A Vortex Model for Studying the Effect of Shock Proximity on Richtmyer-Meshkov Instability at High Mach Number

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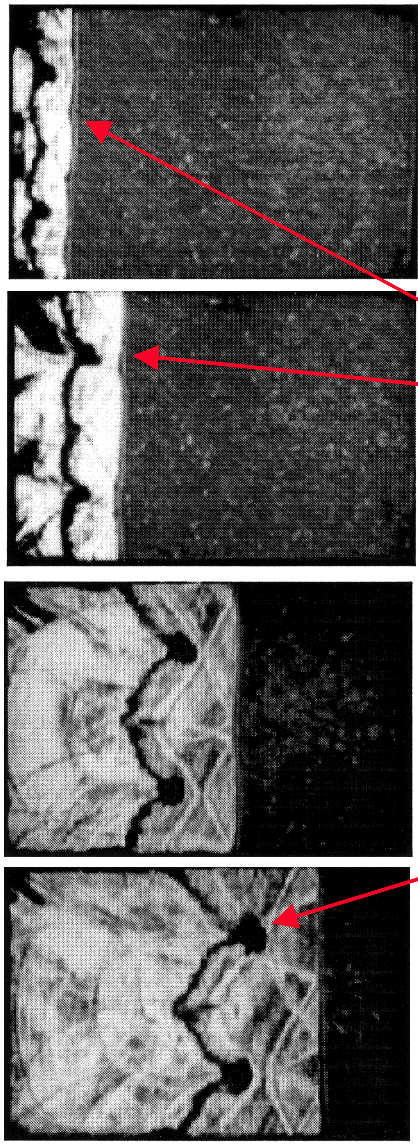
Issue: At high Mach number, the transmitted shock can remain in close proximity to an R-M unstable interface

t = 2 μs

t = 12 μs

t = 37 μs

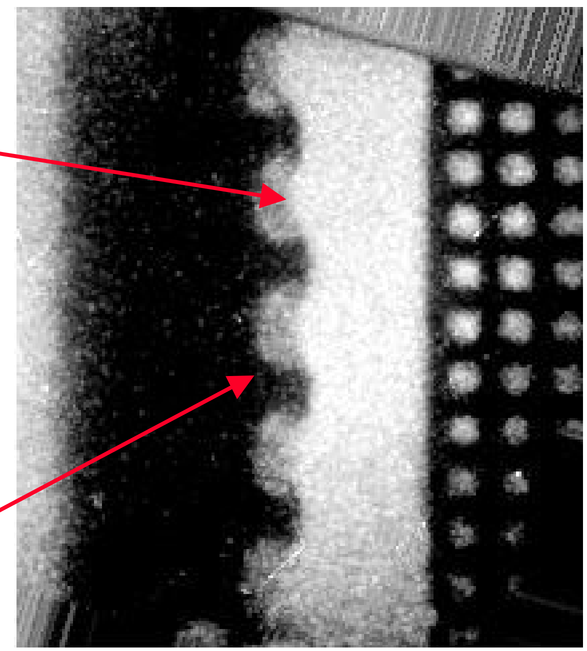
t = 57 μs



Data of Aleshin et al.
$M = 4.5$, $ka_0 = 1.745$, $A = 0.45$,
$\lambda = 36$ mm, $a_0 = 10$ mm

Shock

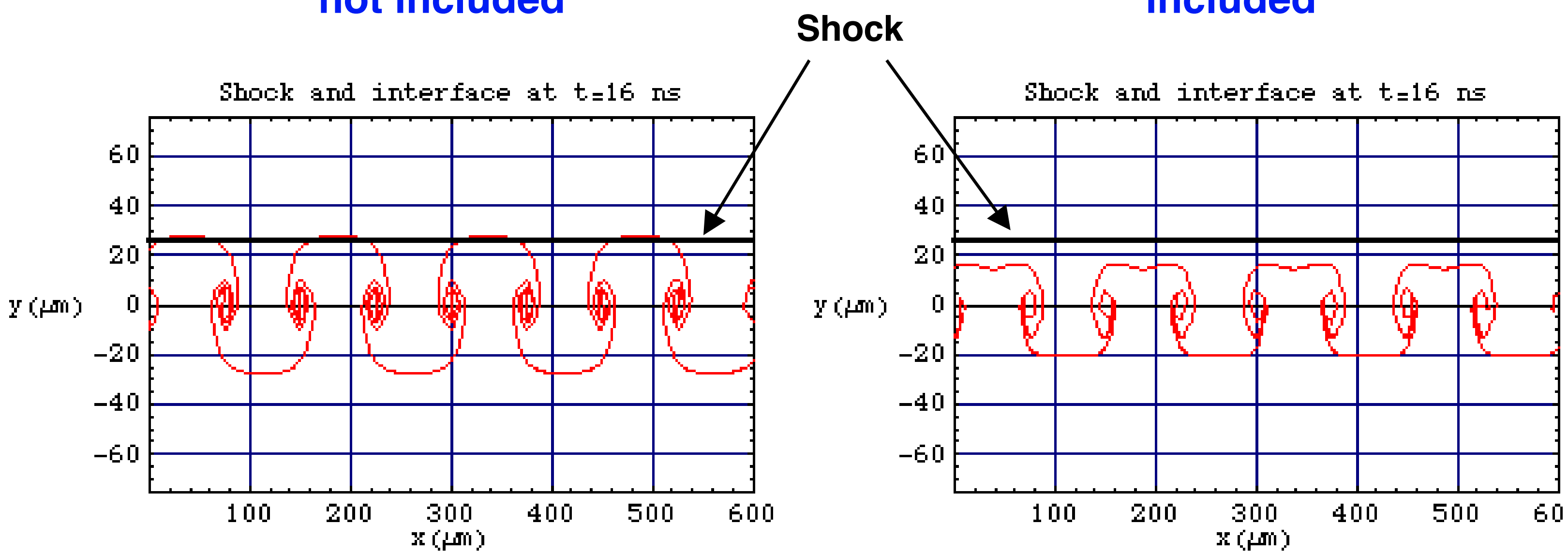
Perturbed interface



Omega data of Glendinning
$M \approx 10$, $ka_0 = 0.9$, $A = 0.47$
$\lambda = 150\ \mu$m, $a_0 = 22\ \mu$m

At these high Mach number conditions, the presence of the shock can affect the Richtmyer-Meshkov growth rate

Shock proximity effect not included | **Shock proximity effect included**

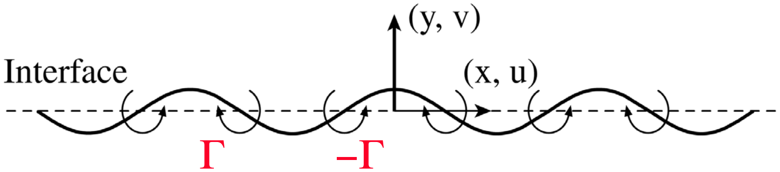


- In certain cases, the predicted linear growth rate can exceed the speed of the transmitted shock relative to the interface

