STABILITY 0F DIVERGING SHOCK WAVES

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Introduction

The problem of a search for cases of unstable evolution of expanding shock waves with non-radial perturbations growing in amplitude was formulated long ago but there was no essential progress in this field until 1980-ths. The first proof of an existence of the unstable regime of perturbation evolution was published in the paper by E.Vishniac (Ref.1) where stability of point blast wave in an ideal gas with a specific heat ratio equal to unit (γ=1) was considered. In this case the gas behind the shock front is compressed into thin dense layer moving under influence of pressure of hot gas in the center region. The calculation of this layer movement can be done simply by application of conservation laws. This calculation was made in Ref. 1 for small blast wave perturbations those being expanded in spherical harmonics. Results of these calculations were as follows:

- small perturbation evolution had oscillating pattern,

-perturbations with big enough harmonic numbers were growing in amplitude with power dependence of time the power exponent being complex number,

- the discovered instability was Raleigh-Taylor type.

Later investigations revealed that all these features are characteristic for the perturbations evolution in the general case of an arbitrary value of γ.

V.Ktitorov[2] and E.Vishniac and D.Ryu[3] first published the solution of the point blast wave stability problem in the general case of an arbitrary γ>1. There was considered the point blast wave in a uniform gas with constant density. There were found values determining the perturbation evolution: the increment of perturbation growth, and the period of oscillations. These values were calculated for a large number of sets: harmonic number *n* and gas specific heat ratio γ.

The critical value of γ determining the blast wave stability was found too (Ref.2): γ_c=1.20. If γ were greater than this value the blast wave would be stable with respect to perturbations of all harmonic numbers; if $\gamma<1.20$, on the contrary, there would exist growing in amplitude harmonics.

Later there was found an experimental validation of these calculation results (Ref.4,5)[5].

This solution was obtained owing to using in Refs. 2,3 a self-similar approach for calculation of perturbation structure. The physical sense of this approach was in considering an asymptotically regime of perturbation evolution. This regime takes place when time of perturbation evolution is large enough to make the influence of initial conditions become negligibly small. The analogy could be made between the role of this solution in describing the perturbation evolution and the role of the well-known Taylor-Sedov solution Ref.6 in describing an evolution of spherically symmetric blasts.

Later on the number of systems with blast waves that were studied using the self-similar approach was extended; and the following cases were considered:

- Cylindrical blast wave (Ref.7),
- Isothermal blast wave (Ref.8),
- Blast wave in gas which has initial density depending on radius as power function (Ref.9),
- Blast wave in a non-ideal gas, which specific heat ratio γ is a function of gas density ρ (Ref.10).

In this paper we use the self-similar approach in order to consider the stability problem of these and some other systems in the unified manner.

1.STABILITY 0F A POINT BLAST WAVE IN AN IDEAL GAS

1.1 Formulating the stability problem

Let us consider a spherical *(s=3)* or cylindrical (*s=2*) blast wave in an ideal gas in the case when the initial gas density ρ_0 is a power function of radius r: $r_0 \sim r^k$. According to[2] we shall write the hydrodynamic system of equations in the Lagrangian form.

The following notation is used below:

$\bar{R}$ - the Eulerian co-ordinates

$\bar{r}$ - the Lagrangian co-ordinates (the initial co-ordinates before shock front are taken as the Lagrangian co-ordinates),

$\frac{d^3(\bar{R})}{d^3(\bar{r})} \equiv \frac{D(R^3, \cos\Theta, \Phi)}{D(r^3, \cos\theta, \varphi)}$ - the Jacobian of the transformation from the Lagrangian to the Eulerian co-ordinates,

P - pressure, r - density, r_0 - the initial density ($r_0 \sim r^k$),

We start considering the stability problem from writing 3D hydrodynamic equations in the Lagrangian form: (here time derivative is to be understood in the Lagrangian sense - it is the derivative along the flow line). The first two equations are obvious:

$$\frac{\partial}{\partial t}\left(\frac{P}{\rho^\gamma}\right) = 0, \tag{1}$$

$$\frac{\rho_0}{\rho} = \frac{d^3(\bar{R})}{d^3(\bar{r})} \tag{2}$$

The equation of motion (the third equation) can be deduced from its Eulerian form (for the sake of simplicity we use here Cartesian coordinates $\bar{R} = (X_1, X_2, X_3)$ and $\bar{r} = (x_1, x_2, x_3)$):

$$\frac{\partial^2 X_k}{\partial t^2} = -\frac{\partial P}{\rho \partial X_k} \tag{3}$$

We change the arguments in the equation of motion written in the Eulerian form using formula:

$$\frac{\partial}{\partial x_i} = \sum_k \frac{\partial X_k}{\partial x_i}\frac{\partial}{\partial X_k} \tag{4}$$

And we get finally:

$$\frac{\partial P}{\rho \partial x_i} = -\sum_k \frac{\partial X_k}{\partial x_i}\frac{\partial^2 X_k}{\partial t^2} \tag{5}$$

For the non-perturbed blast wave the system of equation (1),(2),(5) looks as follows:

$$\frac{\partial}{\partial t}\left(\frac{P}{\rho^\gamma}\right) = 0, \tag{6}$$

$$\frac{\rho_0(r)}{\rho} = \frac{R^{s-1}\partial R}{r^{s-1}\partial r} \tag{7}$$

$$\frac{\partial P}{\rho \partial r} = -\frac{\partial R}{\partial r}\frac{\partial^2 R}{\partial t^2} \tag{8}$$

