Turbulent mixing in RTI as order-disorder process

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LIGHT FLUID   ACCELERATES   HEAVY FLUID

the misalignment of the PRESSURE and DENSITY gradients

the INSTABILITY of the interface

TURBULENT MIXING of the fluids

Rayleigh-Taylor instability	sustained acceleration (gravity)
Richtmyer-Meshkov instability	impulsive acceleration (shock)

Basic Objective: reliable description of turbulent mixing

Fundamental issues:

- the cascades of energy
- <u>the dynamics of the large-scale coherent structure</u>

DYNAMICS of the COHERENT STRUCTURE of
BUBBLES and SPIKES in 3D RTI / RMI

to PUT BACK a REGULARITY into the flow

INTERFACE	**active** regions	**passive** regions
	small scales	**large** scales
	intensive vorticity	simply advected

large-scale coherent motion — scalar fields

spectral approach — group theory

GROUP THEORY

An array of bubbles and spikes periodic in the plane normal to the direction of gravity

- NO travelling waves $\sim e^{iwt}$, NO convection
- The flow dynamics in the direction of gravity is much faster than the evolution of large scales in the normal plane

Periodicity — Equivalence of points and directions

17 plane crystallographic symmetry groups

translations in the plane + rotations + reflections

irreducible representations of crystallographic space groups $\boldsymbol{G}$

wave-vectors $\boldsymbol{k}$ in the reciprocal lattice with subgroups $\boldsymbol{G_k}$

A STRUCTURE is OBSERVABLE

Structure group $\boldsymbol{G}$ wave-vector $\boldsymbol{K}$

- A significant part of the fluid energy is concentrated in the coherent motion

A DOMINANT mode $\boldsymbol{K}$ governs the macroscopic dynamics

- The structure is stable under large-scale modulations

a scalar function of the macroscopic motion

$$\boldsymbol{K}+\xi: \quad \varphi(\boldsymbol{K}+\xi) \approx \varphi(\boldsymbol{K}) + F(\boldsymbol{K})\xi^2, \qquad \boldsymbol{K} \leftrightarrow -\boldsymbol{K}$$

subgroup $\boldsymbol{G_K}$ inversion in the plane

$\boldsymbol{G}$ is a **symmorphic group** with **inversion in the plane**

3D: p6mm, p4mm, p2mm, cmm, p2 2D: pm11

- Rayleigh-Taylor instability: the structure is isotropic in the plane normal to the direction of gravity

3D: p6mm, p4mm 2D: pm11

TURBULENT MIXING and GROUP THEORY

inverse cascade *transitions with the length scale growth*

3D flow symmetry group $\boldsymbol{G}$ representations $\{\boldsymbol{k},\boldsymbol{G_k}\}$

Transitions between **structures** with **various** wave-vectors $\boldsymbol{k}$

Stability under large-scale modulations

$\boldsymbol{k}+\xi$ $\varphi(\boldsymbol{k}+\xi)\approx\varphi(\boldsymbol{k})+f(\boldsymbol{k})\xi^2,$ $\boldsymbol{k}\leftrightarrow-\boldsymbol{k},$ $G_{\boldsymbol{k}}$

wave-vectors $\boldsymbol{k}$ subgroup $\boldsymbol{G_k}$ inversion in the plane

Lifshits[1940] order-disorder transitions

2D flow, $\boldsymbol{G}$=pm11: $\boldsymbol{k}=0,\boldsymbol{G_k}=m,$ length scale $\Lambda=\lambda$

$\boldsymbol{k}=1/2,\boldsymbol{G_k}=m,$ length scale $\Lambda=2\lambda$

2D: scale growing as doubling of the spatial period

3D flow, $\boldsymbol{G}$=p6mm:

$\boldsymbol{k} = 0, \boldsymbol{G_k} = 6mm$ length scale $\Lambda = \lambda$ hexagonal p6mm

$\boldsymbol{k} = (1/2, 0), \boldsymbol{G_k} = 2mm$ $\vec{\Lambda} = (2\lambda, \lambda)$ rectangular p2mm

$\boldsymbol{k} = (1/3, 1/3), \boldsymbol{G_k} = 3m$ 2 ray star triangular p31m

3D flow, $\boldsymbol{G}$=p4mm:

$\boldsymbol{k} = 0, \boldsymbol{G_k} = 4mm$ length scale $\Lambda = \lambda$ square p4mm

$\boldsymbol{k} = (1/2, 0), \boldsymbol{G_k} = 2mm$ $\vec{\Lambda} = (2\lambda, \lambda)$ rectangular p2mm

$\boldsymbol{k} = (1/2, 1/2), \boldsymbol{G_k} = 4mm$ $\vec{\Lambda} = (2\lambda, 2\lambda)$ square p4mm

4 bubble interaction special modulation

3D: scale growing symmetry lowering anisotropy

To keep isotropy of the RT flow, a balance is required between the inverse and direct cascades merging and splitting

An internal coherent structure with hexagonal symmetry and with the length scale $\Lambda = O(\lambda_{\max})$

CONCLUSION

Group theory

- formulation and general requirements
- structural stability order-disorder
- 3D flow with general type of spatial symmetry
- structural transitions
- isotropy, inverse and direct cascades
- internal coherent structure

??? Concept of self-similarity of turbulent mixing

??? Dynamical symmetry