# A General Buoyancy-Drag Model for the Evolution of the Rayleigh-Taylor and Richtmyer-Meshkov Instabilities

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# Abstract

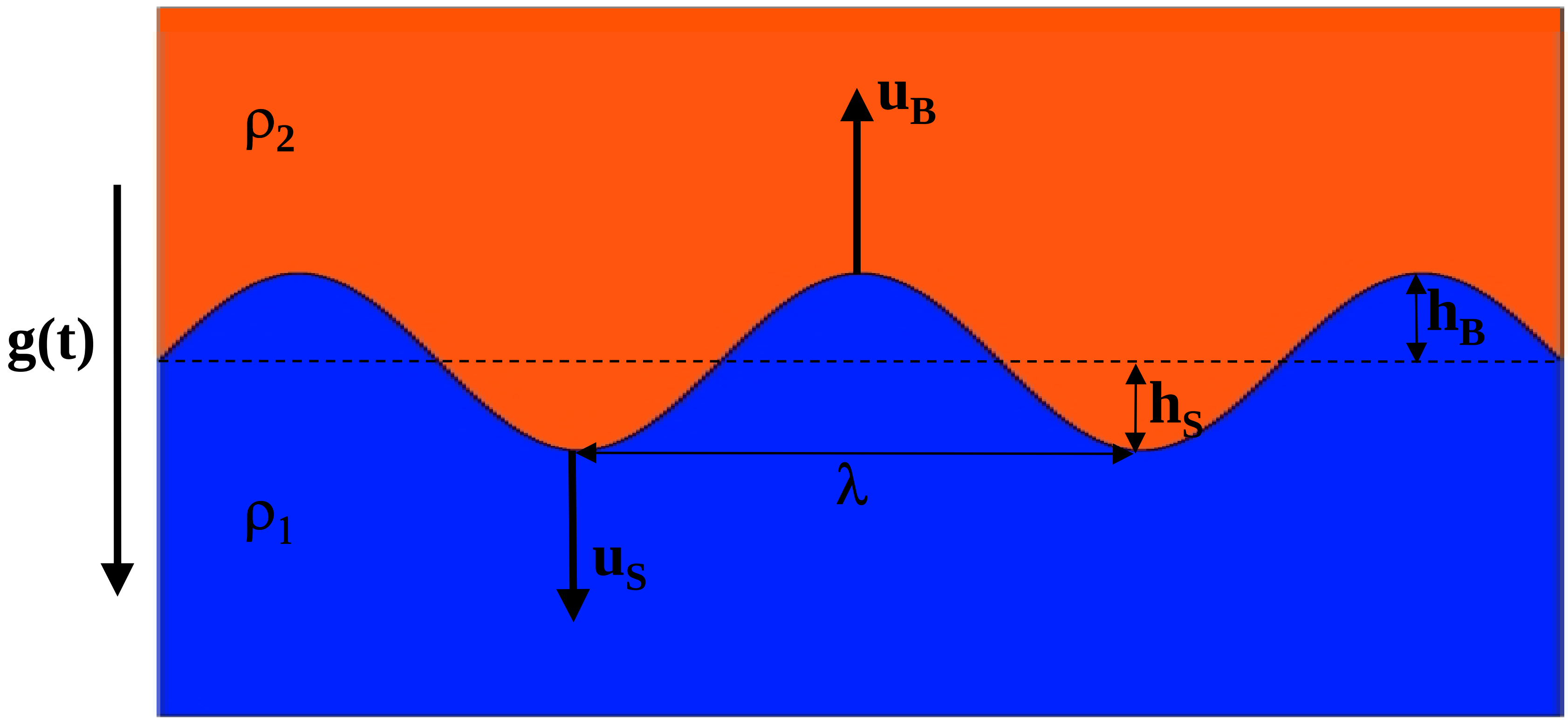
The growth of a single-mode perturbation is described by a buoyancy-drag equation, which describes all instability stages (linear, non-linear and asymptotic) at time-dependant Atwood number and acceleration profile. The evolution of a multi-mode spectrum of perturbations from a short wavelength random noise is described using a single characteristic wavelength. The temporal evolution of this wavelength allows the description of both the linear stage and the late time self-similar behavior. The model includes additional effects, such as shock compression and spherical convergence.

