Scaling in the Shock-Bubble Interaction

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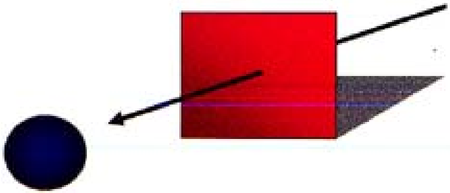
Introduction

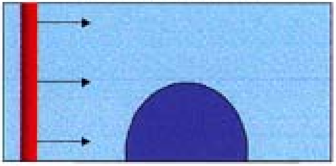
The phenomenon of shock-bubble interaction is of importance in several differently scaled situations, from fragmentation of gallstones or kidney stones by shock waves (*Gracewski et al. 1993*) to the interaction of supernovae shock waves with interstellar clouds (*Klein et al. 2000*). Therefore the scalability of the interaction is important. The passage of a shock wave through a spherical bubble results in the formation of a vortex ring. In the present study, simple dimensional analysis shows that the circulation is linearly dependent on the surrounding speed of sound c_s and the initial bubble radius R. In addition, it is found that the velocities characterizing the flow field are linearly dependent on the speed of sound, and are independent of the initial bubble radius. The dependence of the circulation on the shock wave Mach number M is derived by *Samtaney & Zabusky (1994)* as $(1+1/M+2/M^2)(M-1)$. Using the dimensional analysis the velocities are shown to have a similar dependence. Full numerical simulations and experiments were conducted for slow/fast (air-helium) and fast/slow (air-SF_6) interactions. Good agreement was achieved. From the results it is seen that in both cases, according to the proposed scaling, the vortex ring velocity is bubble radius independent. The numerical results for the slow/fast interaction show that the proposed Mach scaling is valid for $M<2$. Above $M\cong2$ the topology of the bubble changes due to a competition between the upstream surface of the bubble and the incident shockwave.



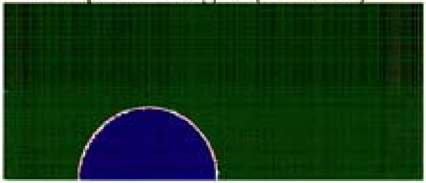
Computational method

- Leeor-2D*: ALE, interface-tracking, finite differences

Initial condition



Computational grid (450x100)

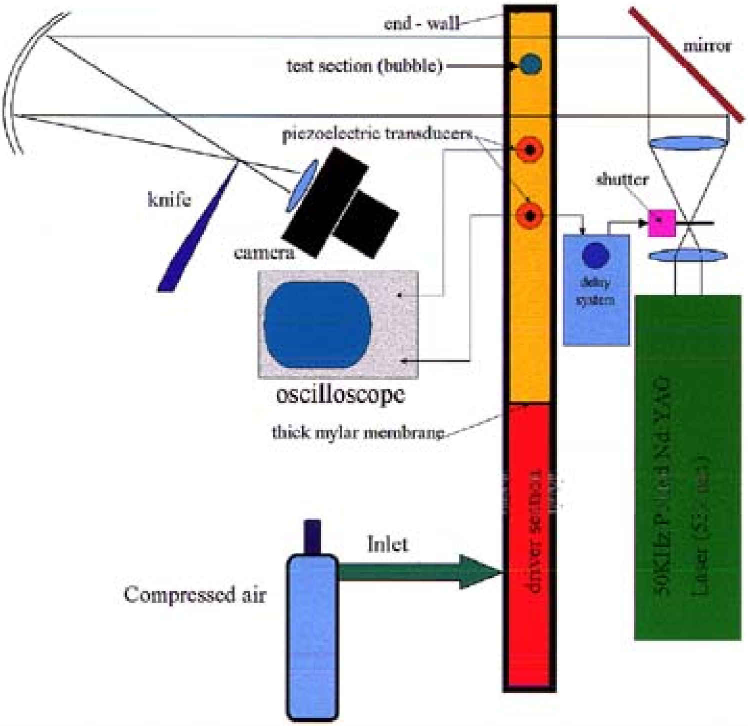


- Ambient gas - air, $E = (\gamma - 1) \cdot P / \rho$
- Heavy bubble (f/s) - Sf_6, $\eta = 5.034$, $r_0 = 1.3cm$
- Light bubble (s/f) - Helium, $\eta = 0.138$, $r_0 = 1.63cm$
- Mach range - 1.05 to 4
- Boundary conditions - free slip
- Shock tube width - 4 cm

