How to inhibit Rayleigh-Taylor mixing

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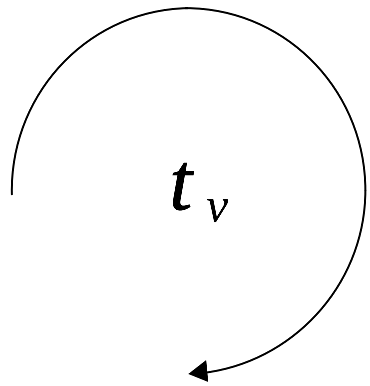
Self-similar flow



Self-similarity means the next rotation period is always proportional to the current one.

$$\tau_v(t + \tau_v(t)) = \text{const.}\ \tau_v(t).$$

In other words, the fractional decrease in rotation period per rotation is a constant, independent of time,

$$[\tau_v(t) - \tau_v(t + \tau_v(t))]/\tau_v(t) = \text{const.} = \beta.$$

Proof that $t_v(t)$ must linear if self similar

Assume

$$\tau_v(t) = \tau_v(0) + a_1 t + a_2 t^2 + \ldots a_n t^n.$$

Then

$$[-a_1(\tau_0+a_1t+a_2t^2+\ldots a_nt^n) + \text{terms up to } t^{n2}]/(\tau_0+a_1t+a_2t^2+\ldots a_nt^n) = \beta.$$

Satisfied iff n^2-n=0. So n = 0 or 1.

Thus $a_j = 0$ for all j >1, and $a_1 = -\beta$.

The only possible self-similar evolution is $\tau_v(t) = \tau_v(0) - \beta t = \tau_0 - \beta t$.

Example - Inertial subrange

One eddy 'turnover time'

$$\tau_\lambda = \lambda/v_\lambda = \lambda^{2/3}/e^{1/3} = (\lambda/\delta)^{2/3}\,\tau_\delta$$

Time interval from scale λ to $(\lambda - d\lambda)$

$$dt = \text{const.}\lambda^{2/3}d\lambda/(e^{1/3}\lambda)$$

(Kulkarny & Broadwell)