- In this study, point vortex methods are used as a simple means of incorporating the effect of a transmitted shock on the instability growth

Point vortex methods can be used to approximate the evolution of interfacial perturbations throughout the non-linear regime

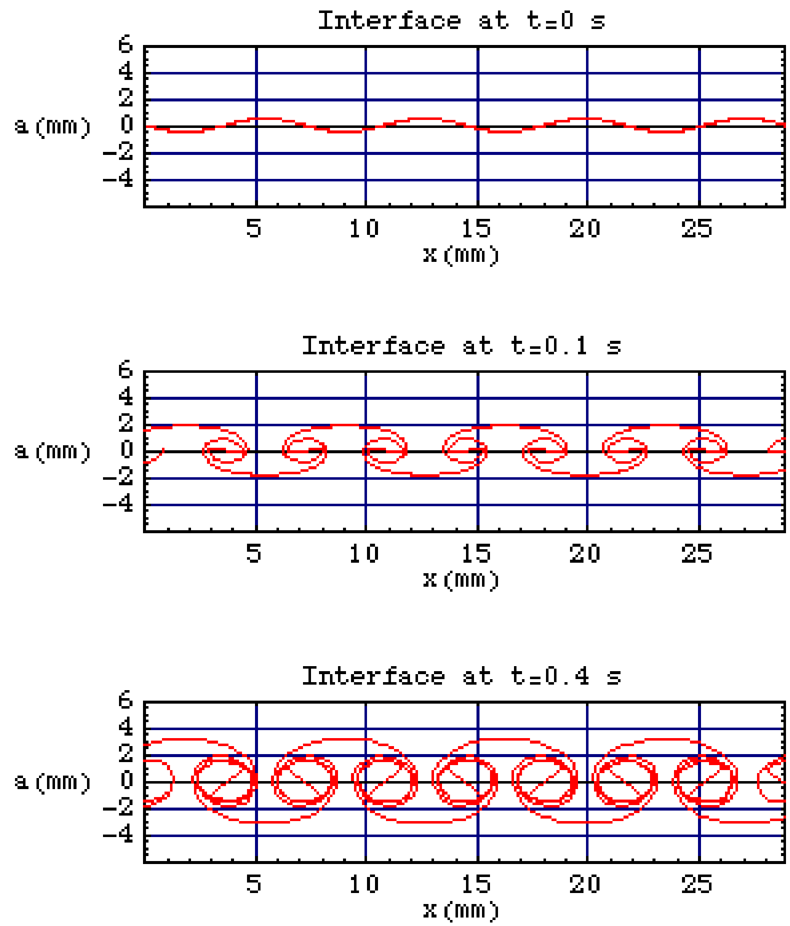
Following Jacobs & Sheeley*, interfacial vorticity is modeled by an alternating array of point vortices of circulation, Γ :



The flow evolution is obtained from a streamfunction of the form:

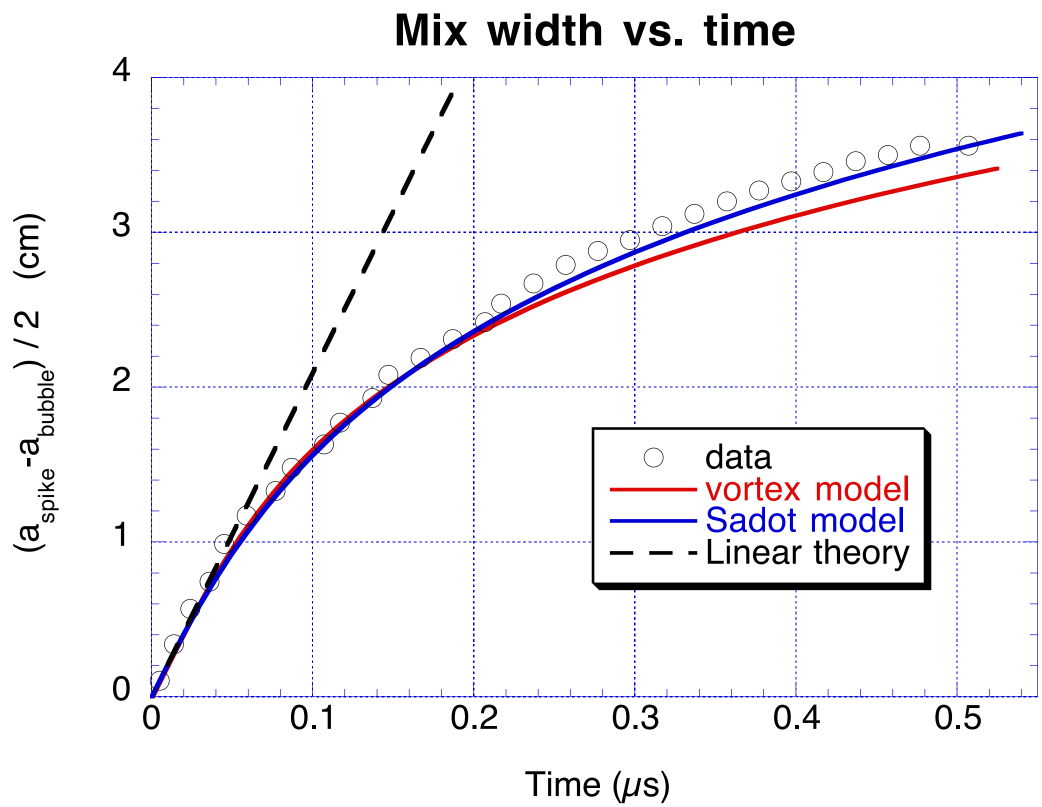
$$\psi = \frac{\Gamma}{4\pi} \ln\left(\frac{\cosh(ky) + \sin(kx)}{\cosh(ky) - \sin(kx)} \right)$$

$$u = \partial\psi / \partial y \quad , \quad v = -\,\partial\psi / \partial x$$



***Phys. Fluids 8(2), 405 (1996)**

The incompressible, A = 0.155 experiments of Jacobs & Sheeley are well modeled by point vortex methods

