NON LINEAR RT and RM SINGLE MODES (ANALYTIC)

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OUTLINE

- ASSUMPTIONS
  + INITIAL 2D/3D PERTURBATION
- ANALYTICAL MODEL
- STRUCTURE OF THE INTERFACE
- SHAPE FACTOR
- COMP. OTHER WORKS.

ASSUMPTIONS:

- $g = f(t)$ arbitrary

$$\begin{cases} g \sim \text{cste} & \text{RTI} \\ g \sim \delta(t) & \text{RMI} \end{cases}$$

- IMCOMPRESSIBLE

(compressible - Linear)

- POTENTIAL FLOW

$$\vec{V} = \vec{\nabla}\phi \implies \boxed{\nabla^2\phi = 0}$$

- $At = \dfrac{\rho_H - \rho_L}{\rho_H + \rho_L} \longrightarrow 1$

- SINGLE MODE STUDY

2D and 3D

- INFLUENCE OF THE LATTICE (PERIODIC)

B3(J6), B4(J4), B6(J3)

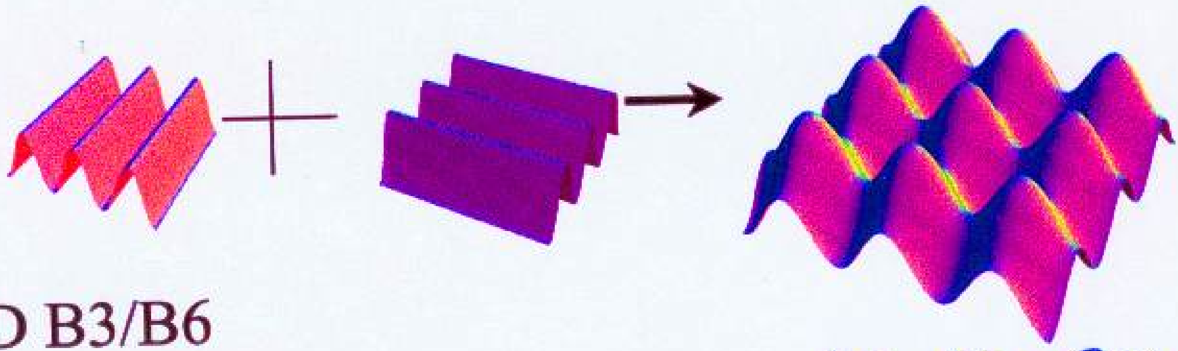
Perturbations

- 2D

RIPPLES

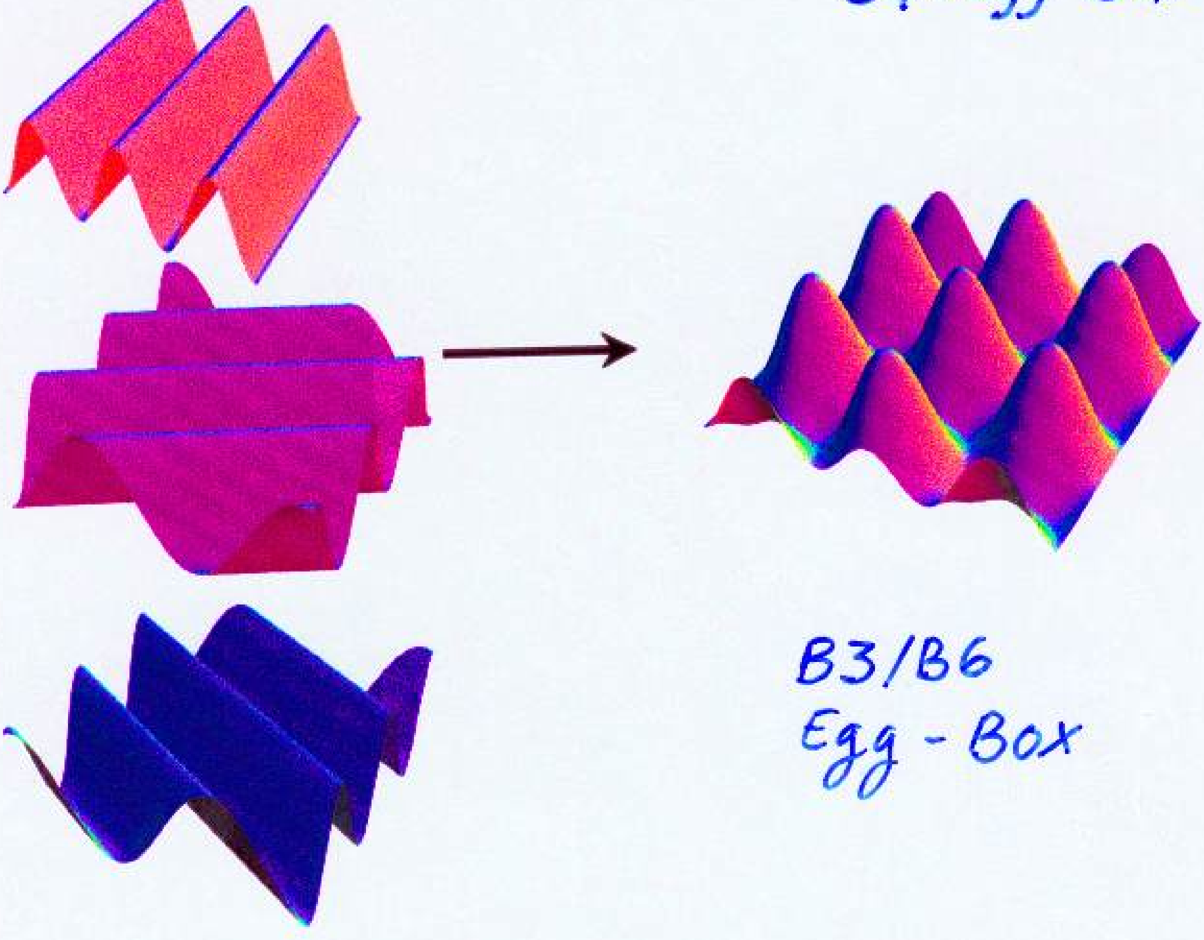
- 3D B4

