

**STABILITY OF REFLECTED FROM THE CENTER SELF-SIMILAR  
CONVERGING SHOCK WAVE**

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In this paper the technique first used in Ref.1,2 for solving the point blast wave stability problem is applied for the one of the self-similar converging shock wave after focusing.

Equations for the case of converging self-similar shock wave

The obtained in Ref.1-4 system of equations can be applied for the case under consideration.

Let us consider the uniform equation for perturbations (we use the notation of Ref.4):

$$\frac{P_1}{\rho} +_1 + \gamma \frac{(R^2 R_1)'}{R^2 R'} - n(n+1) \gamma F_1 = 0 \quad (1)$$

$$\frac{P_1}{\rho} + R_1 \ddot{R} + (R^2 \dot{F}_1) = 0 \quad (2)$$

$$\frac{P_1'}{\rho} + \ddot{R} \frac{R_1}{R} + \ddot{R}_1 + n(n+1) \ddot{R} F_1 = 0 \quad (3)$$

With boundary conditions:

$$\begin{aligned} R_1(R = S, t) &= R_1(1 - R') + \frac{s_1}{3} R'(S) \\ F(S, t) &= \frac{\lambda + 3}{3N} s_1(S) \\ \dot{F}(S, t) &= -\frac{\dot{R}(S)}{\dot{S}} \end{aligned} \quad (4)$$

$$P_1(S, t) = P_1(\text{Shock}) - \frac{s_1}{3} P'(S)$$

According to the self-similar approach we look for the solution in the self-similar form:

$$(F_1, R_1, P_1) \sim r^\lambda f(r/S), \text{ and: } s_1 \sim r^\lambda \quad (5)$$

After inserting of self-similar variables (functions of  $z=r/S$ ) we get:

$$\begin{aligned} P_1 b - (D + \lambda + 3w) r_1 - N w F_1 &= 0 \\ P_1 w + r_1 a + (D + 2w + \frac{1}{\alpha} - 1) D F_1 &= 0 \end{aligned} \quad (6)$$

$$(D + \lambda + 3 - w) P_1 + ((D + 2w + \frac{1}{\alpha} - 1) D - a) r_1 + N a F_1 = 0$$

(Here  $D$  stands for differential operator:  $Df \equiv \frac{d}{d \ln z} f$ )

Because of presence of non-zero initial velocity before shock front (we denote this velocity  $u_0$ ) the mass conservation equation is changing:

$$\frac{1}{\rho} = \frac{1}{1+u_0} \frac{R^{s-1} \partial R}{r^{s-1} \partial r} \quad (7)$$

So coefficients in (Eqs.27) look like as follows:

$$\begin{aligned} w &= \frac{Dx}{x} \\ a &= Dw + (w-1)\left(w + \frac{1}{\alpha} - 1\right) \\ b &= (1+u_0) \frac{wz^3}{\gamma \alpha P} \end{aligned} \quad (8)$$

(Here variables  $x=R/S$ ,  $P=P/S^2$  are functions of  $z$ ), the front value of coefficient  $w$  is  $w(1) = \frac{1+u_0}{\delta}$ , (here  $\delta$  stands for front compression), the front value of coefficient  $a(1)$  is calculated from spherically symmetric solution.

Equation system (6) has the fourth order; there are four boundary conditions on the front edge ( $z=1$ ):

$$\begin{aligned} x_1(1) &= 1 - \frac{1+u_0}{\delta} + \frac{s_1}{3} \frac{1+u_0}{\delta} \\ F_1(1) &= \frac{\lambda+3}{3N} s_1 \\ DF_1(1) &= 1 - \frac{1+u_0}{\delta} \\ P_1(1) &= P_1(Shock) + \frac{s_1}{3} a(1) \end{aligned} \quad (9)$$

These conditions construct full set of boundary conditions necessary to determine a solution. (To derive the conditions one should take into account the characteristics of gas flow before the shock front because values  $P_1(shock)$  and  $s_1$  in (31) depend on values of gradients of pressure and velocity before the shock front).

