Modeling Radiation Effects in Mixing Layers

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Background/Goals

- Models of Rayleigh-Taylor and Richtmyer-Meshkov mixing layers are relatively sophisticated.
  - Multiphase flow models, turbulent models, hybrid models…
- Additional physical phenomena must be included for many practical circumstances.
  - Material strength, heat flux, evaporation….
- Goal: To develop a simple model to describe the effects of radiative heat transfer and ablation on a turbulent mixing layer.
  - Emphasis is on simplicity--fidelity will be assessed by comparison to experiment or simulations, possibly motivating additional complications.
  - Competition between instability growth rate and ablative growth rate; depends on the initial scale of the perturbations.

Strategy/Problem Formulation

- Multi-material flow formulation.
  - Appropriate for multi-material problems...
- Simple drag model for multi-field interaction.
  - May include more sophisticated models later, as needed.
- Radiation diffusion approach.
  - For simplicity...
- Simple heat transport model.
  - Heat exchange occurs in a thin “skin” of the cold material.
  - Heat transfer to cold material leads to ablation, not temperature increase.
  - Ablated “cold” material becomes “hot fluid” (e.g. melting ice in water etc.)
  - Cold material and hot fluid experience PdV work (and temperature change).

Heat Transfer Model

- Prescription for area for heat transfer per unit volume
  - Uniform distribution of "spheres" of varying sizes.
  - (Similar to simple spherical-particle model for multi-fluid drag models?)
- Quasi-steady state for local heat flux at particle surface.
- Model:

$$Q = \left(\frac{12\pi}{7}\right) \frac{\alpha_1 \alpha_2 \kappa(T_2)}{\rho_2 r^2 \left[1 - \left(\frac{3\alpha_1}{4\pi}\right)^{1/3}\right]} \left(T_2^4 - T_1^4\right)$$

- We also have an equation for r ("radius"):

$$\frac{\partial r}{\partial t} + u_{1n} \frac{\partial r}{\partial x_n} = -\frac{r}{3\rho_1} \left\{ \frac{d\rho_1}{dt} + \frac{1}{\alpha_1} \left(\frac{Q}{C_{v1} T_{c1} + L_{H1}} \right) \right\}$$

Two-Field Continuity Equation

- Cold material (1) is heated and becomes hot fluid (2).

$$\frac{\partial \rho_k \alpha_k}{\partial t} + \frac{\partial \rho_k \alpha_k u_{kn}}{\partial x_n} = (-1)^k \frac{Q}{C_{v1} T_{c1} + L_{H1}}$$

- Material exchange rate is related to heat transfer rate, Q.
  - More complicated model could be incorporated, involving temperature increase and then "evaporation."

- Adjustable rate base on $C_{v1}\,T_{c1} + L_{H1}$

Momentum Equation

- Momentum equation
  - Changes in momentum enter through pressure and changes in mass fractions.

$$\frac{\partial \rho_k \alpha_k u_{ki}}{\partial t} + \frac{\partial \rho_k \alpha_k u_{ki} u_{kn}}{\partial x_n} = -\alpha_k \frac{\partial P}{\partial x_i} + \bar{\rho} \kappa_D \left(u_{k'i} - u_{ki} \right)$$

- Drag coefficient is based on a simple spherical particle drag model (courtesy of B. Kashiwa, T-3)(Note Re is very large…).
  - We need additional guidance from simulation or experiment to improve this model for ablating materials.

$$\kappa_D = \frac{3C_D}{8r} \frac{\alpha_1 \alpha_2}{\rho} \left| u_1 - u_2 \right| \qquad C_D = 0.44 + \frac{24.0}{\mathrm{Re}} + \frac{6}{1 + \mathrm{Re}^{1/2}}$$

Equations of State (present example…)

- Simple equation of state for sample calculation.
  - Model does not require a particular thermodynamic EOS.
  - May need better EOS for comparison to experiments.
- Hot fluid (2) has radiation pressure contribution.
  - May require radiation pressure and heat transfer/temperature increase in cold material if T_c is much hotter than T_1.
- Pressure equilibration between phases.
  - Restrictive assumption-- presents difficulties when temperature difference is very large….

$$p_1 = c_{v1}(\gamma_1 - 1)\rho_1 T_1 \qquad p_2 = c_{v2}(\gamma_2 - 1)\rho_2 T_2 + \frac{4\sigma}{3c} T_2^4$$

$$I_1 = c_{v1} T_1 \qquad I_2 = c_{v2} T_2 + \frac{4\sigma}{\rho_2 c} T_2^4$$

Energy Equation

- Energy equations

$$\frac{\partial \rho_1 \alpha_1 I_1}{\partial t} + \frac{\partial \rho_1 \alpha_1 u_{1i} I_1}{\partial x_n} = +P_1 \frac{\alpha_1}{\rho_1} \frac{d\rho_1}{dt}$$

$$\frac{\partial \rho_2 \alpha_2 I_2}{\partial t} + \frac{\partial \rho_2 \alpha_2 u_{2i} C_{v2} T_2}{\partial x_n} = \frac{\partial}{\partial x_n}\left\{\frac{\alpha_2}{\rho_2}\kappa_2\left(T_2\right)\frac{\partial T_2^4}{\partial x_n}\right\} + P_2 \frac{\alpha_2}{\rho_2} \frac{d\rho_2}{dt} - Q$$