3D PERIODIC LATT.



B4 - Egg Box

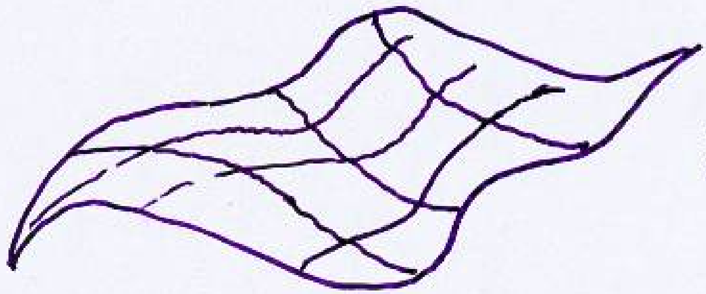
- 3D B3/B6



B3/B6
Egg - Box

EQUATIONS

- INTERFACE $\begin{cases} \text{2D} & z = \eta(x,t) \\ \text{3D} & z = \eta(x,y,t) \end{cases}$



3D

$z = \eta(x,y,t)$

- PARTICLES of FLUID ATTACHED TO THE INTERFACE

$$\frac{\partial \eta}{\partial t} = v_z - v_x \frac{\partial \eta}{\partial x} \underbrace{- v_y \frac{\partial \eta}{\partial y}}_{\text{3D}}$$

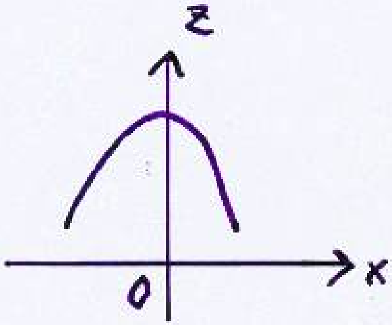
- BERNOUILLI EQUATION

$$\frac{\partial \phi}{\partial t} = \frac{1}{2}\left(\vec{v}\right)^2 + g(t)\,\eta$$

MODEL

Extension of LAYZER Approach

ApJ 122 1 (1955)



Top of a bubble:

1) $\eta(x,t) = \eta_0(t) + \frac{1}{2}\eta_1(t)x^2 + \cdots$

2) monomode potential

$\phi(x,z,t) \sim a(t)(\cos kx)e^{-kz}$

OUR WORK:

2D: $\phi(x,z,t) = \sum_{n=1}^{N} \phi_n(t)\cos(nx)\,e^{-nz}$

$$\eta = \eta_0 + \sum_{n=1}^{N} K_n(t)\frac{x^{2n}}{(2n)!}$$

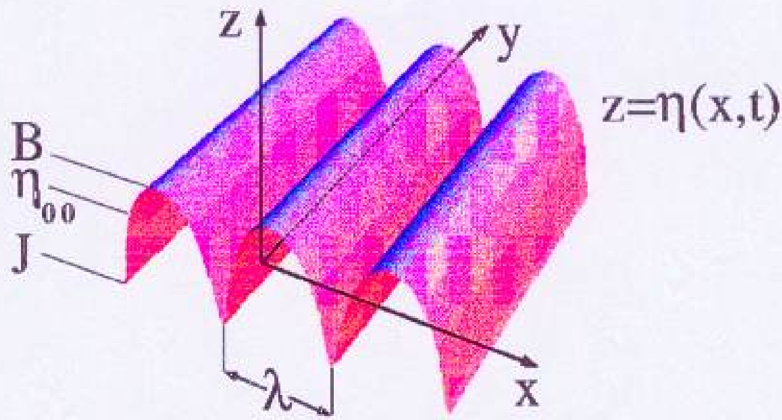
N - TRUNCATION NUMBER
(up to 6 for J, for the first time)

3D
$$\begin{cases} \phi \sim \sum_{nm}^{n+m<N} \phi_{nm}(t)\cos nx \cos my\, e^{-q_{nm}z} \\ \eta \sim \sum K_{nm}(t)\frac{x^{2n}}{(2n!)}\frac{y^{2m}}{(2m!)} \end{cases}$$

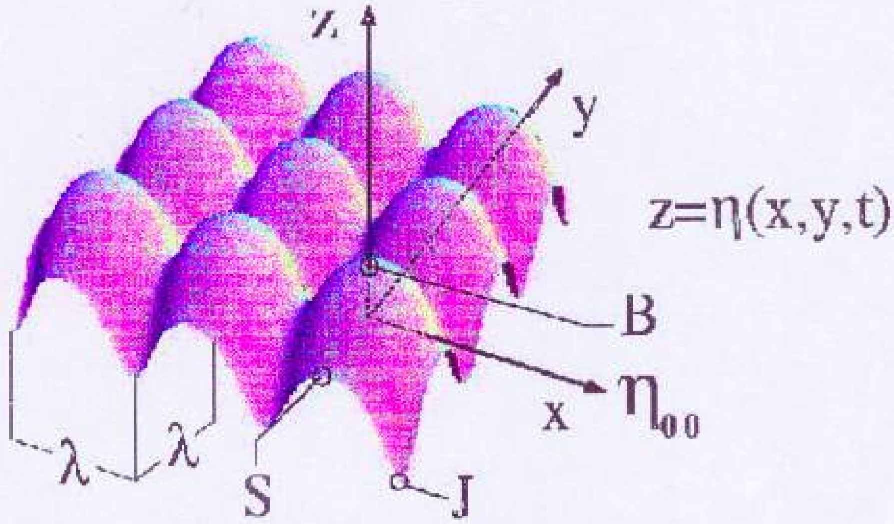
STAGNATION POINTS

- TOP OF BUBBLES
- TIP OF JETS
- SADDLE POINTS

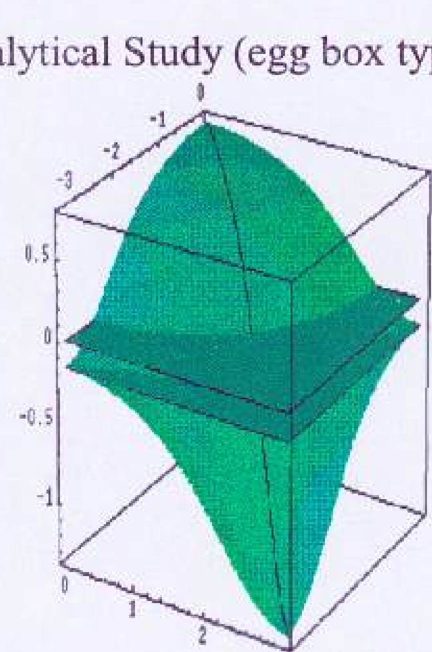
for the first time



2D – Nonlinear Analytical Study (parrallel ripples)