$$\eta = \rho_{bubble} / \rho_{ambient\_gas}$$

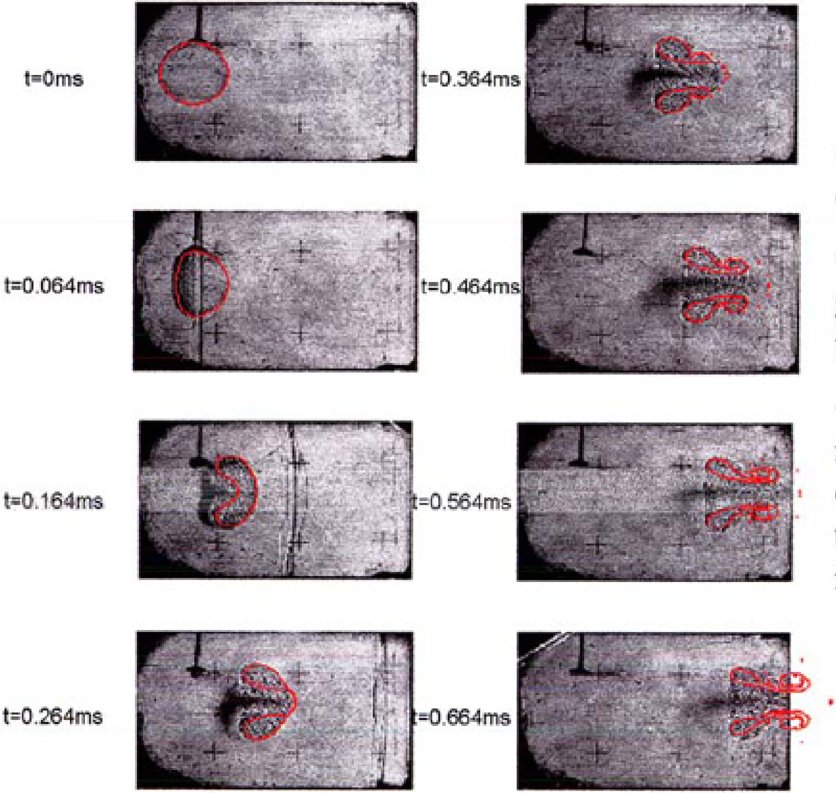
*D. Shvarts et al., Phys. Plasma 2, 2465 (1995)

Experimental Apparatus



Experiment/simulation comparison

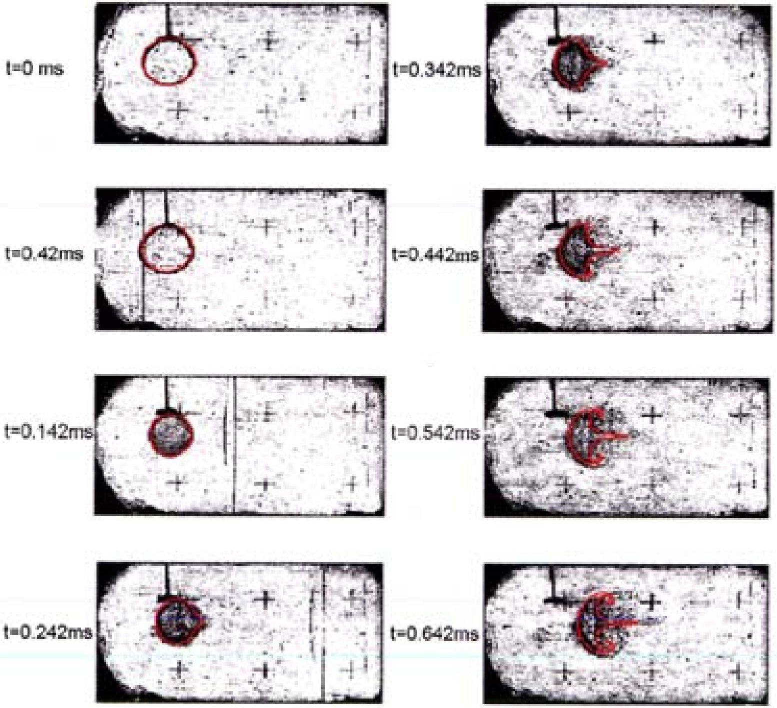
slow/fast (air-helium), M=1.22



- Schliren images of the experiment
- Red contour: gas interface from the simulation
- Good agreement is found during the early and late times of the interaction.