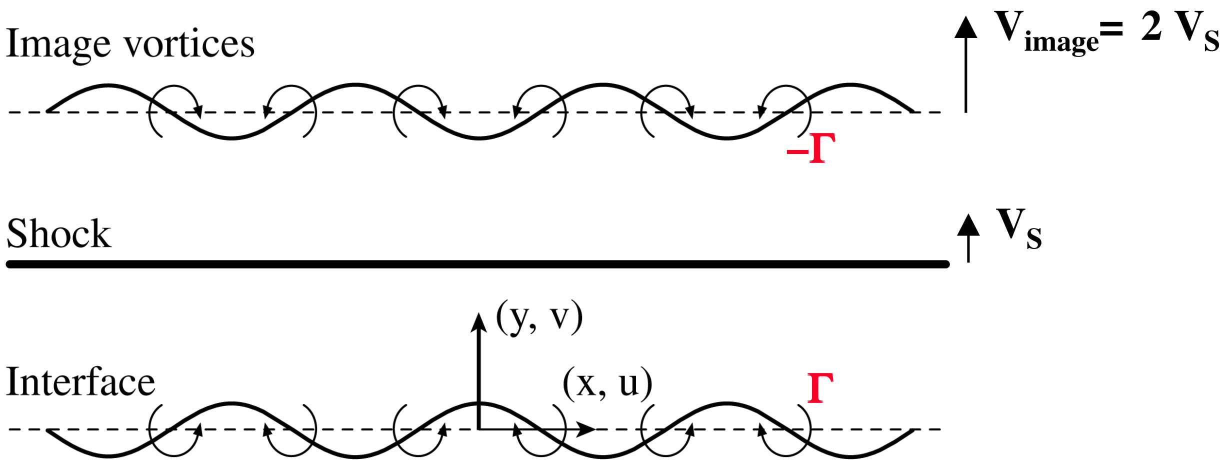


The circulation Γ is defined as that required to reproduce the initial linear growth rate :

$$\Gamma = 2\pi v_{IM} / k$$

- **The model of Sadot et al., PRL 80(8), 1654 (1998) is in excellent agreement with the data**
- **The vortex model predicts an amplitude slightly below the data at later time, but is within 6% of the data and the Sadot model.**

An image vortex model can be used to incorporate the effect of a transmitted shock as a downstream boundary condition



The image vortex array moves away from the interface at twice the shock-to-interface velocity

The streamfunction is now given by :

$$\psi = \frac{\Gamma}{4\pi}\left[\ln\left(\frac{\cosh(ky)+\sin(kx)}{\cosh(ky)-\sin(kx)}\right) - \ln\left(\frac{\cosh(k(2(a_0^*+V_{st}\,t)-y))+\sin(kx)}{\cosh(k(2(a_0^*+V_{st}\,t)-y))-\sin(kx)}\right)\right]$$

Interface vortices — **Image vortices**

Though derived from potential flow theory, this model includes effects due to compressibility and finite Atwood number

Compressibility enters through the circulation which depends on the post-shock Atwood number and compressed perturbation amplitude

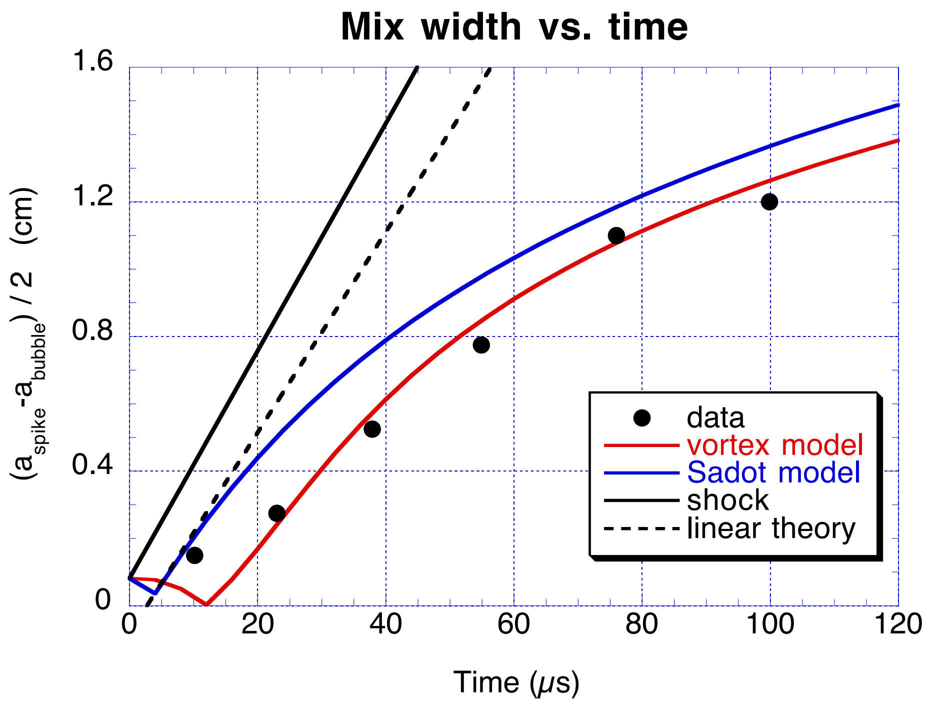
$$\Gamma = \frac{2\pi\, v_{IM}}{k} \quad \& \quad v_{IM} = kA^* \frac{(a_0 + a_0^*)}{2} u_C$$

wher e the post-shock Atwood number is : $A^* = \dfrac{\rho_1^* - \rho_2^*}{\rho_1^* + \rho_2^*}$

and post-shock perturbation amplitude is approximated as : $a_0^* = a_0\,(1 - \dfrac{u_C}{V_{SI}})$

All model parameters are obtained from the solution of the associated Riemann problem for the unperturbed interface

Example 1: The image vortex model has been applied to the M = 4.5 shock tube experiments of Aleshin et al.

