A New Turbulent Two-Fluid RANS Model for KH, RT and RM Mixing Layers

Pascal BAILLY, Antoine LLOR

Commissariat à l'Energie Atomique
B.P.12 91680 Bruyères-le-Châtel Cedex

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Abstract

Our aim is to develop an accurate turbulent mixing model for combined RT, RM and KH types of instabilities, with arbitrarily variable accelerations.

Following the recent analysis of the RT and RM cases by G. Dimonte 2000, and of the self-similar variable acceleration RT flows (SSVARTs) by A. Llor in an other presentation to the present workshop, we have considered as crucial to capture the following physical aspects by using the corresponding model features:
– the directed transport by a two-fluid approach,
– the correct buoyancy force by including mass transfer between the fluids,
– the turbulence diffusion by including most of the standard k-e features,
– the geometrical aspects by consistent closures of the length scales.

Identifying the large scale transport structures and including the above physical aspects, we developped the TTT model (Two-structure Two-fluid Two-turbulence) whose specific and original features will be discussed. Preliminary 1D numerical results of the TTT model will be presented for self-similar RT flows.

I Introduction

I.1 Aim and strategy

Our aim is to craft a RANS model applicable to:

- Kelvin-Helmoltz,
- Richtmyer-Meshkov,
- Rayleigh-Taylor,
- and SSVARTs (self-similar variable g RT, see Llor)

and able to capture consistently:

- TMZ growth laws ($L(t)$),
- global energy balance (K_I, K_D, K_T),
- and length scales
- (Von Kármán numbers $K^{3/2}/EL$, KH≈ 0.63, RT≈ 0.09).

Our strategy is to combine as much as possible
the relevant features from simple and efficient existing models:

Turbulence dynamics (KH and RM)	$\longleftrightarrow$	k–ε model,
Directed transport (RT, SSVARTs)	$\longleftrightarrow$	Two-fluid model (Youngs 1989),
Buoyancy (RT, SSVARTs)	$\longleftrightarrow$	Mass exchange (Youngs 1995).

The model has to be a closure of RANS equations
to permit calibrations using measured correlations
$\longrightarrow$ avoid a phenomenological length equation.

I.2 Starting point

Young's two-fluid model (1989) as starting point:

- mass conservation, $\alpha^{\pm}\rho^{\pm}$,
- 2 eq. momentum conservation, $\alpha^{\pm}\rho^{\pm}U^{\pm}$,
- 1 eq. turbulent kinetic energy, K,
- 1 eq. characteristic length scale, λ,

with main closures:

- drag $\propto (\delta U)^2/\lambda$,
- turbulent integral length scale $\propto \lambda$,
- length scale production term $\propto \|\delta\vec{U}\|$.

Problems:

- no equation for dissipation, ε,
- no extension of turbulence to pure fluids,
- debatable closures and coefficients,
- λ equation not RANS based,
- singular behavior of λ at TMZ edges,
- overestimates Von Kármán number in RT by factor 6.5.

I.3 Preliminary attempt, k–ε–Σ

Ideas:

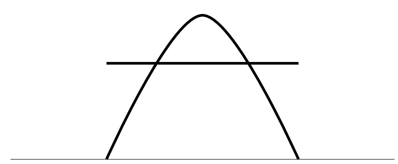
- add ε equation "as in k–ε",
- replace λ by $\Sigma \approx \alpha^+\alpha^-/\lambda$, interfacial area density (RANS),
- add interaction between turbulent and geometrical lengths.

$\longrightarrow$ "k–ε–Σ" model
(Vallet 2000)

Failed because of incompatible behavior at TMZ edges:

$\lambda_i \to 0$ for standard k–ε,

$\lambda_d \to$ Constant in usual two-fluid models.



Schematic mean profiles of drag and integral length scales in typical TMZ.

However, raised the basic issues:

What are the transported/transporting structures in TMZ ? What mechanisms control the length/drag of the structures ? How can length singularities be avoided at edges of TMZ ?

II The TTT model

II.1 The transport structure concept

Large structures ≈ Large turbulent eddies ≈ Bubbles ≈ Plumes ≈ ... ≈ Basic building block for transport and mixing.

