IWPCTM 2001

Three Dimensional Multi-Mode Rayleigh-Taylor and Richtmyer-Meshkov Instabilities at All Density Ratios

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Relevant Publications

Oron et al. PoP 2001.

Alon et al, PRL 1994, 1995.

Hecht et al. PoF 1994.

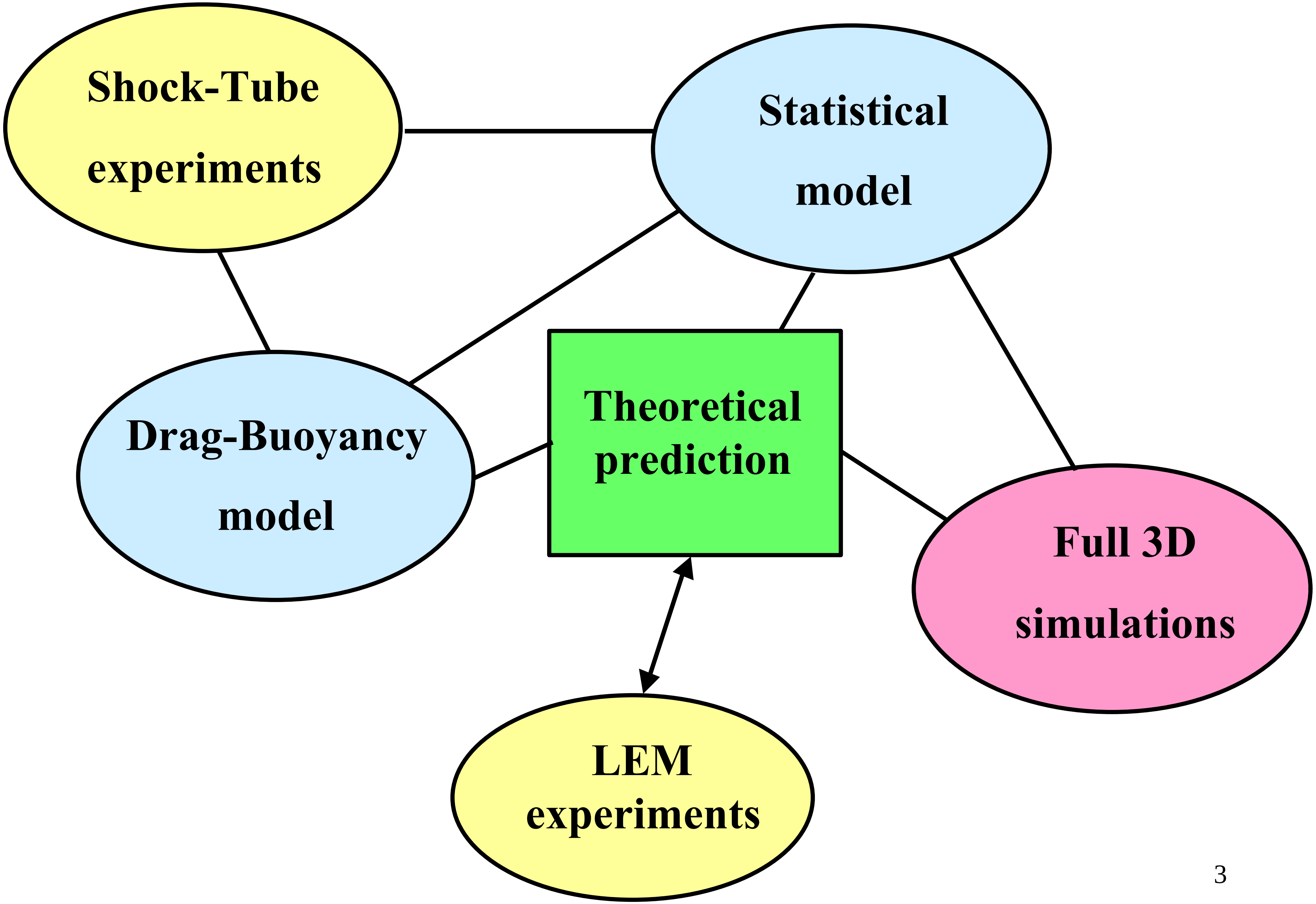
Oron et al. PoP 1998.

Rikanati et al. PRE 1998.

Sadot et al. PRL 1998.

Layzer APJ 1955.

Dimonte PoF, PoP 2000.


Shock-Tube
experiments
Statistical
model
Drag-Buoyancy
model
Theoretical
prediction
Full 3D
simulations
LEM
experiments

Outline

The Drag-Buoyancy Model for Bubbles

2D Statistical Model

Full 3D Numerical Simulations

3D Statistical Model

The Drag-Buoyancy Model for Spikes

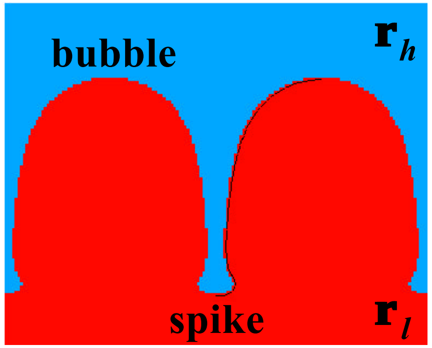
Conclusion

Single Mode Nonlinear Stage

$$(r_l \cdot V + C_a \cdot r_h \cdot V)\cdot \frac{du}{dt} = (r_h - r_l)V \cdot g \quad - \quad r_h \cdot S \cdot u^2$$

(inertia) + (added mass) = (buoyancy) (kinematic drag)

Where V/S $\propto \lambda$



$$(r_l + C_a r_h)\dot{u} = (r_h - r_l)\cdot g - \frac{C_d}{l} r_h \cdot u^2$$

Where C_a and C_d are geometric constants.

Single Mode   Asymptotic Velocities

$$(r_l + C_a r_h)\dot{u} = (r_h - r_l)\cdot g - \frac{C_d}{l} r_h \cdot u^2$$

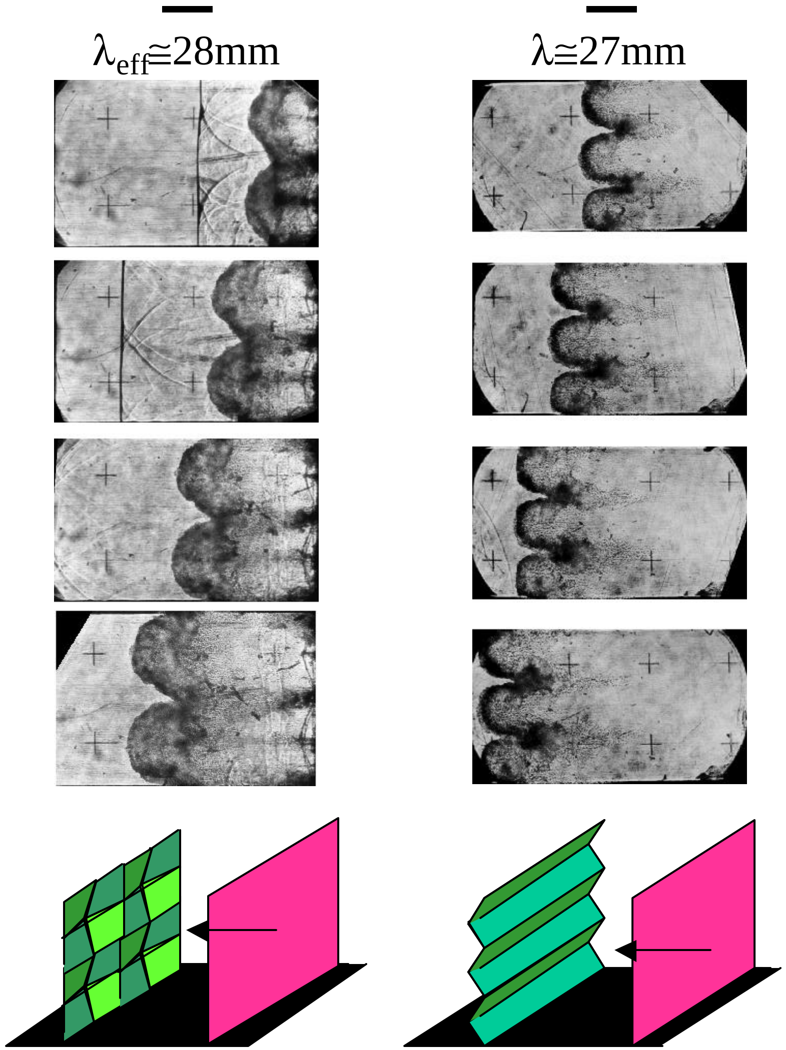
RT: $$u_B = \sqrt{\frac{1}{C_d}\left(\frac{2A}{1+A}\right) g\lambda}$$

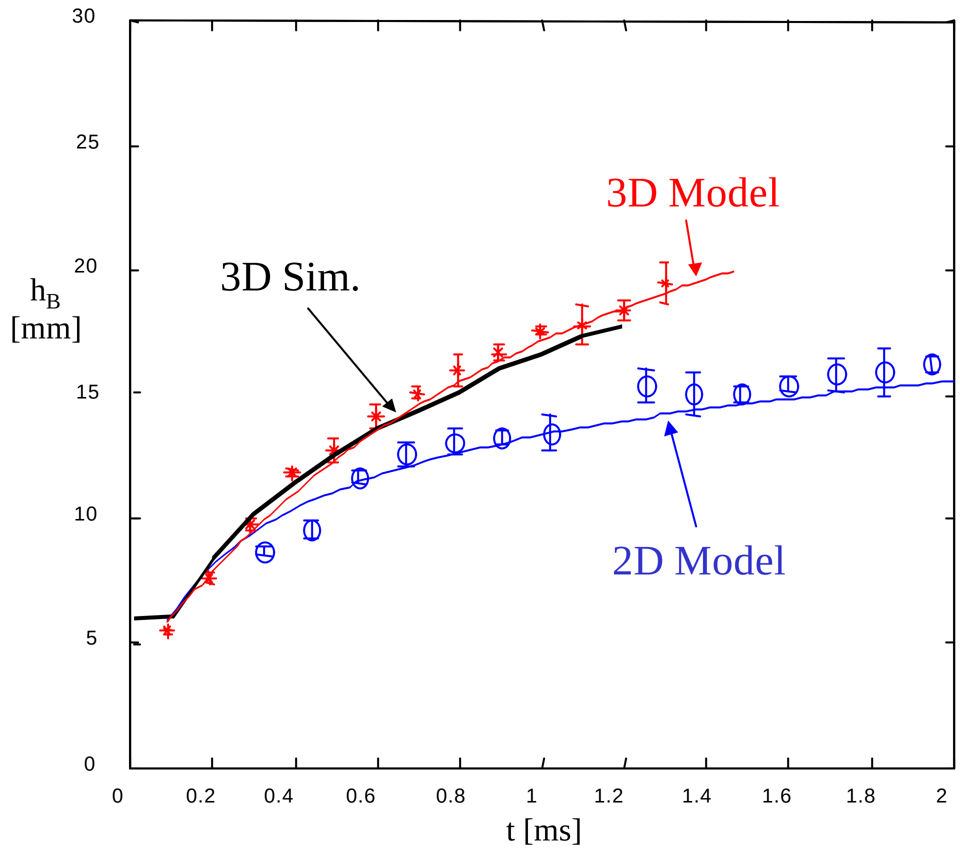
RM: $$u_B = \frac{1}{C_d}\cdot\left(\frac{1-A}{1+A} + C_a\right)\cdot\frac{\lambda}{t}$$

The geometric constants depend on the dimensionality:

$$C_a^{2D} = 2, \qquad C_d^{2D} = 6\pi$$
$$C_a^{3D} = 1, \qquad C_d^{3D} = 2\pi$$

