Nonlinear evolution of unstable fluid interface

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LIGHT FLUID ACCELERATES HEAVY FLUID

misalignment PRESSURE and DENSITY gradients

INSTABILITY TURBULENT MIXING

Rayleigh-Taylor instability	sustained acceleration (gravity)
Richtmyer-Meshkov instability	impulsive acceleration (shock)

- thermonuclear flashes on surface of stars; supernova explosion
- inertial confinement fusion; interaction of laser with matter

Basic objective: reliable description of turbulent mixing

Fundamental issues:

- the cascades of energy
- the dynamics of small-scale structures
- the dynamics of the large-scale coherent structure

<u>Coherent structure</u>

an array of bubble and spikes periodic in the plane

normal to the direction of acceleration (shock)

- ➢ Dynamics of 3D and 2D nonlinear structures in RMI
- ➢ Properties of the 3D-2D dimensional crossover in RMI

INTERFACE	**active** regions	**passive** regions
Aref[1989]	**small** scales	**large** scales
	intensive vorticity	simple advection

large-scale coherent motion	scalar fields
spectral approach	group theory

Abarzhi[1996] (RTI)

Group theory

coherent structure periodicity

group of invariance 17 plane crystallographic symmetry groups

G translations in the plane + rotations + reflections

The COHERENT STRUCTURE is OBSERVABLE

- A significant part of the fluid energy is concentrated in the coherent motion

  a DOMINANT mode **K** governs macroscopic dynamics

- The structure is stable under modulations

$\boldsymbol{K}+\xi$: $\varphi(\boldsymbol{K}+\xi)\approx\varphi(\boldsymbol{K})+F(\boldsymbol{K})\xi^2$ $\boldsymbol{K}\leftrightarrow-\boldsymbol{K}$

a scalar macroscopic function

G is a **symmorphic group** with **inversion** in the plane

3D: p6mm, p4mm, p2mm, cmm, p2 **2D**: pm11

LARGE-SCALE COHERENT MOTION

time t potential $\Phi(x, y, z, t)$ free surface $z^*(x, y, t)$

$$\Delta\Phi = 0, \qquad \nabla\Phi\,|_{z=+\infty} = 0$$

$$\frac{\partial z^*}{\partial t} + \nabla z^* \nabla\Phi - \frac{\partial \Phi}{\partial z}\Big|_{z=z^*} = 0, \quad \frac{\partial \Phi}{\partial t}\Big|_{z=z^*} + \frac{1}{2}(\nabla\Phi)^2 + g(t)z\Big|_{z=z^*} = 0$$

$g \geq 0$ - instability: RT $g > 0$ RM $g = 0$

Initial conditions: $z^*(x, y, t_0)$ $\boldsymbol{v}(x, y, t_0)$

length scale $\lambda\,(\sim \lambda_{max})$ time scale $\tau \sim \lambda/|v_0|$

Symmetry: periodic, symmorphic + inversion in the plane (x, y)

difficulty **SINGULARITY** **interplay of harmonics**

2D RTI: Taylor[1950], Fermi[1952], Layzer[1955], Garabedian[1957], Birkgoff[1957], Zuffiria[1986], Inogamov[1990], Tanveer[1993], Hazak[1997]

3D RTI: Abarzhi[1995]

LOCAL EXPANSIONS *ASYMPTOTIC SOLUTIONS*

2D & 3D RMI: Shvarts[1995], Inogamov[1995], Mikaelian[1998], Zhang[1998], Abarzhi[2000]

Layzer-type approach single-mode approximation

- Expansion in terms of orthogonal functions:

$$\Phi = \sum_{n=1}^{\infty} \Phi_n(t) \left(\frac{1}{\gamma_n} \exp\left(-\gamma_n (z - z_0(t)) + i \sum_j \boldsymbol{k}_j \boldsymbol{r} n_j \right) + c.c \right)$$

irreducible representations of group $\boldsymbol{G}$ | wave-vectors $\boldsymbol{k}$, $\boldsymbol{G_k}$

project operators | Fourier expansion

- Local expansion at a highly symmetric point of the interface

$x \approx 0, y \approx 0, z \approx z_0(t)$:

$$z^*(x, y, t) = z_0(t) + \sum_{i+j=1}^{\infty} \zeta_{ij}(t) x^{2i} y^{2j}, \qquad N = i + j = 1,2,...\infty$$