Experiment/simulation comparison

fast/slow (air-SF_6), M=1.17



- Schliren images of the experiment
- Red contour: gas interface from the simulation
- Good agreement is found during the early and late times of the interaction.
- Emerging jet is highly sensitive to resolution changes.

Circulation Dimensional Analysis

- Circulation: [Γ]=[length]^2/[time]
- Flow parameters: c_s - *speed of sound, R - bubble radius,* dimensionless parameters: *M - Mach number,* η *- density ratio,* γ *- ratio of specific heats of both gases*

$$\Rightarrow \quad \Gamma = f(M,\gamma,\eta)\cdot R\cdot c_s$$

- Using a vorticity deposition model*:

$$\Gamma = (1+\frac{\pi}{2})\cdot\frac{2\gamma^{1/2}}{1+\gamma}(1-\eta^{-1/2})(1+\frac{1}{M}+\frac{2}{M^2})(M-1)\cdot R\cdot c_s$$

*Samtaney & Zabusky, JFM. 269 (1994)
Zabusky & Zeng, JFM. 362 (1998)

Velocity Dimensional Analysis

- Velocity: [U]=[length]/[time]
- Since the flow velocities are determined by the vorticity distribution in the flow:

$$U_i = g_i \cdot \frac{\Gamma}{R} = g_i \cdot f(M, \eta, \gamma) \cdot c_s$$

i - ring, jet, etc...

g_i - is a constant representing the influence of the topology of the problem, the boundary conditions and the specific velocity U_i.

For example: the velocity of an ideal vortex ring*:

$$U_{ring} = \frac{1}{4\pi}\left\{\log(\frac{8R}{\delta}) - 1/4\right\}\frac{\Gamma}{R}$$

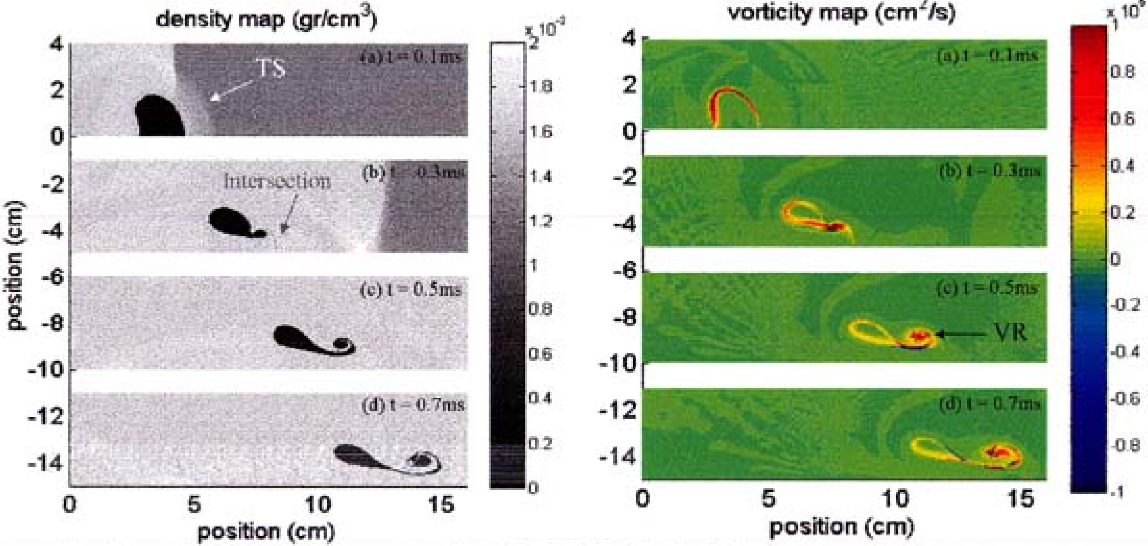
R - vortex ring radius,

δ - radius of core section (or average of ellipse axes)

*Lim & Nickels in "Fluid Vortices" edited by S.I. Green (1995)

Typical simulation

slow/fast, M=1.22



(a) - Transmitted wave (TS) emerges from the bubble ahead of the incident wave.