The corresponding spherically (or cylindrically) symmetric self-similar solution of Eqs. (6-8) is well known. It can be written using the shock wave radius value $S(t) \sim t^{2/(k+s+2}$ and functions $x(z)$, $p(z)$, $r(z)$ of the self-similar argument $z=r/S$,:

$$
\begin{aligned}
&R_0(t,r) = Sx(z),\\
&P_0\ \ (t,r) = \frac{2}{\gamma+1}\rho^0(S)\dot{S}^2 p(z),\\
&\rho_0(t,r) = \frac{\gamma+1}{\gamma-1}\rho^0(S)\rho(z),\\
&\Theta_0 = \theta,\\
&\Phi_0 = \varphi
\end{aligned}
\tag{9}
$$

where *x*, *P*, *r* should satisfy to ordinary differential equations:

$$
\begin{aligned}
&\rho = \frac{\gamma-1}{\gamma+1}\frac{z^{s+k}}{x^s w},\\
&p = \frac{\rho^\gamma}{x^s z^s},\\
&\frac{p'}{p} = -\gamma\frac{ab}{w};
\end{aligned}
\tag{10}
$$

With boundary conditions: *x(1)=p(1)=r(1)=1,*

Here we denote:

$$
\begin{aligned}
&w = \frac{zdx}{xdz} = \frac{x'z}{z},\\
&a = w'z + (w-1)(w+\frac{s+k}{2}),\\
&b = \frac{\gamma+1}{2\gamma}\frac{wz^{s+k}}{px};
\end{aligned}
\tag{11}
$$

We insert to the system of equations (1),(2),(5) hydrodynamic values corresponding to the perturbed blast wave ($\tilde{\bar{R}}, \tilde{P}, \tilde{\rho}$) presented as sums of the non-perturbed blast wave values ($\bar{R}, P, \rho$), and small perturbations ($\bar{R}_1, P_1, \rho_1$):

$$
\begin{aligned}
&\tilde{R}(t,\bar{r}) = R(t,r) + R_1(t,\bar{r}),\\
&\tilde{\Theta}(t,\bar{r}) = \theta + \Theta_1(t,\bar{r}),\\
&\tilde{\Phi}(t,\bar{r}) = \varphi + \Phi_1(t,\bar{r}),\\
&\tilde{P}(t,\bar{r}) = P(t,r) + P_1(t,\bar{r}),\\
&\tilde{\rho}(t,\bar{r}) = \rho(t,r) + \rho_1(t,\bar{r}).
\end{aligned}
\tag{12}
$$

Also we suppose that an angular motion is the potential one:

$$
\Theta_1 = \frac{\partial F_1}{\partial \cos\theta}, \Phi_1 = \frac{\partial F_1}{\partial \varphi}, \tag{13}
$$

and that supposition will make the Jacobian being equal to:

$$
\frac{d^3(\bar{R}+\bar{R}_1)}{d^3\bar{r}} = \frac{R^2\partial R}{r^2\partial r}(1+\Delta_{\theta\varphi}F_1) + \frac{\partial R^2 R_1}{r^2\partial r} \tag{14}
$$

We insert expressions (12,13) to the system of equations Eqs.(1,2,5), and after finishing linearization procedure we get the following equations for perturbations:

Equation of entropy conservation:

$$
\frac{\partial}{\partial t}(\frac{P_1}{P} - \gamma\frac{\rho_1}{\rho}) = 0 \tag{15}
$$

Continuity equation:

$$\frac{\rho_1}{\rho}+\Delta_{\theta\varphi}F_1+\frac{\dfrac{\partial(R^{s-1}R_1)}{r^{s-1}\partial r}}{\dfrac{R^{s-1}\partial R}{r^{s-1}\partial r}}=0 \tag{16}$$

Angular component of the equation of motion (here we denote: $\dot{R}\equiv\dfrac{\partial R}{\partial t}$):

$$\frac{P_1}{\rho}+R_1\ddot{R}+(R^2\dot{F_1}\dot{)}=0 \tag{17}$$

Radial component of the equation of motion:

$$\frac{\partial P_1}{\rho\partial r}+\ddot{R}\frac{\partial R_1}{\partial r}+\ddot{R}_1\frac{\partial R}{\partial r}-\frac{\rho_1\partial P}{\rho^2\partial r}=0 \tag{18}$$

Here and after S stands for the non-perturbed shock front radius, $S(t)\sim t^{2/(k+s+2)}$, and S_1 stands for the value of the shock front radius perturbation.

Boundary conditions for the system of equations Eqs.(15-18) should be put on the shock front ($r=S+S_1$) using the following scheme:

$$\begin{aligned}
&(R+R_1)(r=S+S_1,t)=S+S_1\\
&(R+R_1)(r=S+S_1,t)=R(r=S+S_1,t)+R_1(r=S+S_1,t)\\
&R(r=S+S_1,t)=R(r=S,t)+S_1\frac{\partial R}{\partial r}(r=S,t)=S+S_1\cdot\frac{wx}{z}(z=1)\\
&R_1(r=S+S_1,t)=R_1(r=S,t)
\end{aligned} \tag{19}$$

From Eqs.(10,11) we have:

$$w(1)=\frac{\gamma-1}{\gamma+1},\, x(1)=1\,. \tag{20}$$

Hence we get:

$$R_1(r=S,t)=\frac{2}{\gamma+1}S_1 \tag{21}$$

The next front conditions are calculated in the same way. We get:

$$\begin{aligned}
&F_1(r=S,t)=0\\
&\dot{F_1}(r=S,t)=-\frac{2}{\gamma+1}\frac{\dot{S}}{S^2}S_1\\
&P_1(r=S,t)=\frac{2\rho_0}{\gamma+1}\frac{\dot{S}^2}{S}S_1(2\lambda+s+2+a(1))\\
&\rho(r=S,t)=\frac{(\gamma+1)\rho_0}{\gamma+1}\frac{S_1}{S}a(1)
\end{aligned} \tag{22}$$

According to Ref. 2 the perturbations are expanded in spherical harmonics $Y_{nm}\sim P_{nm}(\cos\theta)e^{im\varphi}$ (or in axial harmonics $e^{in\varphi}$ in the cylindrical case), and the components of expansion are presented in the self-similar form.