Total elapsed time to reach scale λ

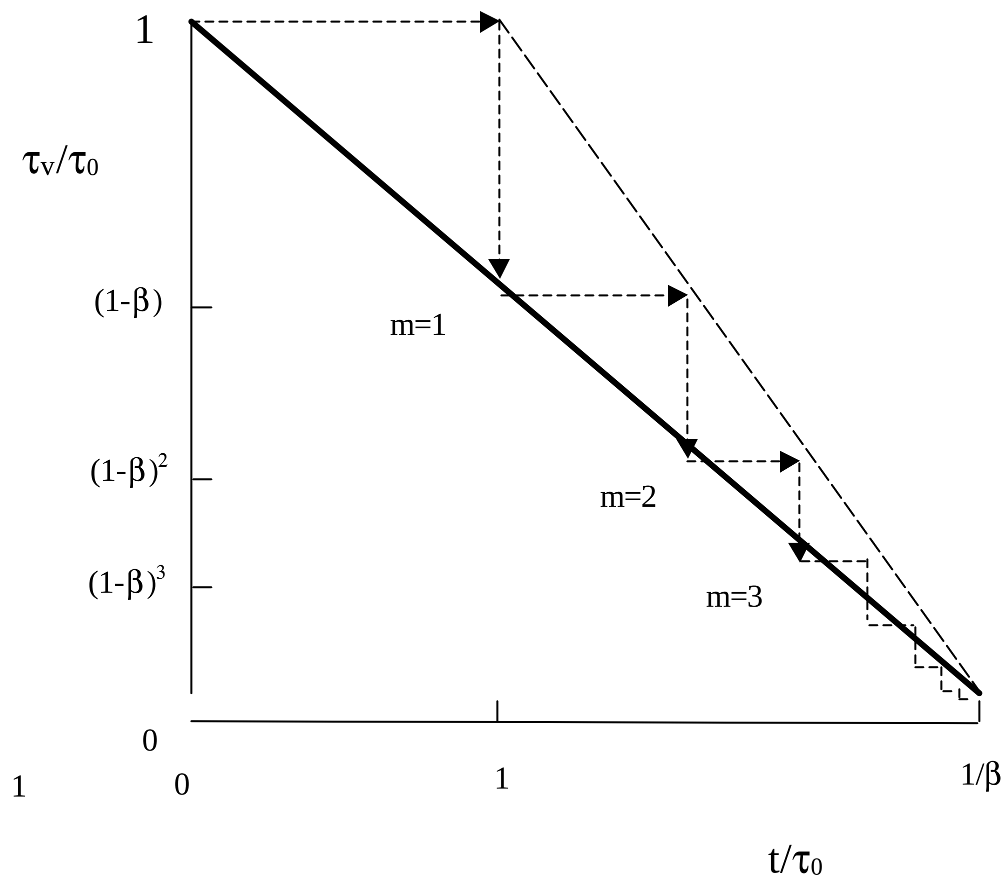
starting from scale δ is

$$t(\lambda) = [1 - (\lambda/\delta)^{2/3}]\,\tau_{\delta},$$

so

$$\tau_\lambda/\tau_\delta = 1 - t(\lambda)/\tau_\delta$$ ... linear

in t



Vortex rotation period $\tau_v(t)$ for $\beta > 0$

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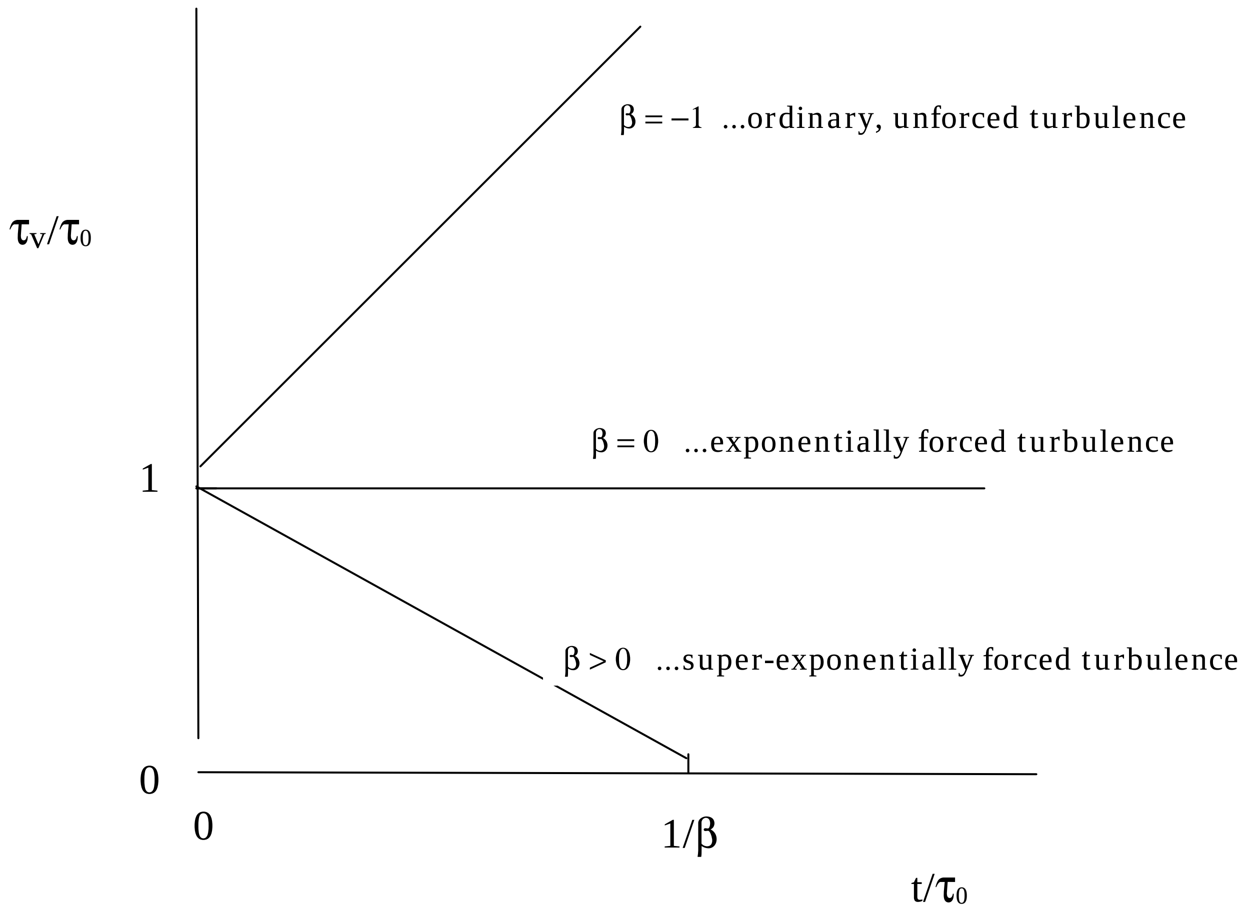