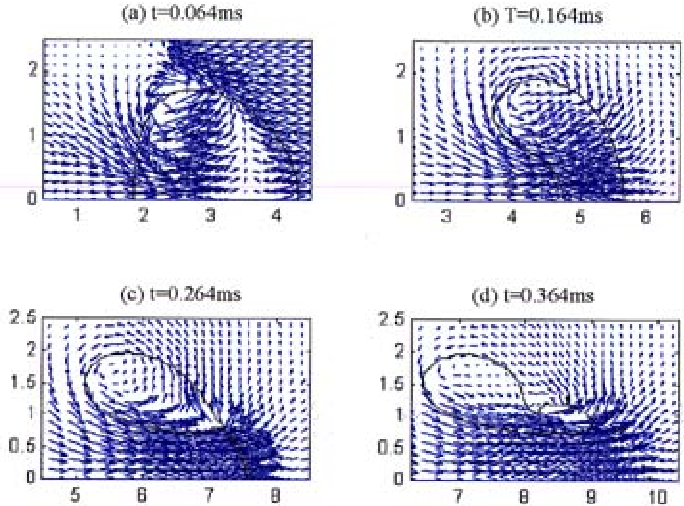
(b) - Upstream side of bubble reaches the downstream side. There is vorticity on the inner side of the bubble because of the shear velocity.

(c) - The vortex ring (VR) has formed at the downstream side.

(d) - The vortex ring is moving at a steady speed. The vortex ring separates from the upstream remains of the bubble.

Vortex ring formation - velocity field

low/fast, M=1.22



The velocity field is drawn in a coordinate moving with the shocked air

(a) The upstream interface of the bubble is initially accelerated and a shear velocity is formed leading to rotational flow.

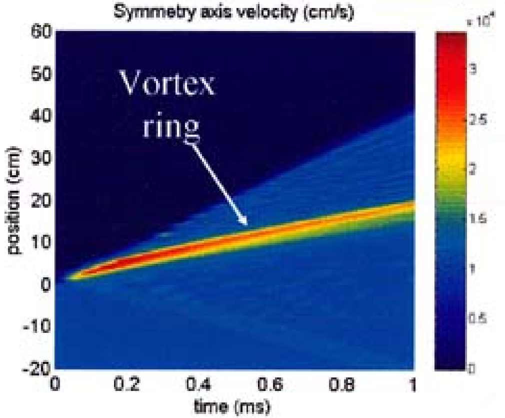
(b) The ambient gas moves in to fill the space left by the bubble.

(c) The bubble is penetrated through the center creating a torus-like shape

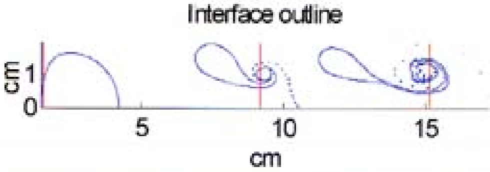
(d) The vortex ring is formed

Vortex ring velocity definition

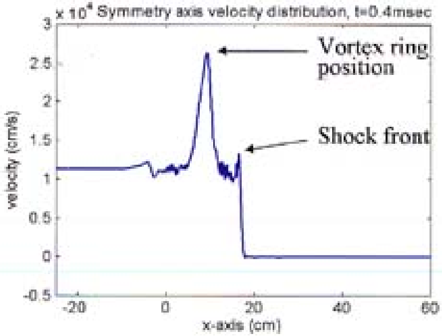
slow/fast, M=1.22



- The motion of the vortex ring is characterized by the region of high velocity.
- The maximum velocity on the symmetry axis is located at the center of the vortex ring.
- The vortex ring velocity is derived from the movement of this point.