In order to construct the eigenvalue problem we add the fifth boundary condition (at the center):

$$P_I(0)=0 \quad (10)$$

We solve the eigenvalue problem (Eqs.6,9,10) and we calculate the values of power exponent  $\lambda$  (see Eq.5) as eigenvalues.

We used the simultaneous solving of spherically symmetric equations and equations for perturbations. The variables  $PI$ ,  $FI$ ,  $xI$  are expressed as functions of  $w$  and coefficients  $a$ ,  $b$ , in Eq.6 are also expressed as functions of  $w$ :

$$\begin{aligned}
a &= \frac{b}{1-b} \left[ (w-1) \left( w-1 + \frac{1}{\alpha} \right) - 3w \left( w-1 + \frac{2/\alpha - 2}{3\gamma} \right) \right] \\
D &\equiv \left[ a - (w-1) \left( w-1 + \frac{1}{\alpha} \right) \right] \frac{d}{dw} \\
\frac{db}{bdw} &= \frac{6 - 3\gamma + w(3\gamma - 1)(b - 1)}{3w \left( w-1 + \frac{2/\alpha - 2}{3\gamma} \right) - b \left( w-1 + \frac{1}{\alpha} \right)} + \frac{\gamma + 1}{w}
\end{aligned} \tag{11}$$

Using this approach the eigenvalue problem was solved numerically. The values of  $\lambda$  were calculated as eigenvalues. The dependence of  $\lambda$  on  $\gamma$  and harmonic number  $n$  is shown on Fig.1, Fig2.

The case under consideration differs from other cases with diverging shock waves considered earlier. Curves on figures look like irregular ones. The reason of it is that self-similar solutions exist only in small region on the plane  $n$ - $\gamma$ . For big harmonic numbers ( $n \geq 15$  for  $\gamma=1.2$ ,  $\gamma=1.4$  and  $n \geq 7$  for  $\gamma=1.667$ ) self-similar solution supposedly does not exist at all.

Fig.3, Fig.4 present the computer validation of self-similar results shown on Fig.1, Fig.2. V.Yu Meltsas performed the computer modeling of the reflected from the center shock wave in the way he did it in Ref.5 . The results of computer modeling agree the results of self-similar calculations: we see the exponential perturbation grow on Fig. 3 and oscillation regime of perturbation evolution for the case  $n=2$ ,  $\gamma=1.667$  on Fig.4.

### Acknowledgments

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### References:

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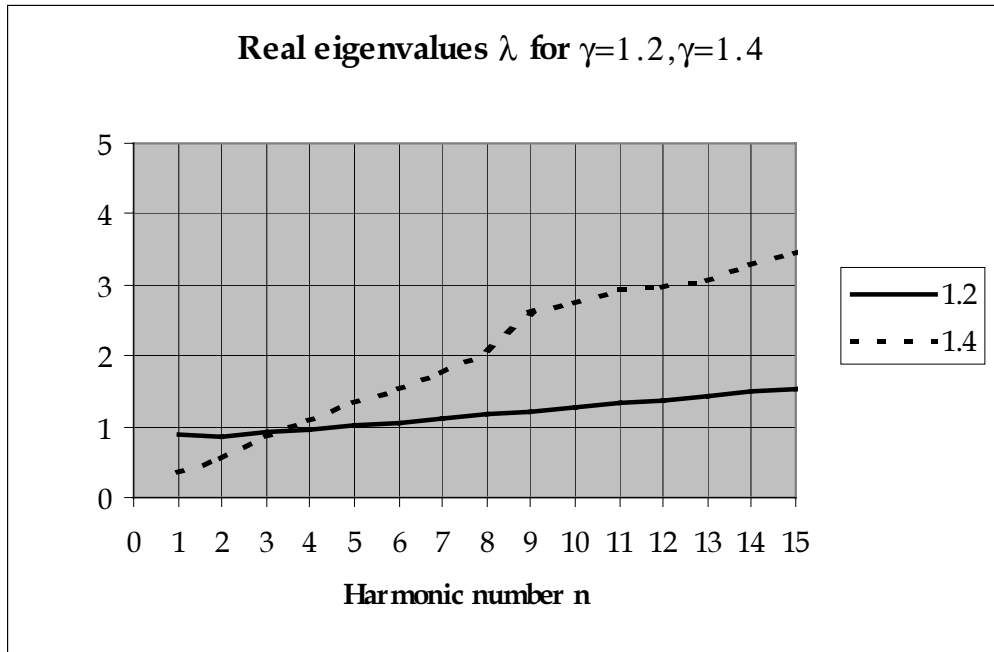