RM Single Mode Experimental Results





Multi Mode   Drag Buoyancy Model

Generalization of the drag-buoyancy equation, using the *self-similarity* assumption:

$$h_B^{(MM)}(t) = b(A) \cdot \langle l \rangle (t)$$

RT: $$u = \dot{h} = \sqrt{c_1(A) \cdot g \cdot h / b(A)} = \alpha \cdot A \cdot g t^2$$

RM: $$u = \dot{h} = c_2(A) \cdot h / b(A) \cdot \frac{1}{t} = c \cdot t^{\theta}$$

$$\alpha = \frac{1}{2(1+A)} \frac{1}{C_d} \frac{1}{b^{RT}(A)}$$

$$\theta = \left(\frac{1-A}{1+A} + C_a \right) \cdot \frac{1}{C_d} \cdot \frac{1}{b^{RM}(A)}$$

3D Multi Mode the **a** **q** *b* Relations

Using the 3D coefficients in the expressions relating α, θ and b, and assuming that $b^{(RM)}=b^{(RT)}$, the differences between 2D and 3D bubble front growth are obtained:

	2D ($C_a=2$, $C_d=6\pi$)		**3D** ($C_a=1$, $C_d=2\pi$)	
assumed **a**	0.05		0.05	
q	0.1(A+3)	(=0.3-0.4)	0.2	
b	$\frac{1}{2(A+1)}$	(=0.25-0.5)	$\frac{3}{2(A+1)}$	(=0.75-1.5)

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2D Statistical Model

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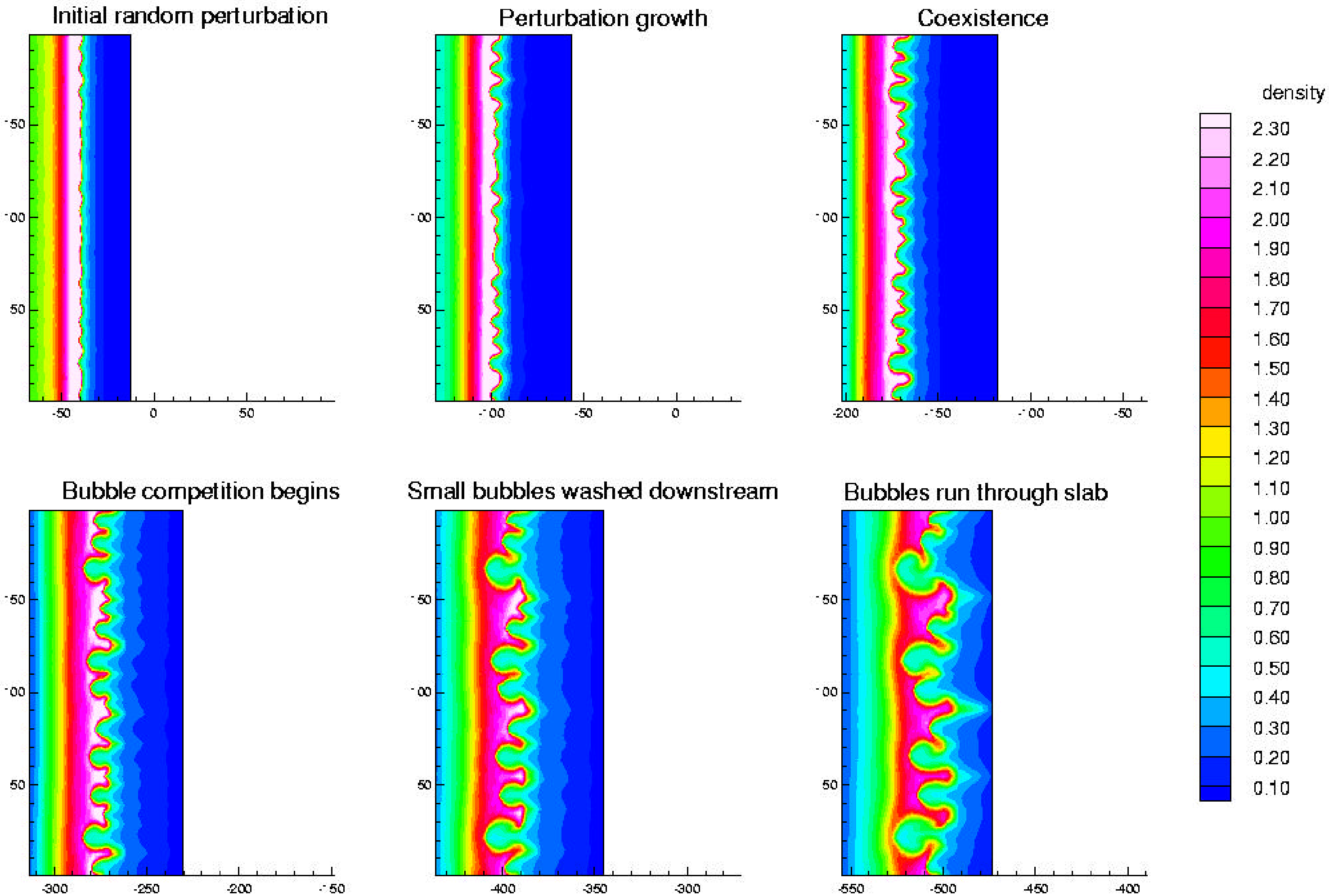
3D Statistical Model

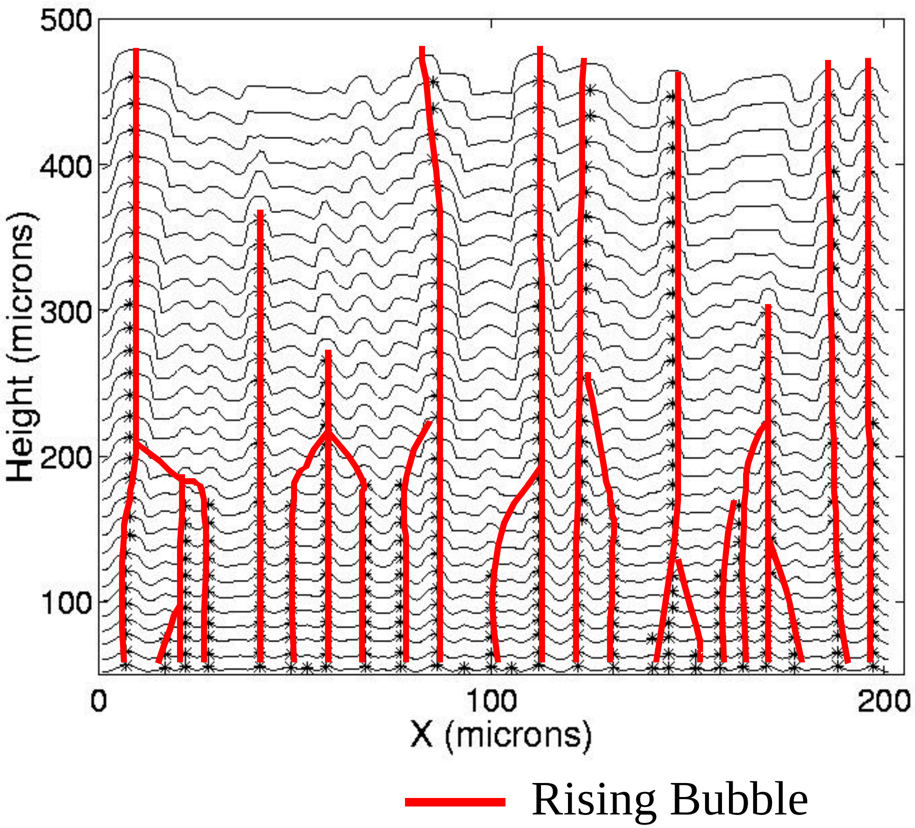
The Drag-Buoyancy Model for Spikes

Simulation of 2D Multimode Perturbation Evolution



Multi Mode 2D Statistical Model (Alon et. Al.)

Bubble Envelope (Simulation)

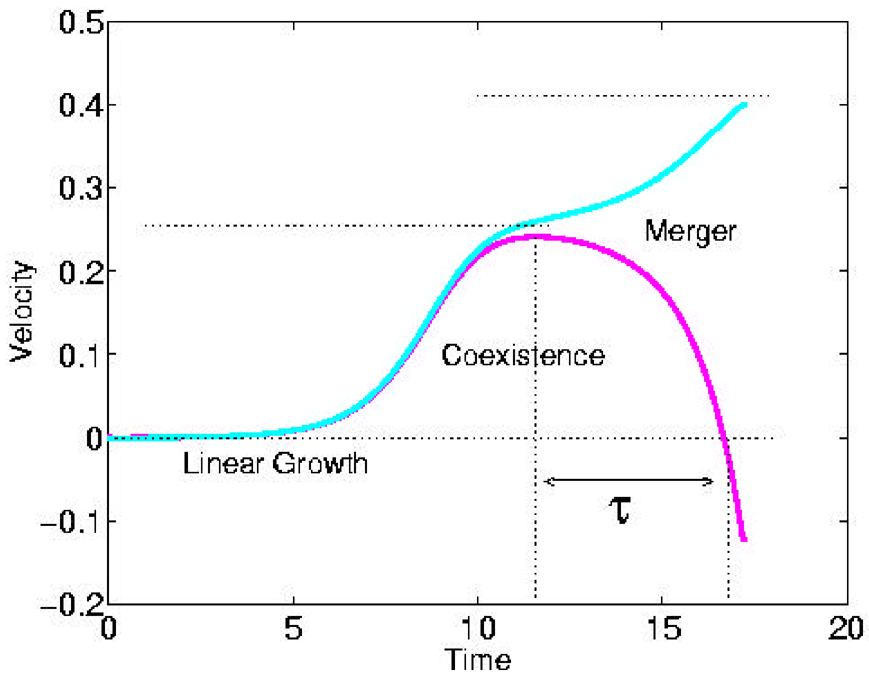


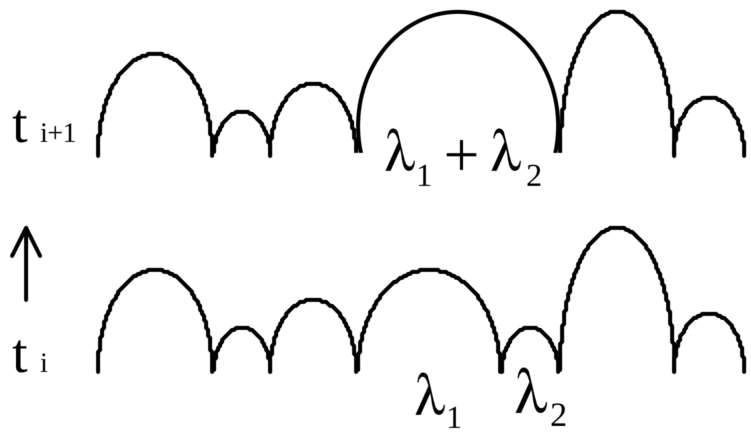
Each bubble grows with its asymptotic velocity, according to its wavelength:

$$u_i = u_{2D}^{(asy)}(l_i, t)$$

Multi Mode Bubble Merger

Velocity evolution of two non-identical adjacent bubbles:





Merger Rate:

$$(\, l_i ,\, l_{i+1})$$

$$\downarrow \quad w_{merge}(\, l_i ,\, l_{i+1}, t) \sim 1/t$$

$$(\, l_i + l_{i+1})$$

Multi Mode　2D Statistical Model

$$N(t)\cdot\frac{\partial g(\lambda,t)}{\partial t}=-2g(\lambda,t)\int_0^{\infty}g(\lambda',t)\omega(\lambda,\lambda')d\lambda' \qquad \text{death}$$