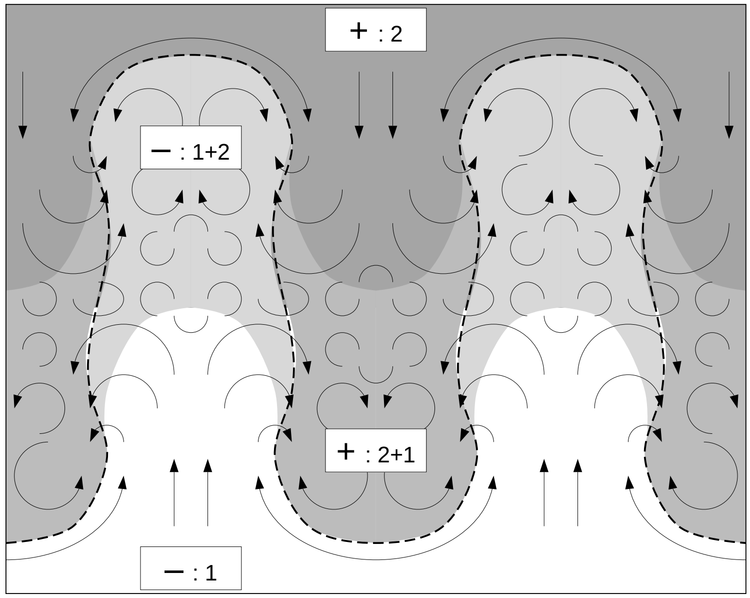
Assumption holds because:
- separation of fluids inside large eddies cannot occur (small size, large drag, high turbulence),
- experimentally, bubble sizes match Von Kármán number, (Dimonte 2000...),
- in simulations, visual correlation of turbulence and density, (Dalziel 1999, Inogamov 2001...).

Solves length scale paradox at edges:
$\lambda_i/\lambda_d \to$ Constant in structures at edges,
but $\langle\lambda_i\rangle \to 0$ because of intermittency
(structures at edges enter laminar zones when TMZ grows).

Simple modeling should be accessible
since plumes are appropriatly captured by adapted k–ε models.

Averaging over the TMZ, modeling demands:
- two velocities (plume shearing),
- mass exchange (entrainment in plumes),
- two turbulence fields (intermittency between plumes).

II.2 Two-structure RANS equations



Schematic representation of the Two-structure, Two-fluid, Two-turbulent TMZ;
dashed line: $b^{\pm}$ boundary; gray shading: $c^{1,2}$ levels.

Structures + and − defined by presence functions $b^{\pm}$ (=0 or 1).

Structure evolution defined by velocity field $\vec{w}$ for $b^{\pm}$:

$$\frac{\partial}{\partial t} b^{\pm} + w_j \, b^{\pm}_{,j} = 0. \qquad (1)$$

Evolution equations for hydro quantities a:

$$\frac{\partial}{\partial t}(\rho a) + (\rho a u_j)_{,j} = s - (\theta_j)_{,j}, \qquad (2)$$

where $a = c^{1,2}$ (fluid mass fractions), u_i, k, ε, e...

$c^1 = b^-$ and $c^2 = b^+$ in initial state (separate fluids).

Conditional averaging by $b^{\pm}$ yields two-structure RANS :

$$\frac{\partial}{\partial t}(\alpha^{\pm}\rho^{\pm}A^{\pm}) + (\alpha^{\pm}\rho^{\pm}A^{\pm}U_j^{\pm})_{,j} = S^{\pm} - (\Theta_j^{\pm})_{,j} - (\Phi_j^{\pm})_{,j} \mp X \mp \Psi \qquad (3)$$

where:

$\alpha^{\pm} = \overline{b^{\pm}}$	structure volume fractions,
$\rho^{\pm} = \overline{b^{\pm}\rho}/\alpha^{\pm}$	structure densities,
$A^{\pm} = \overline{b^{\pm}\rho a}/(\alpha^{\pm}\rho^{\pm})$	structure quantities,
$U_i^{\pm} = \overline{b^{\pm}\rho u_i}/(\alpha^{\pm}\rho^{\pm})$	structure velocities,
$S^{\pm} = \overline{b^{\pm}s}$	structure source terms,
$\Theta_i^{\pm} = \overline{b^{\pm}\theta_i^{\pm}}$	structure fluxes,
$\Phi_i^{\pm} = \overline{b^{\pm}\rho a u_i^{\pm}}$	structure turbulent fluxes,
$u_i^{\pm} = u_i - U_i^{\pm}$	structure velocity fluctuations,
$\boxed{X = \pm\, \overline{b_{,j}^{\pm}\theta_j^{\pm}}}$	inter-structure flux term ($\rightarrow$ buoyancy + drag),

$\boxed{\Psi = \pm\, \overline{b_{,j}^{\pm}\rho a(w_j - u_j)}}$

inter-structure turbulent-flux and exchange term.