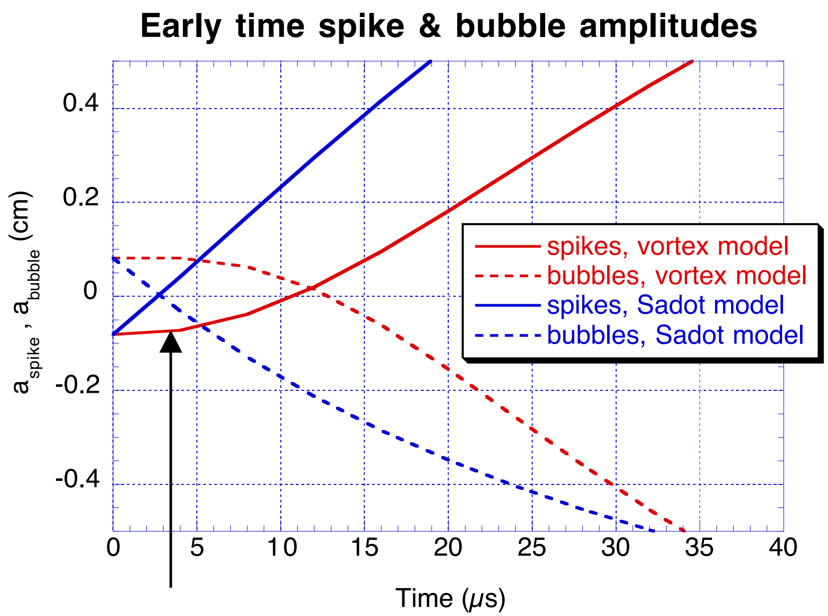


Aleshin et al., run #630B

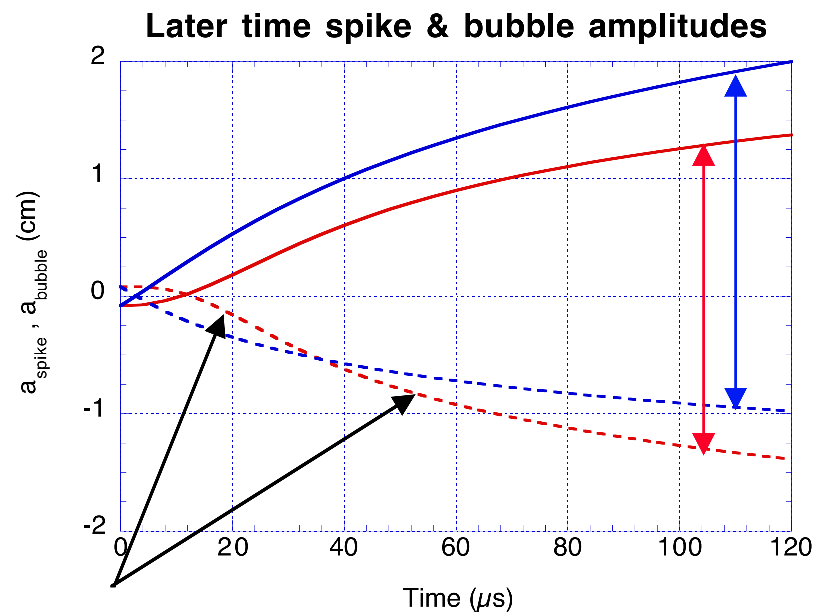
Xe -> Ar, M = 4.5,
A = 0.45, λ = 36 mm
$2a_0$ = 20 mm (P-V)
ka_0 = 1.745

- **The data falls well below the linear theory for the entire experiment**
- **After phase inversion, the vortex model agrees well with the data**
- **The Sadot model predicts an amplitude consistently above the data.**

A look at the development of individual spike and bubble amplitudes reveals further differences



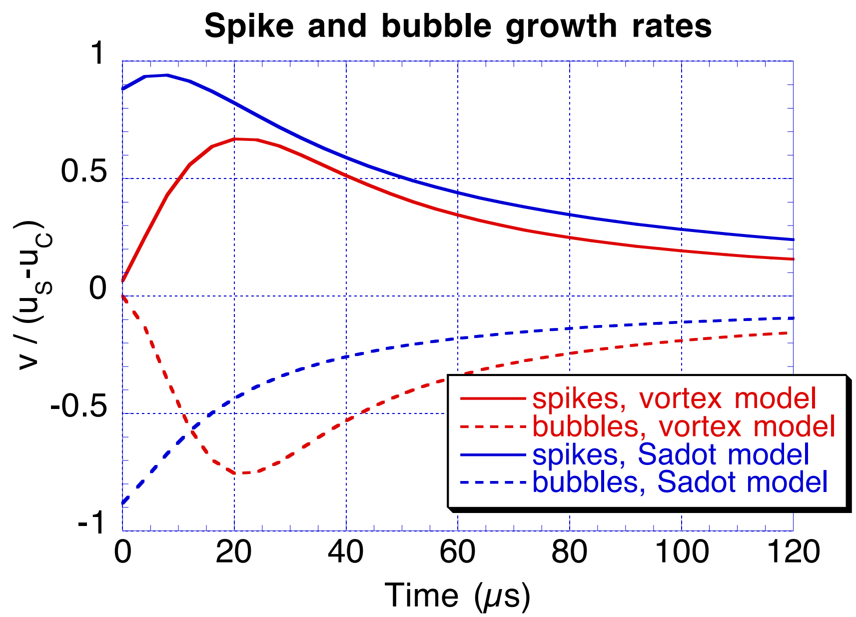
Suppressed growth early in time

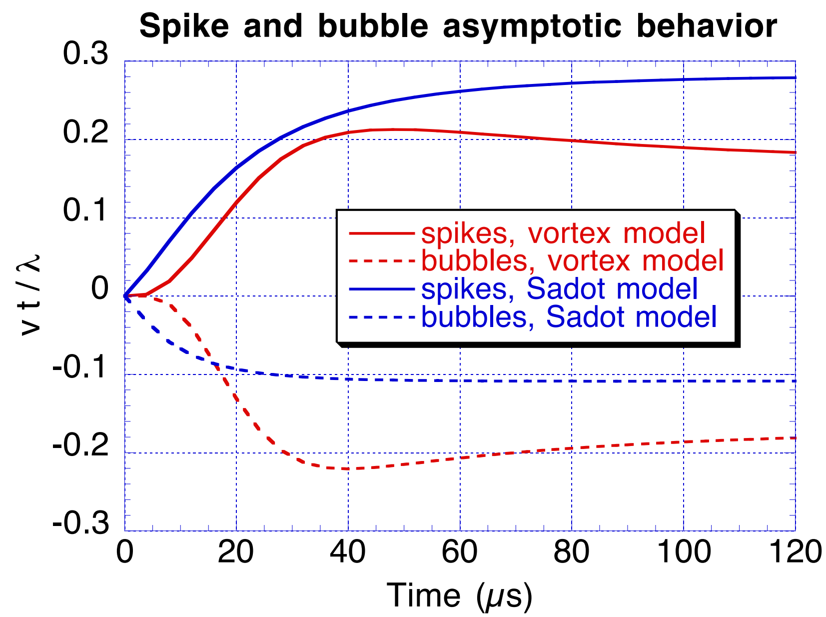


Bubble growth is initially suppressed, but later rebounds

- **The vortex model exhibits a suppressed growth early in time when the shock (and therefore the image vortex system) are close to the interface**
- **Later in time, the spike growth continues to be suppressed since the spikes remain in close proximity to the shock, whereas the bubble growth rebounds. This results in a more symmetrical bubble-to-spike development.**

The spike and bubble growth rates and asymptotic behavior also show the effect of shock proximity