- Opacity: $\mu\left(T_2\right) = \mu_0\left(\frac{T_0}{T_2}\right)^3 + \mu_{scattering}$
- Transmissivity: $\kappa\left(T_2\right) = \frac{1}{\mu\left(T_2\right)}$

Turbulence Model?

- Current formulation does not have a detailed model for turbulence.
  - Need more comparison to simulations/experiments for motivation
- We could add hybrid multi-phase model.
  - Cranfill’s hybrid model, Youngs model etc.
- Modification of turbulence model for ablated materials?
  - Fluctuating velocity is not solenoidal, pressure fluctuations tied to radiation (i.e., opacity….), and material transfer, et cetera.
- Such modifications would require simulations and experiment for guidance...

Sample Problem

- Problem is statistically one-dimensional.
- Hot fluid (2) is 1.5 keV, cold fluid (2) is 0.2 keV
- Boundary conditions:
  - Hot side (left) is constant temperature, no mass flux or velocity.
  - Cold side (right) is simple out-flow boundary (dp/dx = 0).
- Pressure equilibration in mixed zone requires some artful choices.
  - Assume that pressure in mixing zone has a smooth transition from hot side to cold side.

- Choice of constants based on iron properties (and expedience)...

Sample Problem Parameters

- Constants:

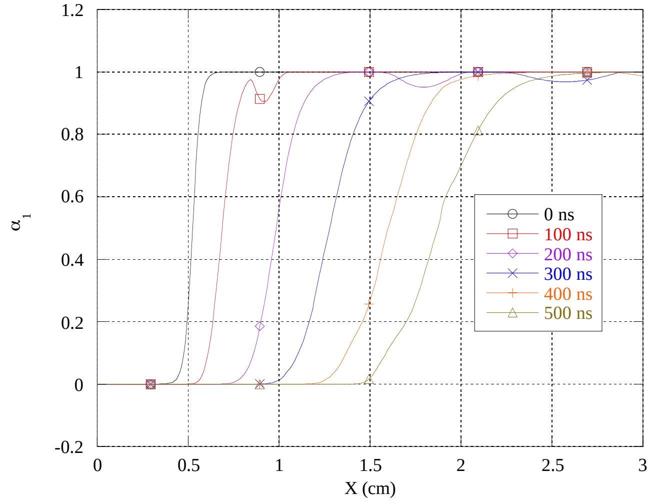
$$\Sigma_0 = 10^4 \frac{cm^2}{gm}, \qquad \Sigma_{scatter} = 0.2 \frac{cm^2}{gm}$$