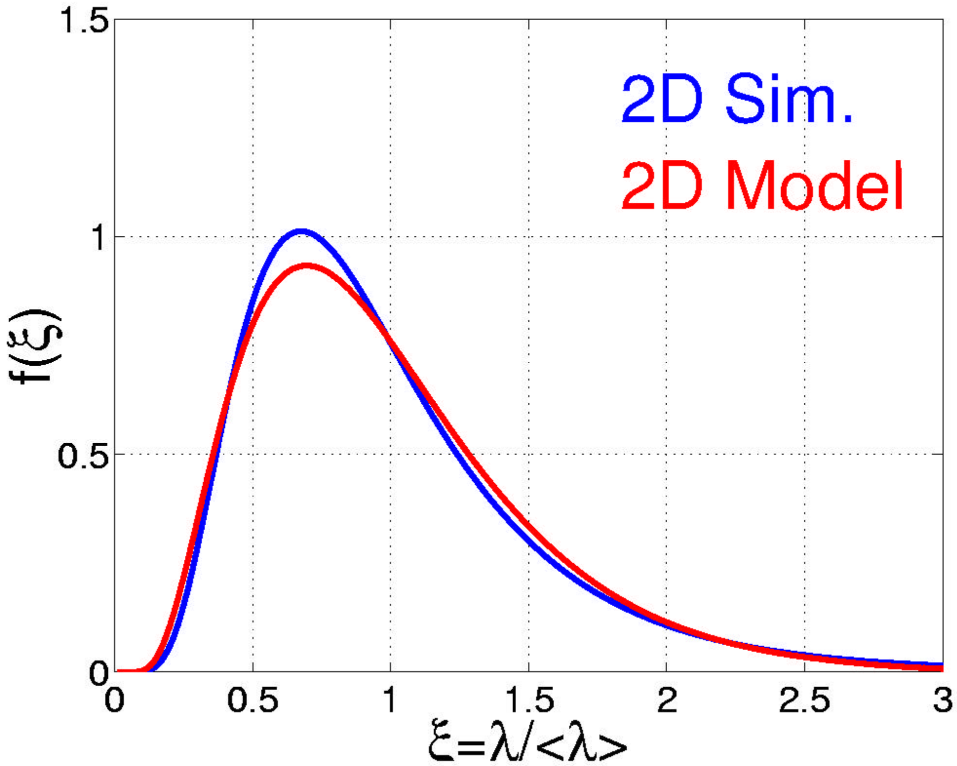
$$+\int_0^{\infty}g(\lambda-\lambda',t)g(\lambda',t)\omega(\lambda-\lambda',\lambda')d\lambda' \qquad \text{birth}$$

Where g(λ,t) is the number of bubbles with wavelength λ within interval dλ at time t, and N(t) is the total number of bubbles:

$$\frac{\partial N(t)}{\partial t}=-\langle \omega \rangle_g \cdot N(t)$$

Multi Mode 2D Statistical Model Results

1. The λ distribution reaches an asymptotic function:



RT simulation,
A=0.5

Multi Mode 2D Statistical Model Results

2. The average bubble and spike heights are obtained for both the RT and the RM case:

RT: $u_B^{asy} \propto \sqrt{g\lambda} \xrightarrow{\text{self-similarity}} \dot{h}_B \propto \sqrt{gh} \rightarrow h_B = \alpha_B \cdot gt^2$

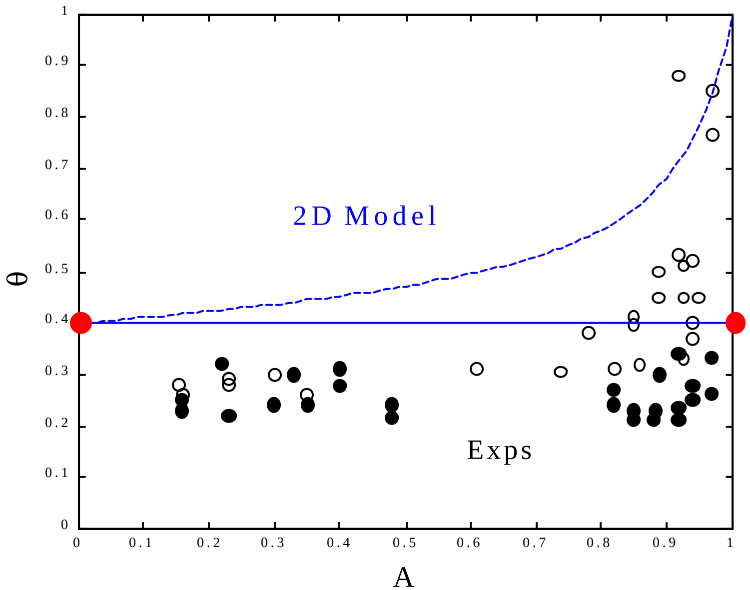
RM: $u_B^{asy} \propto \dfrac{\lambda}{t} \xrightarrow{\text{self-similarity}} \dot{h}_B \propto \dfrac{h_B}{t} \rightarrow h_B = c \cdot t^{\theta_B}$

<u>Self-similarity parameter:</u> $b^{RT} = b^{RM} = \dfrac{h_B}{\langle\lambda\rangle}$

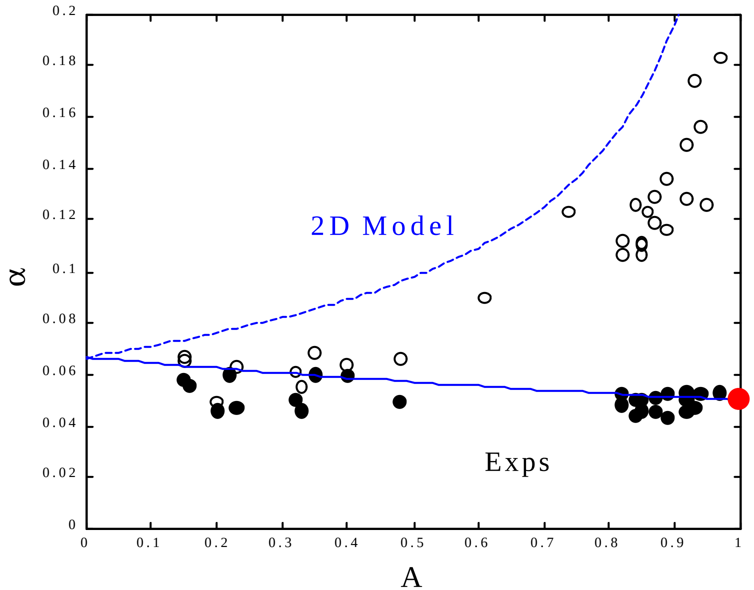
2D Statistical Model Results:

$$\alpha_B \cong 0.05 \qquad \theta_B \cong 0.4 \qquad b \cong 0.2$$

LEM Experimental Results



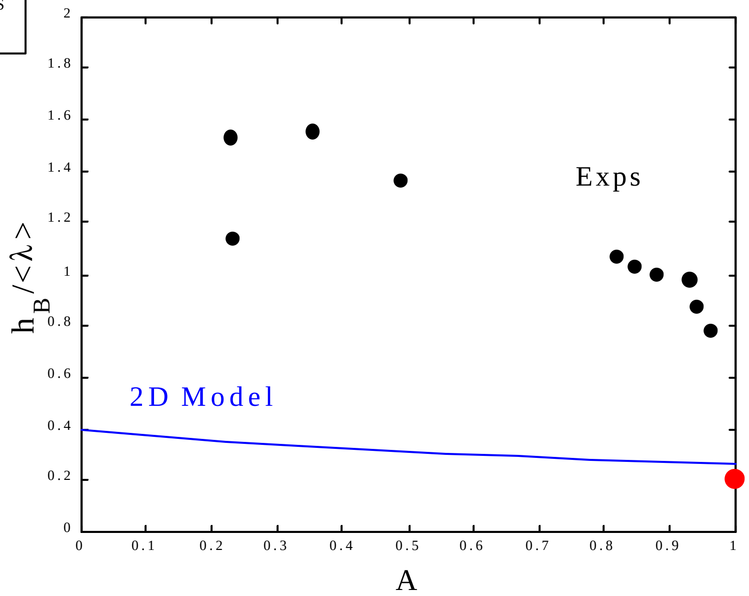
RM: $h_{B/S} = a_0 \cdot t^{\theta_{B/S}}$



RT: $h_{B/S} = \alpha_{B/S} \cdot Agt^2$

Drag-Buoyancy Model
Effective 1-Mode

2D Statistical Model



Outline

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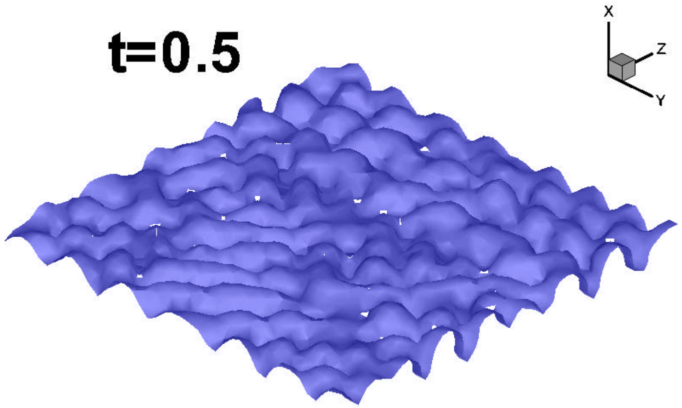
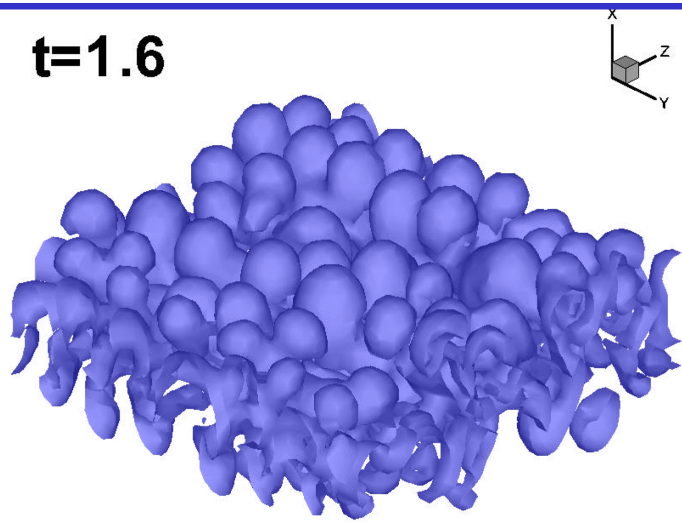
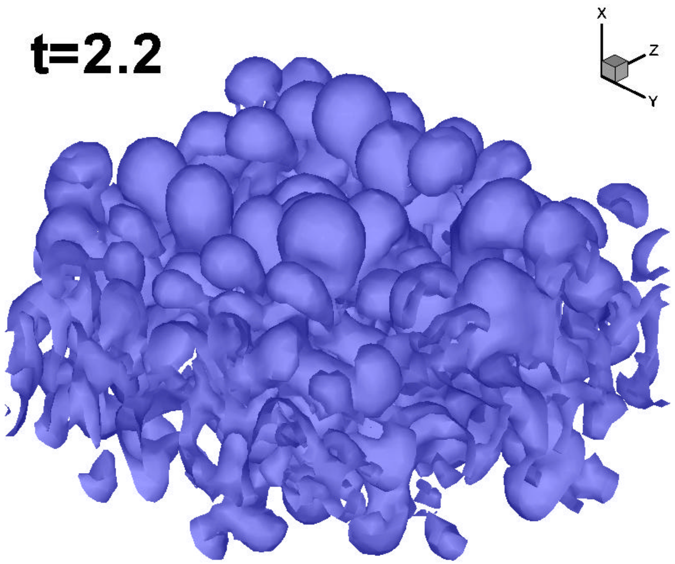
3D Statistical Model