Closures of standard terms are adapted from usual single fluid approaches and not discussed here.

Closures of exchange terms X and Ψ are crucial.

Presently, $b^{\pm}$ is loosely defined to match the turbulent/laminar areas in flow.
Eventually, $b^{\pm}$ will be more precisely given by analysis of density/velocity fields in simulations.

However, closure of $b^{\pm}$-involving average terms is possible now.

II.3 TTT: the closed modeled equations

TTT = Two-structure, Two-fluid, Two-k–ε. "Minimal".

Incompressible version (extension to compressibility and shocks to be published).

Single pressure is assumed (two pressure version also possible, but weak effect).

$$\left\{\begin{aligned}
(\alpha^\pm\rho^\pm)_{,t} \quad + \quad (\alpha^\pm\rho^\pm U_j^\pm)_{,j} &= \mp\,\Psi \\
(\alpha^\pm\rho^\pm C^{m\pm})_{,t} + (\alpha^\pm\rho^\pm C^{m\pm} U_j^\pm)_{,j} &= -\,\Phi^{m\pm}_{j,j} \mp \Psi^m \\
(\alpha^\pm\rho^\pm U_i^\pm)_{,t} \;+\; (\alpha^\pm\rho^\pm U_i^\pm U_j^\pm)_{,j} &= -\,\alpha^\pm P_{,i} + \alpha^\pm\rho^\pm g_i \\
&\qquad - R^\pm_{ij,j} \mp D_i \mp M_i \mp X_i^U \mp \Psi_i^U \\
(\alpha^\pm\rho^\pm K^\pm)_{,t} \;+\; (\alpha^\pm\rho^\pm K^\pm U_j^\pm)_{,j} &= +\,\Pi^\pm + \xi^\pm\Pi^d + \chi^\pm\Pi^\Psi \\
&\qquad - \alpha^\pm\rho^\pm\varepsilon^\pm - \Phi^{K\pm}_{j,j} \mp \Psi^K \\
(\alpha^\pm\rho^\pm\varepsilon^\pm)_{,t} \;+\; (\alpha^\pm\rho^\pm\varepsilon^\pm U_j^\pm)_{,j} &= \\
&\qquad + C_{\varepsilon 1}\tfrac{\varepsilon^\pm}{K^\pm}(\Pi^\pm + \xi^\pm\Pi^d + \chi^\pm\Pi^\Psi) \\
&\qquad - C_{\varepsilon 2}\,\alpha^\pm\rho^\pm\,\tfrac{(\varepsilon^\pm)^2}{K^\pm} - \Phi^{\varepsilon\pm}_{j,j} \mp \Psi^\varepsilon
\end{aligned}\right. \tag{4}$$

Production of $K^\pm$ by structure shear, $\Pi^\pm$, drag dissipation, Π^d, and momentum exchange due to mass exchange, Π^Ψ. Weighing of productions on $\pm$ according to $\xi^\pm$ and $\chi^\pm$.

Single-fluid ε equation is assumed valid in each structure: $K^\pm$ production and dissipation mirrored with $C_{\varepsilon 1}$ and $C_{\varepsilon 2}$.

Reduces to standard k–ε for identical structures/fluids.

II.4 k–ε like closures

Turbulence characteristics:

$$\nu_t^\pm = C_\mu (K^\pm)^2/\varepsilon^\pm, \qquad \sigma^\pm = \varepsilon^\pm/K^\pm, \qquad \lambda^\pm = (K^\pm)^{3/2}/\varepsilon^\pm.$$

First gradient assumption for turbulent fluxes in structures:

$$\Phi_j^{A\pm} \overset{\mathrm{m}}{=} \alpha^\pm \rho^\pm \left(\frac{\nu_t^\pm}{\sigma_a} + D_W \right) A_{,j}^\pm + \text{correction for } \rho_{,j}^\pm, \tag{5a}$$

$$R_{ij}^\pm \overset{\mathrm{m}}{=} \alpha^\pm \rho^\pm \left[\frac{2}{3} (K^\pm + \nu_t^\pm U_{k,k}^\pm)\delta_{ij} - \nu_t^\pm (U_{i,j}^\pm + U_{j,i}^\pm) \right]. \tag{5b}$$