Vortex rotation period $\tau_v(t)$ for all self-similar flows

Vortex stretching

$$Dw/Dt = (Ñ\mathbf{u})\ w$$

Symmetry arguments imply

$$mag((Ñ\mathbf{u})\ w) = c\ \omega^2$$

$$D\omega/Dt = c\ \omega^2.$$

$$\omega(t)/\omega(0) = 1/(1 - ct/\tau_v(0)),$$

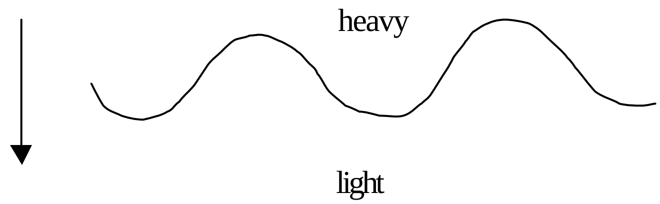
identical to

$$\tau_v(t)/\tau_v(0) = 1 - \beta t/\tau_v(0)$$

for $\omega = 1/\tau_v$ and $c = \beta$.

Rayleigh-Taylor

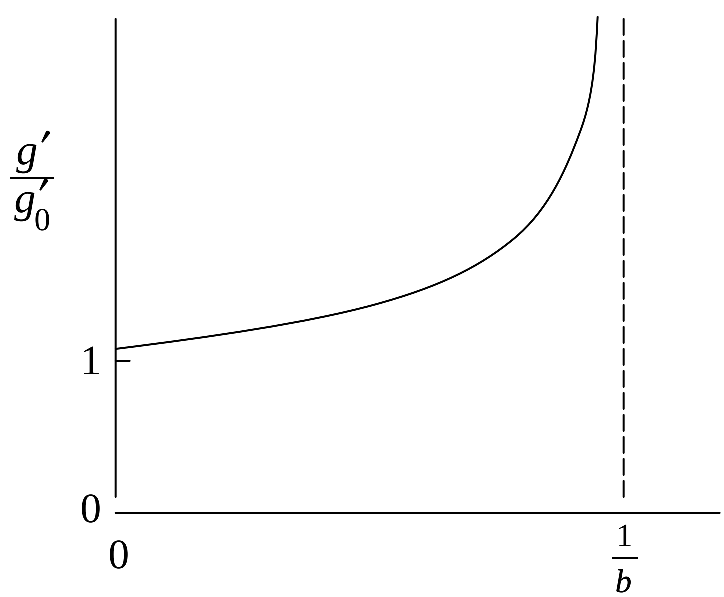
$$g' \equiv \frac{\Delta r}{\bar{r}} g$$



Impose an e-folding time scale $\tau_v(t) = \tau_0 - \beta t$.

So force with "super-exponential"

acceleration $$\frac{g'}{g'_0} = e^{\left(\frac{t}{t_0 - bt}\right)}.$$



$\frac{t}{t_0}$

How to inhibit Rayleigh-Taylor entrainment?

Instead of constant acceleration, use super-exponential, e.g.