Dynamical system of ordinary differential equations

$$\sum_{i+j=1}^{\infty} D_{ij}\left(\dot{M}, M, \zeta\right) x^{2i} y^{2j} = 0 \qquad \sum_{i+j=1}^{\infty} K_{ij}\left(\dot{\zeta}, M, \zeta\right) x^{2i} y^{2j} = 0$$

$\zeta = \{\zeta_{ij}\}$; $\qquad M = \{M_n\}$ moments $M_n = \sum_{m=1}^{\infty} \Phi_m (km)^n$

- Local dynamics, any time t; the length scale(s) λ is invariable
- Multiple harmonics presentation
- 3D flows with general type of symmetry and 2D flows
- Desired accuracy, $x \approx 0, y \approx 0, z \approx z_0(t)$, $N = i + j = 1,2,...\infty$

REGULAR ASYMPTOTIC SOLUTIONS

$$\sum_{i+j=1}^{\infty} D_{ij}\left(\dot{M}, M, \zeta\right) x^{2i} y^{2j} = 0 \qquad \sum_{i+j=1}^{\infty} K_{ij}\left(\dot{\zeta}, M, \zeta\right) x^{2i} y^{2j} = 0$$

regular asymptotic solutions $t/\tau >> 1$

Richtmyer-Meshkov **bubble**: $v(t) \sim \lambda/t,\ \zeta(t) \sim 1/\lambda$

Layzer-type expansion: regular asymptotic solutions are absent in general case

- non-linearity is non-local
- singularities determine the interplay of harmonics

At a fixed length scale(s) λ, shape of the regular bubble is free and is parameterised by the principal curvature(s)

number of the parameters N_p *symmetry of the* 3D (2D) *flow*

$$N_p \leq 3$$

2D pm11, 3D p4mm, p6mm $N_p = 1$ 3D p2mm $N_p = 2$

✓ to capture the interplay of harmonics

✓ to show existence and convergence for solutions in the family

✓ to involve all bubbles allowed by symmetry of the flow

✓ to choose the physically dominant (i.e. the fastest stable) solution

RICHTMYER-MESHKOV bubbles

Curvature radius R (radii $R_{x,y}$) $kR_{cr} \le kR \le \infty$

Velocity $v = L(k,R)/t$ surface variables $\zeta_n = \zeta_n(k,R)$

Fourier amplitudes $\Phi_n = \varphi_n(k,R)/t$

$$\left|\Phi_n / \Phi_1\right| \sim \exp(-pn)$$

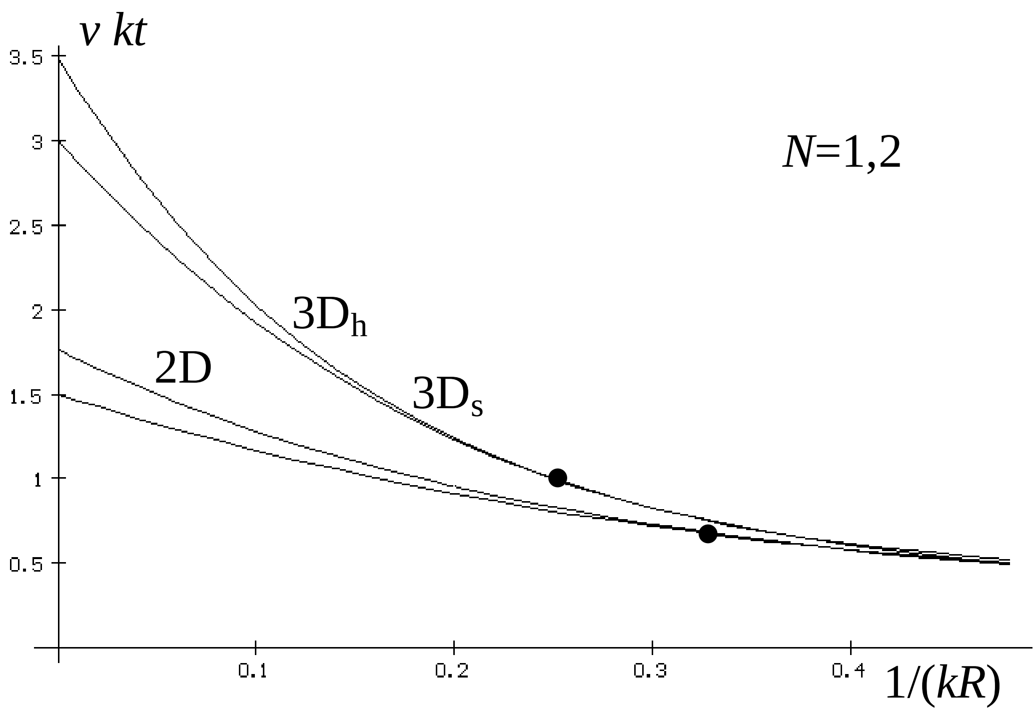
Asymptotic stability $v - L(k,R)/t \sim t^{\beta-1}$, $\zeta_n(t) - \zeta_n(k,R) \sim t^{\beta}$