Model results are compared to full 2D numerical simulations and shock-tube experiments of random perturbations, studying the various stages of the evolution.

# Ideal Model Requirements

- Calculate mix region for:
  - general acceleration profile (RT and RM).
  - all instability stages (linear, early nonlinear, asymptotic)
  - general geometry (planar, cylindrical, spherical)
  - compressibility and coupling to 1D flow.
  - ablation.
- Describe internal structure of mixing zone:
  - density, temperature and pressure of every material.
  - degree of mixing.
- Feedback to 1D simulation:
  - material flow.

# Definitions




Atwood number

$$A = \frac{\rho_2 - \rho_1}{\rho_2 + \rho_1}$$

$$k = \frac{2\pi}{\lambda}$$

# Layzer model

$$(2D)\quad \frac{du_B}{dt} = \left(\frac{1-E}{2+E}\right)\cdot g(t) - \left(\frac{6\pi}{2+E}\right)\cdot \frac{u_B^2}{\lambda} \quad , \qquad E = e^{-3kh_B}$$

$$(3D)\quad \frac{du_B}{dt} = \left(\frac{1-E}{1+E}\right)\cdot g(t) - \left(\frac{2\pi}{1+E}\right)\cdot \frac{u_B^2}{\lambda} \quad , \qquad E = e^{-2kh_B}$$

- Single mode (periodic array of bubbles and spikes).
- Describes all instability stages.
- Valid for a general acceleration profile.
- Limited to A=1.

# Buoyancy-drag equations

$$\left(\rho_1 + C_a \rho_2\right)\frac{du_B}{dt} = \left(\rho_2 - \rho_1\right)\cdot g(t) - \frac{C_d}{\lambda}\rho_2 \cdot u_B^2$$

$$\left(\rho_2 + C_a \rho_1\right)\frac{du_S}{dt} = \left(\rho_2 - \rho_1\right)\cdot g(t) - \frac{C_d}{\lambda}\rho_1 \cdot u_S^2$$

- Single mode (periodic array of bubbles and spikes).
- Describes only asymptotic stage.
- Valid for a general acceleration profile.
- Valid for every A.

# New model for single-mode perturbation

- We combine Layzer model with buoyancy-drag equations.
- $C_a$, $C_d$, $C_e$ are determined from Layzer model for A=1, and assumed to be Atwood independent.

$$\left[\left(C_a \cdot E(t) + 1\right)\rho_1 + \left(C_a + E(t)\right)\rho_2\right]\frac{du_B}{dt} =$$

$$\left(1 - E(t)\right)\cdot\left(\rho_2 - \rho_1\right)\cdot g(t) - \frac{C_d}{\lambda}\rho_2 \cdot u_B^2$$

$$\left(E(t) = e^{-C_e \cdot k \cdot h_B}\right)$$

# Multimode evolution

**Mixing fronts (bubbles and spikes) are described by one characteristic wavelength: <λ>=<$\lambda_{BUB}$>.**

- Linear stage: $$\frac{d\langle\lambda\rangle}{dt} = 0$$

- Asymptotic self-similar behavior:

$$\frac{h_B}{\langle\lambda\rangle} = b(A) \quad\Rightarrow\quad \frac{d\langle\lambda\rangle}{dt} = \frac{u_B}{b(A)}$$

- Transition from linear to asymptotic is at:

$$h_B = \langle\lambda_0\rangle \cdot b(A)$$

# Model properties

- Linear stage:

reproduces theoretical result (first order):

$$\ddot{h}(t) = Akgh(t)$$

- Early nonlinear stage:

for A→1, correct to second order (Layzer model)

- Asymptotic stage:

buoyancy-drag equation for all A.