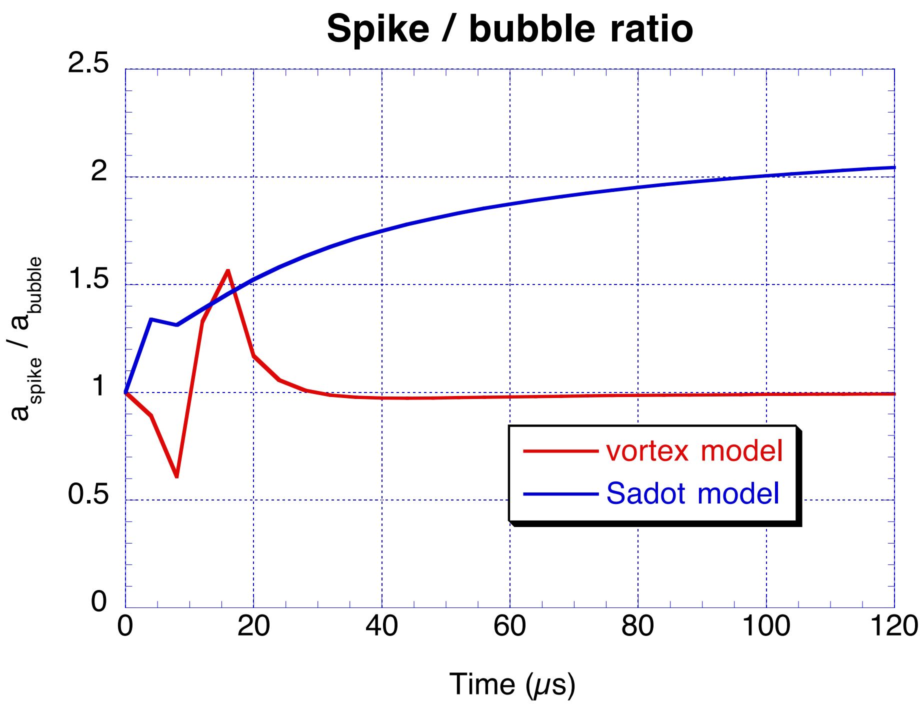


- **The growth rate of the vortex model exhibits a peak which is both reduced in magnitude and delayed in time.**

- **The delayed peak growth is qualitatively consistent with the fully compressible linear theory of Yang, Zhang, & Sharp, Phys. Fluids 6, 1856 (1994)**

- **At late time, all models asymptote to a t^{-1} behavior.**

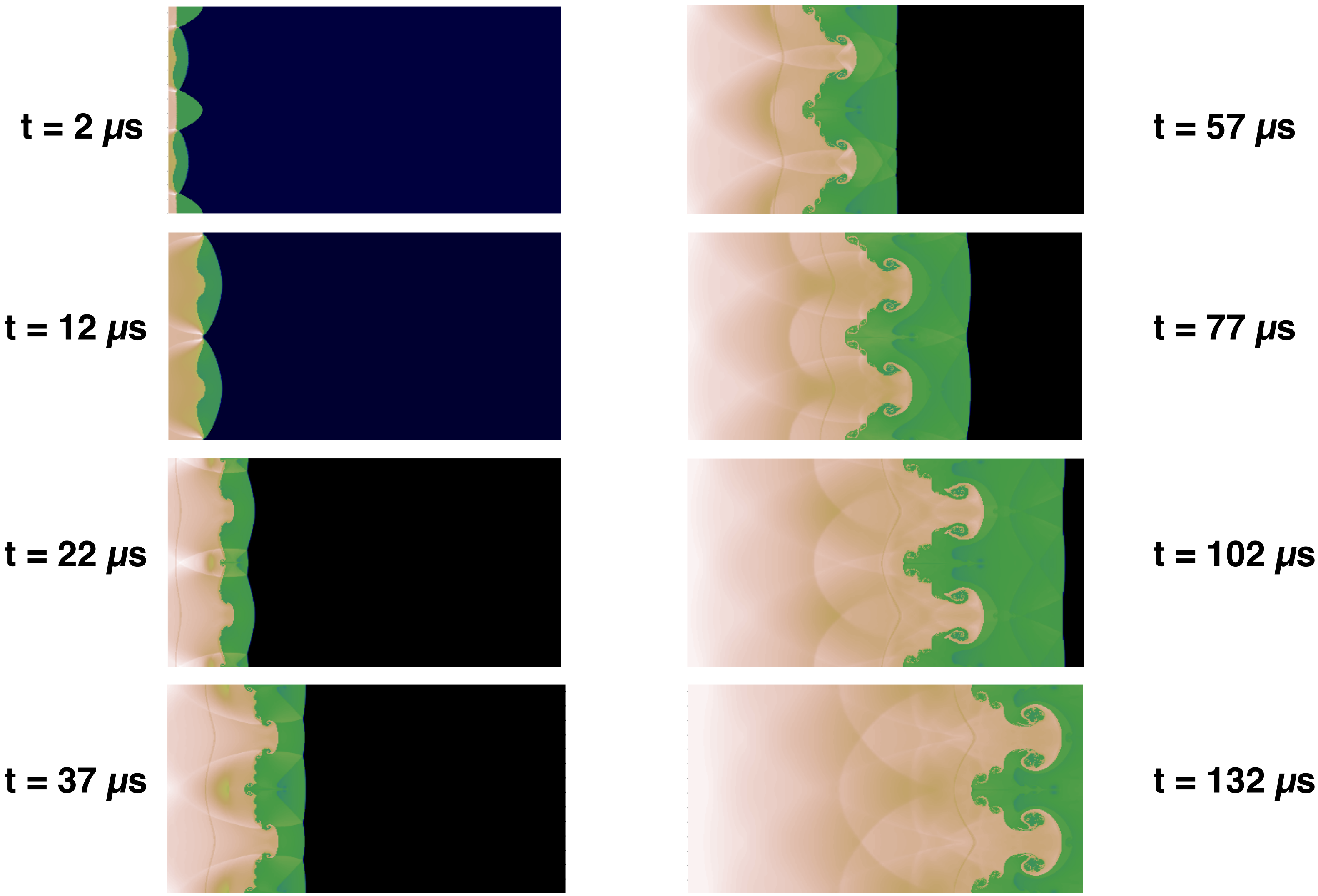
The ratio of spike-to-bubble amplitudes quantifies a very important difference resulting from shock proximity



- **Clearly, the spike to bubble ratio of the vortex model is due to the single fluid (A=0) assumption and is therefore wrong, right?**