$\beta = \beta(kR)$ for stable solutions $\mathrm{Re}[\beta] < 0$

Properties

1. The physically dominant solution in the family corresponds to a bubble with a flattened surface, $kR \to \infty$
2. The bubble flattens in time as $kR \sim (t/\tau)^{|\beta_\infty|}$
3. For highly symmetric 3D flows: $kR_{3D} \sim (t/\tau)^{|\beta_\infty|}$, $v_{3D} \sim 4/k\,t$
4. The local dynamics of 3D highly symmetric flows is universal; near-circular contour $z^* \sim \zeta_1\left(x^2 + y^2\right)$
5. 3D anisotropic bubbles tend to conserve a near-circular contour
6. 3D anisotropic bubbles are unstable
7. 3D Layzer-type “square” solution is the point of bifurcation
8. The dimensional crossover is discontinuous, $\beta_{3D-2D} > 0$
9. NO 2D flows

Family of regular asymptotic solutions in RMI



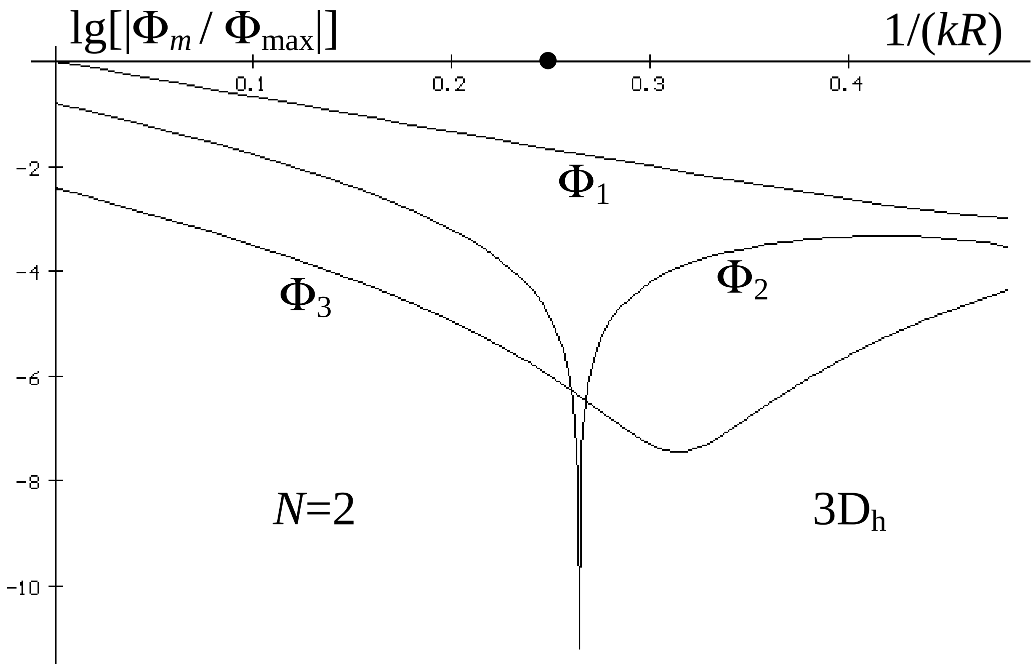
Velocity v as the function on the radius of curvature R

Three-dimensional flows with hexagonal ($3D_h$) and square ($3D_s$) symmetry and two-dimensional flow (2D); k is the wave-vector, t is time, N is order of approximation.