Shock wave radius perturbations are expanded too: ($S_1=\sum S_{1mn}Y_{mn}$). We suppose that the components of expansion are power functions of time for each harmonic number:

$$S_{1mn}\sim S^{\lambda_{mn}+1}\sim t^{\frac{2(\lambda_{mn}+1)}{k+s+2}}\,, \tag{23}$$

Here λ_{mn} is a complex number.

So we write:

$$\tilde{R}(t,\bar{r}) = R(t,r) + \frac{2}{\gamma+1}\sum_{nm} S_{1nm}(t)x(z)x_{1nm}(z)z^{\lambda_{nm}}Y_{nm}(\theta,\varphi),$$

$$\tilde{P}(t,\bar{r}) = P(t,r) + \frac{2}{\gamma+1}\rho_0(r)\frac{\dot{S}^2(t)}{S}\sum_{nm} S_{1nm}(t)p_{1nm}(z)z^{\lambda_{nm}+S+k}Y_{nm}(\theta,\varphi),$$

$$\tilde{\rho}(t,\bar{r}) = \rho(t,r) + \frac{\gamma+1}{\gamma-1}\rho_0(r)\frac{1}{S}\sum_{nm} S_{1nm}(t)\rho_{1nm}(z)z^{\lambda_{nm}}Y_{nm}(\theta,\varphi), \qquad (24)$$

$$\Theta(t,\bar{r}) = \theta + \frac{2}{\gamma+1}\frac{1}{S}\sum_{nm} S_{1nm}(t)f_{1nm}(z)z^{\lambda_{nm}}\frac{\partial Y(\theta,\varphi)}{\partial\cos\theta},$$

$$\Phi(t,\bar{r}) = \varphi + \frac{2}{\gamma+1}\frac{1}{S}\sum_{nm} S_{1nm}(t)f_{1nm}(z)z^{\lambda_{nm}}\frac{\partial Y(\theta,\varphi)}{\partial\varphi};$$

1.2 Formulating an eigenvalue problem.

At first, using boundary conditions we integrate adiabatic equation (15) . We get (here and after we omit indices *n,m* of p_1, r_1, x_1, f_1, *l*, S_1):

$$\frac{P_1}{P} - \gamma\frac{\rho_1}{\rho} = (2\lambda+s+2)z^{\lambda}\frac{S_1}{S} \qquad (25)$$

After that using the assumption of the self-similarity (24) we convert the system of equations (16), (17), (18), (25) into the system of ordinary differential equations. We write it having excluded ρ_1 :

$$bp_1 + (\frac{zd}{dz} + \lambda + sw)x_1 - n(n+s-1)wf_1 = \frac{\gamma+1}{2\gamma}(2\lambda+s+k+2)w, \qquad (26)$$

$$wp_1 + ax_1 - (\frac{zd}{dz} + 2w + \frac{s+k-2}{2})\frac{zd}{dz}f_1 = 0, \qquad (27)$$

$$[\frac{zd}{dz} + \lambda + s + k - (s+k-2)w]p_1 + \left[(\frac{zd}{dz} + 2w + \frac{s+k-2}{2})\frac{zd}{dz} - (s+k-2)a\right]x_1 + \qquad 28)$$
$$+ n(n+s-1)af_1 = 0;$$

With boundary conditions as follows:

- On the right edge *z=1* (shock front):

$$f_1(1) = 0,$$
$$f_1'(1) = 1,$$
$$x_1(1) = 1, \qquad (29)$$
$$p_1(1) = 2\lambda + \frac{6\gamma+2-s(3\gamma-1)+k(\gamma+1)}{(\gamma+1)^2};$$

- On the left edge *z=0* (at the center):

$p_1(1)=0$; (30)

The system of equations Eqs.(26-28) is of the forth order one, so four edge conditions Eqs.(29) construct the complete set of boundary conditions necessary for solution. The fifth edge condition Eq.(30) at *z=0* converts the problem of solving the Eqs.(26-29) to an eigenvalue problem. Solving this problem includes calculation of the complex exponent λ as eigenvalue. Coefficients in (26)-(30) are independent of index *m*, so $\lambda \equiv \lambda(n)$.

1.3 Numerical solution results

In general the case $\gamma > 1$ the eigenvalue problem Eqs.(26-30) was solved numerically. The values of the complex exponents λ_n were calculated in the wide region of γ , and *n* for spherical

(s=3) and cylindrical (s=2) blast waves. The values of k were k=0,-1,-2. The results are presented on Figs.1,2.

The instability region on the plane n - γ is shown on Fig.3. Blast wave is unstable in the right lower corner of the chart. If the function $\gamma_0(n)$ corresponds to the boundary line of the instability region then the critical value of specific ratio γ_c can be defined:

1.