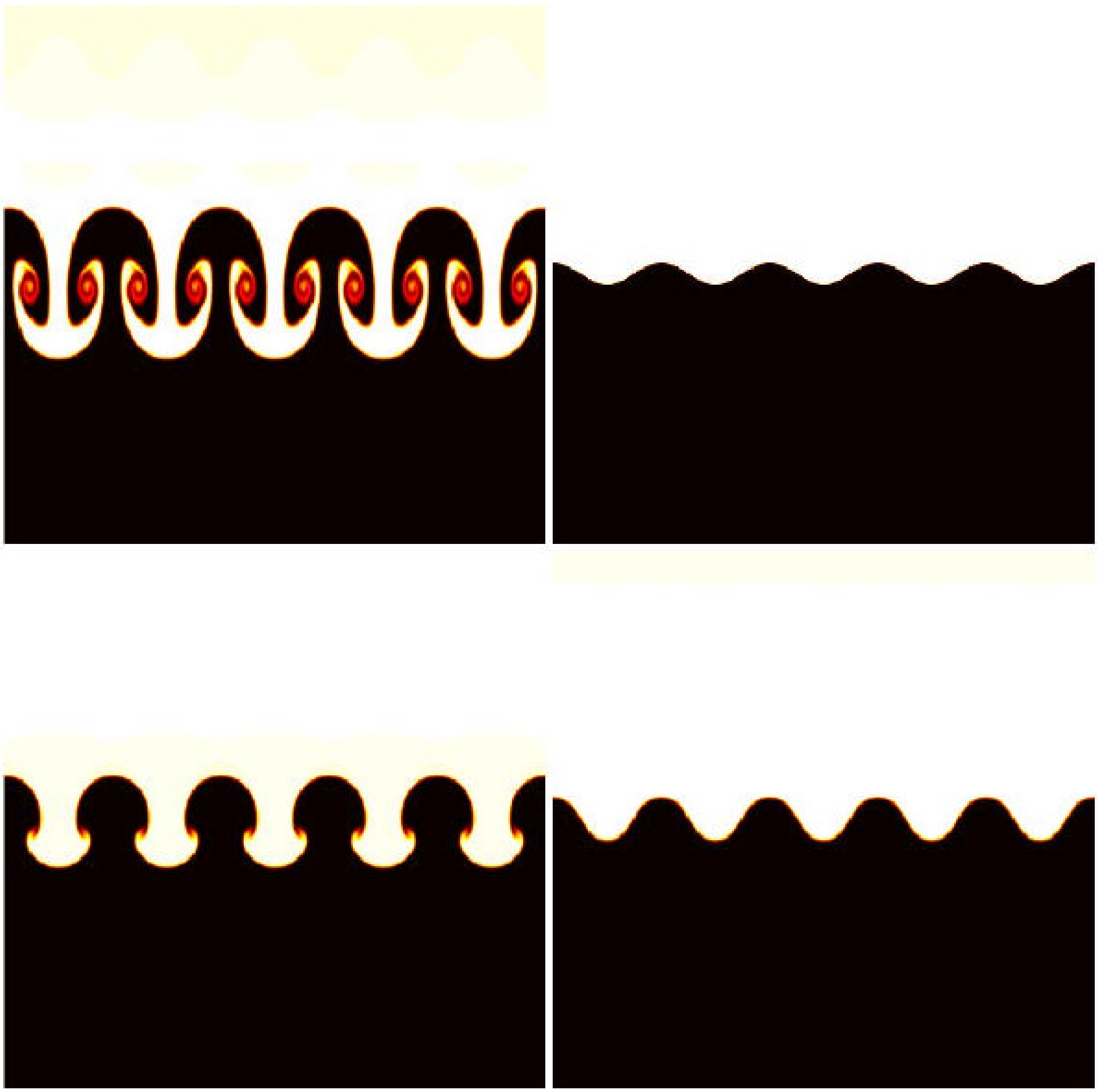
$$\frac{g'}{g'_0} = \exp\left(\frac{t}{t_0 - bt}\right).$$

As $b \to 1$, entrainment is inhibited for all perturbation wavelengths $\lambda > g_o'\tau_o^2$.