$$C_{v1} = C_{v2} = 6.856 \times 10^{-3} \frac{j}{gm \;\; keV}$$

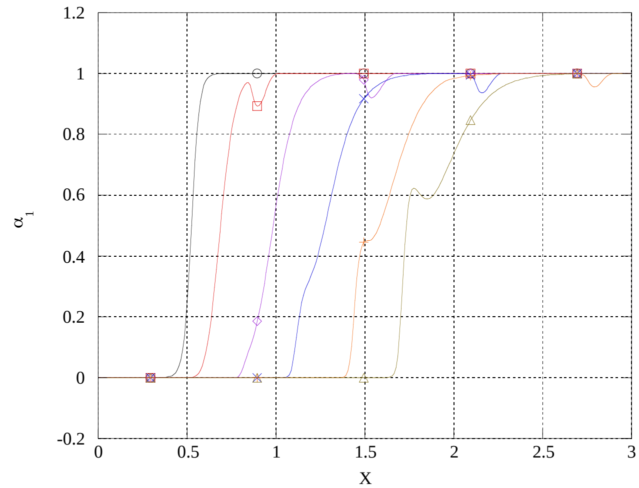
$$\gamma_1 = \gamma_2 = 0.6667$$

$$T_c = 0.2 \;\; keV, \qquad L_H = 0.2 \frac{j}{gm}$$

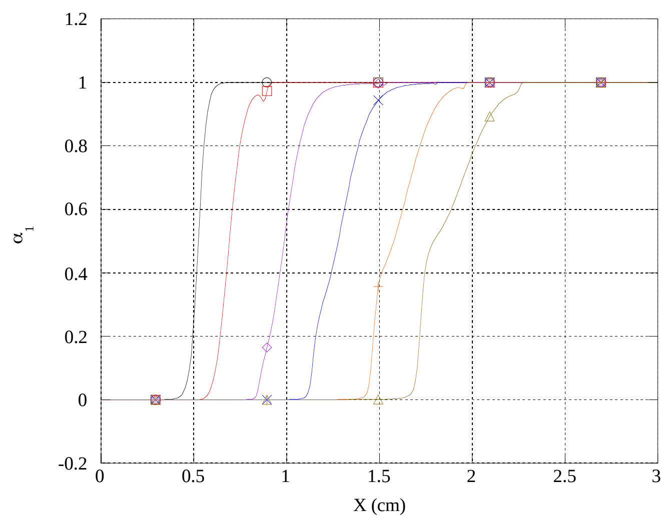
$$\text{Pressure is in} \left(\frac{j}{cm^3} \right), \quad \text{Energy, } I_k \text{ is in} \left(\frac{j}{gm} \right)$$



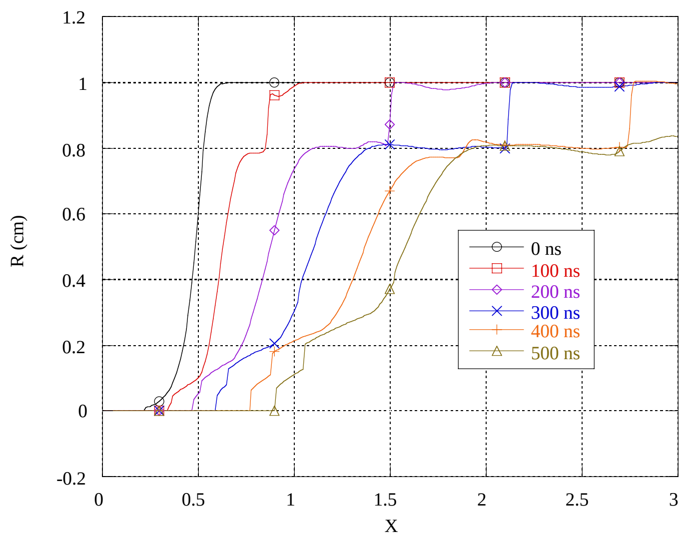
Volume Fraction of Cold Fluid
R_0=1.0 cm



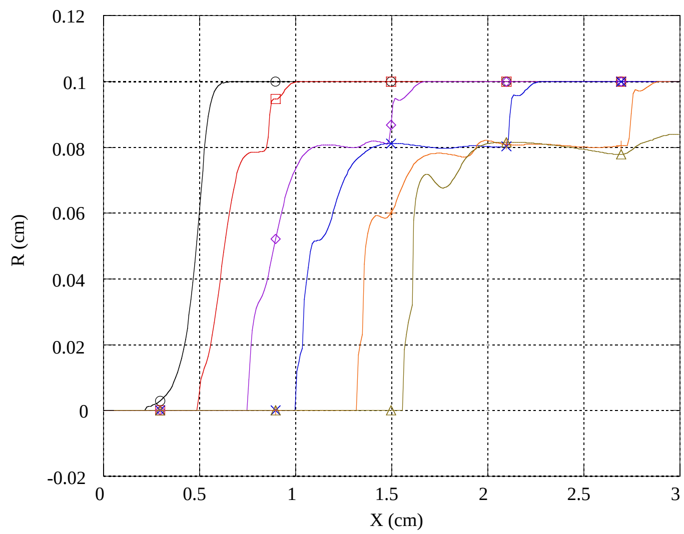
Volume Fraction of Cold Fluid
R_0=1.0x10^{-1} cm



Volume Fraction of Cold Fluid
R_0=1.0x10^{-2} cm



Radius of Characteristic "Droplet"
R_0 = 1.0 cm



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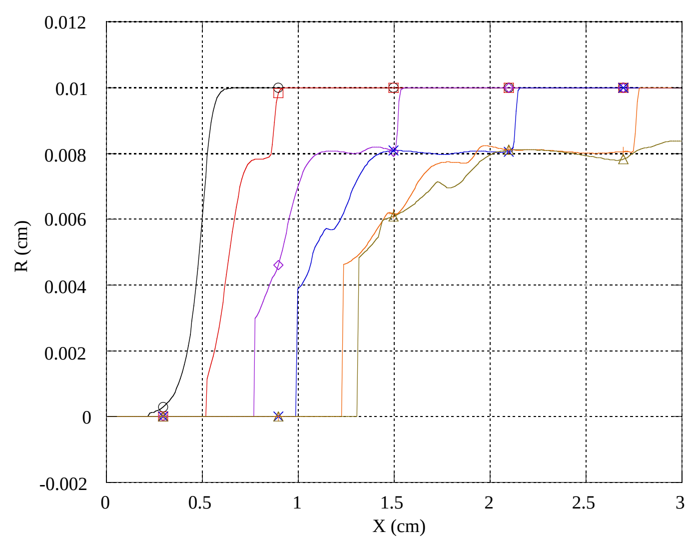