2D – Nonlinear Analytical Study (egg box type initial perturbation)

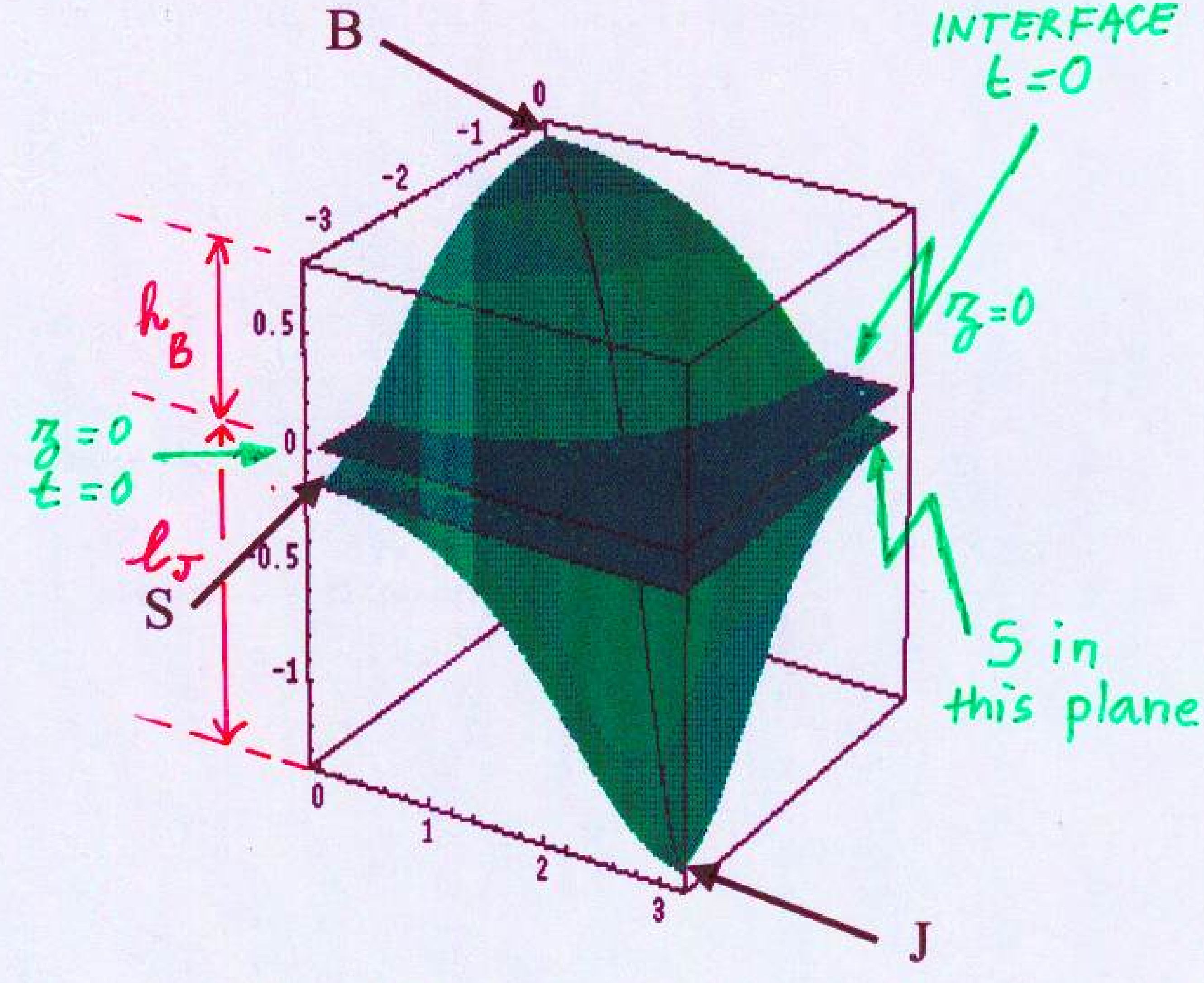


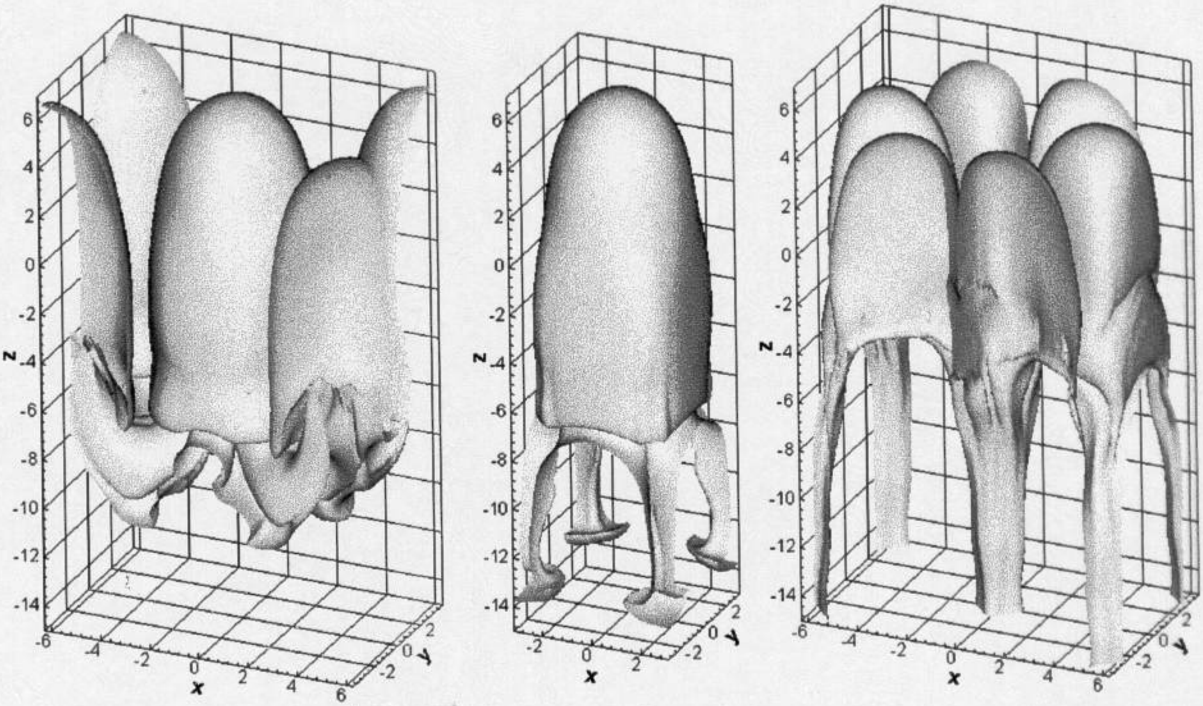
Detail of a Local System « Bubble+Jet+Saddle Point »