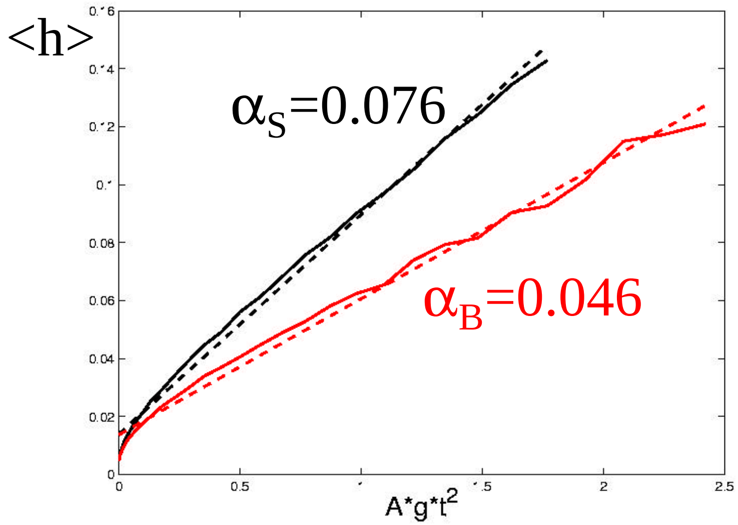
The Drag-Buoyancy Model for Spikes

Full 3D Numerical Simulation of the RT A=0.5 Case





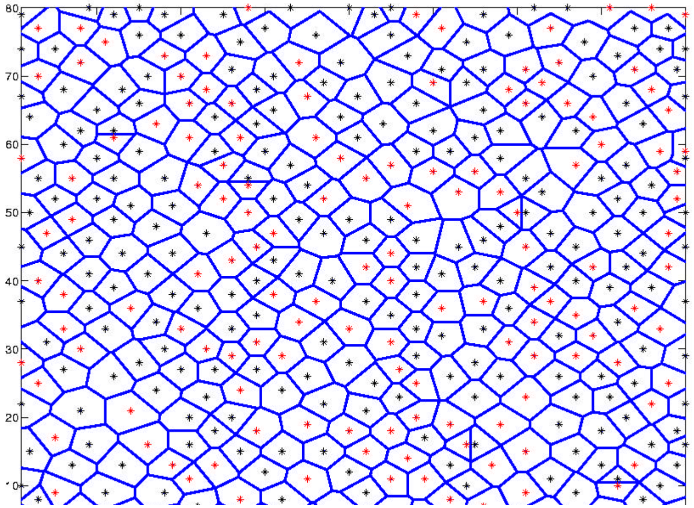




Voronoi Cell Structure of the Bubble Front Demonstrates the 3D Bubble Merger

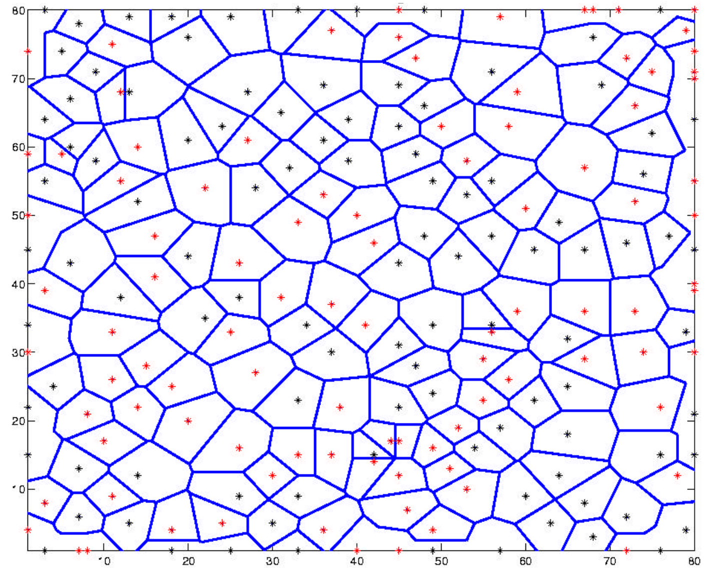
t=0.28

n=260



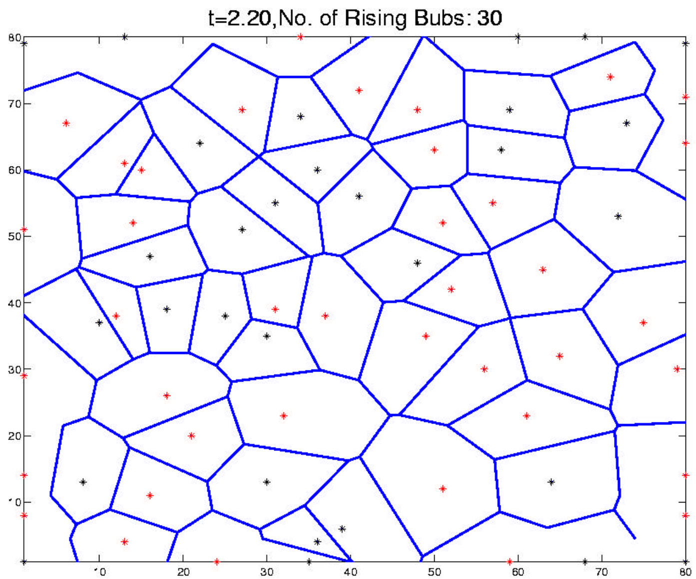
t=1.4

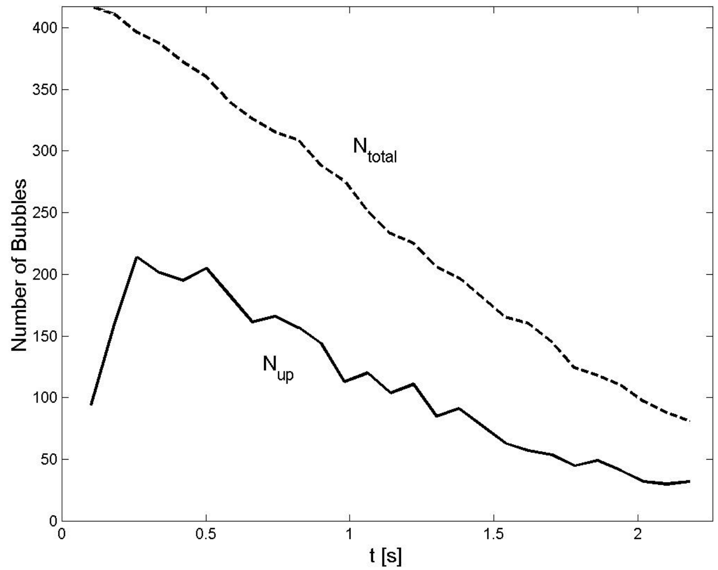
n=112



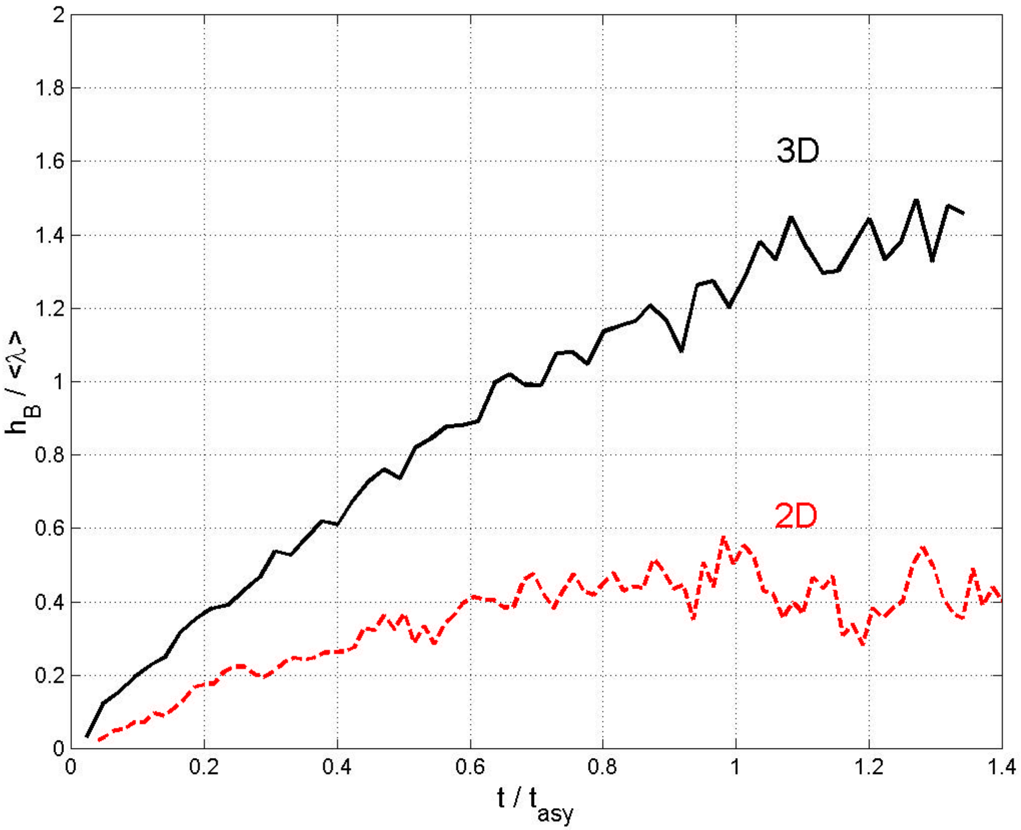
t=2.2

n=30

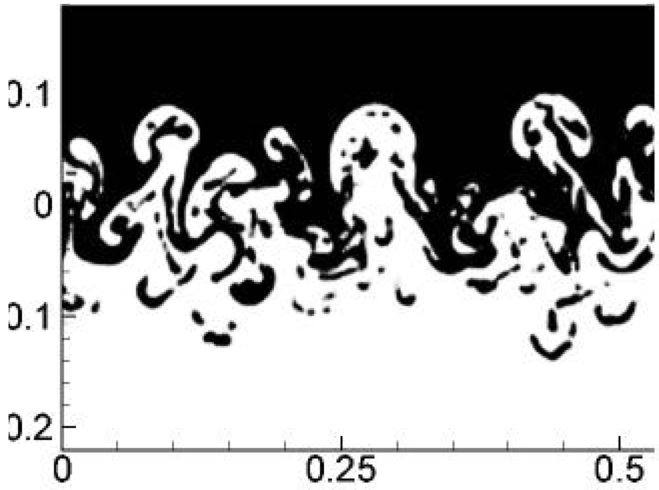




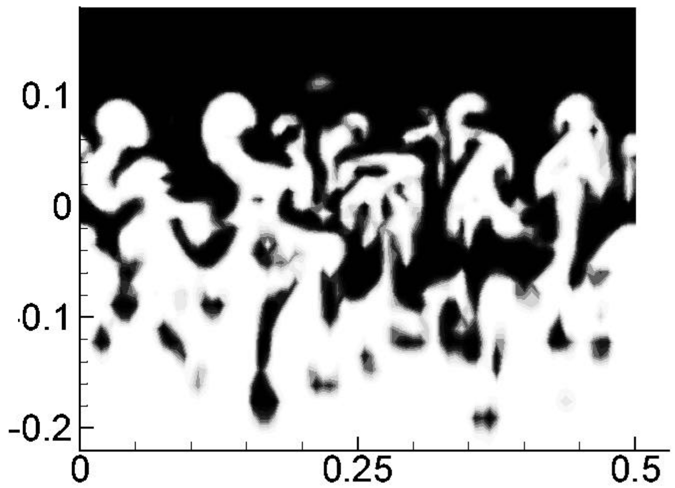
3D Simulation Results



2D Simulation



3D Simulation slice



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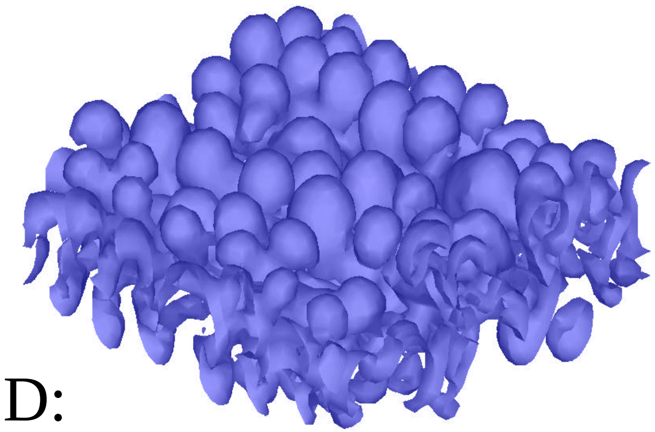
Multi Mode   3D Statistical Model