Turbulent kinetic energy production by Reynolds stresses:

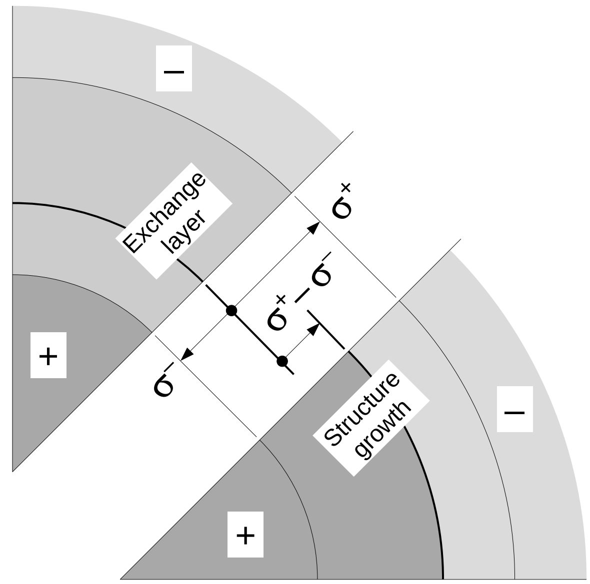
$$\Pi^\pm = -R_{ij}^\pm \,(U_{i,j}^\pm - U_{k,k}^\pm \delta_{ij}/3). \tag{6}$$

Conpensation of spurious production by turbulent pressure demands presence of momentum exchange term:

$$X_i^U = -\tfrac{2}{3} \left[\alpha_{,i}^+ (\alpha^+ \rho^+ K^+ + \alpha^- \rho^- K^-) \right. \\ \left. + (\alpha^+ \alpha^- (\rho^+ K^+ - \rho^- K^-))_{,i} \right] \tag{7}$$

here called turbulent buoyancy.

II.5 Mass exchange and structure growth closure



Schematic representation of inter-structure mass transfer and structure growth.

Closure of Ψ^a obtained by extension of Youngs' approach (1995).

Ψ^a is proportional to interface density $\approx \alpha^+\alpha^-$.
Affected volume fractions per time unit are proportional to $\sigma^\pm$.
Mixing layer thus defined is assumed homogeneous.
Rate of volume fraction transfer is proportional to $\sigma^+ - \sigma^-$.
Conserved quantities in layer are then attributed to $\pm$ structures.

Therefore:

$$\begin{aligned}
\Psi &\overset{\text{m}}{=} C_\psi\, \alpha^+\alpha^-\,(\xi^-\sigma^-\rho^+ - \xi^+\sigma^+\rho^-) && \text{(8a)}\\
\Psi^m &\overset{\text{m}}{=} C_\psi\, \alpha^+\alpha^-\,(\xi^-\sigma^-\rho^+C^{m+} - \xi^+\sigma^+\rho^-C^{m-}) && \text{(8b)}\\
\Psi_i^U &\overset{\text{m}}{=} C_\psi\, \alpha^+\alpha^-\,(\xi^-\sigma^-\rho^+U_i^+ - \xi^+\sigma^+\rho^-U_i^-) && \text{(8c)}\\
\Psi^K &\overset{\text{m}}{=} C_\psi\, \alpha^+\alpha^-\,(\xi^-\sigma^-\rho^+K^+ - \xi^+\sigma^+\rho^-K^-) && \text{(8d)}\\
\Psi^\varepsilon &\overset{\text{m}}{=} C_\psi\, \alpha^+\alpha^-\,(\xi^-\sigma^-\rho^+\varepsilon^+ - \xi^+\sigma^+\rho^-\varepsilon^-) && \text{(8e)}
\end{aligned}$$

where $\xi^\pm = \sigma^\pm\,/\,(\sigma^+ + \sigma^-)$.

Dissipation of directed energy by momentum exchange
results in total turbulent kinetic energy production:

$$\begin{aligned}
\Pi^\psi &= [\Psi_i^U - \Psi(U_i^+ + U_i^-)/2]\,\delta U_i\\
&= C_\psi\, \alpha^+\alpha^-\,(\xi^-\sigma^-\rho^+ + \xi^+\sigma^+\rho^-)\,\delta U_i \delta U_i/2, && (9)
\end{aligned}$$

distributed on $\pm$ structures according to $\xi^\pm$.

Matching the dissipation rate of density fluctuations
in homogeneous isotropic turbulence yields $C_\psi \approx 1$.