Fig. 1 Real eigenvalues  $\lambda(n)$  for  $\gamma=1.2, \gamma=1.4$ . Shock wave is unstable with respect to perturbations of all harmonic numbers.

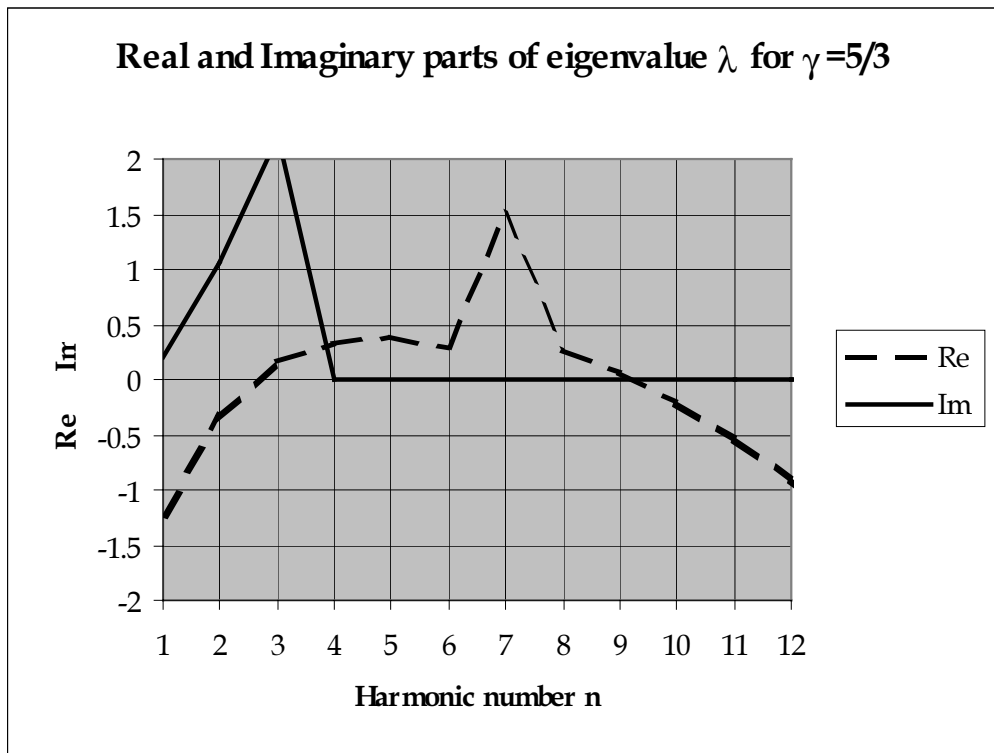


Fig.2 Complex (for  $n=1,2,3$ ) and real (for  $n>3$ ) power exponents (eigenvalues)  $\lambda(n)$  for gases with  $\gamma=1.667$ . There appears to be no self-similar perturbations for  $n>6$ .