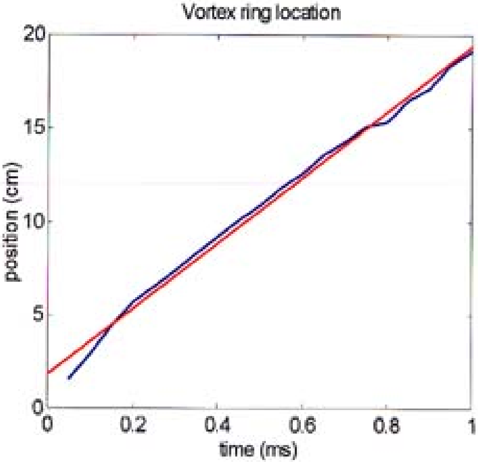


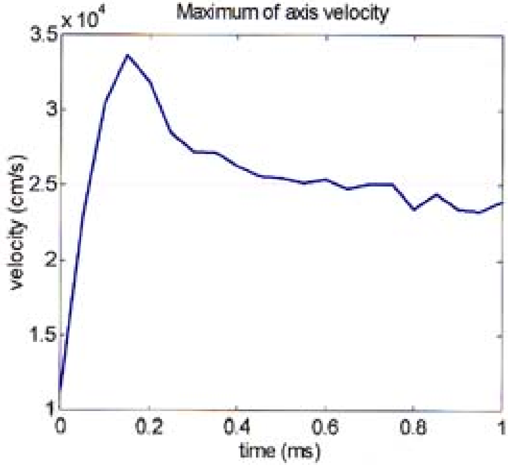
Vertical lines represent position of maximum velocity



Vortex ring velocity development

slow/fast, M=1.22

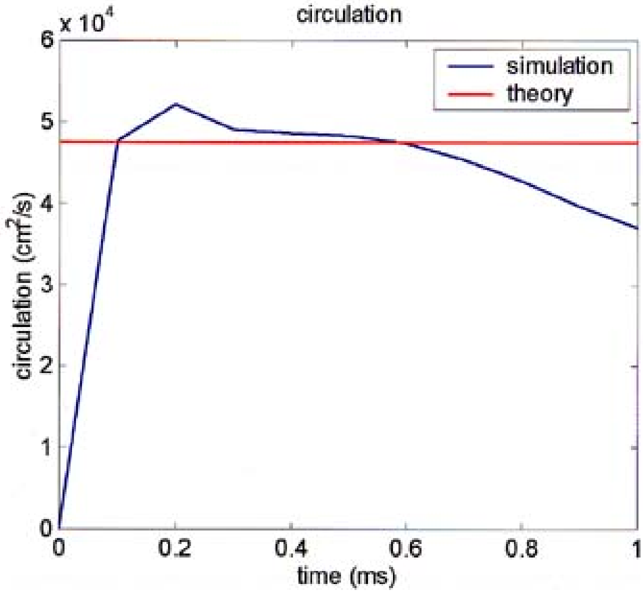




- From $t = 0.2\ ms$ the maximum axis velocity and the vortex ring velocity stabilize.
- Resulting vortex ring velocity is approximately 176 m/s.

Circulation

slow/fast, M=1.22



Circulation from simulation:

$$\Gamma = \int_A \omega \cdot dA$$

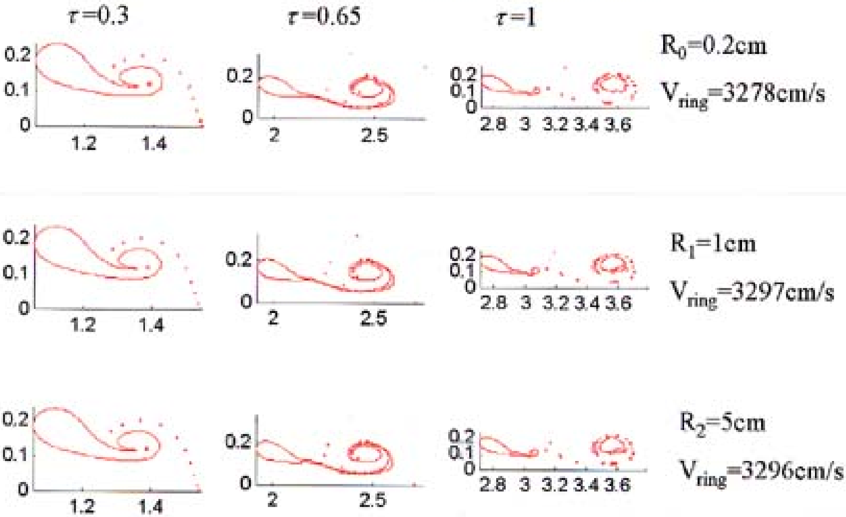
where ω is the vorticity:

$$\omega = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$$

- The circulation is in good agreement with Zabusky's model in early times.
- The decrease after t = 0.6ms is a result of numerical viscosity