II.6 Drag closure

Analogy with Π^{ψ} suggest to choose drag as:

$$D_i \stackrel{\mathrm{m}}{=} C_d\,\alpha^+\alpha^-\,(\sigma^+\rho^+ + \sigma^-\rho^-)\,(\delta U_i - W_i), \tag{10}$$

where $C_d \approx 0.3$ for invicid bubble of size $2\lambda^{\pm}$ in viscous liquid.

Is not a Newton regime drag in $(\delta\vec{U})^2$!

Is a Stokes regime drag in $\delta\vec{U}$ because
structure size and turbulent viscosity yield Re ≈ 1.7.

Thus, drag length scale is implicit combination of λ^+ and λ^-.

Can be interpreted by assuming that
shear due to δU_i is evenly spread on structures,
where structure viscosities induce dissipation.

Total turbulent kinetic energy production by drag,
$\Pi^d = D_i \delta U_i$, spread on structures according to:

$$\chi^{\pm} = \sigma^{\pm}\rho^{\pm}\,/\,(\sigma^+\rho^+ + \sigma^-\rho^-). \tag{11}$$

Dispersion drift velocity is given by

$$W_i \stackrel{\mathrm{m}}{=} D_W \left(\frac{(\alpha^+\rho^+)_{,i}}{\alpha^+\rho^+} - \frac{(\alpha^-\rho^-)_{,i}}{\alpha^-\rho^-} \right). \tag{12}$$

Since $D_W \propto (\delta\vec{U})^2$, drag vanishes if $\delta\vec{U} = \vec{0}$.

II.7 Added mass closure

Various motivations for including added mass energy, K_A:
- significant in energy balance since $K_A \approx K_D$,
- important at TMZ edges (dominant in invicid laminar flow),
- may affect evolution in demixing and SSVARTs at $n \to -2$,
- makes $k^\pm - k_a$ and $(U^\pm)^2/2 + k_a$ continuous,
- represents large scale contribution to $k^\pm$ spectra,
- provides rationale of drag closure.

In two-field RANS equations, k_a is included in $k^\pm$,
and should be separated and modeled by evolution equation.

Here, algebraic closure will be assumed:

$$K_A \stackrel{\mathrm{m}}{=} C_a\, \alpha^+\alpha^-\,(\rho^+ + \rho^-)\,\delta U_i \delta U_i/2. \tag{13}$$

where $C_a \approx 0.5$ for bubbles in liquid.

Drag can now be interpreted as
dissipation of added mass energy in each structure.

An associated term in momentum equation can be found as:

$$M_i \stackrel{\mathrm{m}}{=} C_a\, \alpha^+\alpha^-\,(\rho^+ + \rho^-)\,\Big[\Big(\frac{\mathrm{d}^+}{\mathrm{d}t}U_i^+ - \frac{\mathrm{d}^-}{\mathrm{d}t}U_i^-\Big) - \Big(\alpha^+ U^+_{i,j} + \alpha^- U^-_{i,j}\Big)^S \delta U_j - \Big(\frac{\overline{\rho}_{,j}}{\rho^+ + \rho^-} + \alpha^+_{,j}\Big)\,\delta U_j \delta U_i/2\Big] \tag{14}$$

without $M_i \delta U_i$ dissipation in $K^\pm$ equations.

Will be rederived in near future from least action principle.

II.8 Diffusion from added mass velocity fluctuations

Added mass energy is contained in velocity fluctuations
which induce diffusion, described by D_W:

$$D_W \stackrel{\mathrm{m}}{=} C_W \, \alpha^+ \alpha^- \frac{(\delta \vec{U})^2}{\alpha^+ \sigma^- + \alpha^- \sigma^+} \tag{15}$$

D_W obtained assuming:

- characteristic velocity fluctuations are $\|\delta \vec{U}\|$,
- characteristic "interaction time" is average of $\sigma^\pm$,
- average is weighed by less present structure (avoids singularities at TMZ edges).

Added mass diffusion dominates turbulent diffusion,
increases transport slightly, and damps elliptic instabilities.

II.9 Summary of constants

Standard k–ε						TTT extension			
C_μ	σ_c	σ_k	σ_ε	$C_{\varepsilon 1}$	$C_{\varepsilon 2}$	C_ψ	C_d	C_a	C_W
0.09	0.7	1.0	1.3	1.47	1.92	0.8	0.15	0.17	3

C_ψ, C_d, C_a and C_W ajusted on RT to match respectively:

- molecular mixing fraction Θ (Youngs 1995),
- growth rate coefficient $\mathcal{Y}_0$,
- estimated added mass energy in internal shear,
- damping of elliptical modes (Youngs 1989...).