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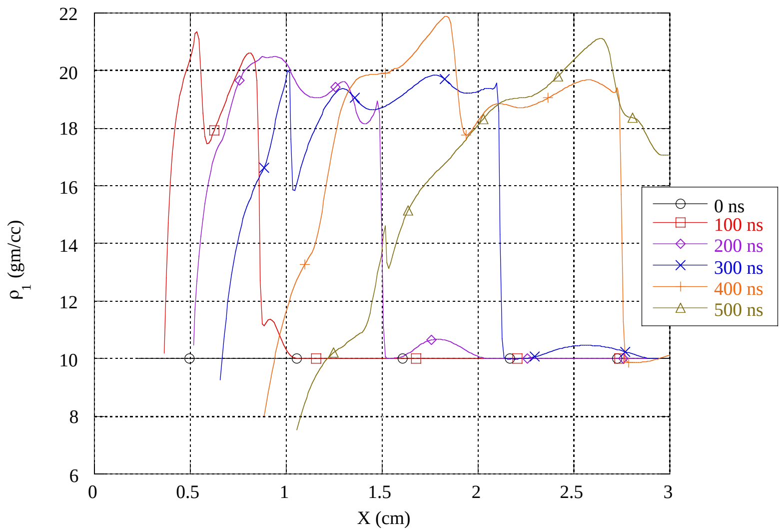
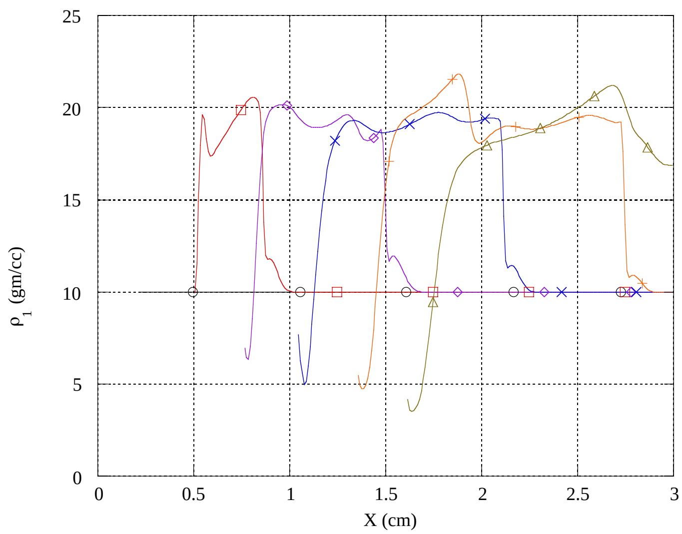


Density of Cold Fluid
R_0=1.0 cm



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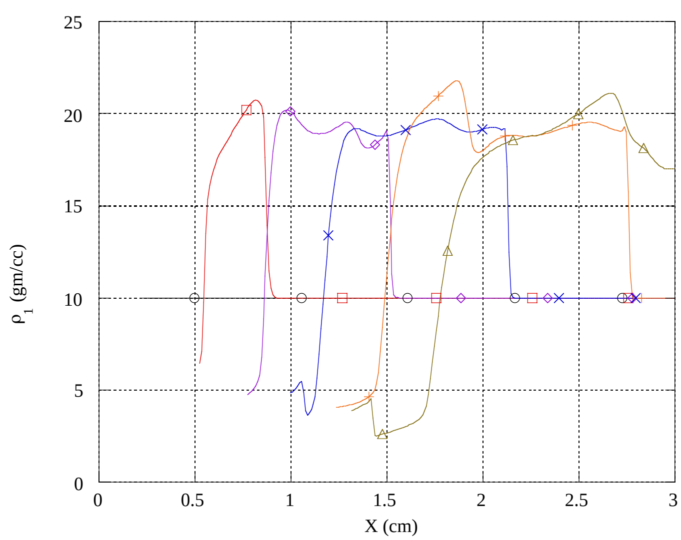