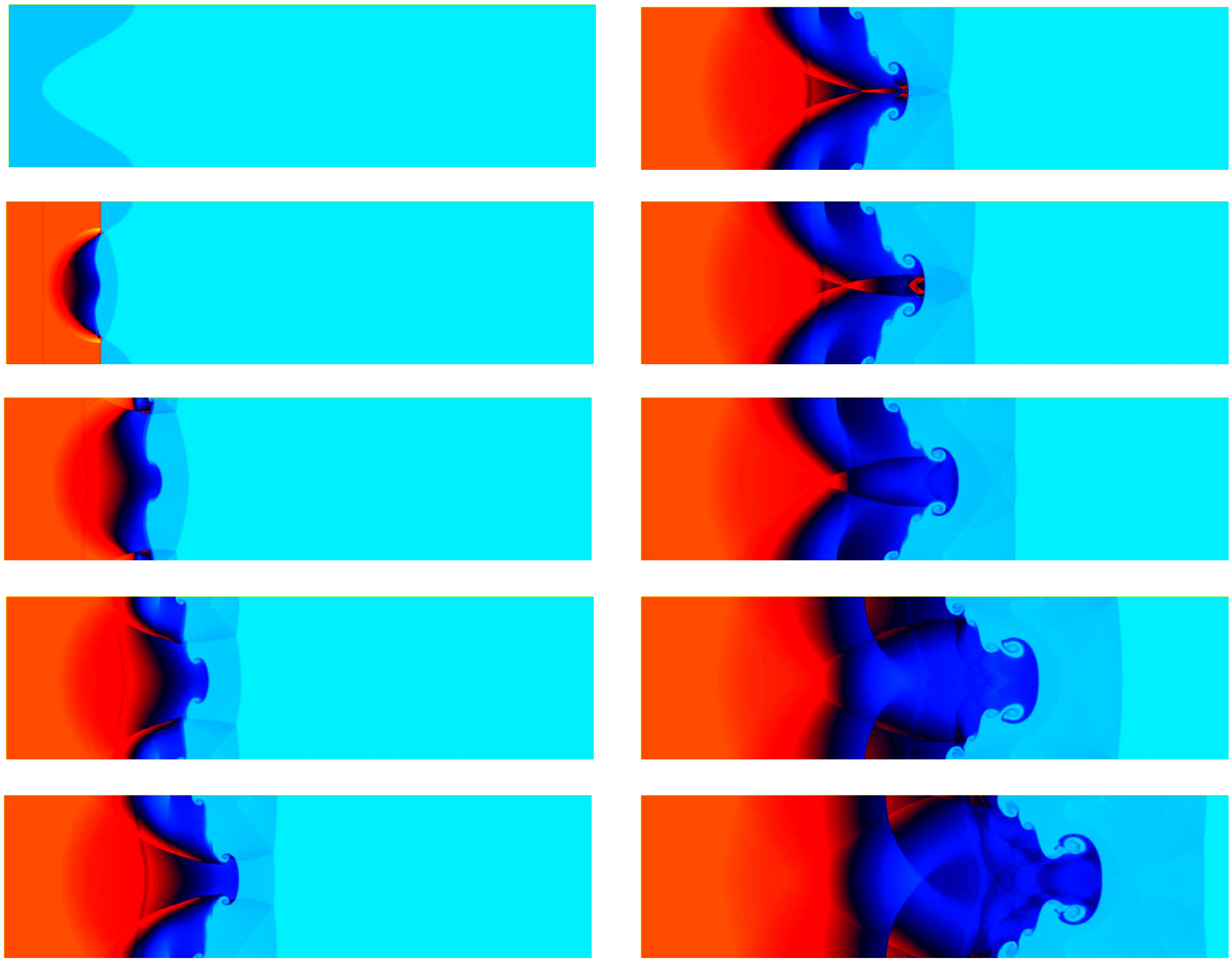
- **To answer this question, we turn to numerical simulation**

Numerical simulations of Aleshin experiment N630B have been performed using a 2D ALE code, HYDRA



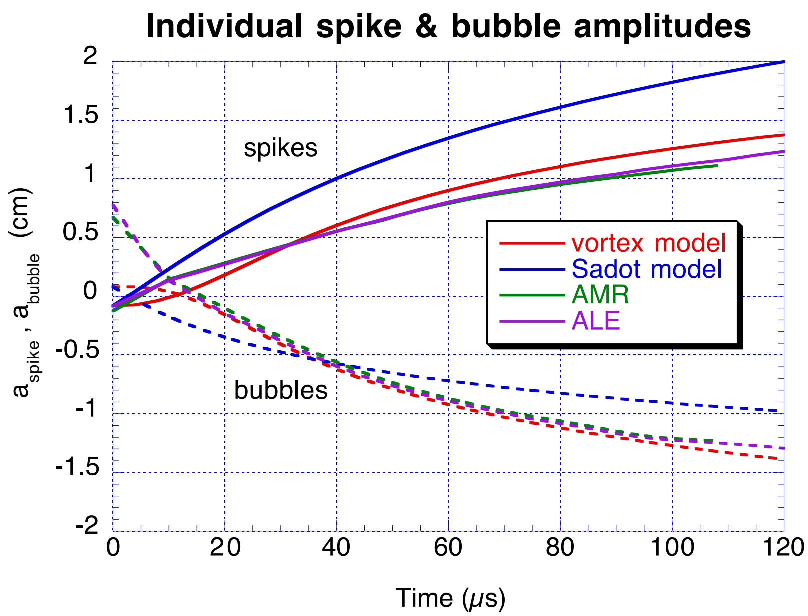
Simulations of S. V. Weber, resolution = 512 zones / wavelength

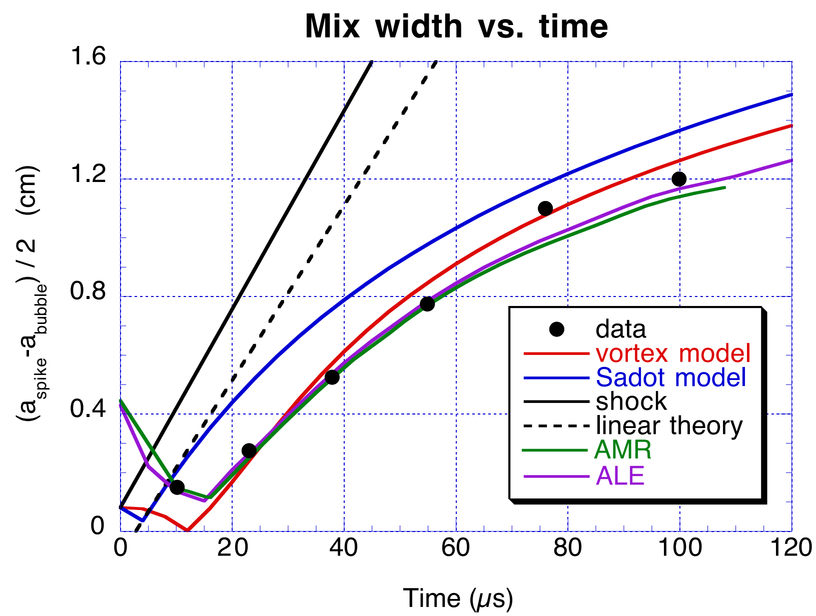
Numerical simulations of Aleshin experiment N630B have also been performed using a 2D AMR code