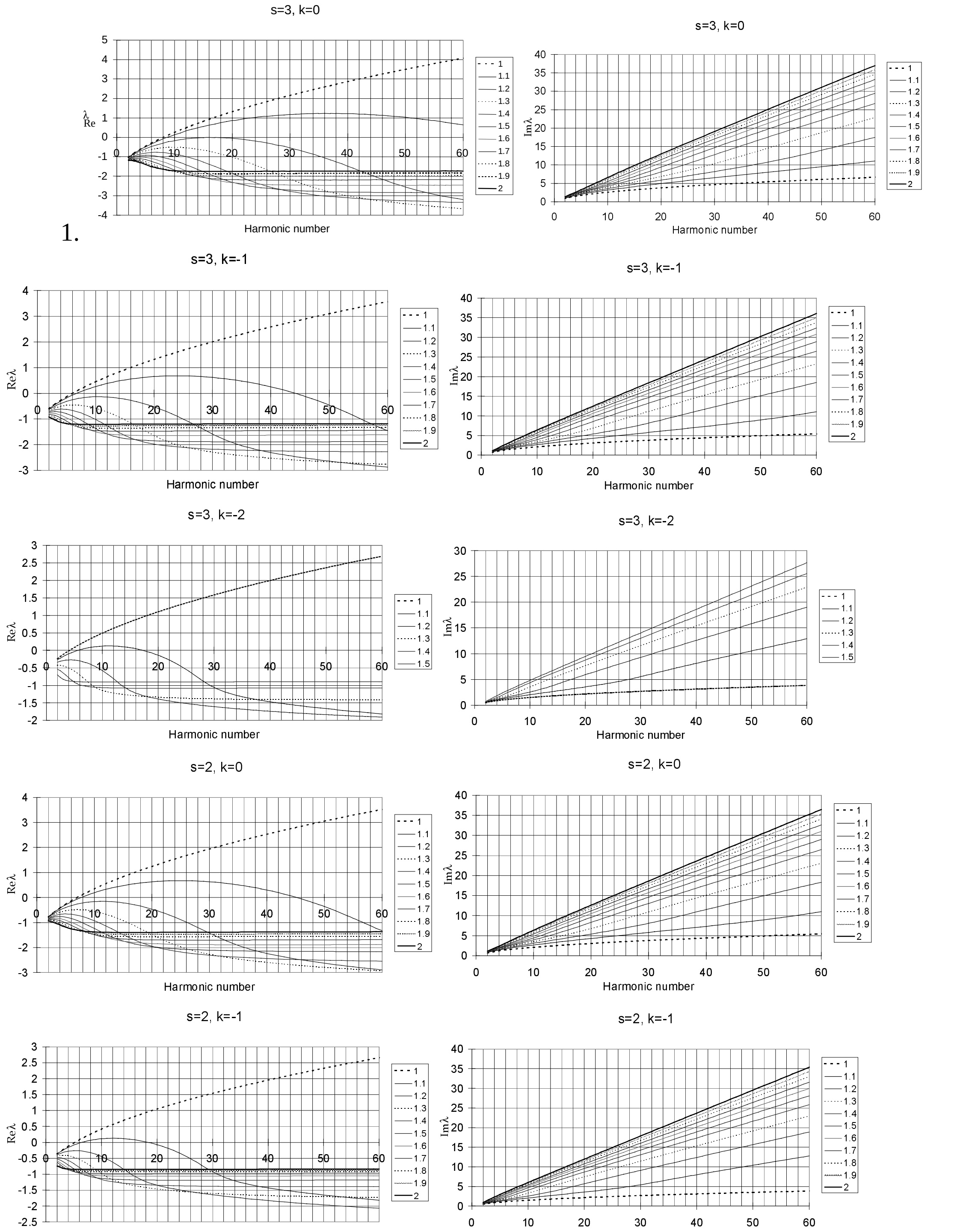


Fig.1 Values of complex exponent λ in the cases of spherical *(s=3)* and cylindrical *(s=2)* blast wave in a gas which initial density ρ_0 is power function of radius: $\rho_0 \sim r^k$ for a number of values of gas specific ratio γ. For the case γ=1 values are calculated using analytic expressions Eq.(36).

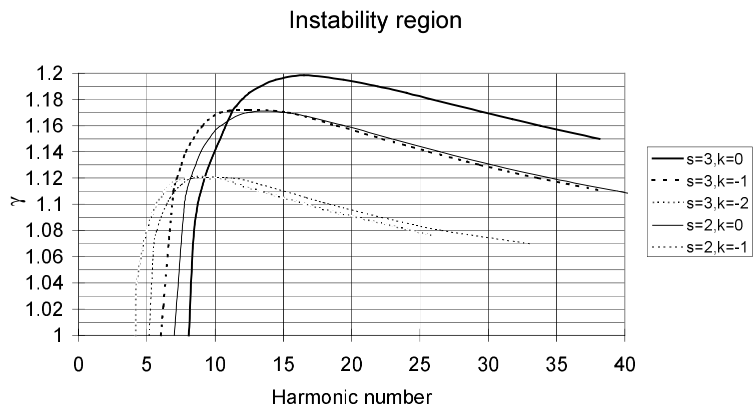


Fig.2 Instability region on the plane n-γ

1.4 Analytical solution results

Let us consider some cases when eigenvalue problem has an analytical solution .

1.4.1 Thin shell approximation: γ -1 << 1

The problem has an analytic solution in the case when the value of *g* is close to unit. In this case we let in the equations (26)-(30): $\gamma = 1$, $w = 0$, $a=-(s+k)/2$, $b=0$.