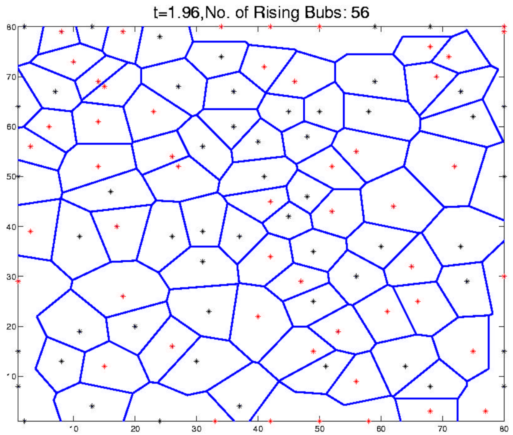
Dimensionality effects on the statistical model:

Asymptotic velocity of each bubble 1.5-2 times higher than in the 2D case:

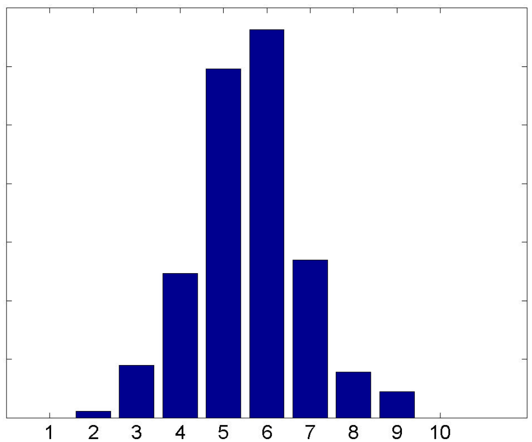
$u_{3D}=1.5\text{-}2u_{2D}$

Average number of neighbors per bubbles≈6 in 3D, rather than 2 in 2D:





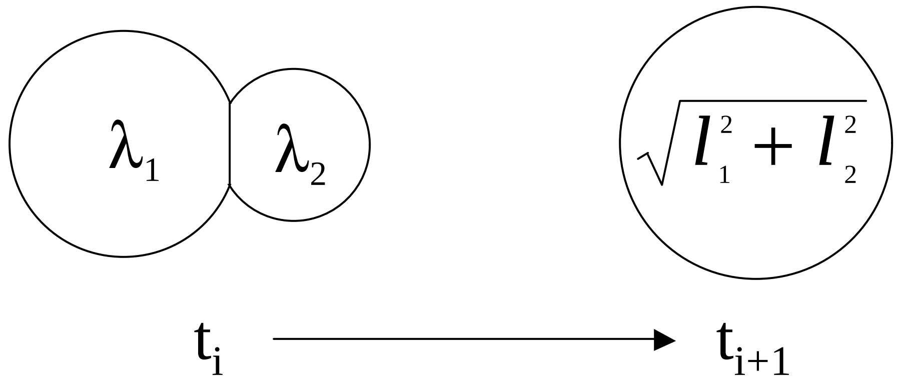
No. of Neighbors Distribution



Multi Mode 3D Statistical Model

Dimensionality effects on the bubble merger:

Bubble merging in 3D conserves <u>area</u>, rather than length in 2D:

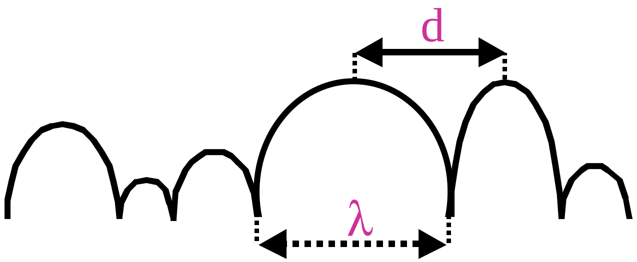


The merges occur with rate $\omega(\lambda_1, \lambda_2)$. At first step ω^{3D} was taken to be equal to $\left(u_{3D}^{asy} / u_{2D}^{asy}\right) \cdot \omega^{2D}$

Because of the area conservation, a 3D bubble has to merge with more of its neighbors in order to reach the same l. This effect reduces dl/dt, which in turn reduces both a and q (and increases b).

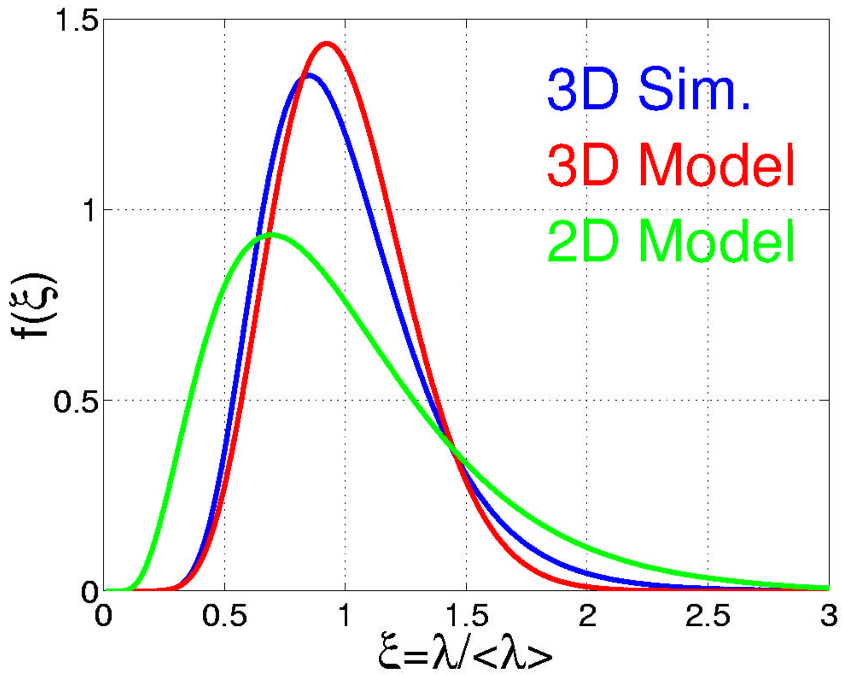
Multi Mode 3D Statistical Model Results

The segment distribution g`(d) is obtained from the simulation:



The relation between g`(d) and g(λ) is given by:

$$g`(d) = 2\int_0^d g(2d - 2l)g(2l)dl$$



Multi Mode 3D Statistical Model Results

3D Statistical model results agree well with 3D Simulations.

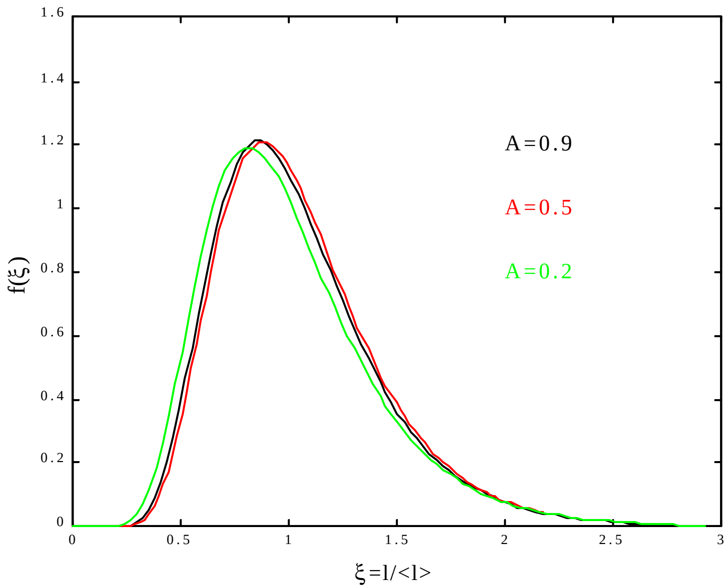
$$\alpha_B = 0.055 \qquad \theta_B = 0.18 \qquad b = 0.67$$

The 3D wavelength distribution is narrower than the 2D distribution. The narrowing of the λ distribution is due to:

a) Reduction of $d\lambda/dt$.

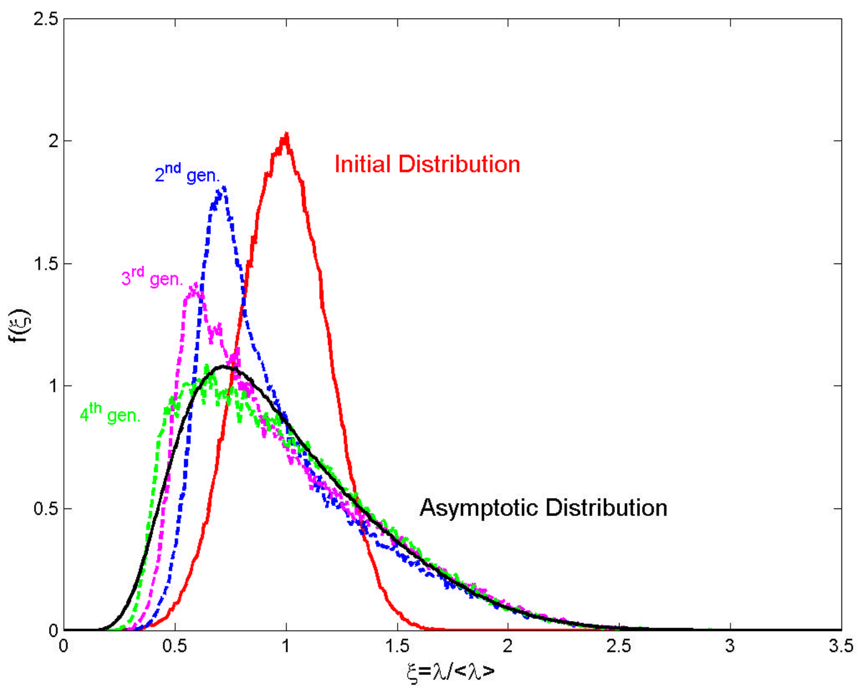
b) Increased number of neighbors.