Simulations of J. A. Greenough, resolution = 2560 zones / wavelength

Both numerical simulations are in reasonable agreement with the image vortex model

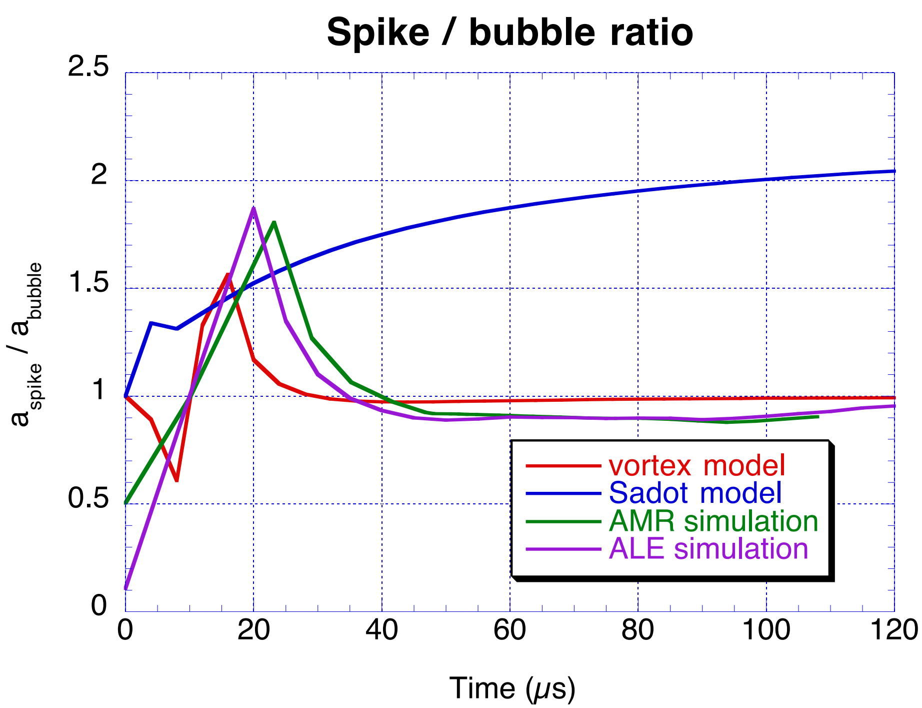




Both simulations show :

- **A delayed phase inversion due to reduced growth early in time**
- **Reduced spike growth throughout the duration of the experiment**
- **more symmetrical spike-to-bubble development**

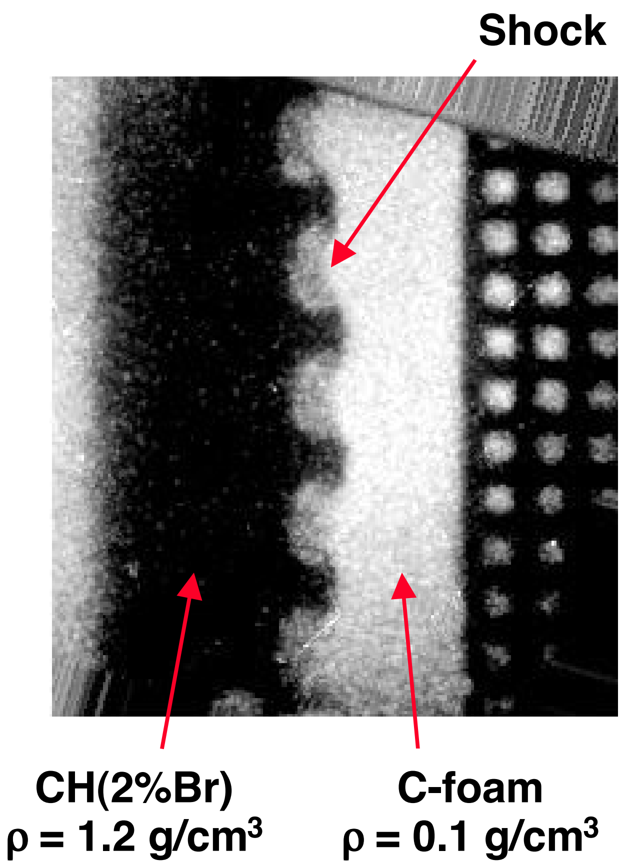
The spike / bubble ratio obtained from the numerical simulations also agrees with the image vortex model



- **Differences are again observed at early time as the interface inverts phase, but later the amplitude of the spikes remains less than that of the bubbles.**

- **This effect is not observed at lower Mach number and is an essential effect due to shock proximity.**

Example 2: The image vortex model has also been applied to the Omega experiments of Glendinning et al.

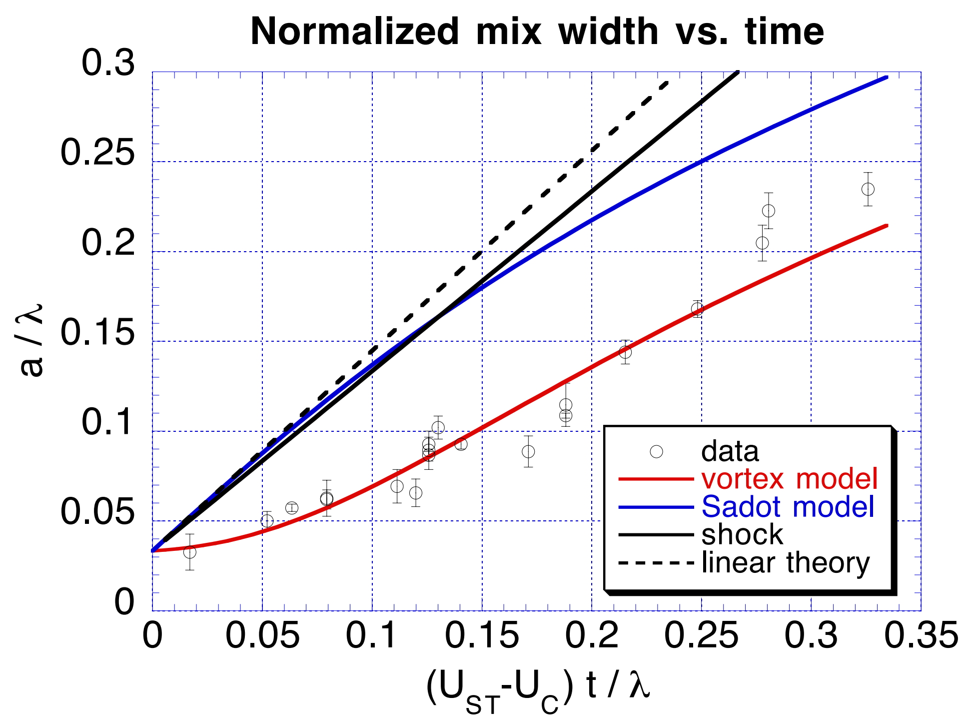


X-ray radiograph
@ t = 22 ns

This experiment differs from that of Aleshin et al. in the following :