Radius scaling

slow/fast, M=1.22



Scaling:

$$\tau = t \cdot R_0 / R_i$$

$$\zeta = x \cdot R_0 / R_i$$

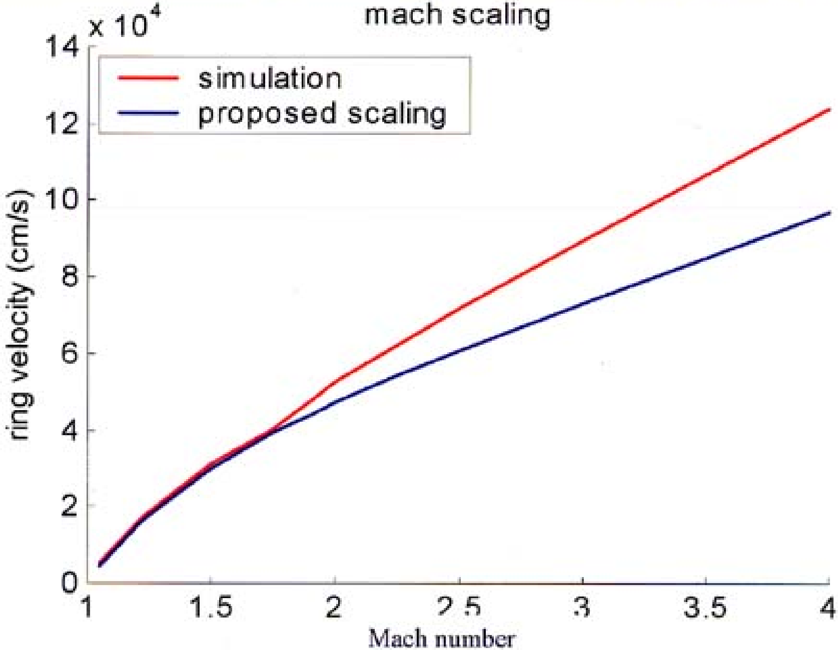
$$\xi = y \cdot R_0 / R_i$$

$$v \neq f(R)$$

The ring velocity in the three cases differs by less than 1%. Furthermore, the full evolution of the ring is also velocity independent.

Mach scaling, slow/fast

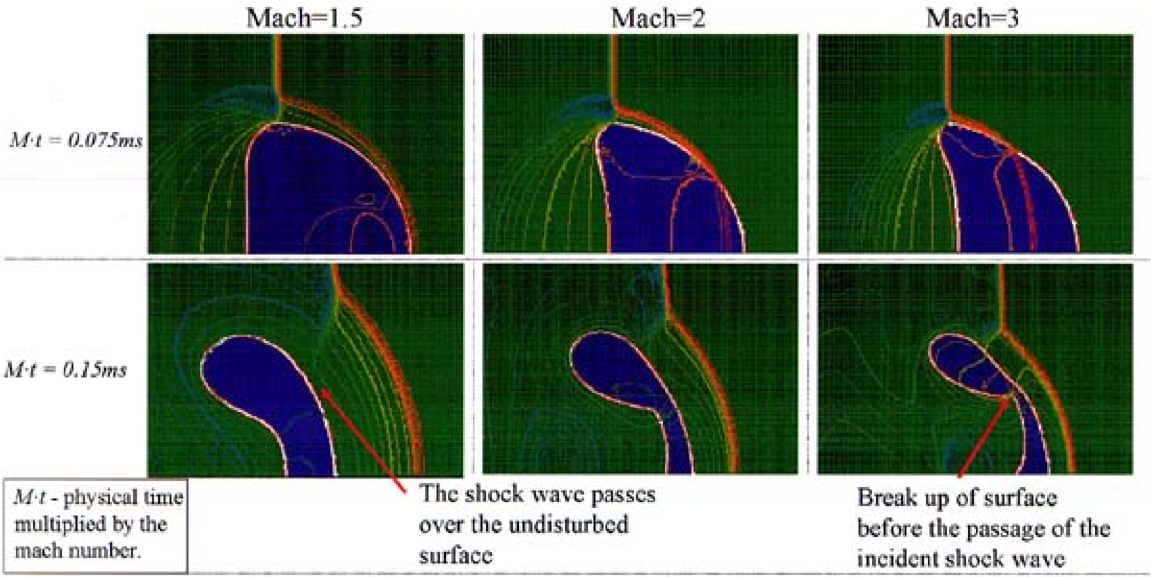
scaling factor: $(1+1/M+2/M^2)(M-1)$



- Good agreement is found for M<2.