III Results of TTT on SSVARTs

III.1 Numerical conditions

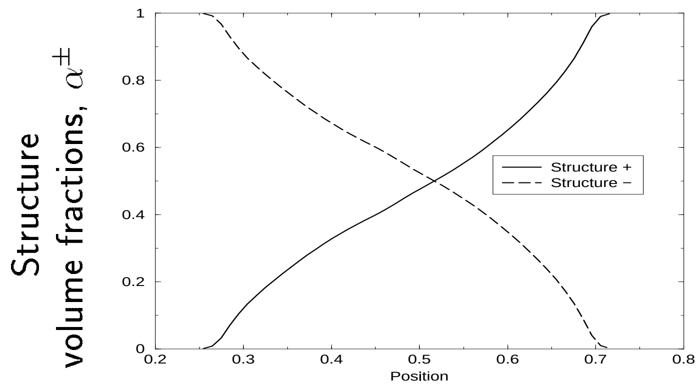
Used 1D all-purpose test code :

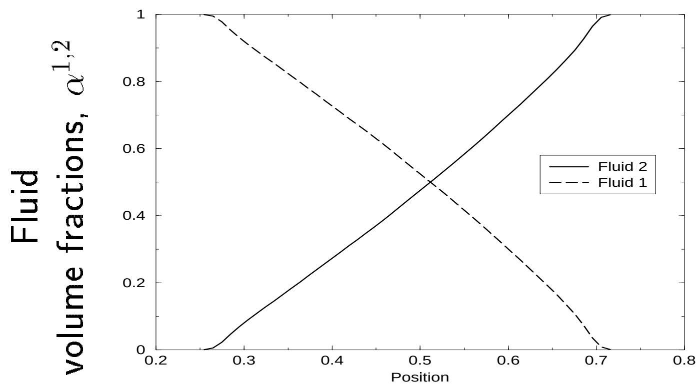
compressible, artificial viscosity,
explicit, no splitting, second order RK2,
bi-lagrangian + TVD projection (ALE),
non-diffusive evolution of TMZ edges,
(P.-H. Cournède 2001)

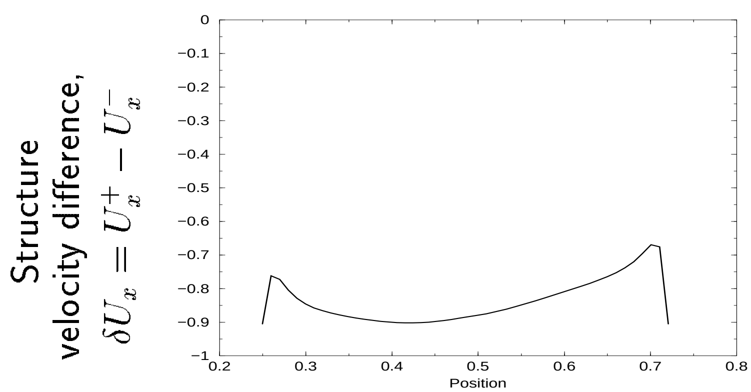
TTT model adapted for compressibility effects.