**Limited to planar geometry and incompressible flow.**

# 1D Hydrodynamic coupling

The dynamic front equation is solved coupled to the 1D lagrangian motion:

- Change in Atwood number:

$$\rho_i = \left| \int_{h_{1d}}^{h_i} \rho_i V dx \right| \Big/ \left| \int_{h_{1d}}^{h_i} V dx \right| \qquad i = 1,2$$

- 1D Lagrangian “drift” of the mixing zone boundaries:

$$\mathrm{u_B} \rightarrow \mathrm{u_B} + \mathrm{U_{1d}}(\mathrm{h_B})$$

$$\mathrm{u_S} \rightarrow \mathrm{u_S} + \mathrm{U_{1d}}(\mathrm{h_S})$$

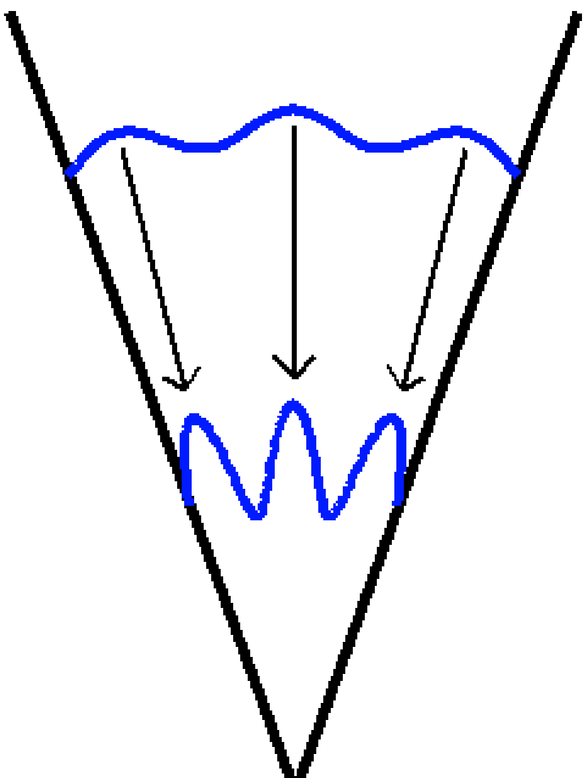
# Corrections required for non-planar geometry

## Non-planar geometry introduces two effects:

- change in amplitude due to 1D motion (Bell-Plesset)

  - included in 1D coupling to lagrangian flow.

- Change in wavelength (conservation of wavenumber, $\ell = \lambda / R$).