We get:

$$(\frac{zd}{dz}+\lambda)x_1 = 0, \tag{31}$$

$$-\frac{s+k}{2}x_1 + (\frac{zd}{dz}+\frac{s+k-2}{2})\frac{zd}{dz}f_1 = 0, \tag{32}$$

$$\begin{gathered}(\frac{zd}{dz}+\lambda+s+k)p_1 + ((\frac{zd}{dz}+\frac{s+k-2}{2})\frac{zd}{dz}+(s+k-2)\frac{s+k}{2})x_1 - \\ -n(n+s-1)\frac{s+k}{2}f_1 = 0\end{gathered} \tag{33}$$

with boundary conditions:

$$\begin{gathered}f_1(1) = 0, \\ f_1'(1) = 1, \\ x_1(1) = 1, \\ p_1(1) = 2\lambda + \frac{4-s+k}{2}, \\ (p_1 z^{s+k})(z \to 0) = 0\end{gathered} \tag{34}$$

Solution of (31)-(34) is as follows:

$$x_1 = z^{-\lambda},$$

$$f_1 = \frac{s+k}{2\lambda(\lambda - \frac{s+k-2}{2})} z^{-\lambda} - \frac{2}{s=k-2}\frac{\lambda+1}{\lambda - \frac{s+k-2}{2}} z^{-\frac{s+k-2}{2}} + \frac{2}{s=k-2}\frac{\lambda + \frac{s+k}{2}}{\lambda},$$

$$p_1 = [\frac{\lambda(\lambda - \frac{s+k-2}{2}) - \frac{(s+k-2)(s+k)}{2}}{s+k} + \frac{n(n+s-1)(s+k)}{2\lambda(\lambda - \frac{s+k-2}{2})}] z^{-\lambda} - \tag{35}$$

$$- \frac{(s+k)(\lambda+1)n(n+s-1)}{(\lambda - \frac{s+k-2}{2})(\lambda + \frac{s+k+2}{2})} z^{-\frac{s+k-2}{2}} + \frac{(s+k)(\lambda + \frac{s+k}{2})n(n+s-1)}{\lambda(\lambda - +s+k)},$$

-and after simple calculations we get the equation for λ (note that it is possible to get this equation in a simpler way applying conservation laws for calculating thin layer motion[1]):

$$(\lambda+s+k)^2(\lambda + \frac{s+k+2}{2})^2 - \frac{s(s+k)}{2}(\lambda+s+k)(\lambda+\frac{s+k+2}{2}) + \\ + n(n+s-1)\frac{(s+k)^2}{4} = 0; \tag{36}$$

One of two pairs of the complexly conjugated roots of this equation with the greater real part is the solution of the problem in the case *n>1*.

In the case *n=1* the equation (34) has four real negative roots. The two of them satisfying boundary condition at the center are solutions:

One of these solutions is determined with formula:

$$\lambda = -\frac{s+k}{2}, \tag{37}$$

this case corresponds to explosion direct motion with momentum conservation. This value of l can be calculated from dimension consideration.

The other is the greater root of equation:

$$\lambda^2 + \lambda\frac{3s+3k+2}{2} + \frac{(s+k)(k+3)}{2} = 0 \tag{38}$$

it is equal to *-1 if k=0*; this case corresponds to the blast being displaced.

.We can continue analytical calculations if we take into account next terms of expansion of solution of eigenvalue problem (26)-(30) by powers of the small parameter *(g-1)*:

$$\lambda=\lambda^{(0)}+(\gamma-1)\,\lambda^{(1)}, \tag{39}$$

where $\lambda^{(0)}$ is determined by (36)

In this case we get for the first approximation terms the equation system with constant coefficients analogous to (31) but with non-uniform equations. The right parts of these equations are of the first order of smallness. This equation system is integrated quite similar to that was done for (31,32), and (omitting cumbersome transformations) we arrive to the following formula[2,7] for the first order correction of *l* (to avoid bulky expressions here we present only the special case *s=3, k=0)* ; here *b(a)* stands for integral $\beta(\alpha) = \int_0^1 \frac{t^\alpha dt}{1+t}$, and *l* is from (36)):

$$\lambda^{(1)} = \frac{1}{3}\frac{(\lambda+1)(\lambda+\frac{3}{2})(\lambda+\frac{5}{2})(\lambda+3)(\lambda+4)}{(\lambda+\frac{11}{4})(\lambda^2+\frac{11}{2}\lambda+\frac{21}{4})}\{(1-2\ln 2\frac{(\lambda+\frac{3}{2})(\lambda+\frac{5}{2})}{\lambda(\lambda-\frac{1}{2})} - -\frac{4}{3}(\lambda+\frac{5}{2})[\frac{(\lambda^2+6\lambda+\frac{9}{2})}{\lambda}\beta(\frac{\lambda+3}{3}) - \frac{(\lambda^2+\frac{11}{2}\lambda+3)}{\lambda-\frac{1}{2}}\beta(\frac{\lambda+\frac{5}{2}}{3})]\} \quad (40)$$

The results of (39),(40) are in a good agreement with the results of numerical calculation for the imaginary parts of l and in the worse one for the real parts. For example, in the case: *s=3, k=0, n=8,* γ*=1.2,* we have[2]: $\lambda_{analyt} = 0.15 + 3.14i$, and $\lambda_{numer} = -0.30 + 3.07i$.