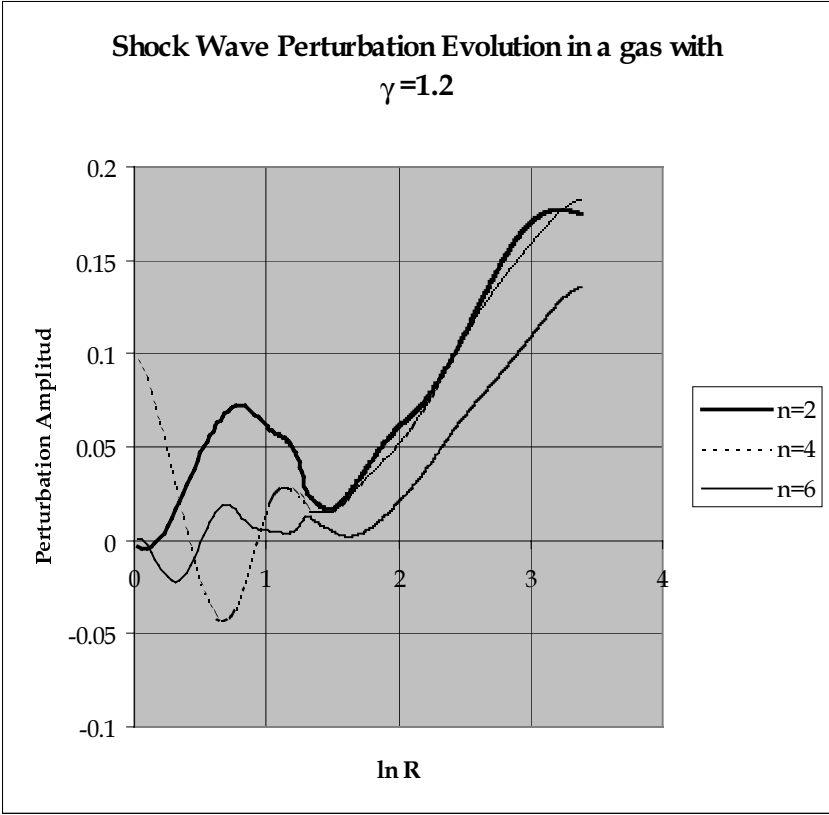


Fig.3 Perturbation evolution for shock wave in a gas with  $\gamma=1.2$

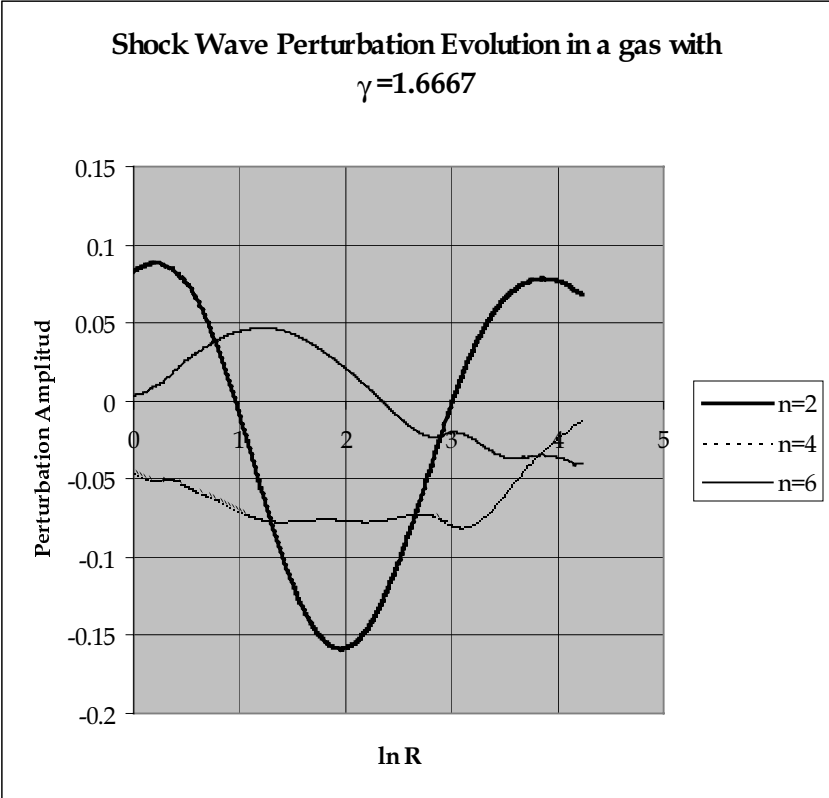


Fig.4 Perturbation evolution for shock wave in a gas with  $\gamma=1.667$