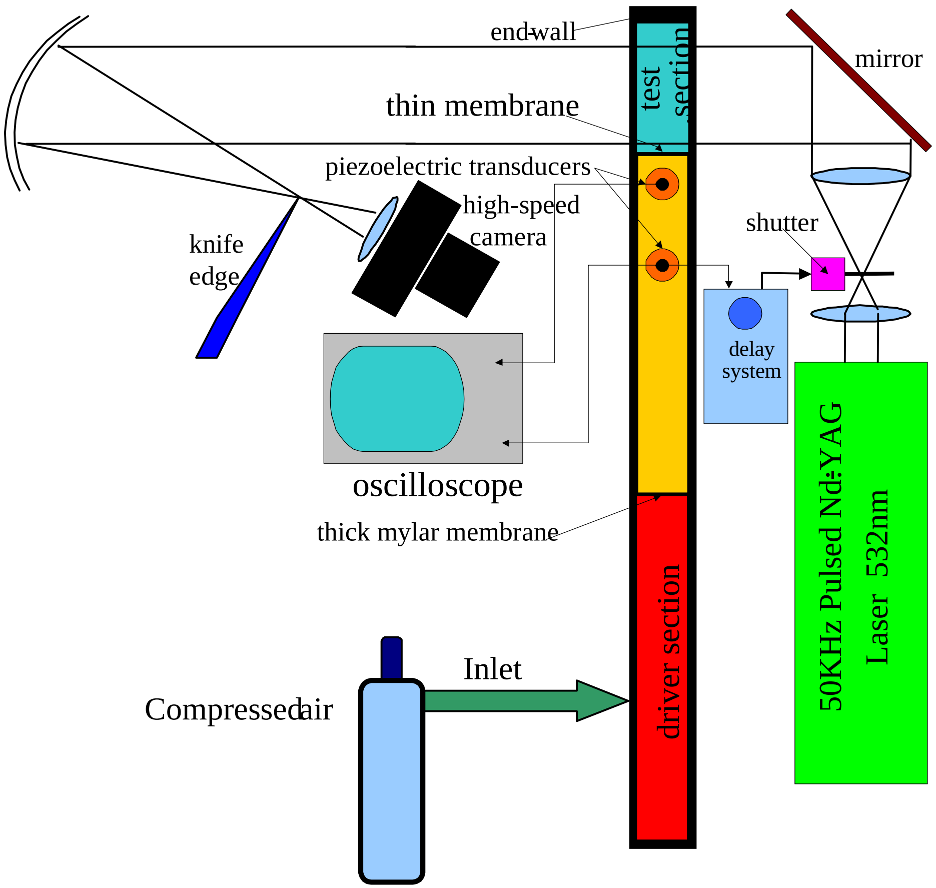


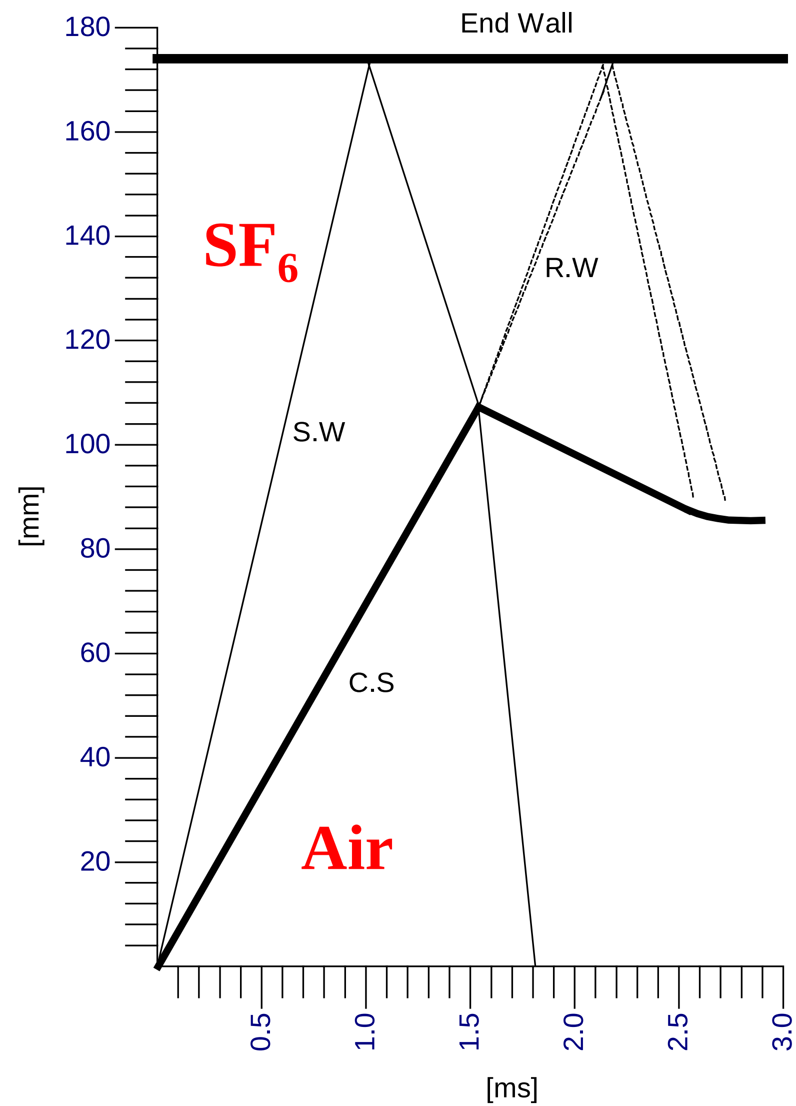
- geometric term added to wavelength equation:

$$\frac{d\langle\lambda\rangle}{dt} \rightarrow \frac{d\langle\lambda\rangle}{dt} + \left(\frac{d\langle\lambda\rangle}{dt}\right)_{geometry}$$

$$\left(\frac{d\langle\lambda\rangle}{dt}\right)_{geometry} = \langle\lambda(t)\rangle \frac{U_{1d}(t)}{R_{1d}(t)}$$