Density of Cold Fluid
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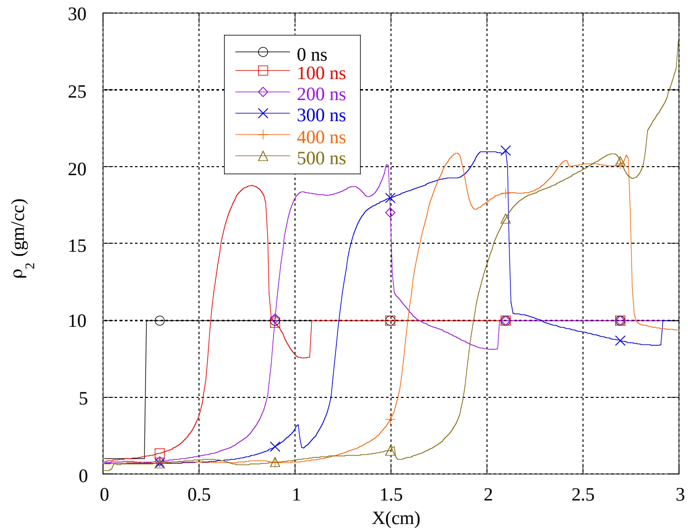
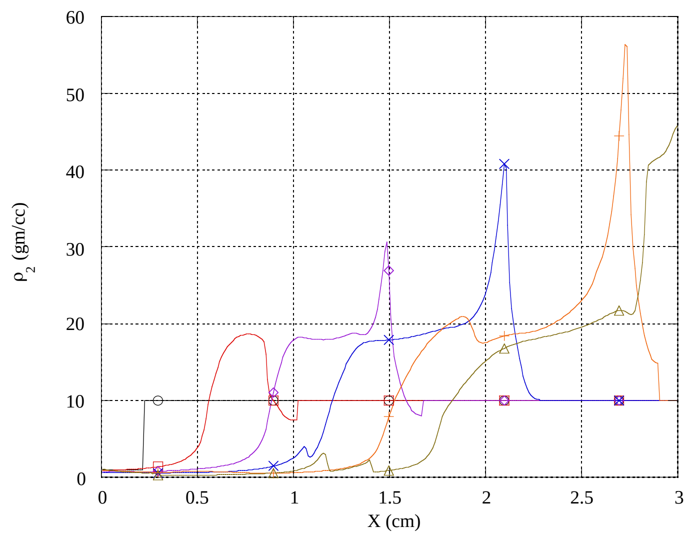


Density of Hot Fluid
R_0 = 1.0 cm



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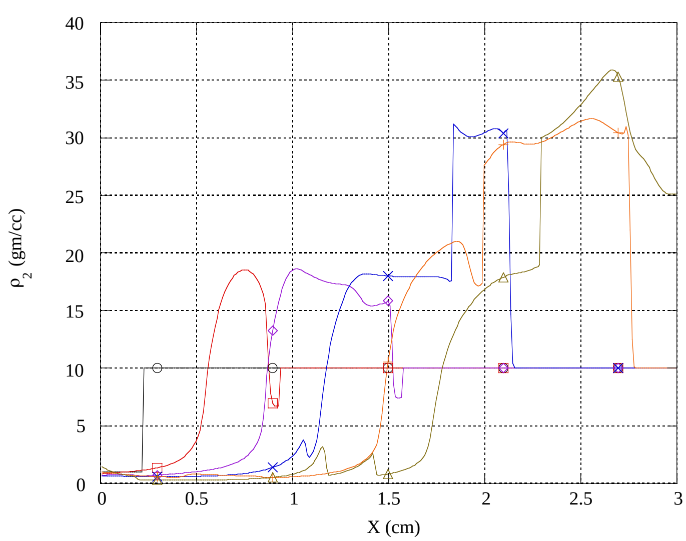
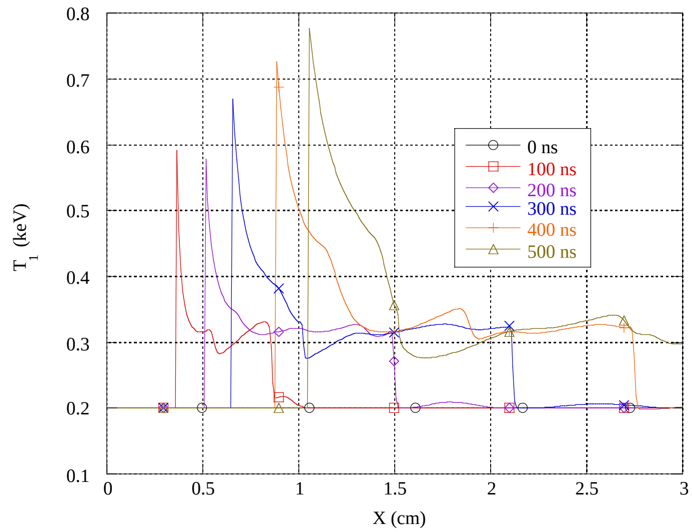
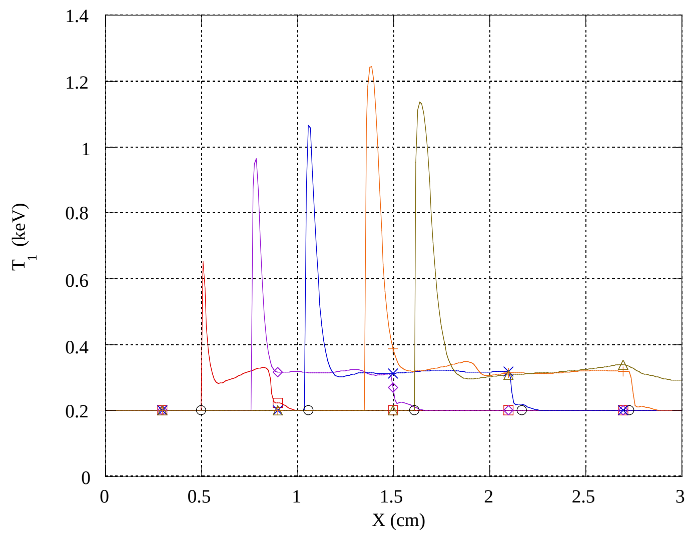