2D inviscid simulations

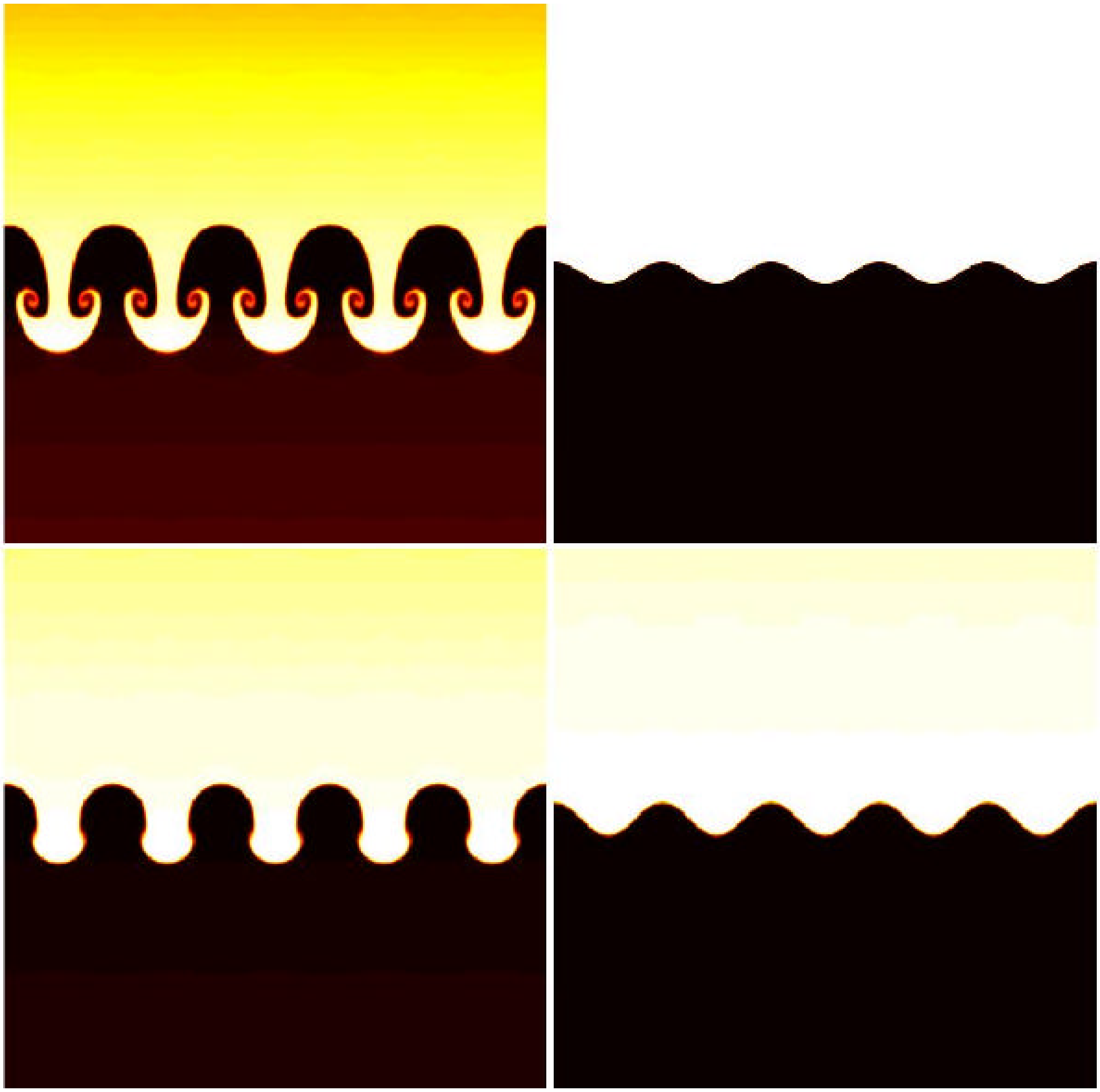
Christian Anitei

Randy Leveque's CLAWPACK code
(suggested by Hamid Johari)
Derek Bale
James Rossmanith