Black circles mark the Layzer-type solutions with

$R_L = 4/k, v_L = 1/kt$ in 3D and $R_L = 3/k$, $v_L = 2/3kt$ in 2D.

Family of regular asymptotic solutions in RMI



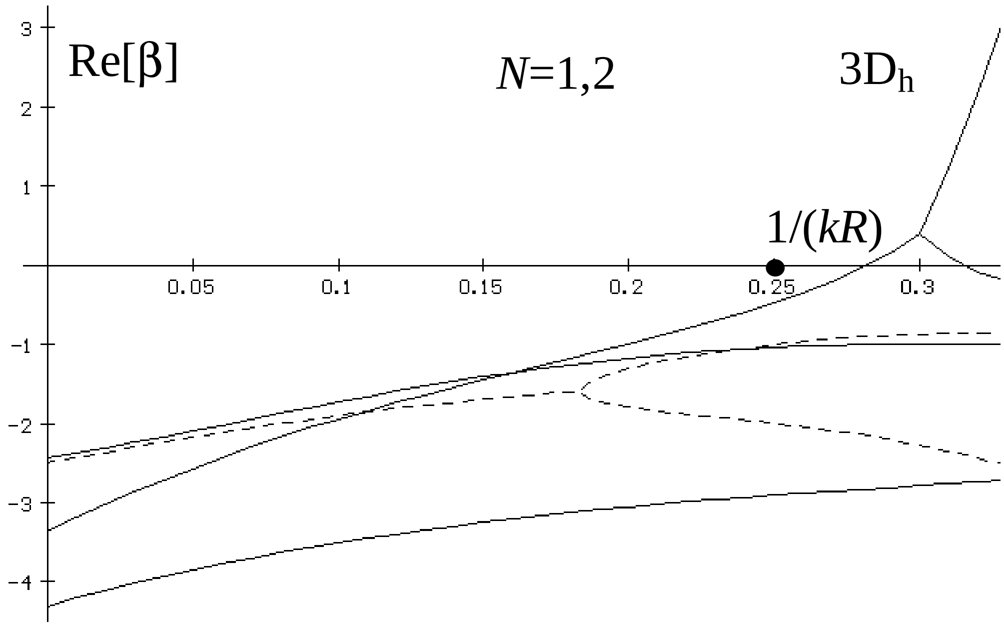
Exponential decay of the Fourier-amplitudes

with an increase in their number.

Three-dimensional flows with hexagonal symmetry p6mm ($3D_h$);

$\Phi_{max} = \Phi_1(kR \equiv \infty)$; black circle corresponds to the Layzer-type bubble.

Family of regular asymptotic solutions in RMI



Stability analysis for the family of regular asymptotic solutions

Real parts of exponents β as functions on the radius of curvature *R*

Dashed lines correspond to *N*=1, solid lines – to *N*=2, black circle corresponds to the Layzer-type solution.

Evolution of the bubble front in RMI

Highly symmetric 3D and 2D coherent structures

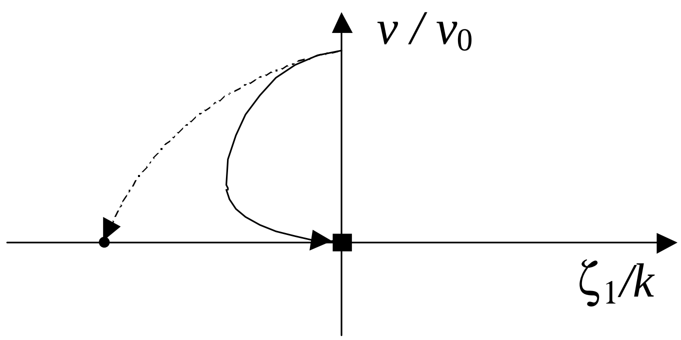
time scale $\tau \sim 1/v_0 k$

$t \ll \tau$: curvature $\zeta_1(t) \sim -k\, t/\tau$, velocity $v(t) - v_0 \sim v_0\, t/\tau$