1.4.2 The first harmonic: *n=1*

Note that both solutions (35,38): $\lambda = -1$, and $\lambda = -(s+k)/2$, are independent of γ, and they are solutions of eigenvalue problem (26)-(30) in the general case of an arbitrary $\gamma > 1$*:*

1) The displacement solution. λ=-1

We can verify it by a direct substitution of the following expressions to the system of equations (26)-(28):

$n = 1, k = 0, \lambda = -1,$

$$f_1 = \frac{\gamma+1}{2}(\frac{z}{x}-1)$$
$$x_1 = \frac{\gamma+1}{2}(\frac{z}{x}-w) \quad (42)$$
$$p_1 = \frac{\gamma+1}{2}a$$

So λ*=-1* is the correct solution for the case: *n=1, k=0*

2) $n = 1, \lambda = -\frac{s+k}{2}$,

We subtract equation (27) from equation (28) multiplied by *(s-1)*, and integrate the result. Then we get:

$$p_1 + (\frac{zd}{dz} + w + \frac{s+k}{2})x_1 + (\frac{zd}{dz} + w + \frac{s+k}{2})f_1 = 0 \quad (41)$$

This equation together with Egs.(25,27) forms equation system. It can easily be shown that all solutions of this system have the required convergence at the center independently of shock front boundary conditions. So $\lambda = -\frac{s+k}{2}$ ' is the correct solution of the eigenvalue problem for *n=1*.

1.4.3 Investigation of the features of the solution in the limit *n*$\to\infty$.

The next case of analytical approach to solving the eigenvalue problem is a short wavelength approximation: *n>>1*. In this case we seek for the solution, which looks like:

$\exp(\int\frac{q(z)dz}{z})$, where $|q| \sim |l| \sim n >> 1$. (43)

We substitute so defined values to Eqs. (26-28), remove all terms of the lowest order, and get the following equation system:

$$bp_1 + (q+\lambda)x_1 - n^2 w f_1 = \frac{\gamma+1}{\gamma}\lambda w,$$
$$wp_1 + q^2 f_1 = 0, \tag{44}$$
$$(q+\lambda)p_1 + q^2 x_1 = 0,$$

and boundary conditions:

$$x_1(z=1) = 1,$$
$$f_1(z=1) = 0,$$
$$\frac{zd}{dz} f_1(z=1) = 1, \tag{45}$$
$$p_1(z=1) = 2\lambda,$$
$$\mathrm{Re}\,al(q) > 0,$$

Solution of (44),(45) is as follows:

$$x_1(z) = \frac{\gamma^2-1}{2\gamma^2}(\exp(\int_1 \frac{q(z)dz}{z}) - 1) - \frac{\lambda}{\gamma}\ln z + 1,$$
$$f_1(z) = -\frac{\gamma^2-1}{2\gamma^2\lambda}(\exp(\int_1 \frac{q(z)dz}{z}) - 1) - \frac{1}{\gamma}\ln z, \tag{46}$$
$$p_1(z) = 2\lambda\exp(\int_1 \frac{q(z)dz}{z})$$

where

$$\lambda = in\sqrt{\frac{\gamma-1}{\gamma+1}}, \tag{47}$$

and $q(z)$ is the root with a positive real part of the following equation:

$$q^2(1-b) + 2\lambda q + \lambda^2 - n^2 w^2 = 0, \tag{48}$$

Data of Fig.1 show that formula Eq.(47) is in a reasonable agreement with numerical results. We should note also that Eq.(47) has the right limit of $n \to \infty$, it is in an agreement with the corresponding formulas describing perturbation oscillations in plane shock waves.[10]

2. Evolution of shock front radius small perturbations in the case when the gas density before shock front has small perturbation.

We considered above free oscillations of blast wave . We can develop an analogy between the blast wave perturbations and other oscillating systems and consider the perturbations generated by perpetually operating external sources. One of these cases is when perturbations of density before shock front $r_{o1} \sim r^{l+1}$ take place. This problem was considered earlier in Ref. 2. In this case hydrodynamic equation system for perturbations coincides with (16),(17),(18),(22) except for continuity equation which has a non-zero right part:

$$\frac{\rho_1}{\rho} + \Delta_{\theta\varphi}F_1 + \frac{\frac{\partial(R^{s-1}R_1)}{r^{s-1}\partial r}}{\frac{R^{s-1}\partial R}{r^{s-1}\partial r}} = \frac{\rho_{01}(r)}{\rho_0(r)} \tag{49}$$

and for boundary conditions:

$$P_1(r=S,t) = \frac{2\rho_0}{\gamma+1}\frac{\dot{S}^2}{S}S_1(2\lambda+s+2+a(1))$$
$$\rho(r=S,t) = \frac{(\gamma+1)\rho_0}{\gamma+1}\frac{S_1}{S}a(1) \tag{50}$$

Using that we get the self-similar equation system and boundary conditions analogous to Eqs. (26)-(30). The difference between eigenvalue problem in this case and stability problem considered above is that the role of eigenvalues is played by the ratio of shock front radius perturbation to initial density perturbation d:

$$d = d(\lambda, n) = \frac{\frac{S_1}{S}(r)}{\frac{\rho_{10}}{\rho}(r)}; \qquad (51)$$

The eigenvalue problem is linear of 1/d. The system of equations is the following::

$$\begin{aligned}
&bp_1 + (\frac{zd}{dz} + \lambda + sw)x_1 - n(n+s-1)wf_1 = \frac{\gamma+1}{2\gamma}(2\lambda + s + k + 2 + \frac{1}{d})w, \\
&wp_1 + ax_1 - (\frac{zd}{dz} + 2w + \frac{s+k-2}{2})\frac{zd}{dz} f_1 = 0, \\
&(\frac{zd}{dz} + \lambda + s + k - (s+k-2)w)p_1 + ((\frac{zd}{dz} + 2w + \frac{s+k-2}{2})\frac{zd}{dz} - (s+k-1))ax_1 + \\
&+ n(n+s-1)af_1 = -\frac{\gamma+1}{2}\frac{a}{d};
\end{aligned} \qquad (52)$$

Boundary conditions are the same as in (29,30), except:

$$p_1(1) = 2\lambda + \frac{6\gamma + 2 - s(3\gamma-1) + k(\gamma+1)}{(\gamma+1)^2} + \frac{1}{d};$$

2.1 The forced oscillations of the shock front radius.

Earlier the case of $\lambda = 0$ was considered in Ref. 2.