Early shock/upstream interface competition

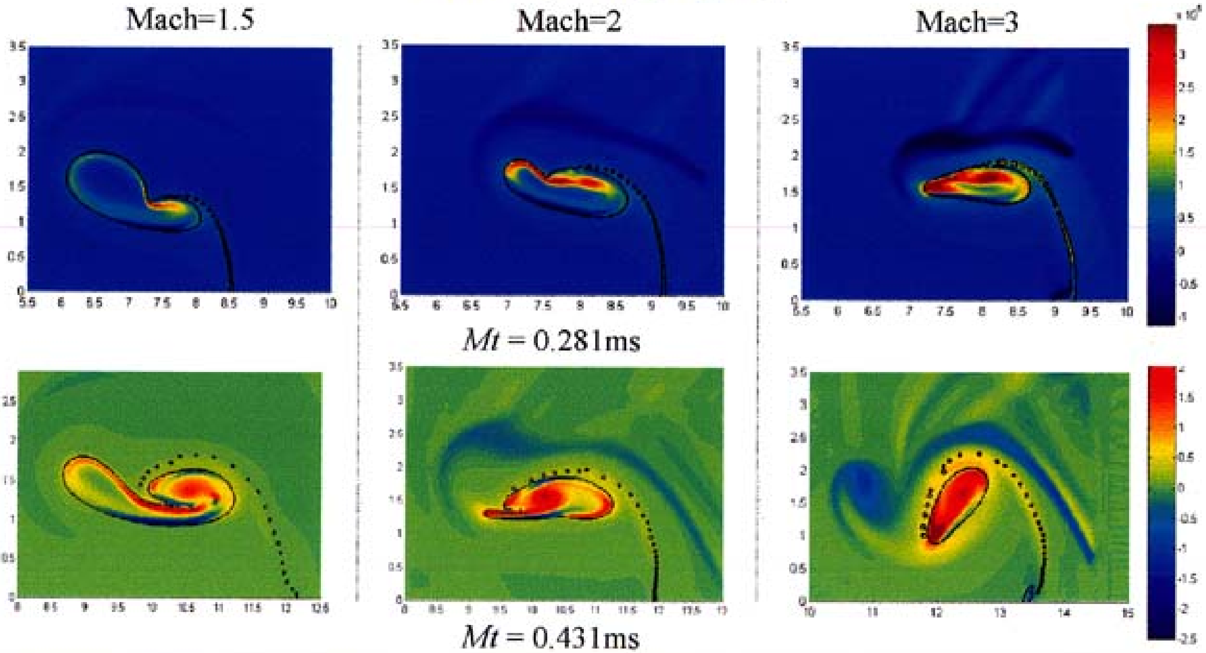
pressure contours



The results show that for M>2 the upstream surface reaches the downstream side of the bubble before the incident shock wave. In this case the incident wave deposits the vorticity on an interface different from the M<2 case. This phenomenon is not considered in the scaling rendering it invalid for M>2.

Late shock/upstream interface competition

vorticity maps (1/s)



The main difference between the results is that for higher mach numbers the vorticity is more concentrated at the vortex ring, whereas for mach=1.5 there is a distribution of the vorticity on the remains of the upstream side of the bubble. In addition, for M=3 the vortex ring is spinning around its axis. These differences are a possible cause for the increase in velocit

Summary

- The phenomenon of a shock wave bubble interaction was investigated using shock tube experiments and simulations. A comparison of the bubble interface shows very good agreement.
- Using dimensional analysis and a previously suggested Mach scaling, a new velocities scaling of the shock bubble interaction is proposed.
- The velocities in the interaction were found to be independent of the initial bubble radius.
- For slow/fast interactions (air/helium) the mach scaling factor was found to be valid for $M<2$.
- It is shown that the scaling is invalid for $M>2$ due to a change in the topology of the ring evolution.