Density of Hot Fluid
R_0 = 1.0x10^{-2} cm



Temperature of Cold Fluid

R_0 = 1.0 cm



Temperature of Cold Fluid

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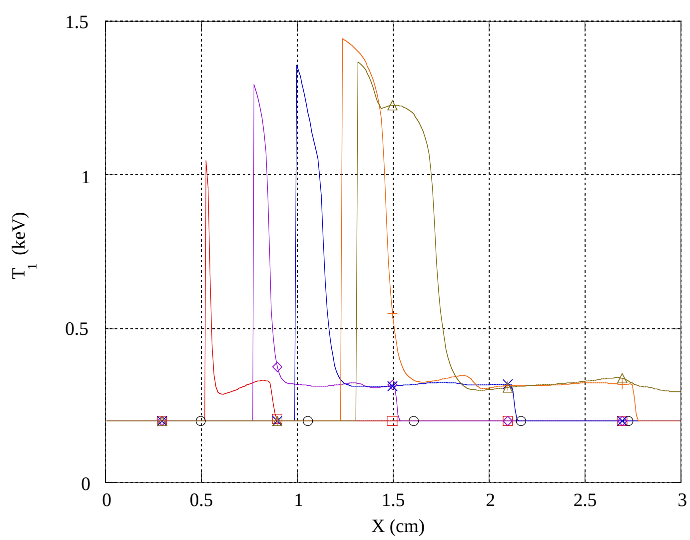
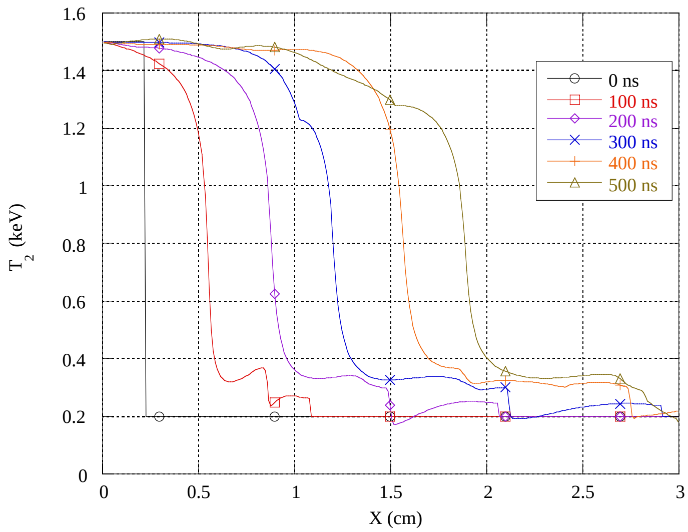
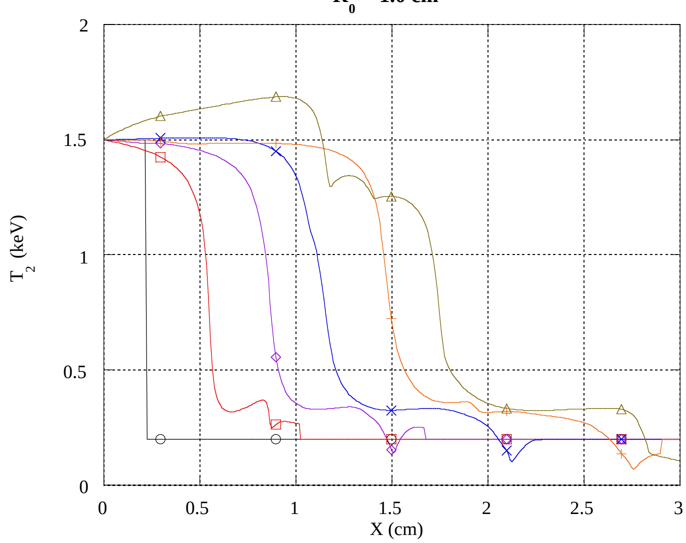