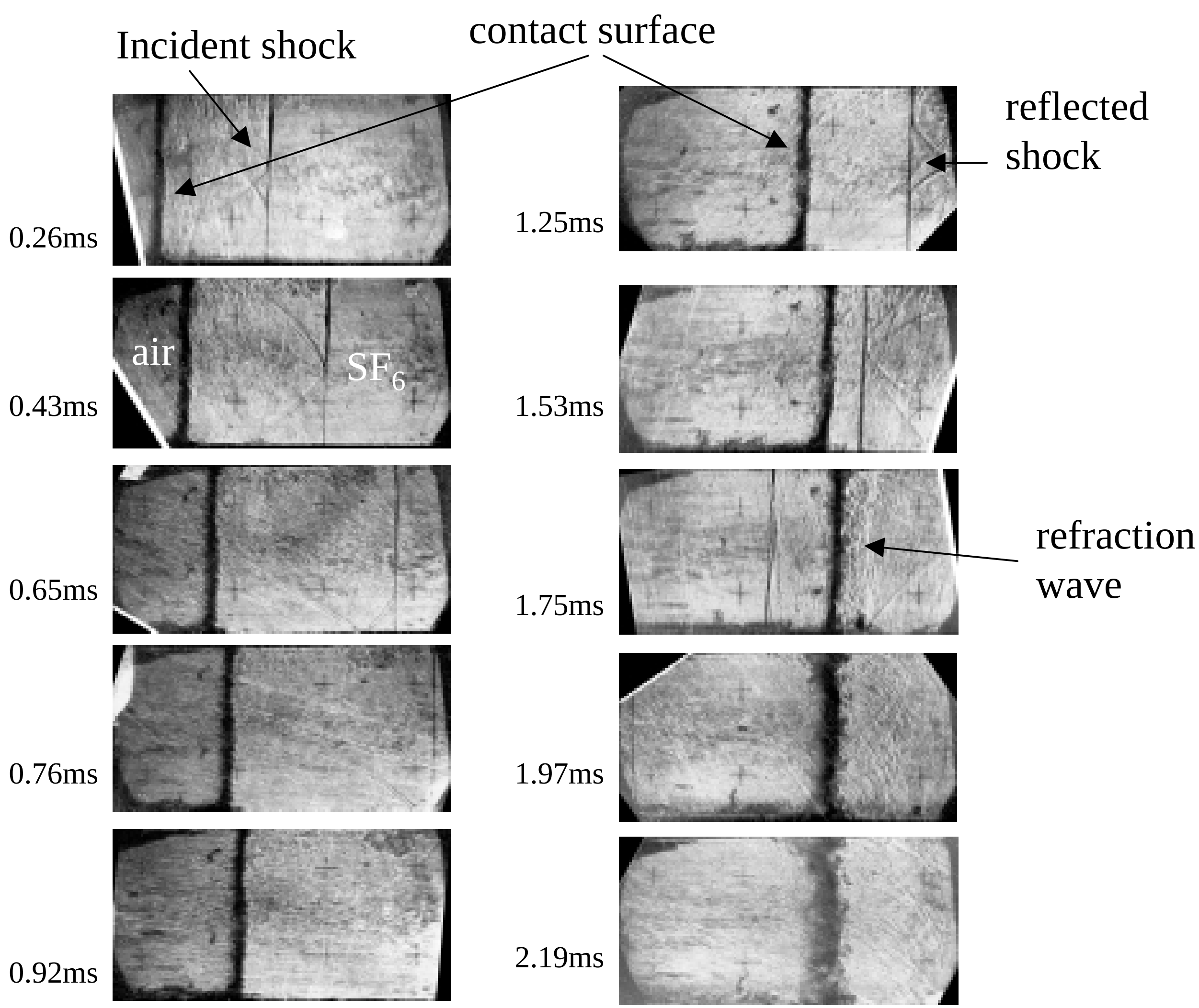
# Shock tube experiments



