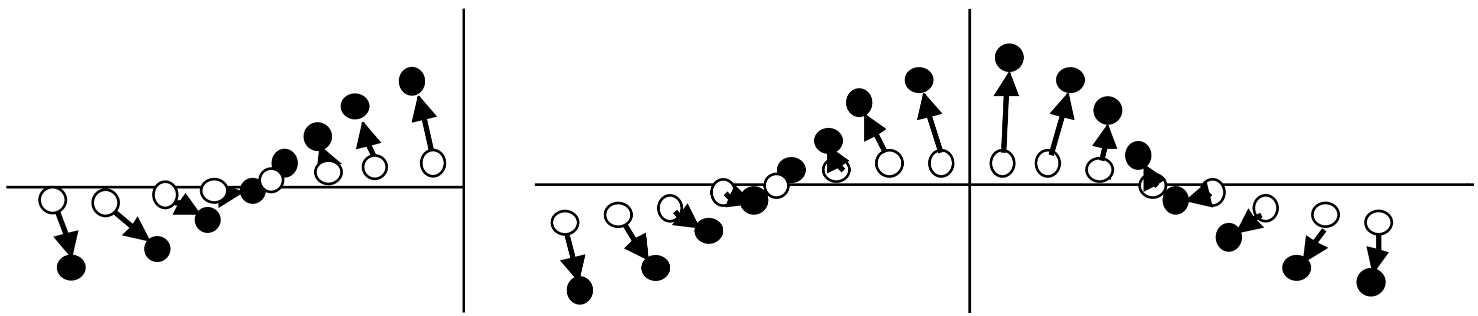
Larger corragation amplitude lead to rapid increase of circulation



Lagrangian markers move both x and y directions.
Stretch and shrink of vortex sheet occur locally.

Theoretical results agrees well with simulations

simulation (shocked interface) | **simulation** (vortex sheet) | **theory** $(x(\theta, t) , y(\theta, t))$



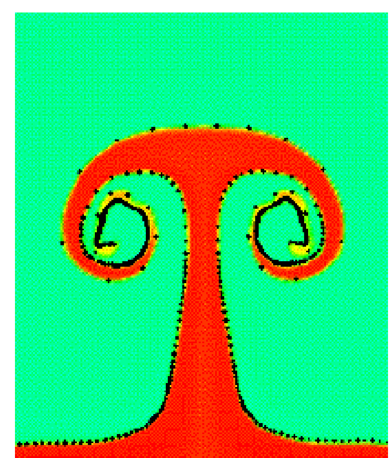
● $Kv_{lin}t = 0.80$

○ $Kv_{lin}t = 0.05$

$A_0' = 0.376$

$\xi'_0 / \lambda = 0.02$

$M = 2$



Theory shows local increase of vorticity (tip of spiral), and local decrease of vorticity (joint of mushroom umbrella).

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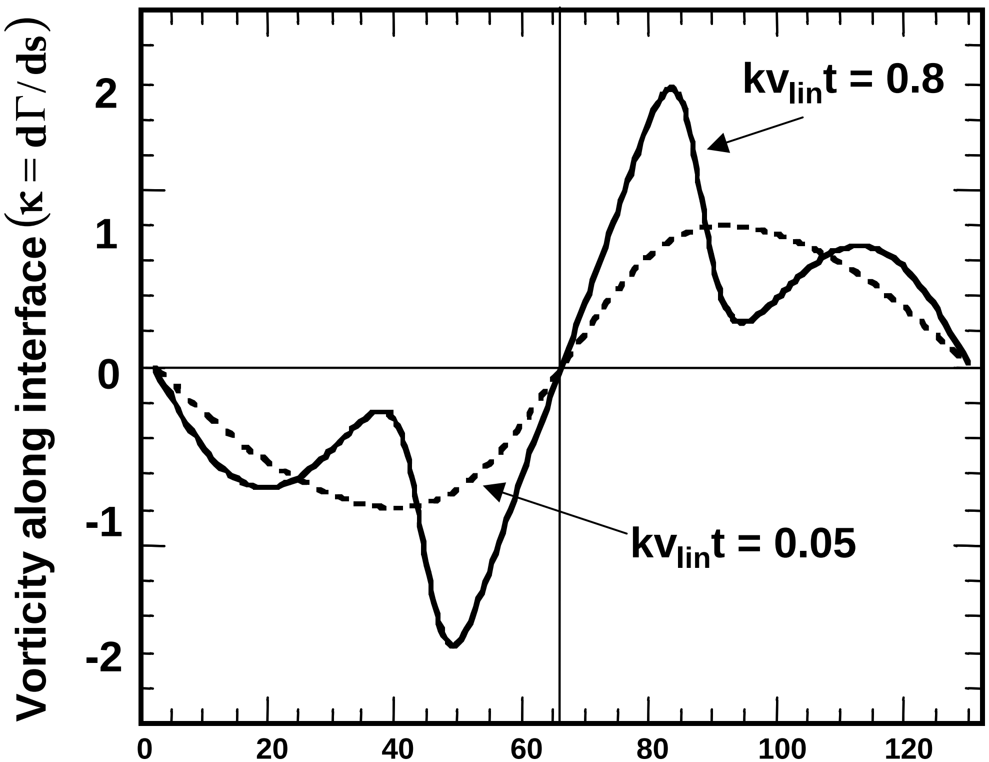