Temperature of Cold Fluid

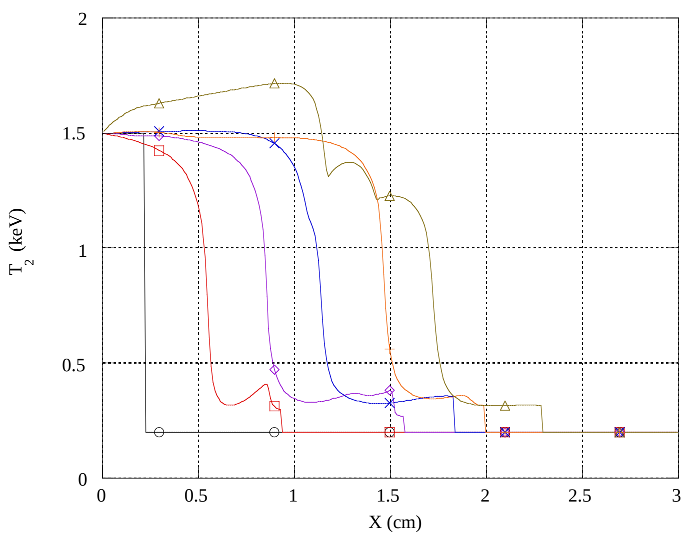
R_0 = 1.0x10^{-2} cm



Temperature of the Hot Fluid
R_0 = 1.0 cm



Temperature of the Hot Fluid
$R_0 = 1.0\times10^{-1}$ cm



Temperature of the Hot Fluid
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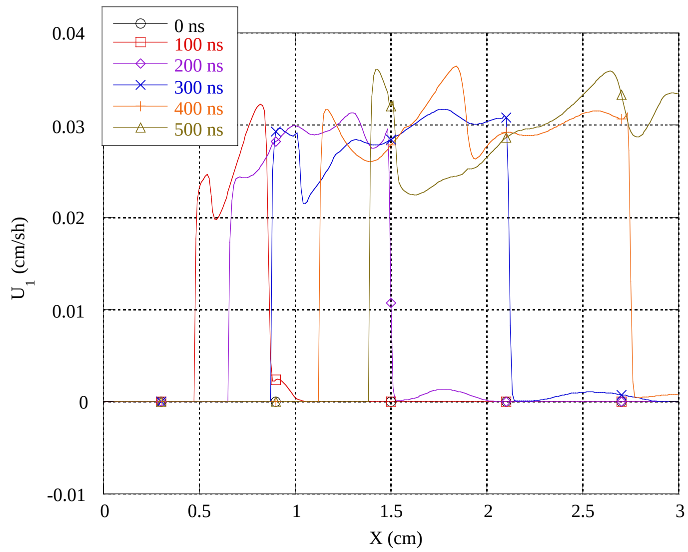
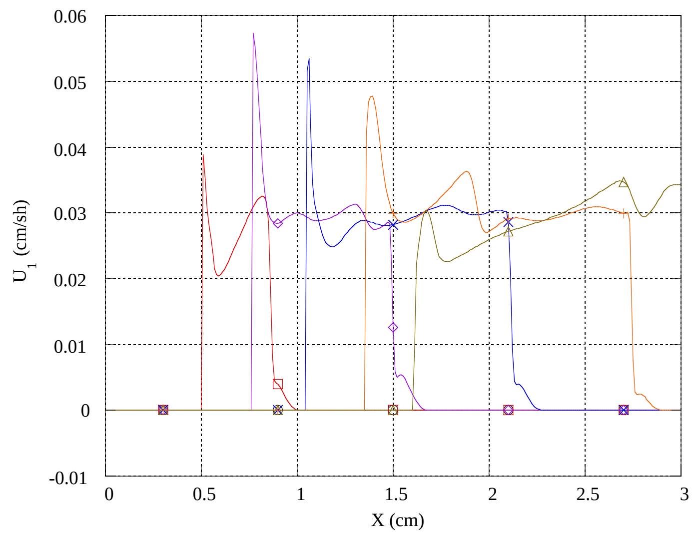