In this paper we consider pure imaginary values of l: $l=iw$. The problem is solved numerically. The calculated values of $|d(w,n)|$ are presented on Fig.3. The sharp resonance pattern (in the vicinity of $\omega = \sqrt{\frac{\gamma-1}{\gamma+1}}n$) is demonstrated for $\gamma=1.2$ and the smeared one is demonstrated for $\gamma=5/3$.

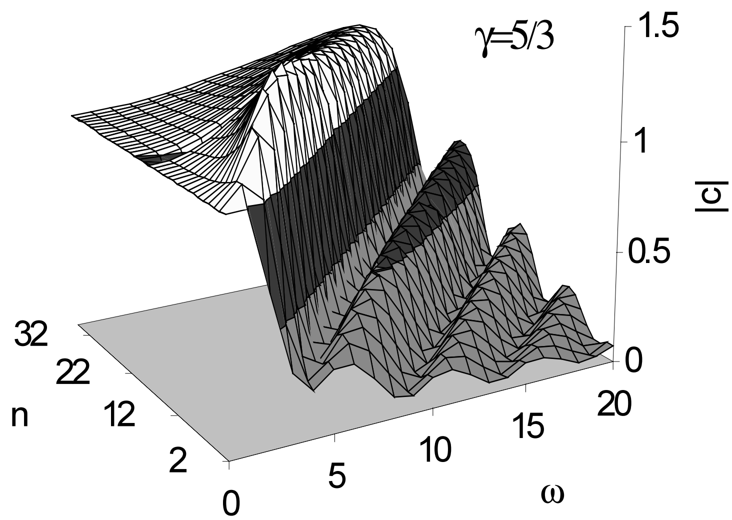


Fig.4a. Resonance patterns $|c(\omega,n)|$ for γ=1.667. Results are normalized to $q(0,n)$: $c(\omega,n) = \left|\frac{q(\omega,n)}{q(0,n)}\right|$. (Note that $q(0,n) \cong \frac{1}{n}\sqrt{\frac{\gamma+1}{2\gamma}}$ for $n>>1$)

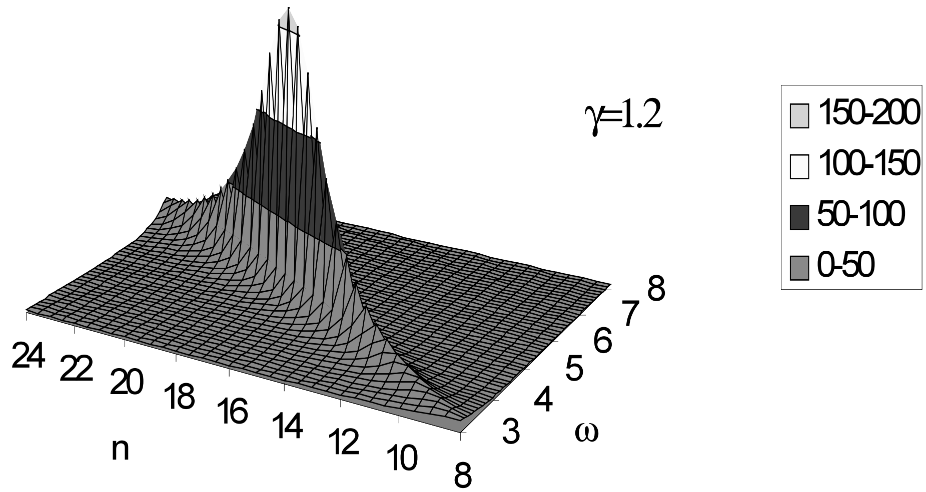


Fig.4b. Resonance patterns $|c(\omega,n)|$ for γ=1.2. Results are normalized to $q(0,n)$:
$c(\omega,n)=\left|\dfrac{q(\omega,n)}{q(0,n)}\right|$.

Analytical solution can be obtained for *n>>1, k=0.*

We again seek solution of (43) type, reduce equation system (52), and get finally the following solution:

$$q = nw\sqrt{\frac{1}{1-b}},\qquad d = -\frac{1}{n}\sqrt{\frac{\gamma+1}{2\gamma}} \tag{53}$$

This result is in a good agreement with the well-known formulas for front perturbations in a stationary shock wave (Ref.11).

3. Stability of the blast wave in a non-ideal gas.

For the sake of convenience of comparison with an experiment we however need a solution of the stability problem for the case of non-ideal gas. In actual practice gas adiabatic exponent is a function of density and temperature: it's getting lower while degree of ionization is increasing, that is while density is lowing or temperature is rising. In Ref.12 the authors considered the stability problem for one of the cases of blast wave in non-ideal gas, namely, the only case when the blast wave in the non-ideal gas is self-similar: we supposed that the gas adiabatic exponent γ was a function of the gas density (i.e. γ was independent from the gas energy). In this case we can use the self-similar technique for solving the blast wave stability problem.