$t \sim \tau$: curvature $\zeta_1(t) \sim -k$

$t \gg \tau$: curvature $\zeta_1(t) \sim -k\,(t/\tau)^{-|\beta_\infty|}$, velocity $v(t) \sim C_\infty / k\, t$

Dynamic trajectories



Solid line corresponds to multiple harmonic solution, and black square - to the flattened bubble. Dashed line corresponds to Layzer-type single-mode solution, and black circle – to the Layzer-type bubble.

Evolution of the bubble front in RMI

RM bubbles flatten | **RM bubbles decelerate**

- Qualitative agreement with experiments
- Bubble velocity $v_\infty \sim C_\infty / kt$ $\quad v_L \sim C_L / kt$

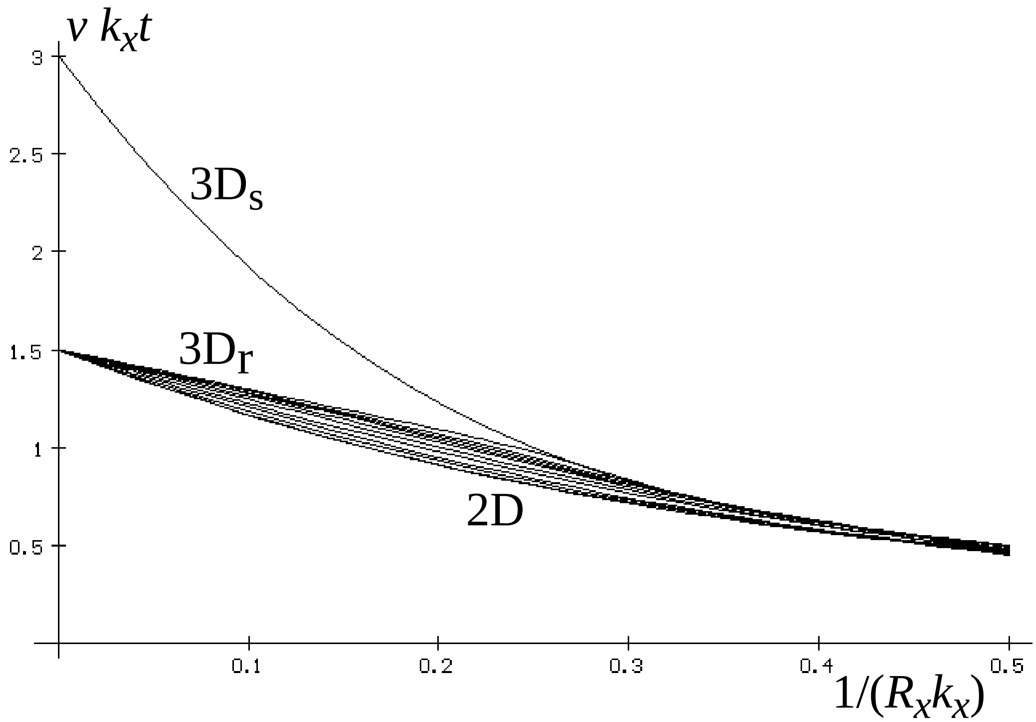
  $C_\infty / C_L \sim 3 - 4$ $\quad \Delta h \sim C \ln(t/\tau)$
- Bubble shape $\zeta_1(t) \sim -k\,(t/\tau)^{-|\beta_\infty|}$ reliable parameter
- Existence of an exact analytical solution

- a rigid body curvature $\sim 1/R$ drag force = $\rho v^2 R^2$

For a two-fluid system, Atwood number < 1

- the Layzer-type approach requires MASS FLUX through the interface
- Flattened RM bubble is a multiple-harmonic solution with NO MASS FLUX through the interface