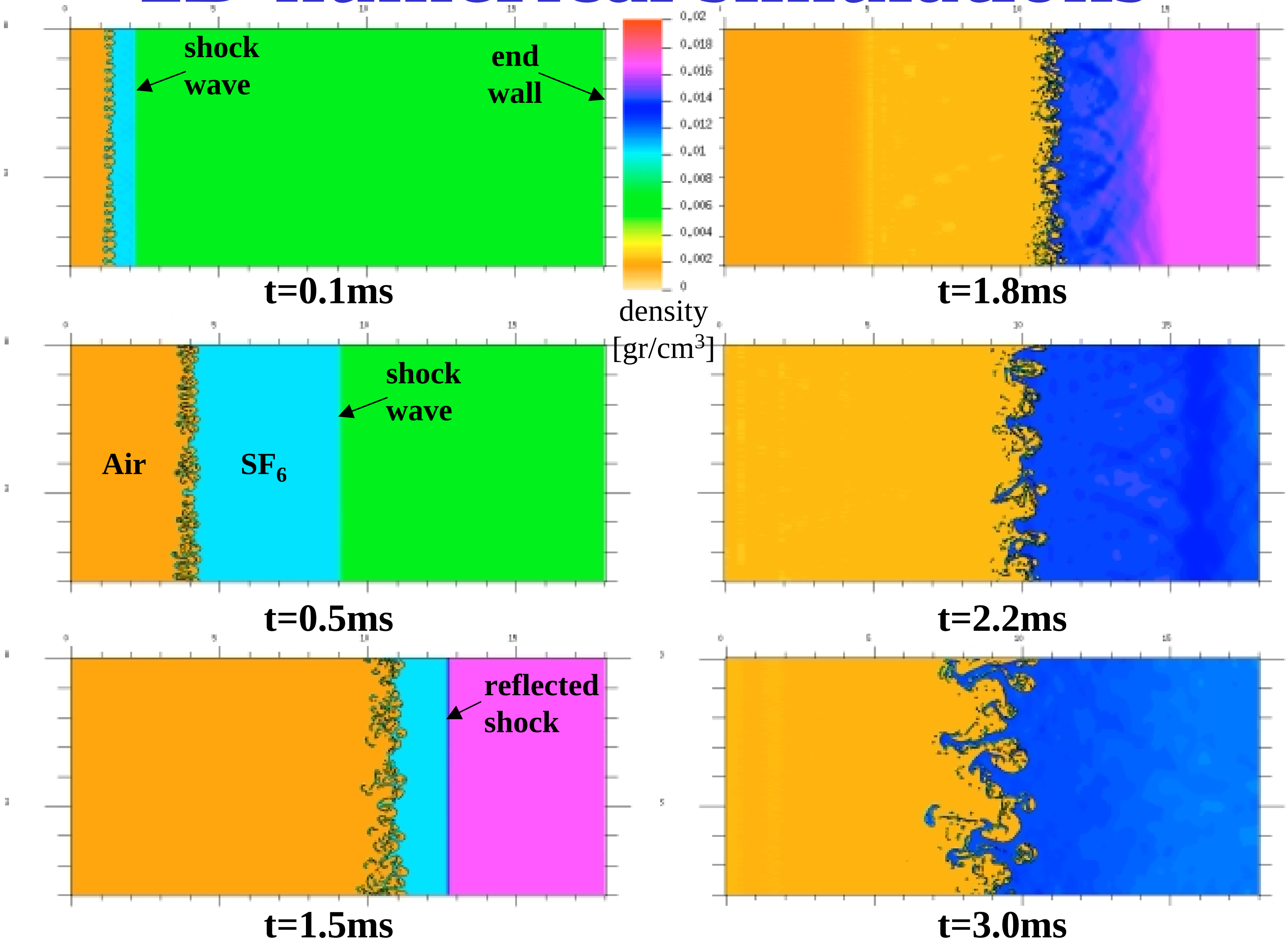




**Mach=1.2**

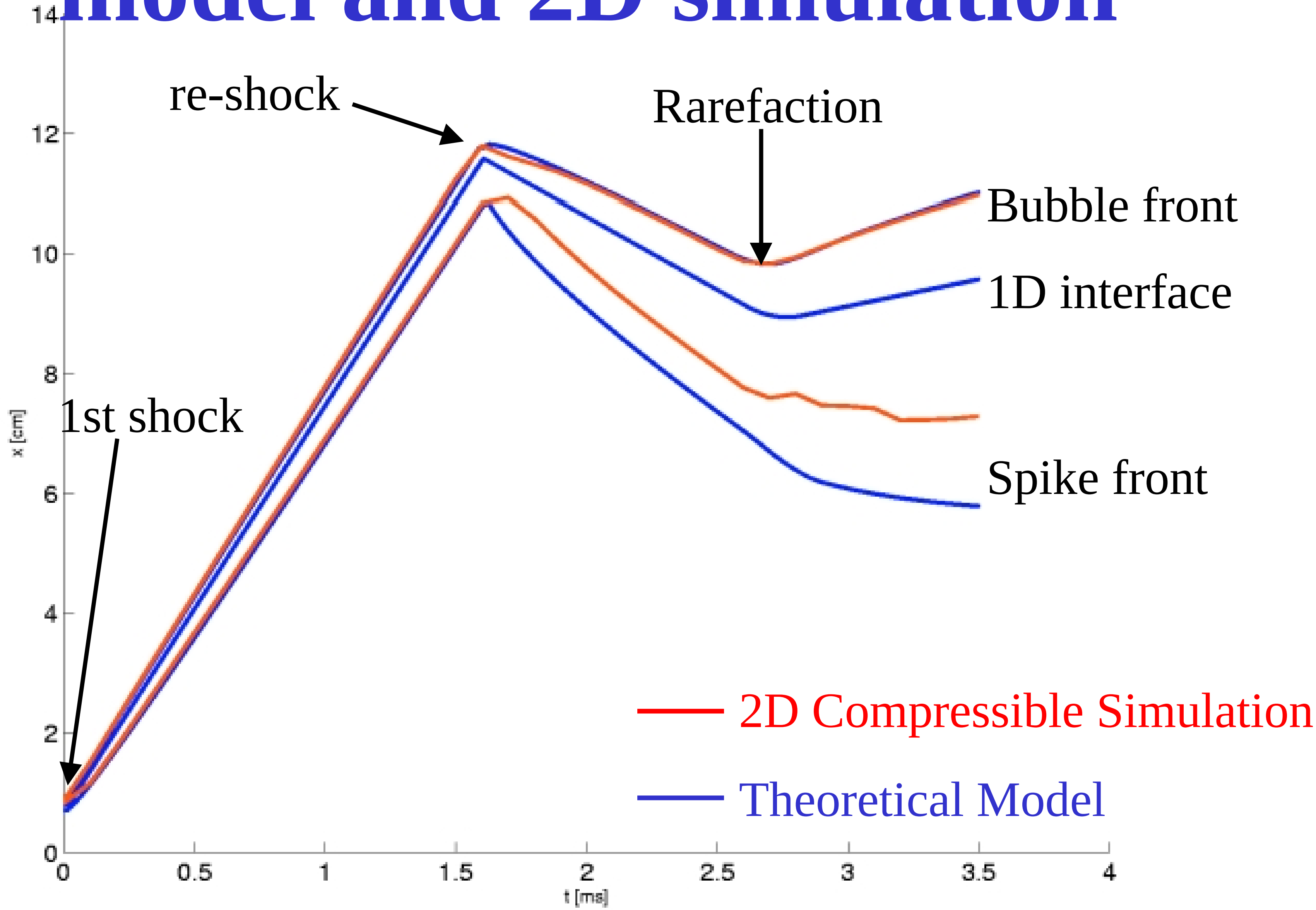
# Experimental results
## (random initial conditions)