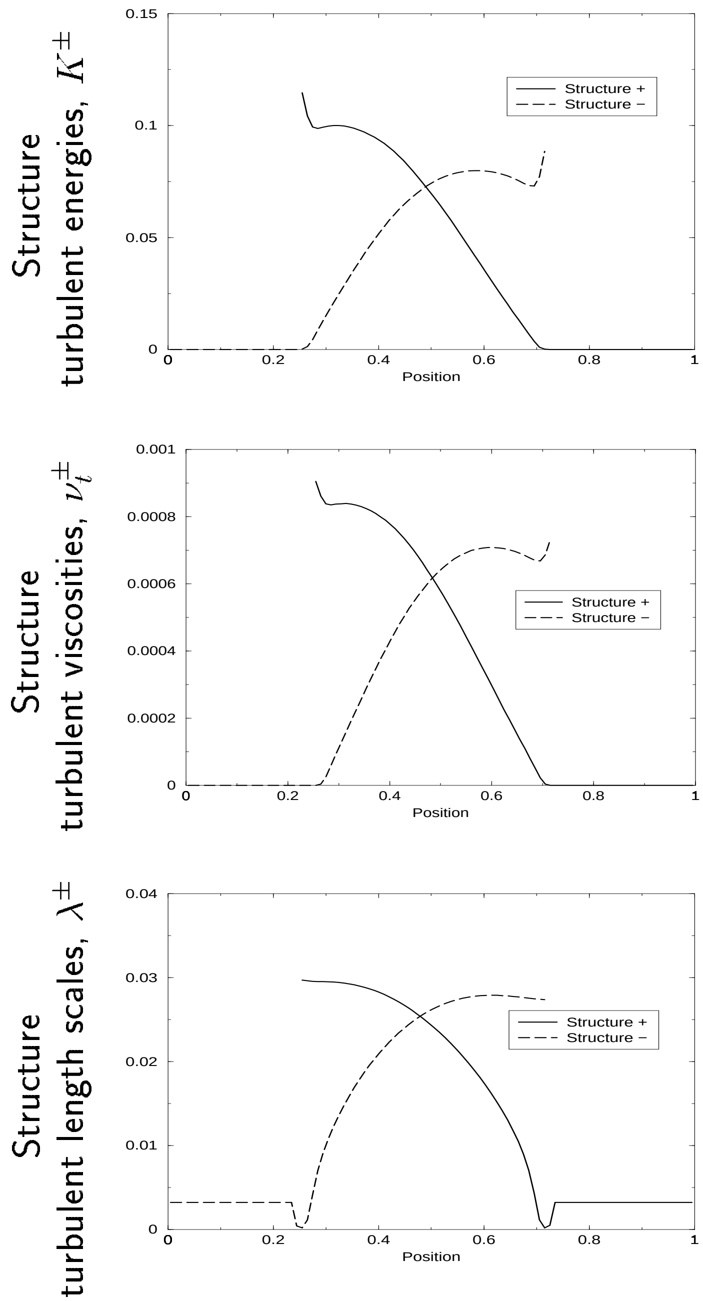
Calculation conditions :
- $x \in [0, 1]$,
- 100 cells,
- $g(t) \propto t^n$,
- energy source term to compensate $g(t)$ (see Youngs, this conference)
- t_0 and $g(t_0)$ ajusted so $L(t_0) \approx 0.4\Delta x$ and $L(1) \approx 1$ for selected n,
- isentropic stratification of perfect gases at $\gamma = 5/3$,
- maximum Mach number below 5%,
- At $= 0.2$ at interface for $t = t_0$,
- Initial turbulence in pure fluids with $\sigma \ll \sigma_{\mathsf{TMZ}}$ and $K \ll K_{\mathsf{TMZ}}$.

III.2 Typical TTT profiles in RT TMZ









Profiles are very similar for explored SSVARTs at $n = -1$ to 1.

Slight artifacts at TMZ edges for $\alpha^{\pm} \to 0$ are numerical.

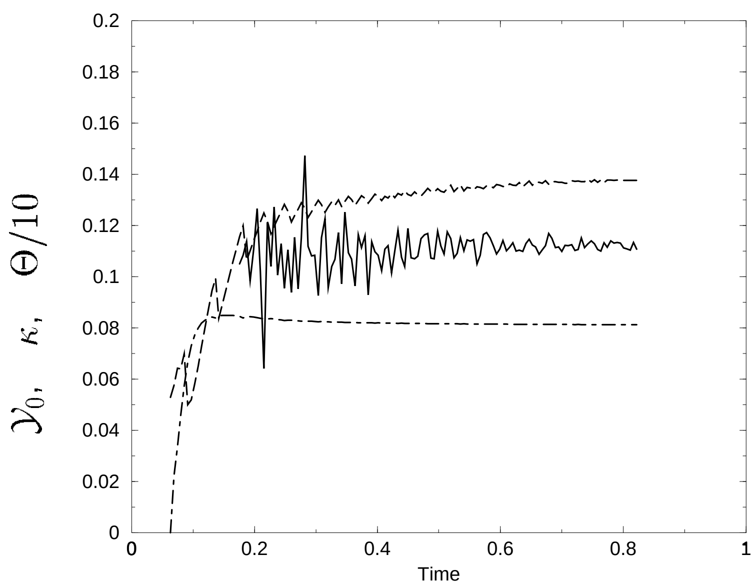
III.3 Typical TTT "0D" parameters in RT TMZ

Three main "0D" parameters :