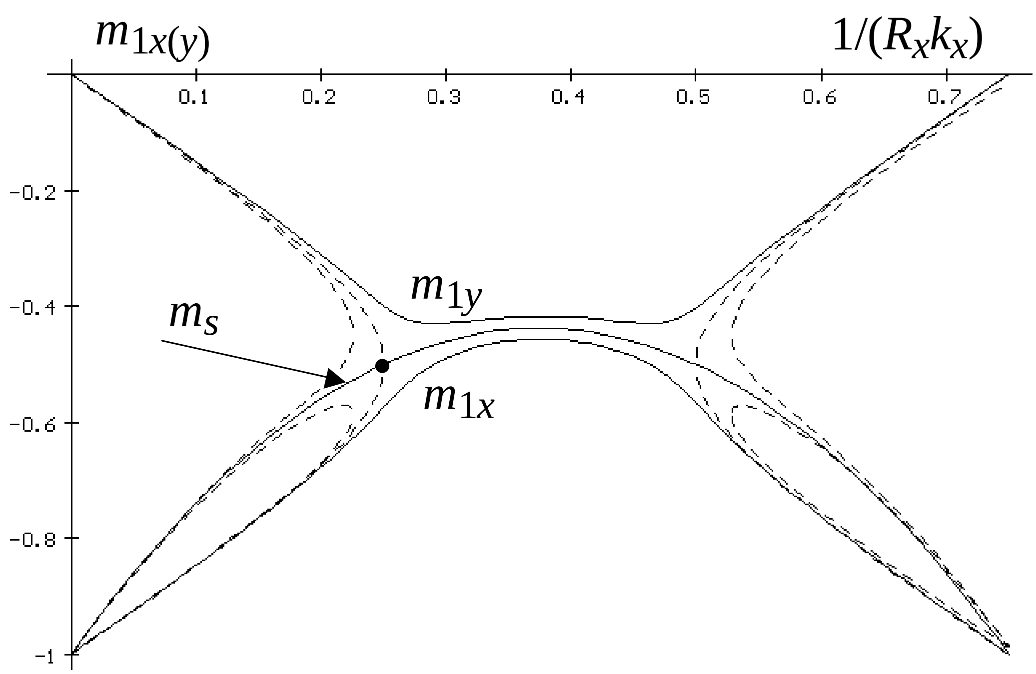
Family of regular asymptotic solutions in RMI



Dependence of velocity $v = L(R_{x,y}, k_{x,y})/t$ on the bubble shape

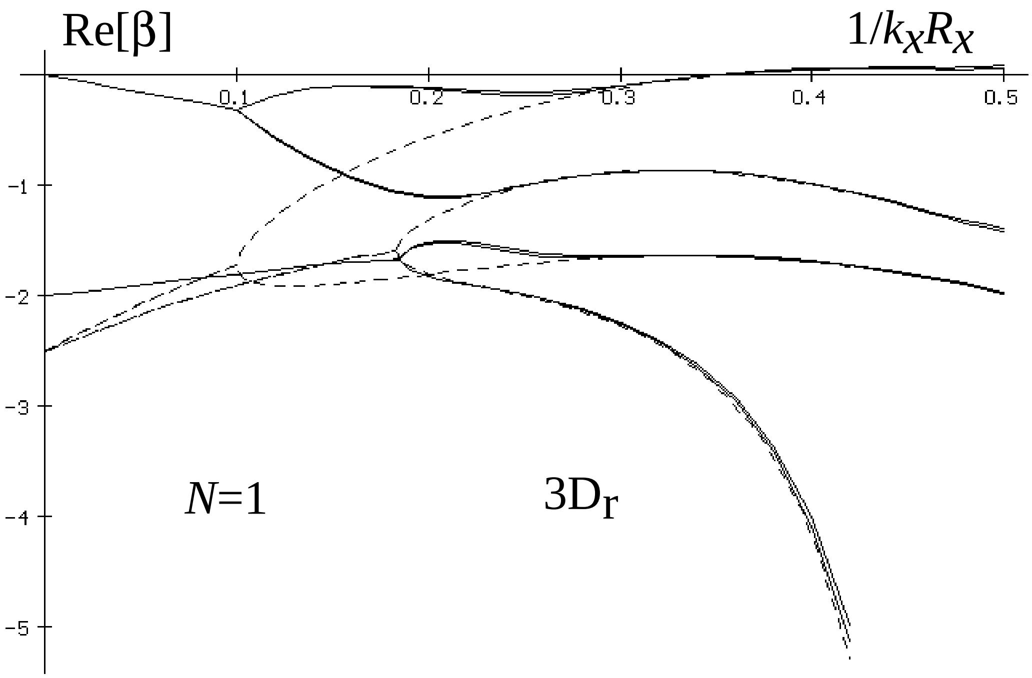
Low-symmetric bubbles with rectangular symmetry $3D_r$, two-parameter family; various values of the aspect ratio; the highest curve $3D_s$ is the family of solutions for 3D square bubbles with $R_y = R_x$ and $k_x/k_y = 1$; the lowest curve 2D is the family of solutions for 2D bubbles flat in the *y*-direction with $R_y \equiv \infty$