In Ref.13 the authors supposed that the gas equation of state (EOS) was as follows:

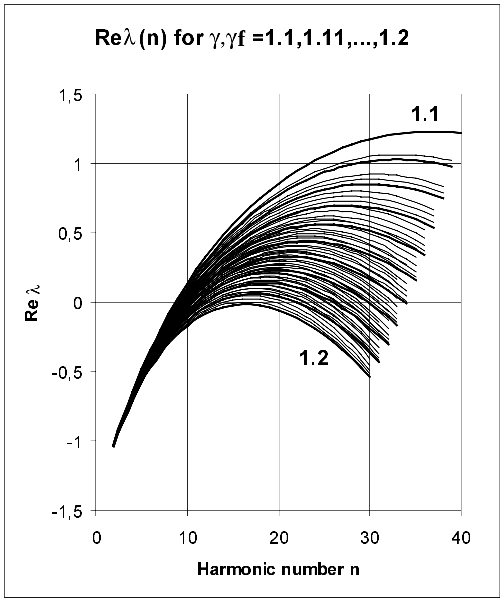
$$P=(\gamma^*(\rho)-1)\rho E \tag{54}$$

where coefficient γ* was a two-parameter function of ρ:

$$\gamma^*(\rho)-1=\frac{\gamma_c-1}{1-\dfrac{\gamma_f-\gamma_c}{\gamma_f+1}\rho} \tag{55}$$

Here parameters: γ_c,γ_f stand for values of the gas adiabatic exponent in the blast center and at the shock front.

The eigenvalue problem in this case is the same (See Ref.13) as in Eqs.(26-31). Fig.4 presents results of numerical solution of the eigenvalue problem.



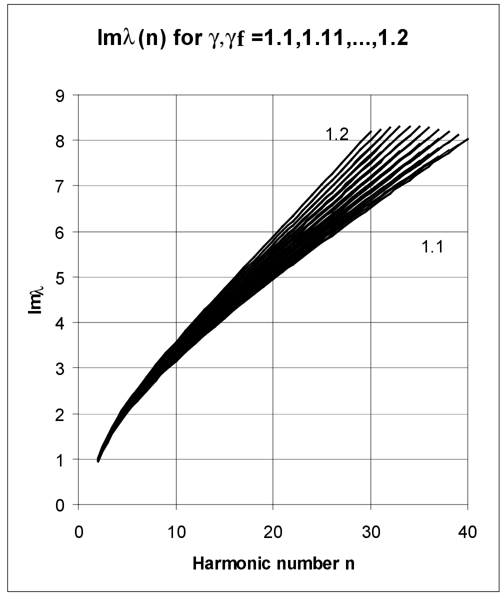


Fig.4 The components of complex eigenvalue λ for some sets of values of parameters of gas EOS. Heavy curves correspond to the cases of ideal gas with $\gamma_f=\gamma_c=\gamma=1.10,1.11,1.12,\ldots 1.20$. the other lines correspond to the cases when $\gamma_f>\gamma_c$.

Conclusion

Blast wave perturbation evolution was considered. The solutions obtained have applications in laboratory physics and astrophysics; they also can serve the tests for elaboration of the 2D and 3D hydrodynamic codes.

Stability of some cases of spherical or cylindrical blast wave was considered using self-similar approach in the unified manner. Namely, stability of the cases in which gas density before shock front was a power function of radius (with power exponent being equal to k) was considered. Blast wave small perturbations were expanded in spherical harmonics. The perturbation value was supposed to be a power function of time (with complex exponent), stability problem was reduced to eigenvalue problem, and the increments of perturbation growth and periods of oscillation were calculated as the eigenvalues for each harmonic number. The calculations were made numerically both for spherical and cylindrical blasts for a large set of blast wave parameters: gas specific heat ratio γ, harmonic number n, value of k. The instability region on the plane n - γ was determined. The critical value of specific heat ratio γ_c was calculated for each case considered.

The evolution of perturbations generated by perpetually operating external sources was considered too. Namely, there were considered the forced oscillations of shock front radius perturbations caused by a presence of spatially oscillating perturbations of initial gas density. The eigenvalue problem in this case was formulated using the self-similar technique.

4. The short wavelength approximation was used to obtain an analytical solution of eigenvalue problem. It was shown that the obtained solutions agreed with the corresponding solutions describing plane shock wave perturbations.

5. The analytical solutions of stability problem were also obtained in the case of values of g being close to unit. In this case expansion of the solution by powers of γ-1 was used. The terms of the zero order, and the first order of negligibility were calculated. It was shown that they were in a reasonable agreement with the numerical results.

6. The spherical (or cylindrical) blast wave stability was considered using the self-similar approach. Using this technique helped us to reduce rather complicated 3D hydrodynamic

problem of small perturbation evolution to the simpler eigenvalue problem. Blast wave should satisfy two conditions to be studied using this approach: it should be self-similar and 1D solution for this blast wave should be smooth at the center. A sound speed should tend to infinity, and a particle velocity should be negligible compared with the sound speed at the center.

The main assumptions of the self-similar approach are:

- Pattern of the perturbation evolution is a self-similar one.
- The value of pressure small perturbations tends to zero at the center.

In the spherical case one more assumption was made. It is:

- Pattern of an angular motion is a potential one.

It seems to us that all three these assumptions were proved in experiments (see Ref. 4 where perturbation oscillation period was measured and the period value proved to be in a good agreement with the self-similar theory results). But for the better proof of the third assumption we need direct computer simulation of blast waves using 3D hydrodynamic codes.

Acknowledgments

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