- growth rate, $\mathcal{Y}_0 \approx 0.12$,
- molecular mixing fraction, $\Theta \approx 0.83$,
- Von Kármán number, $\kappa \approx 0.13$,

are defined by:

$$L(t) = 6\int \alpha^+\alpha^- dx, \tag{16a}$$
$$\Theta(t) = \int \alpha^+\alpha^{2+}\alpha^{1+}+\alpha^-\alpha^{2-}\alpha^{1-} dx \,/\, \int \alpha^2\alpha^1 dx, \tag{16b}$$
$$E(t) = \int \alpha^+\rho^+\varepsilon^+ + \alpha^-\rho^-\varepsilon^- dx \,/\, \int \alpha^+\rho^+ + \alpha^-\rho^- dx, \tag{16c}$$
$$K(t) = \ldots = K^+ + K^- + K_D + K_A, \tag{16d}$$
$$\kappa(t) = K^{3/2} \,/\, E\, L. \tag{16e}$$



Time evolution of "0D" parameters: $\mathcal{Y}_0$ (continuous), Θ (dot–dashed), and κ (dashed).

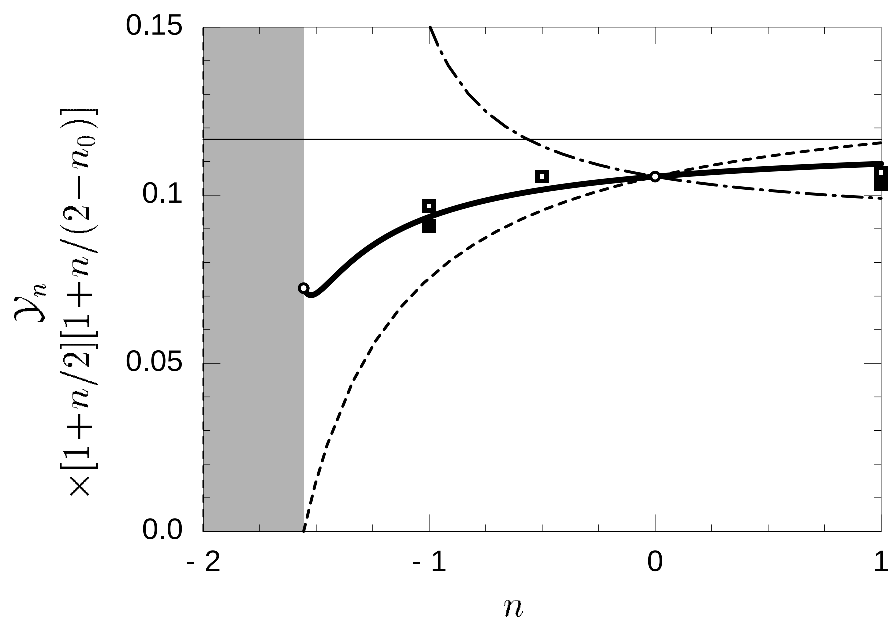
Von Kármán number captured within 20% without ajustments.

For Richtmyer-Meshkov, TTT yields $n_0 \approx 0.3$ as k–ε.

III.4 Growth rate compared to other models

Estimate of growth coefficient
corrected for transient and relaxation effects:

$$L(t) = \mathcal{Y}_n \, \mathrm{At} \, g(t) \, t^2 \times \left[1 + \frac{\delta t}{t} + \frac{\tau^2}{t^2} \right]^{n+2} \tag{17}$$



Normalized growth rates of various models: TTT (squares), Youngs' two-fluid (dot-dashed), free fall (dashed) Read's formula (dotted), and Youngs' preliminary simulations presented at this conference (filled squares).

Molecular mixing fraction and Von Kármán number, Θ_n and κ_n, are practically independent of n for $-1 < n < 1$.

IV Conclusions

A two-structure, two-fluid, two-k–ε model (TTT)
has been developped which:

- derives from two-field RANS closure,
- provides physically consistent picture of RT/SSVARTs TMZs,
- contains single fluid k–ε ($\rightarrow$ KH, RM),
- captures Von Kármán numbers,
- is potentially accurate on a range of SSVARTs and demixing,
- involves few unknown ajustable constants.

TTT is currently being improved regarding:

- Atwood number influence,
- added mass closure,
- response to KH,
- dispersion by added mass velocity fluctuations,
- shocks and compressibility effects.
- confirmation of closures from simulated correlations.

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