BLOW-UP

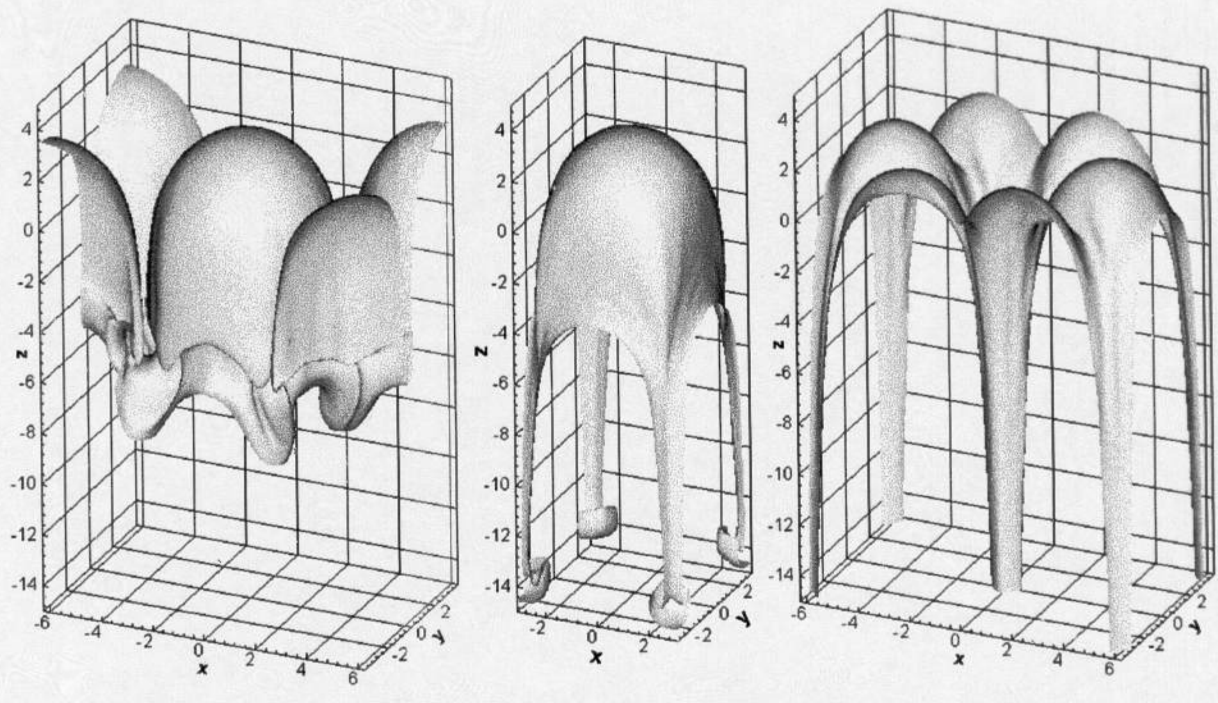
B + S + J

$$\left.\begin{array}{l} h_B \simeq 0.7 \\ l_J \simeq 1.4 \end{array}\right\} l_J \simeq 