Rayleigh-Taylor instability at a tilted interface

in incompressible laboratory experiments and compressible numerical simulations

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Outline

- Introduction

  RT instability at a tilted interface

  Mixing, available energy and mixing efficiency

- Laboratory experiments

  At DAMTP, in the Fluid Dynamics Laboratory

  Incompressible water, NaCl to create density contrast

- Numerical simulations

  At AWE, using Turmoil3D (with David Youngs)

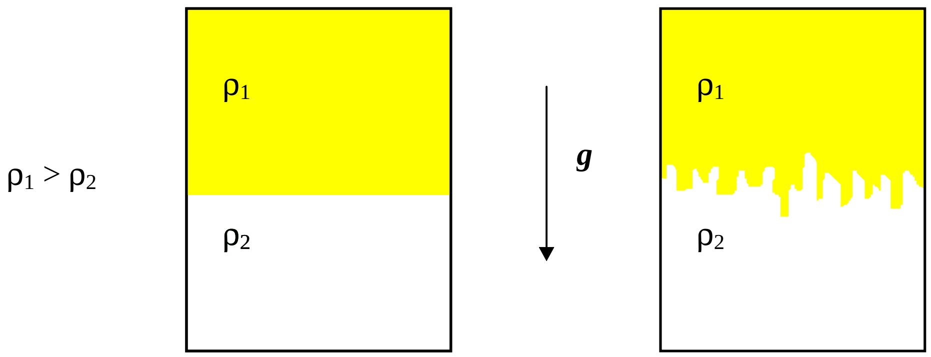
  Compressible code, for a mixture of two ideal gases

- Conclusions and further work

Introduction

- RT instability

Instability of dense fluid accelerated into less dense fluid



Non-dimensional parameter Atwood number $A = \left(\dfrac{\rho_1 - \rho_2}{\rho_1 + \rho_2} \right) = \dfrac{g'}{2g}$

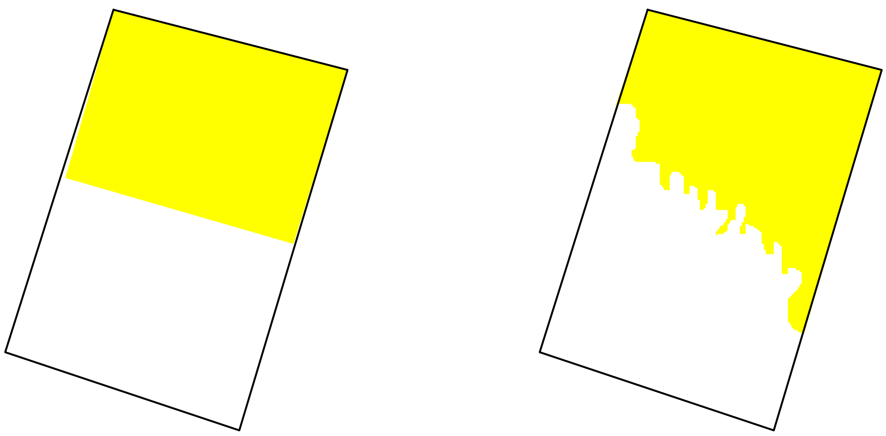
For an external lengthscale H, timescale $\tau = \sqrt{\dfrac{H}{Ag}}$

Much more efficient mixing than other mechanisms (shear instability, mechanical stirring)

An important mixing process within larger-scale flows (3D instability of 2D shear billows)

In environment, R-T instability has non-ideal initial conditions

At a tilted interface, there is competition between local instability and large-scale overturning



• Definitions of mixing

Distinguish between reversible and irreversible mixing:

Reversible mixing - interleaving of fluid with different properties - “reversible mixing = stirring”

Irreversible mixing - homogenisation of fluid properties at the molecular scale - “irreversible mixing = stirring + diffusion”

Irreversible mixing is important for
- chemical reactions
- removal of available energy when mixing density gradients across a gravitational field

• How do we measure mixing?

Mixing can be measured by a molecular mixing fraction

For two fluids, volume fractions f and $(1-f)$:

$$\vartheta(\boldsymbol{x},t) = f(\boldsymbol{x},t)(1-f(\boldsymbol{x},t))$$

Alternatively, for fluids of varying density in a gravitational field, can measure the mixing efficiency η

For a fluid at rest, stirred by an energy input and returning to rest,

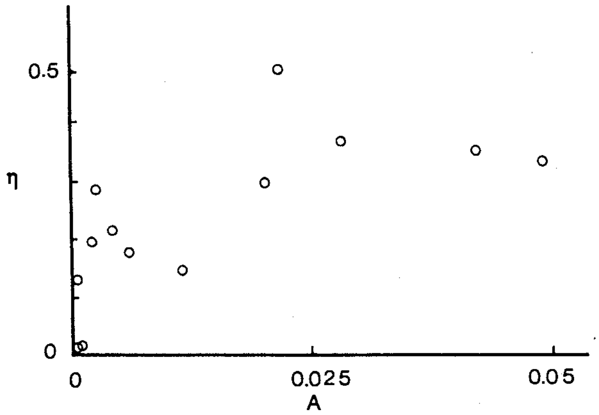
$$\eta = \frac{\text{increase in potential energy}}{\text{amount of energy added}}$$

fraction of energy lost to fluid motion doing work against gravity

- # Mixing in R-T instability

Measurements of η in laboratory experiments - high values with some dependence on A