Velocity of Cold Fluid
R_0 = 1.0 cm



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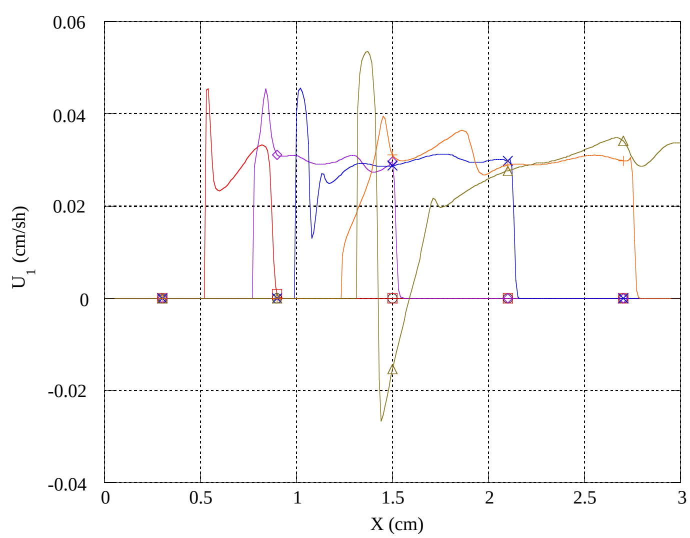
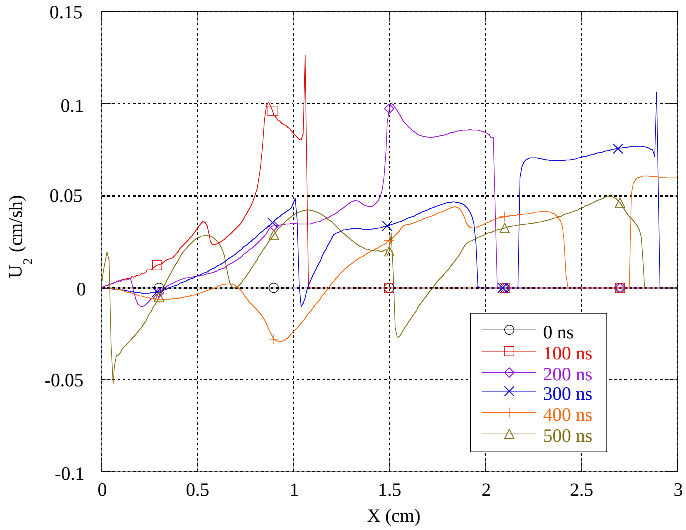


Velocity of Cold Fluid
R_0 = 1.0x10^{-2} cm



Velocity of Hot Fluid
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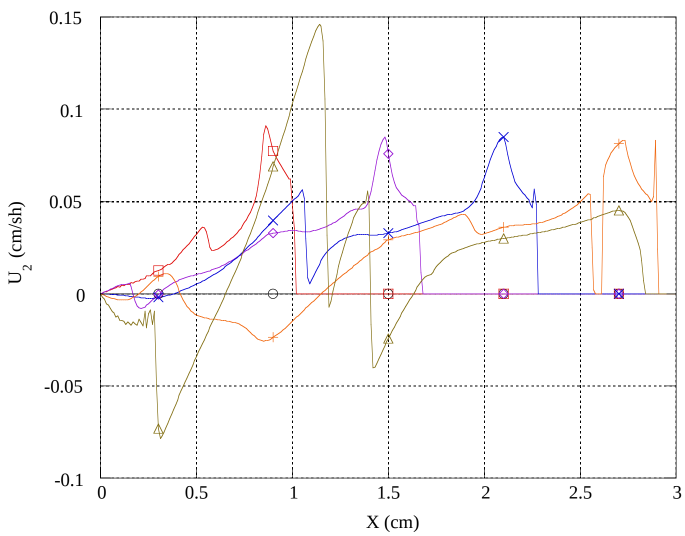

Velocity of Hot Fluid
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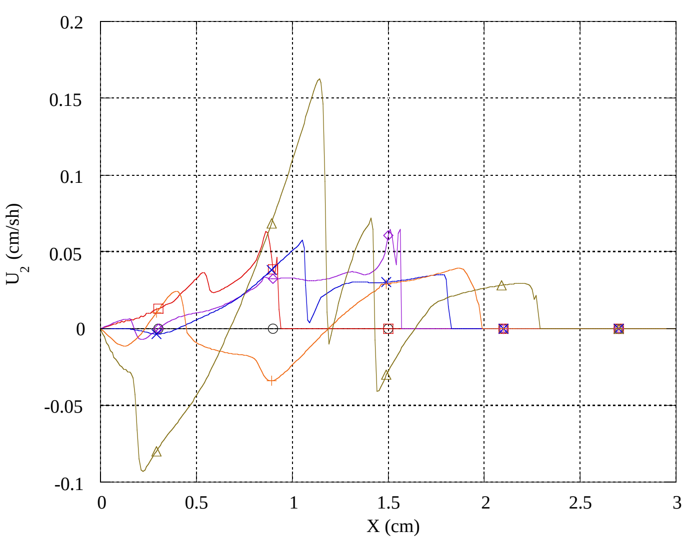

Velocity of Hot Fluid
$R_0 = 1.0\text{x}10^{-2}$ cm

Conclusions

- Model represents a simple, tractable approach to account for radiation and phase change in a two-phase flow.
  - Length scale for radiation transfer (spherical model) is consistent with the drag model.
  - Can be extended to multiple fluids, and more complicated prescriptions for drag, turbulence, heat exchange.
- Demonstrates ablative phenomenon.
  - Have not fully explored parameter regimes.
- We now need detailed comparisons to experiments, simulations and observations.
  - Laser-driven flyer plates, Cepheid variables, Computer simulations.