2 h_B \quad \text{Highly NL}$$





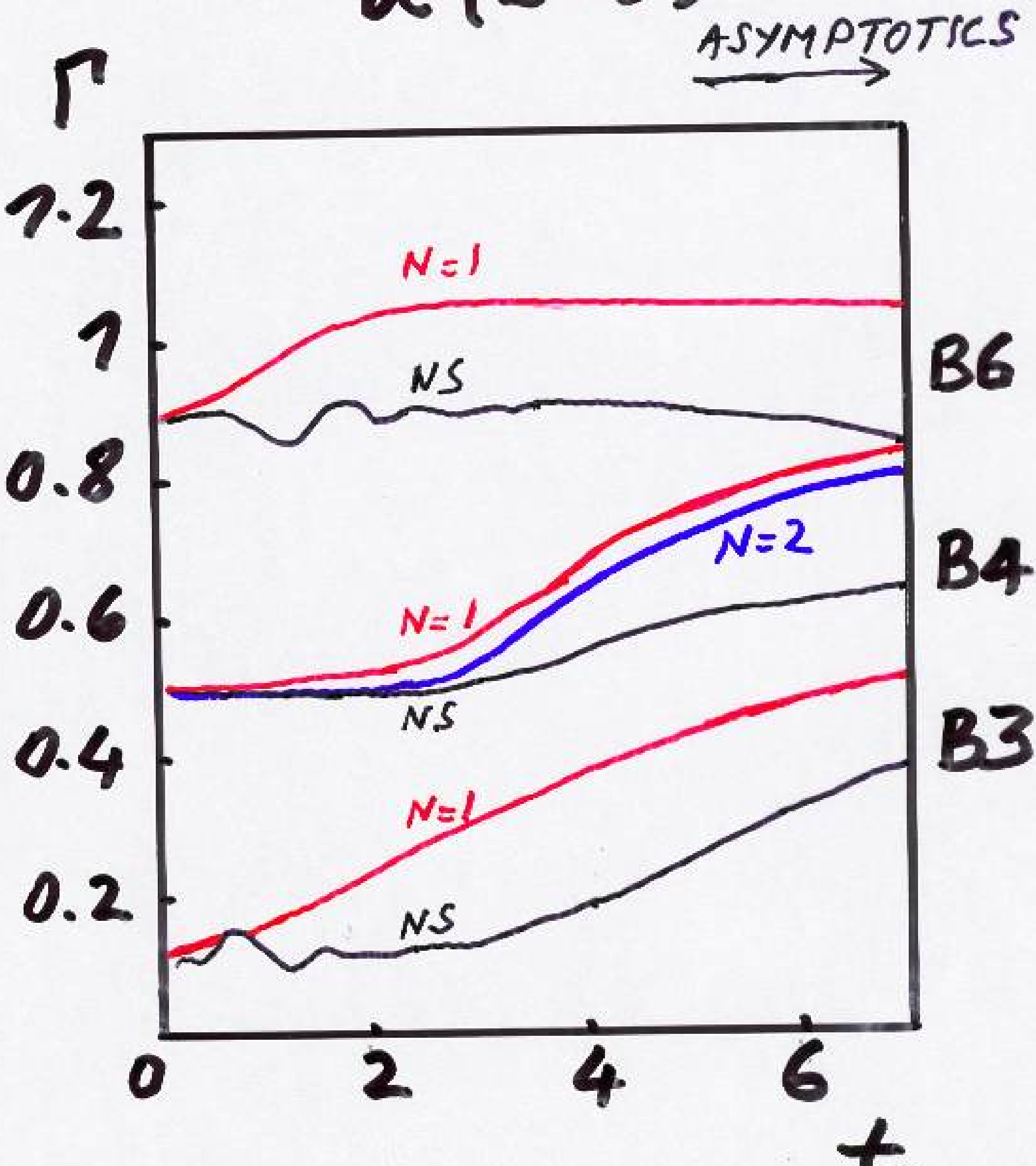
Numerical Simulation of the Rayleigh-Taylor Instability (B6, B4, B3)