# 2D numerical simulations

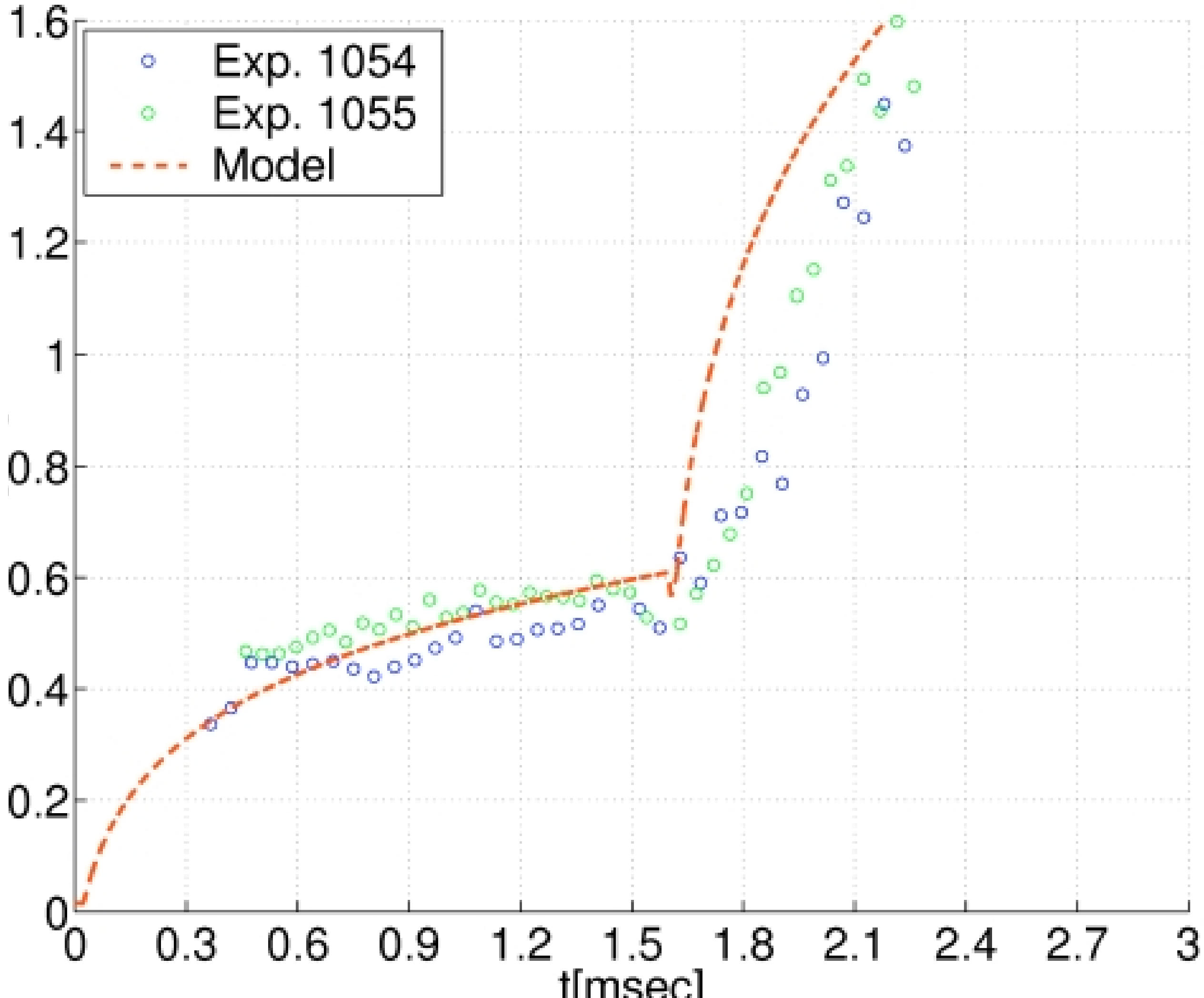


# Good agreement between mix model and 2D simulation



# Model agrees with experimental results

mix region [cm]



# Summary

• Layzer model and buoyancy-drag equation have been combined to describe all instability stages for all Atwood numbers and a general acceleration profile.

• Multi-mode spectrum is described by one characteristic wavelength.

• 1D compressibility and scale change effects are introduced through Lagrangian “drift” of the mixing zone boundaries and by time dependant Atwood number.

• Model results have been compared to experiments and to full 2D numerical simulations.

• Non-planar geometry may be introduced by modifying characteristic wavelength.