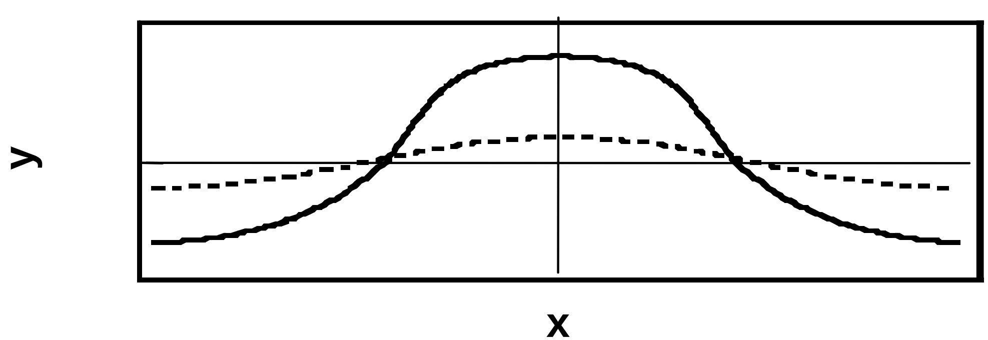

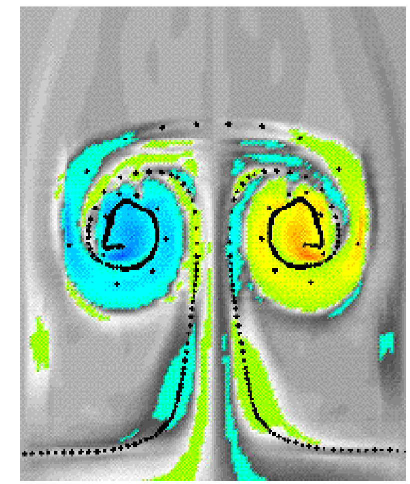

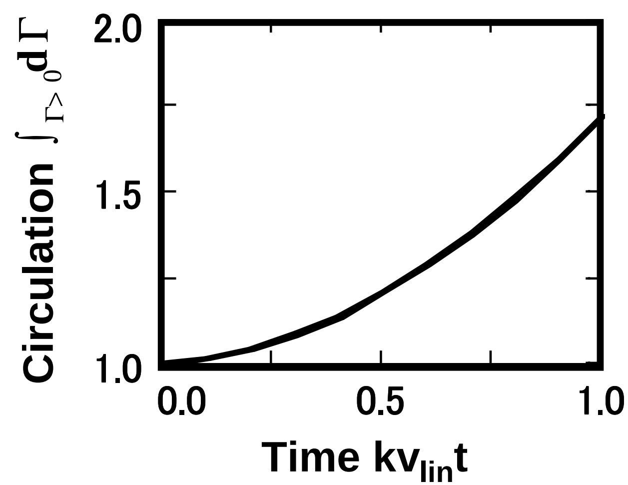

Conclusion

- Nonlinear evolution of Richtmyer-Meshkov instability is described with nonlinear self-interaction of a vortex sheet with a density jump across the sheet.

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