Family of regular asymptotic solutions in RMI



Bifurcation of the Layzer-type square solution (black point) for nearly symmetric flows with $k_x \sim k_y$ and $R_x \sim R_y$

Family of regular asymptotic solutions in RMI



Stability analysis for low-symmetric RM bubbles

Dashing lines corresponds to highly symmetric 3D square solutions with $k_x = k_y$ and $R_x = R_y$. Solid lines correspond to nearly symmetric solutions with $k_x \sim k_y$ and $R_x \sim R_y$. Non-symmetric solutions are unstable.

Evolution of the bubble front in RMI

Low-symmetric 3D coherent structures

The dimensional 3D-2D crossover

2D bubbles under 3D modulations

time scale $\tau \sim 1/|v_0|k_x$

$t << \tau \quad \zeta_{1x}(t) \sim -k_x\, t/\tau \qquad \zeta_{1y} \sim -k_y\, t/\tau$

$t >> \tau \quad \zeta_{1x}(t) \sim -k_x\,(t/\tau)^{-|\beta_\infty|} \qquad \zeta_{1y} \sim -k_x\,(t/\tau)^{|\beta_{3D-2D}|}$

the dimensional crossover is discontinuous, $\beta_{3D-2D} > 0$

Secondary instabilities

- Secondary instabilities in RMI are "slow" in contrast to RTI

SINGULAR ASYMPTOTIC SOLUTIONS

Richtmyer-Meshkov **spikes** small-scale structure dynamics

Singular asymptotic solutions to dynamical system

Zhang[1998], Abarzhi[2000]

$t >> \tau$: shape $z^*(t) \sim \sum_{n=1} C_n \left(\exp(t/\tau) r^2\right)^n$, velocity $v(t) \sim -v_0$

- Tanveer 1993, Baker and Meiron 1989, Pullin[2001]…
- For a two-fluid system, Atwood number < 1, the singular asymptotic solutions requires mass flux through the interface

Conclusion

- ✓ Large-scale coherent motion in RMI
- ✓ Separation of scales active regions passive regions
- ✓ Group symmetry large-scale coherent motion
- ✓ Local dynamics of regular bubbles and singular spikes
- ✓ Consideration of 3D flows with general type of symmetry
- ✓ Singularity – interplay of harmonics – shape of the bubble
- ✓ Family of regular asymptotic solutions – symmetry of the flow
- ✓ The physically dominant solution in the family
- ✓ Multiple harmonic solution
- ✓ Universality of local dynamics for 3D highly symmetric flows
- ✓ Conservation of a near-circular contour of 3D bubbles
- ✓ Discontinuous 3D-2D dimensional crossover
- ✓ Singular asymptotes
- ✓ Comparison between the local dynamics in RTI and RMI
- ✓ Different types of the bubble front evolution in RTI and RMI
- ✓ Layzer-type bubbles in RTI and RMI
- ✓ New type of the evolution of the bubble front in RMI
- ✓ Integral (velocity) and internal (shape) diagnostic parameters
- ✓ Theory works effectively for a two-fluid system

Discussion

??? turbulent mixing in RMI and RTI