Numerical Simulation of the Richtmyer-Meshkov Instability (B6, B4, B3)

RTI

$$\Gamma = \frac{d(B-\overset{S}{\cancel{J}})}{d(B-J)}$$



— Num. Sim. (OPARIN)

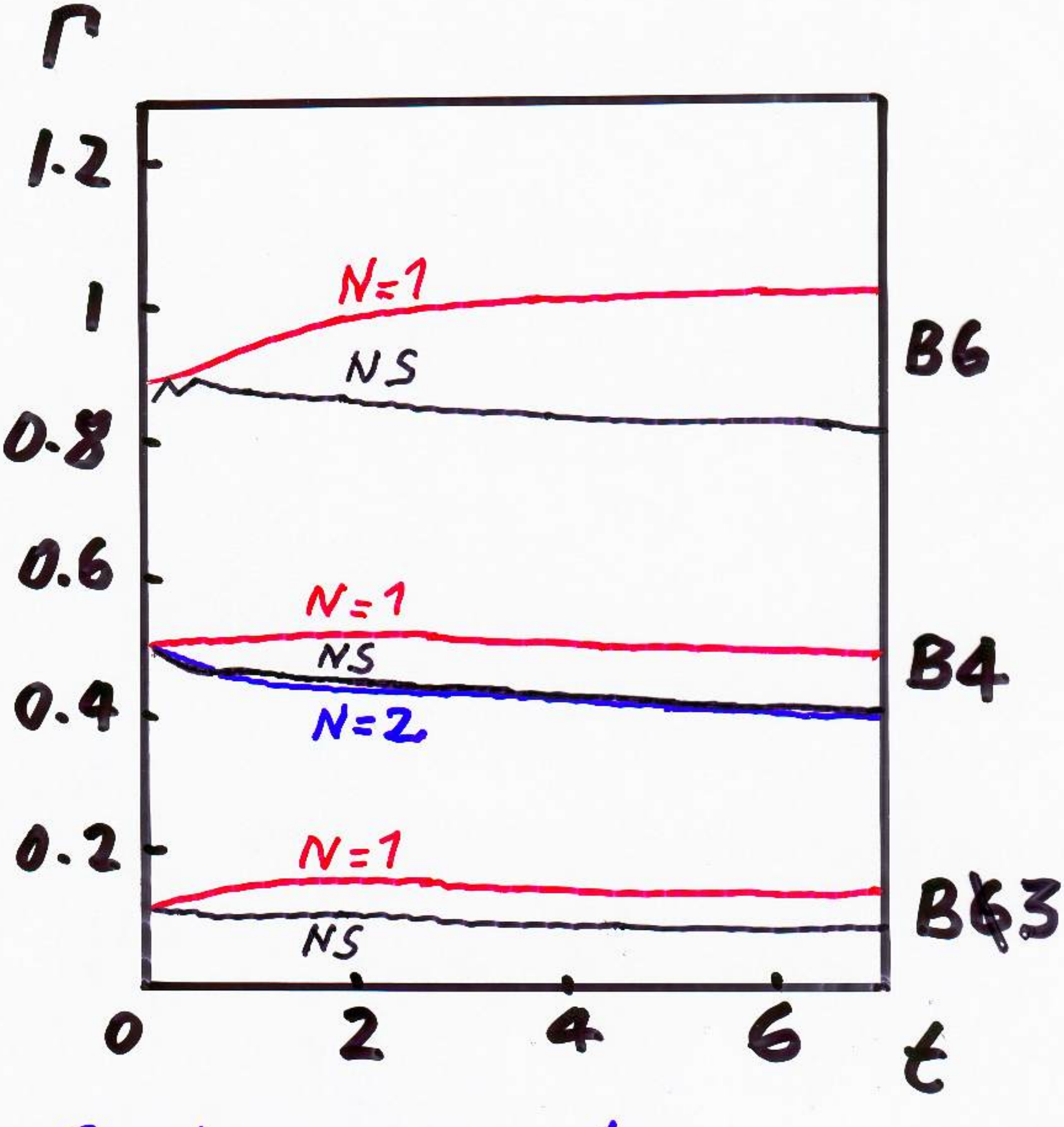
— Theory N=1

— Theory N=2

GOOD TENDENCY (Γ is global)

RMI

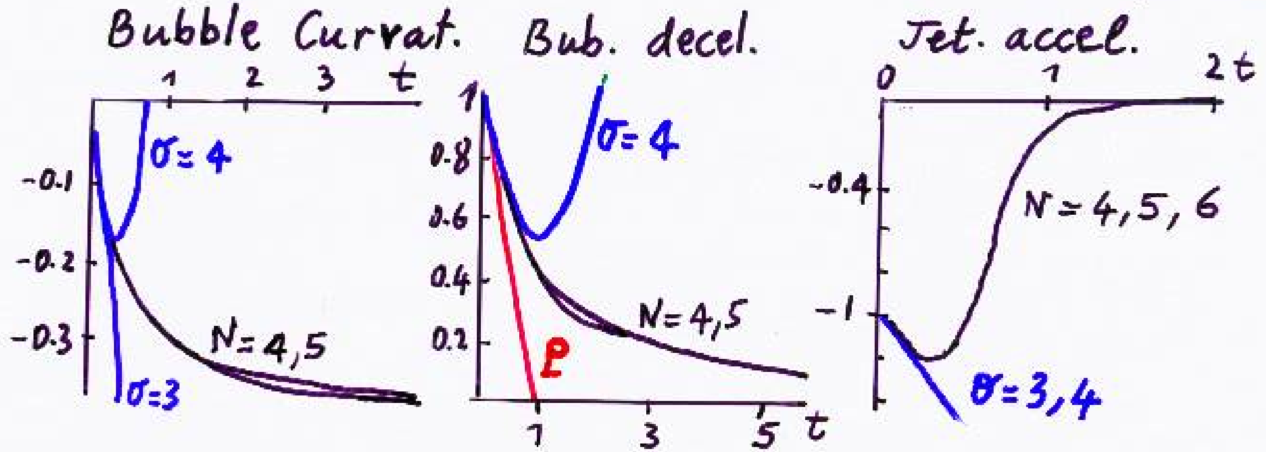
$$\Gamma = \frac{d(B-S)}{d(B-J)}$$



N=2 Good agreement.

Rather good " for asymptotic values.

COMP. WNL APPROACH $\underbrace{a_0 k \alpha t}_{\text{expansion parameter about } t=0} \lesssim 1$
-RMI-



— WNL — Pade — iTOB

RIGHT MODELING OF THE LINEAR PHASES.

$R_b = 2.6$

$V_b = \dfrac{\alpha}{t}$

$\alpha = 0.60$

WNL approach can only follow bubble curvature up to 40% of the asymtotic value

CONCLUSION

- Analytical NL asymtotic stages (+ transients)
- comparisons with 2D/3D CODES wich compute hydro in such NL regimes