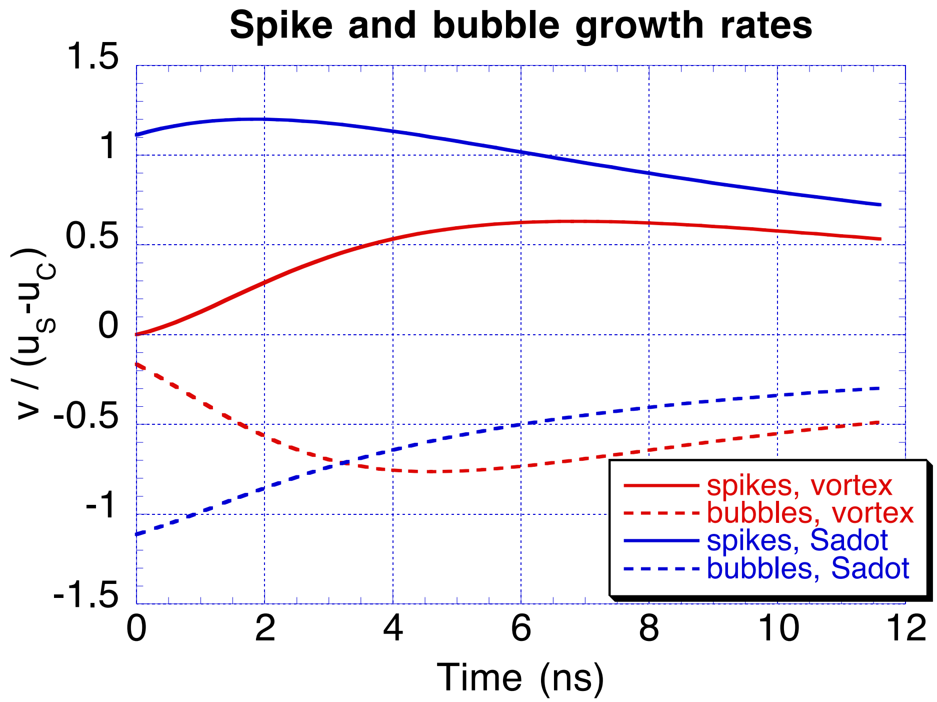
- Higher Mach number, M $\approx$ 10
- Lower initial perturbation amplitude $ka_0 = 0.92$ (vs. $ka_0 = 1.745$)
- Linear theory (Meyer-Blewett) predicts a growth rate which exceeds the shock-to-interface velocity
- Phase inversion of the perturbation is completed by the end of shock refraction
- The effect of shock proximity is more pronounced than before.

The image vortex model does a reasonable job of predicting the perturbation amplitude vs. time



- **The data is well below the linear theory at all times.**
- **The data shows a distinct increase in the growth rate later in time, when the normalized amplitude is small ($a / \lambda = 0.1$)**
- **The shock separation distance from the interface is only 0.33 λ at the latest time observed in the experiment.**

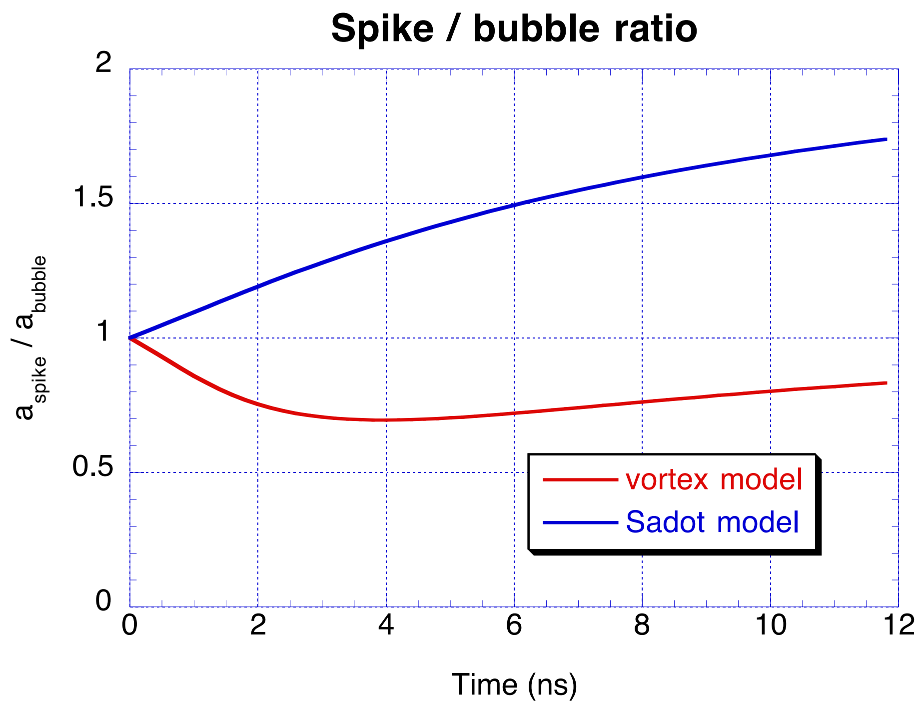
Large differences are again seen in the spike and bubble growth rates due to shock proximity



- **In this case, the linear theory predicts a spike growth rate which is faster than the velocity of the transmitted shock.**

- **The vortex model again predicts a spike growth rate which is at all times lower than that of the Sadot model. The peak growth rate does not occur until ~ 6ns after passage of the shock**

The spike / bubble ratio again shows large differences



- The large supression of the spike growth results in a spike amplitude which remains considerably lower (20-30%) than the amplitude of the of the bubbles.

Conclusions

- An image vortex model has been presented as a simple means of incorporating the effect of a transmitted shock as a downstream boundary condition on the growth of a Richtmyer-Meshkov unstable interface.

- At low Mach number, the vortex model agrees well with the incompressible experiments of Jacobs and Sheeley and also agrees well with the model of Sadot et al.

- At high Mach number, the image vortex model agrees well with shock tube experiments of Aleshin et al. (M=4.5) and laser-driven experiments of of Glendinning et al. (M>10).

Conclusions, continued

The effect of shock proximity is distinguished from saturation effects due to large perturbation amplitude in the following:

- **For shock propagation from heavy to light, the Atwood number dependence observed at lower Mach number is significantly altered due to the presence of the shock boundary. For the two cases discussed, the spike amplitude remains slightly less than that of the bubbles throughout the experiment.**

- **The perturbation growth immediately following passage of the shock is significantly smaller than that given by the linear theory. As the shock departs from the interface, the growth rate increases. Later in time, as the perturbation amplitude increases, normal growth rate saturation effects are seen.**