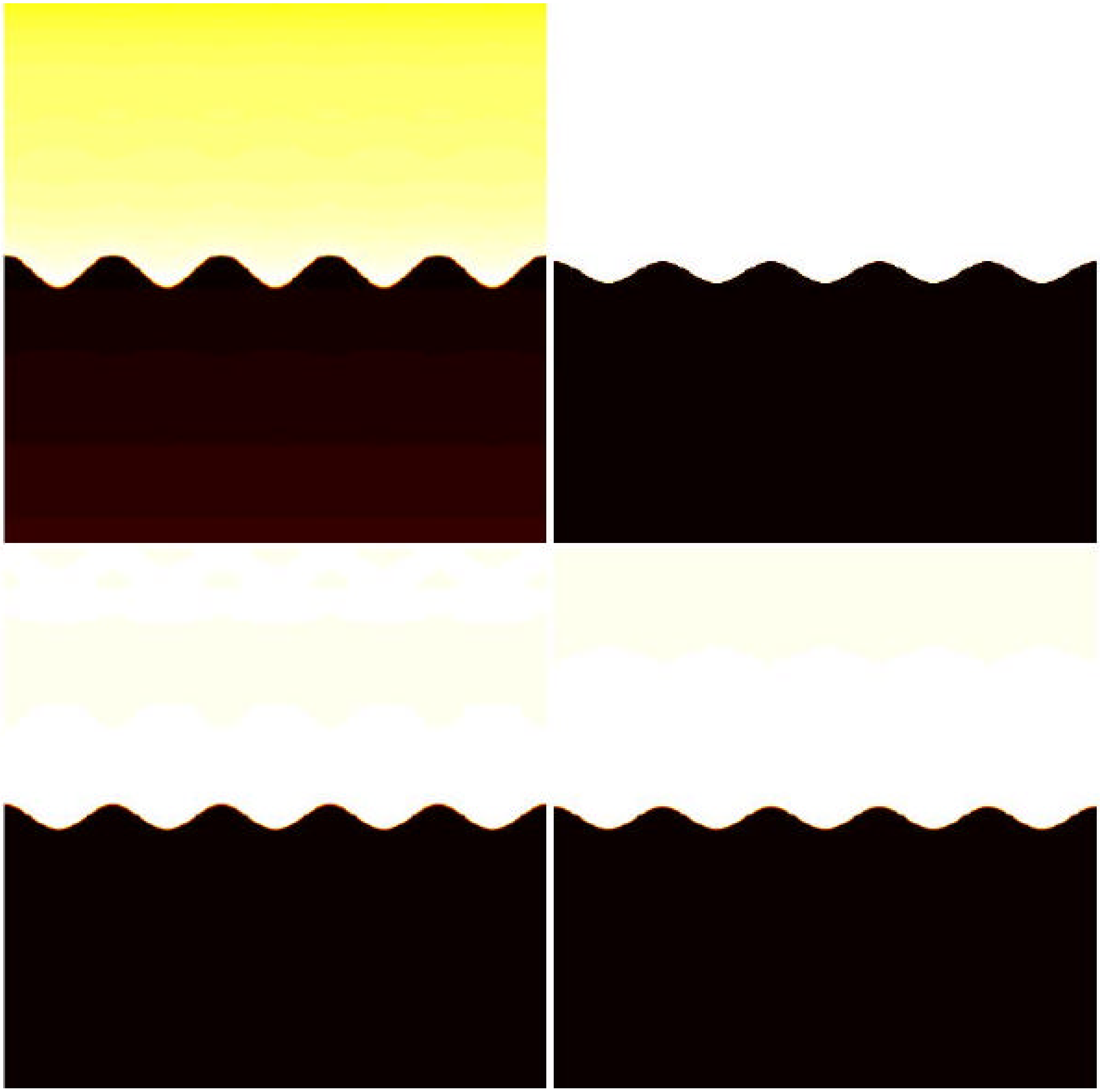


$$\beta = -\infty$$

Time progresses clockwise from upper right

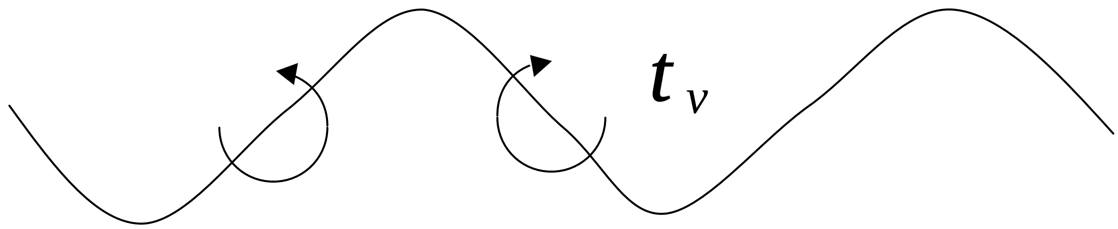


$\beta = 0$



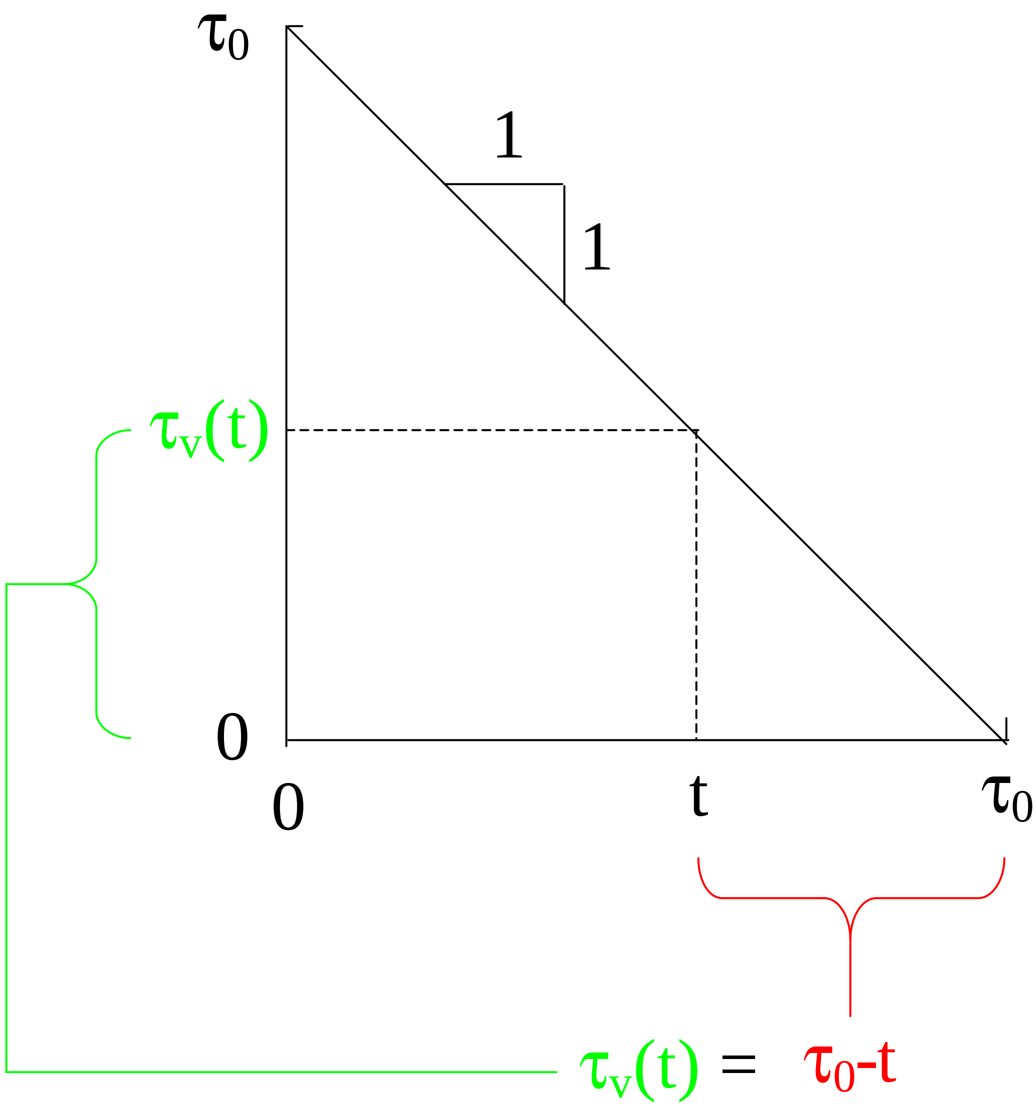
$\beta = 1$

Physical interpretation



It takes time for a vortex sheet to roll up. If at every instant, the required roll-up time is just equal to the remaining available time (β=1), there can be no roll up. Even more that this, the acceleration provides a stabilization.

$\beta = 1$



At every instant, the current rotation period is just equal to the remaining time.

Conclusions

- For all self-similar turbulence, the vortex rotation period is a linear function of time,

  $\tau_v = \tau_0 - \beta t$.

- Entrainment and dissipation are inhibited as $\beta \rightarrow 1$.

- For β=1, Rayleigh-Taylor entrainment is suppressed for all wavelengths

  $\lambda > g_0'\tau_0^2$.