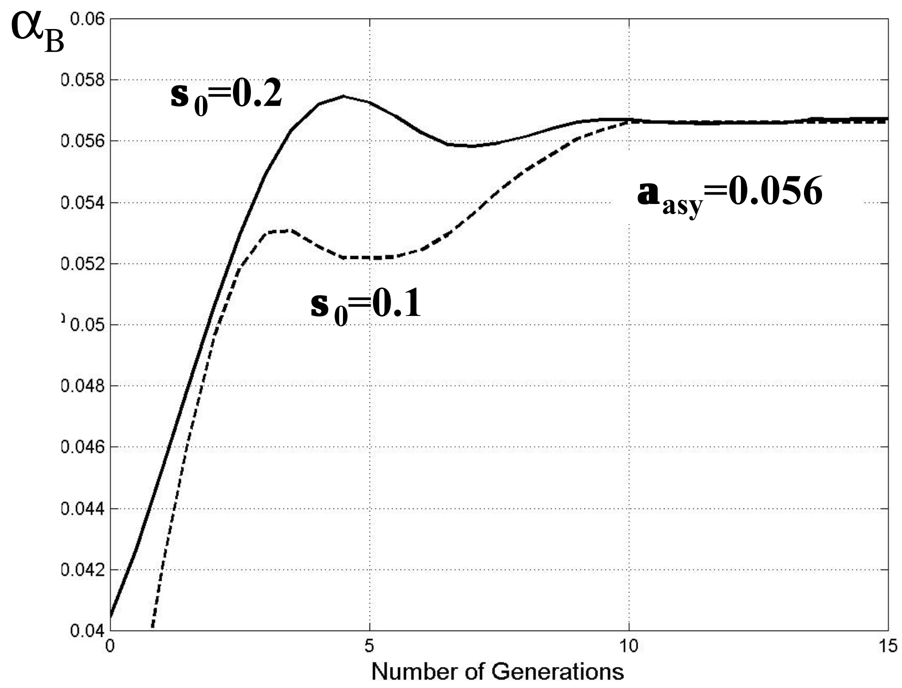
Simulations results indicate that the 3D statistical model may be applicable to a wide range of A:



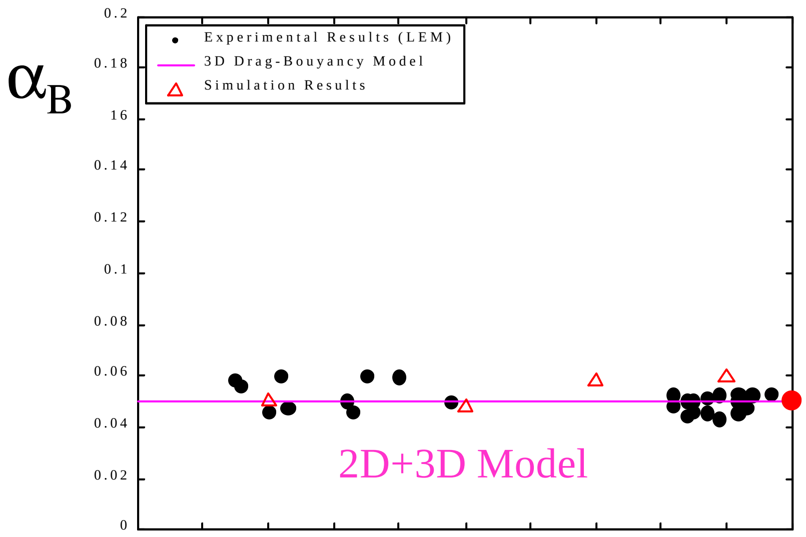
3D Statistical Model Dependence on Initial Distribution

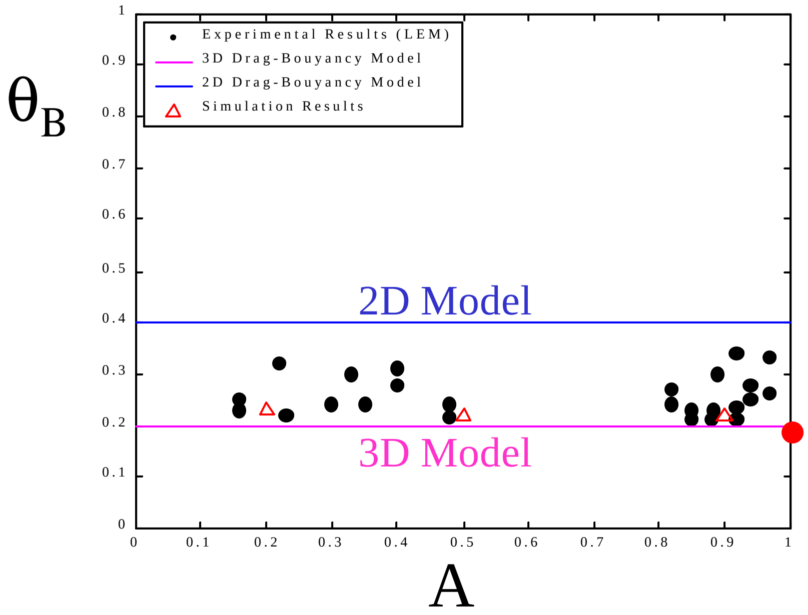
Using the initial wavelength distribution derived from the voronoi diagram in the statistical model gives the α dependence on the generation number:

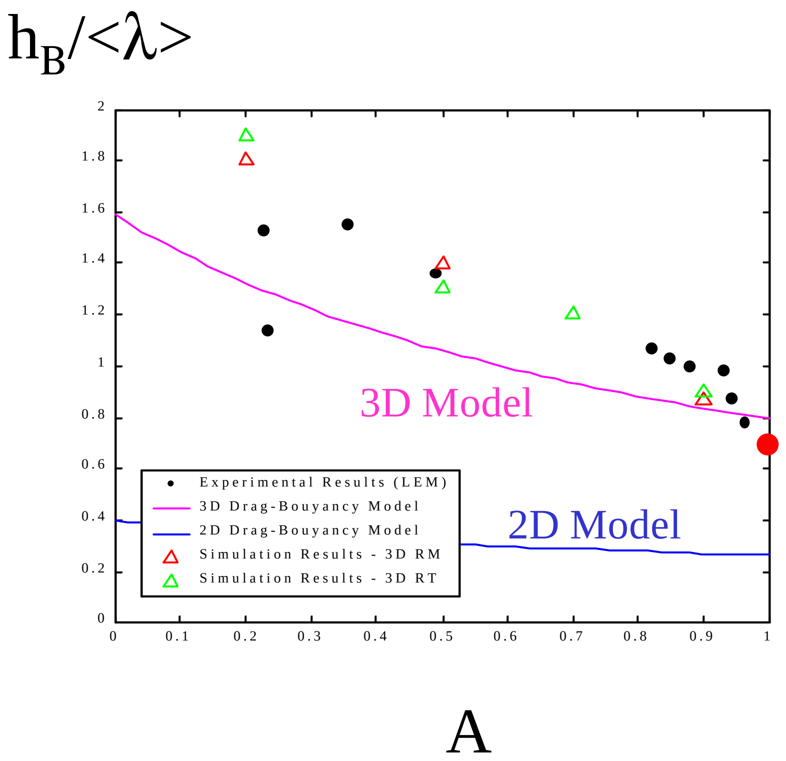




LEM Experimental Results Vs. 3D Models and Simulations

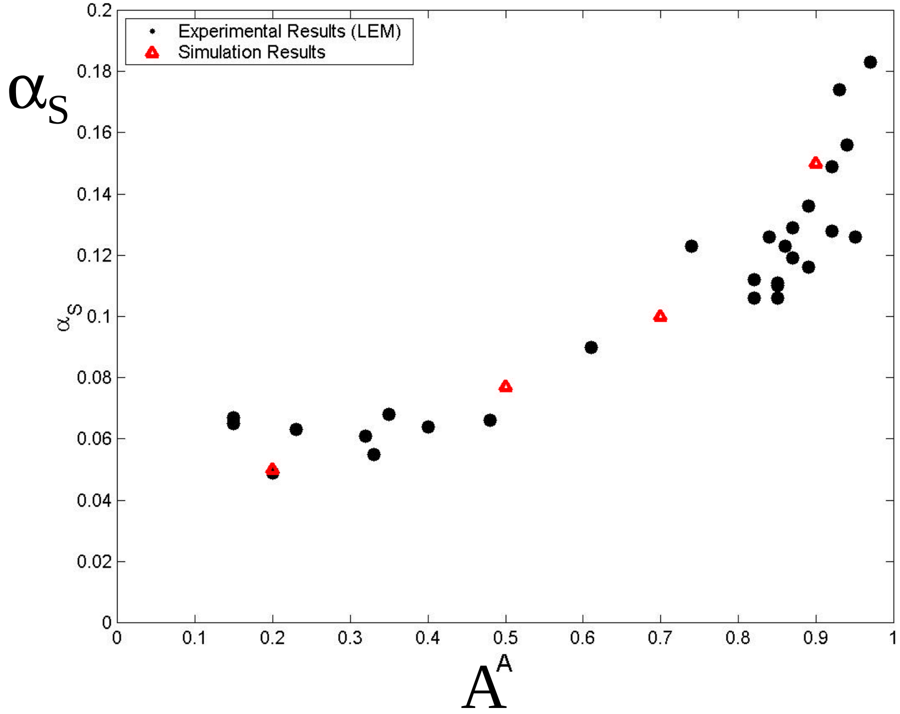




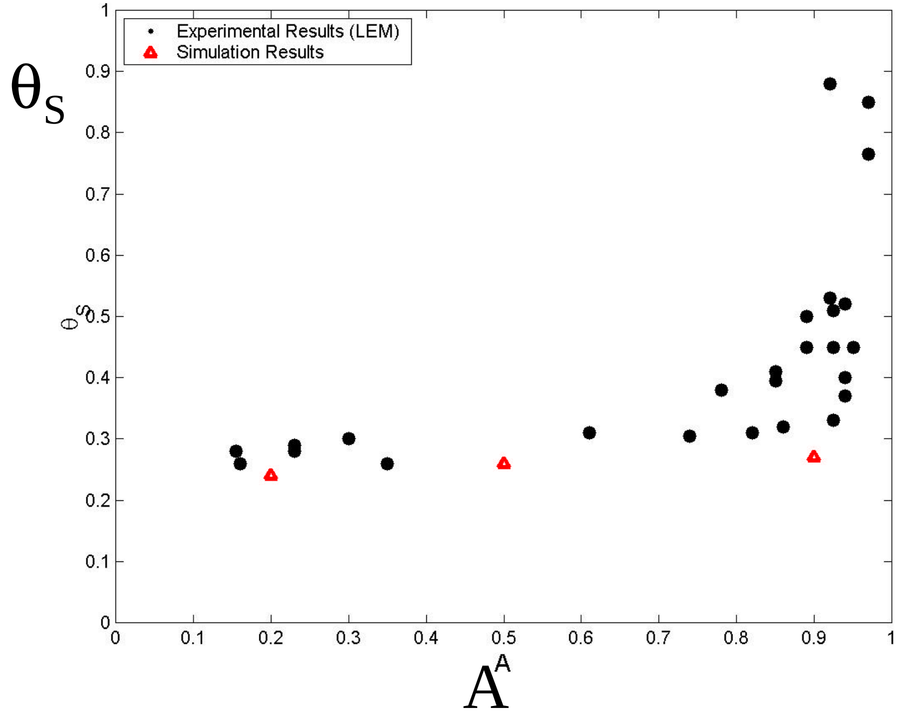


LEM Experimental Results Vs. 3D Simulations

Alpha - Spike

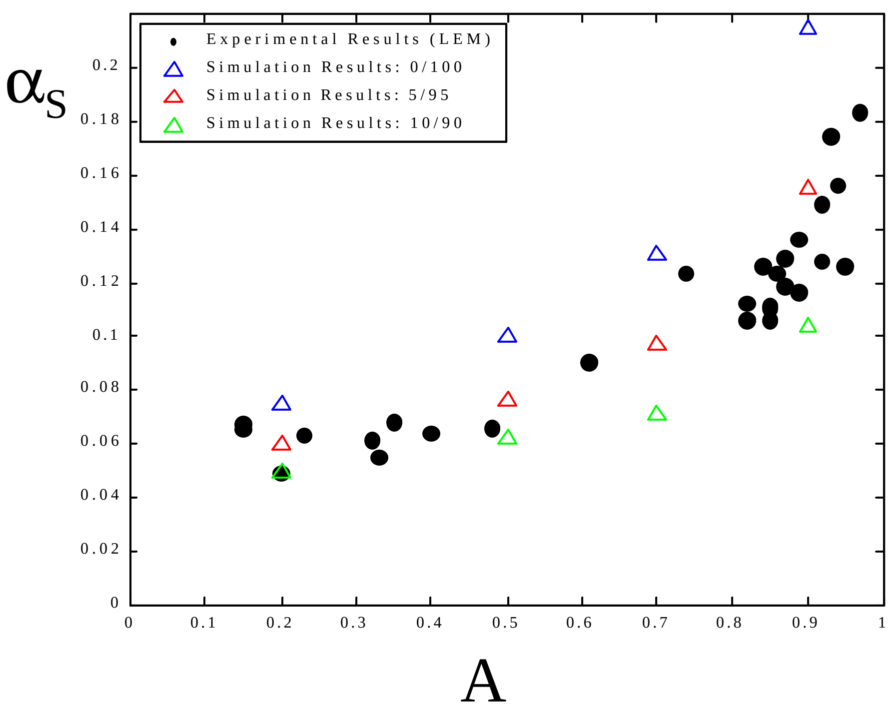


Theta - Spike

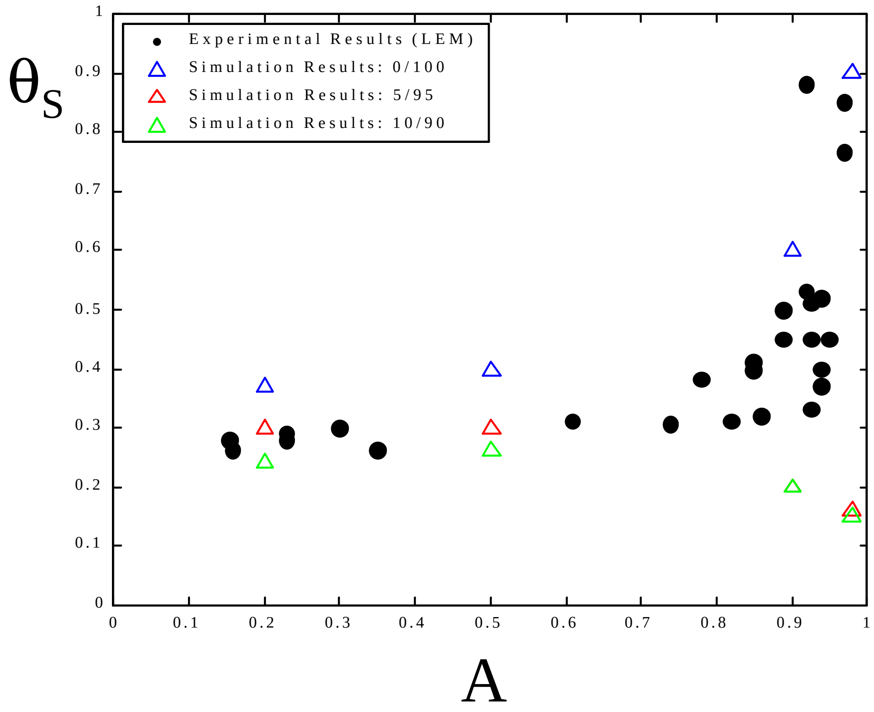


Sensitivity of q_S to percentage criterion

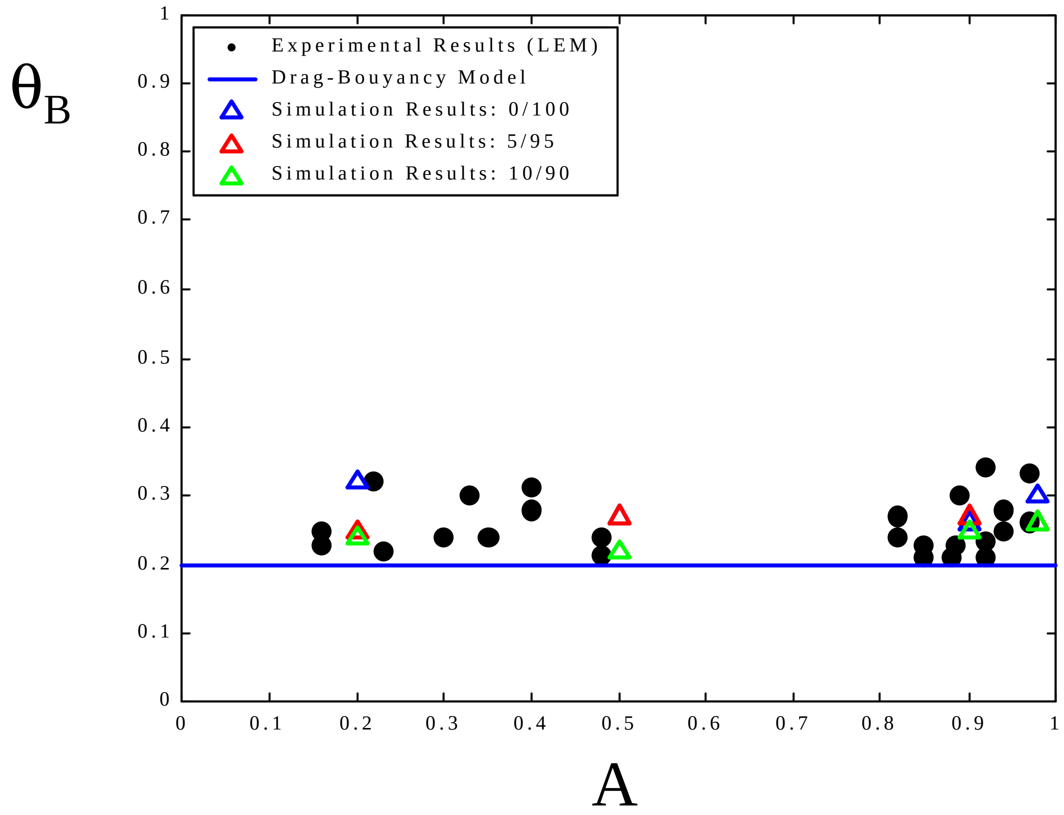
Alpha - Spike



Theta - Spike



θ_B is not sensitive to percentage criterion



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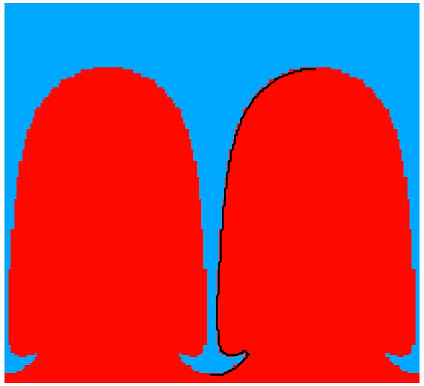
Drag-Buoyancy Model for the Spike Front - I. Single Mode



Since the spikes develop a rounded tip, one can apply the drag-buoyancy equation to them:

$$(\rho_h + C_a \rho_l)\dot{u}_S = (\rho_h - \rho_l)\cdot g - \frac{C_d}{\lambda}\rho_l \cdot u_S^2$$

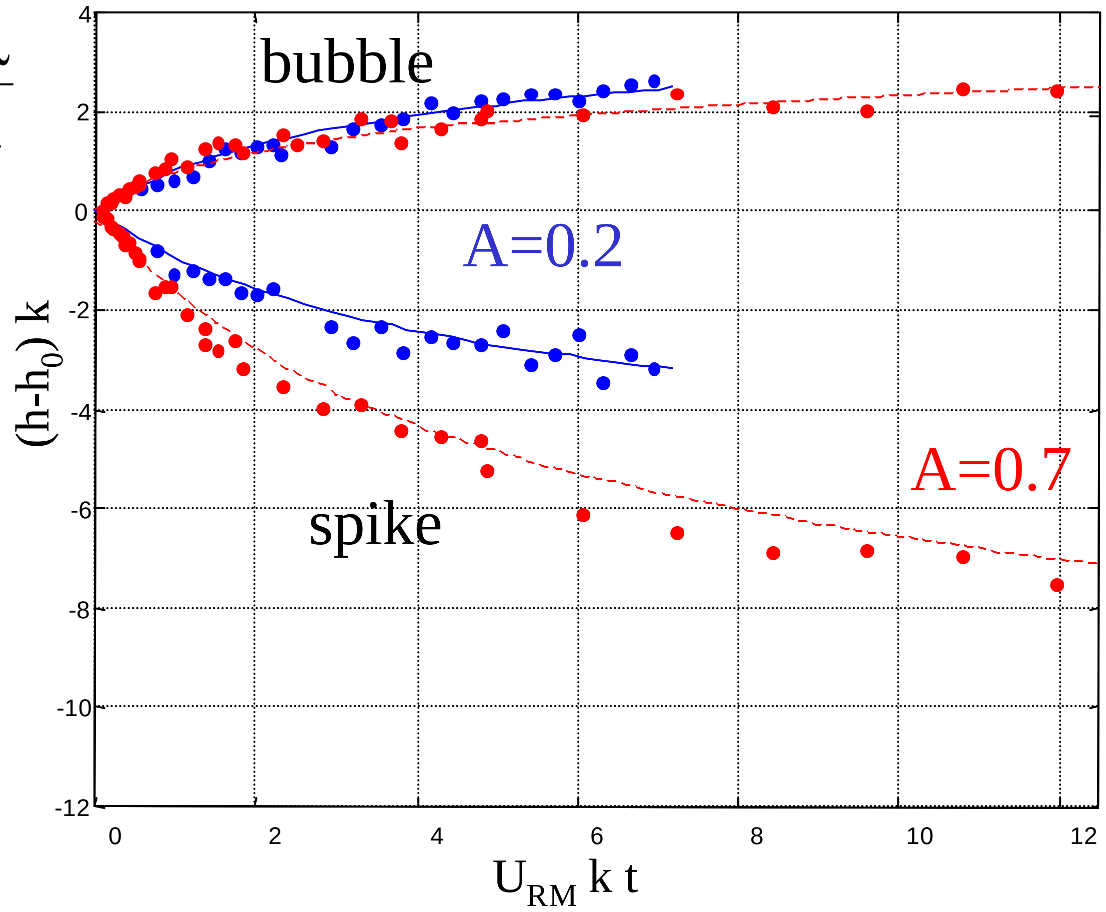
The spikes velocity is obtained using the assumptions:

$$C_{dS}(A) = C_{dB}(A) = \begin{cases} 6\pi & 2D \\ 2\pi & 3D \end{cases}$$

$$C_{aS}(A) = C_{aB}(A) = \begin{cases} 2 & 2D \\ 1 & 3D \end{cases}$$

Drag-Buoyancy Model for the Spike Front - I. Single Mode Shock Tube Experimental Results

$$U_{asy} = C \cdot \frac{\lambda}{t}$$

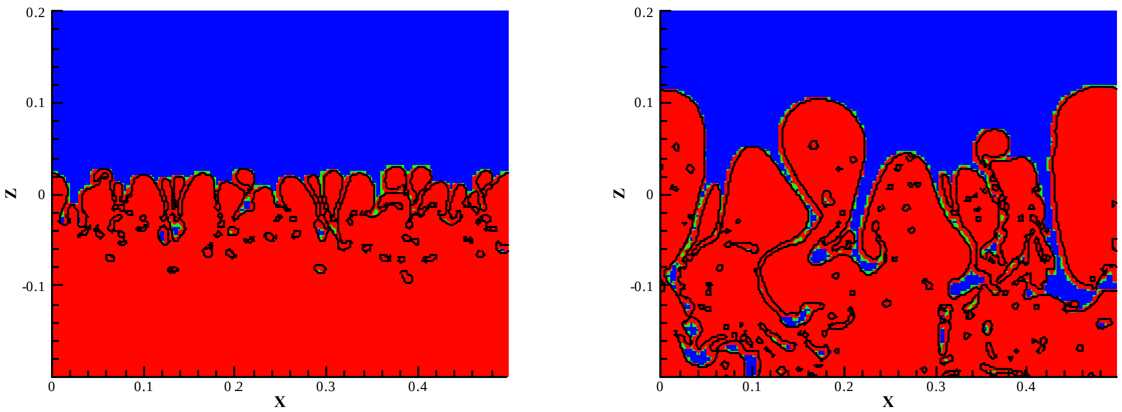


Drag-Buoyancy Model for the Spike Front - II. Multi Mode

<u>1^{st} assumption:</u>

Periodicity of the spikes ≡ Periodicity of the bubbles

$$\lambda_S(t) \equiv \lambda_B(t)$$



2D RT Simulation with A=0.9

Drag-Buoyancy Model for the Spike Front - II. Multi Mode - RT

Na ve approach:

The ratio between the momentary velocities of the spikes and the bubbles equals the ratio between their asymptotic velocities:

$$\frac{u_S(t)}{u_B(t)} = \frac{2\alpha_S Agt}{2\alpha_B Agt} = \frac{\alpha_S}{\alpha_B} = \frac{u_S^{asy}}{u_B^{asy}} = \sqrt{\frac{1+A}{1-A}} = \sqrt{\frac{\rho_h}{\rho_l}}$$

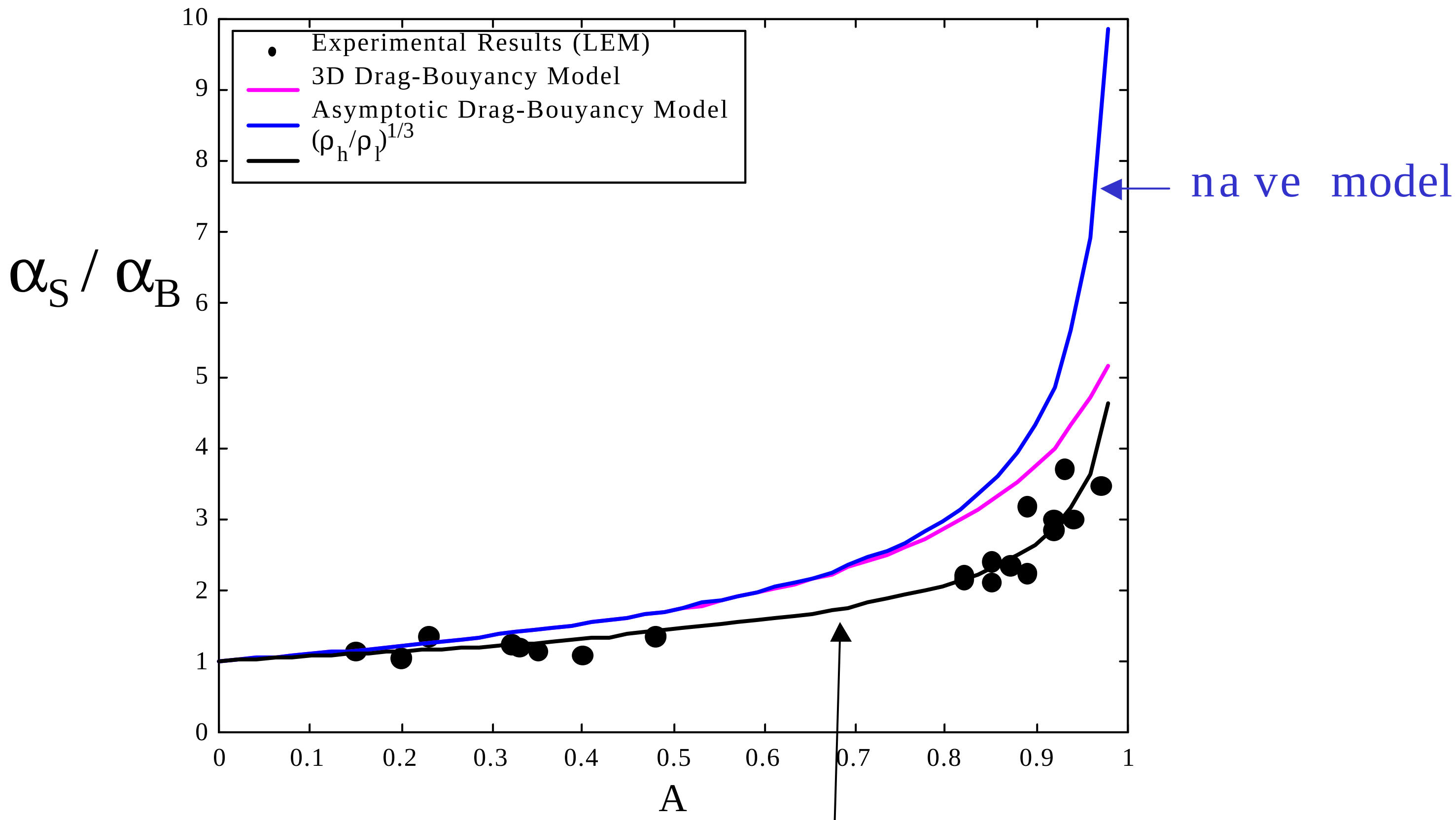
2nd assumption:

The ratio between the momentary velocities of the spikes and the bubbles equals the ratio between their velocities at the time t_b in which the bubbles height reaches its self-similar value: $h_B(t_b)=b\lambda$

$$\frac{u_S(t)}{u_B(t)} = \frac{2\alpha_S Agt}{2\alpha_B Agt} = \frac{\alpha_S}{\alpha_B} = \frac{u_S(t_b)}{u_B(t_b)} \cong \frac{u_S(t_b)}{u_B^{asy}}$$

Drag-Buoyancy Model for the Spike Front - II. Multi Mode - RT Results

3D: $$\frac{\alpha_S}{\alpha_B} = \sqrt{\frac{1+A}{1-A}} \cdot \tanh\left[\sqrt{\frac{1-A}{1+A}} \cdot \cosh^{-1} \cdot \exp(\pi b(1+A))\right]$$



$\alpha_S/\alpha_B \sim (\rho_h/\rho_l)^{1/3}$: Best fit by Dimonte et. Al.

Drag-Buoyancy Model for the Spike Front - II. Single Mode - RM

At late times, the bubble velocity in RM goes like λ/t.

At finite time t its velocity can be expressed by a time-dependent coefficient $\gamma(t)$:

$$\frac{u_B(t)}{u_0} \propto \left(\frac{u_0 t}{\lambda}\right)^{\gamma_B(t)} \qquad \gamma_B(t \to \infty) = -1 \quad \textit{for all A s}$$

$$\frac{u_S(t)}{u_0} \propto \left(\frac{u_0 t}{\lambda}\right)^{\gamma_S(t)} \qquad \gamma_S(t \to \infty) = \begin{cases} -1 & A < 1 \\ 0 & A = 1 \end{cases}$$

Where, from assumption 1: $\langle \lambda(t) \rangle = \frac{h_B(t)}{b} \propto t^{\theta_B}$

Drag-Buoyancy Model for the Spike Front - II. Multi Mode - RM

$$u_S^{MM} \propto t^{\gamma_S(t_b)\cdot(1-\theta_B)} \qquad \textit{also:} \quad u_S^{MM}(t) = \frac{dh_S(t)}{dt} \propto t^{\theta_S - 1}$$

$$\boxed{\theta_S = 1 + \gamma_S(t_b)\cdot(1-\theta_B)}$$

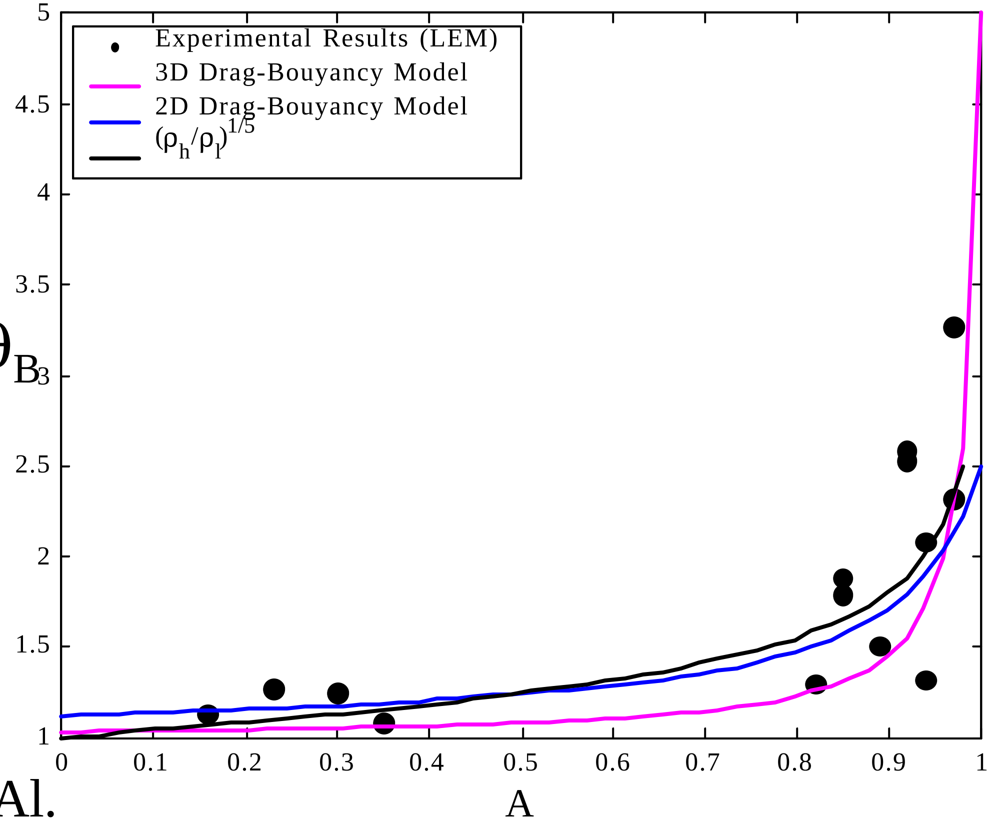
Na ve approach:

$$\gamma_S^{asy} = \begin{cases} 0 & A = 1 \rightarrow \quad \theta_S = 1 \\ -1 & A < 1 \rightarrow \quad \theta_S = \theta_B \end{cases}$$

Drag-Buoyancy Model for the Spike Front - II. Multi Mode - RM Results

3D: $$\theta_S = 1 + \gamma_S(t_b)\cdot(1-\theta_B) = 1 - \frac{\left(\frac{1-A}{1+A}\right)\cdot\left(\exp(\pi b(1+A))-1\right)}{1+\left(\frac{1-A}{1+A}\right)\cdot\left(\exp(\pi b(1+A))-1\right)}\cdot(1-\theta_B)$$

θ_S / θ_B



$$\alpha_S/\alpha_B \sim (\rho_h/\rho_l)^{1/5}$$

Best fit by Dimonte et. Al.

Summary

Good agreement between 3D drag-buoyancy model, statistical model, full 3D simulations and experimental results.

2D and 3D RT and RM scaling laws:

RT: $h_{B(S)} = \alpha_{B(S)} A g t^2$

RM: $h_{B(S)} = a_{B(S)} t^{\theta_{B(S)}}$

Geometrical effect results in different scaling parameters:

α_B	≈ 0.05	in 2D and 3D
θ_B	≈ 0.2-0.3	in 3D
	Vs. 0.4	in 2D
b	≈ 0.75-1.5	in 3D
	Vs. 0.25-0.5	in 2D

Spikes scaling laws are obtained from the drag-buoyancy model.