Linden & Redondo (1991)



Numerical simulations show sensitivity to initial conditions

Linden, Redondo & Youngs (1991), Cook & Dimotakis (2001)

- # Diffusion and viscosity in incompressible fluids

Mechanical energy density per unit volume $E_v = \frac{1}{2}\rho|u|^2 + \rho g z$

$$\frac{\partial}{\partial t} E_v(\boldsymbol{x},t) + \nabla . \boldsymbol{f}_v(\boldsymbol{x},t) = -\varepsilon_v(\boldsymbol{x},t),$$

$\boldsymbol{f}_v$ energy flux

ε_v energy dissipation

Water/salt system - $\nu = 1.0\times10^{-2}\text{cm}^2\text{s}^{-1}$ kinematic viscosity

$\kappa = 1.4\times10^{-5}\text{cm}^2\text{s}^{-1}$ diffusivity

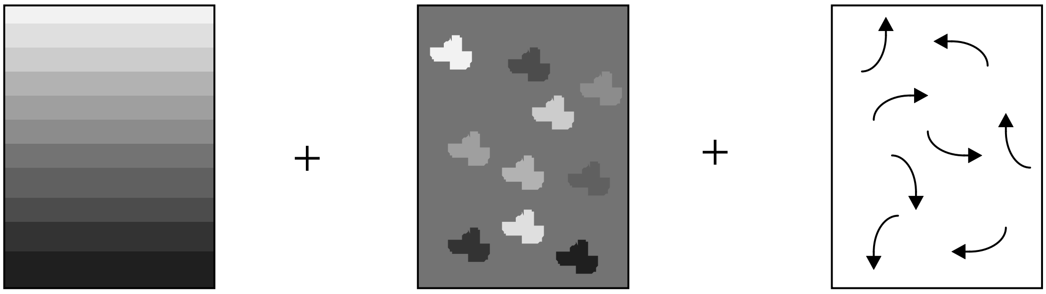
concentration fluctuations persist at smaller scales than velocity fluctuations - $Pr = \frac{\nu}{\kappa} = 700$

Turbulent flows - eddy viscosity = eddy diffusivity

effectively $Pr = 1$

• Available energy in incompressible flow

Mechanical energy in whole fluid $E = \int_V E_V \, dV$ decomposes:



$$\underbrace{PE_{back}}_{E_{back}} \quad + \quad \underbrace{PE_{avail} \quad + \quad KE}_{E_{avail}}$$

Lorenz (1955), Thorpe (1977), Winters et al. (1995)

In unforced, decaying flow

$$\frac{d}{dt}\left(E_{back} + E_{avail}\right) = -\varepsilon$$

loss of E due to turbulent dissipation

$$\frac{d}{dt}\left(E_{back}\right) = q$$

gain in E_{back} due to molecular mixing

Define cumulative mixing efficiency

$$\eta_{cumulative} = \frac{\int_{t_0}^{t} q \, dt}{\int_{t_0}^{t} q + \varepsilon \, dt} = \frac{\Delta PE_{back}}{-\Delta E_{avail}}$$

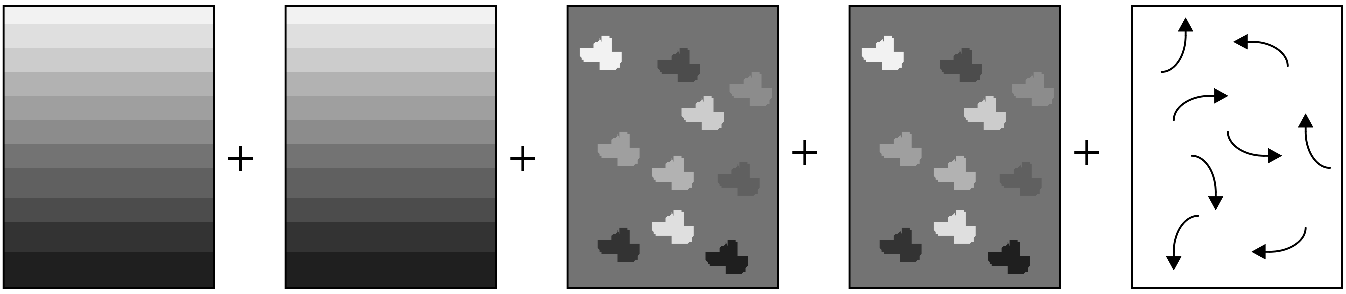
and instantaneous mixing efficiency

$$\eta_{instantaneous} = \frac{q}{q+\varepsilon} = \frac{\delta PE_{back}}{-\delta E_{avail}}$$

• Available energy in compressible flow

Now concerned with total energy (mechanical + internal) so

$E_v = \frac{1}{2}\rho|u|^2 + \rho g z + \rho e$, e internal energy. In whole fluid:



PE_{back} IE_{back} PE_{avail} IE_{avail} KE

$\underbrace{PE_{back} + IE_{back}}_{E_{back}}$ $\underbrace{PE_{avail} + IE_{avail} + KE}_{E_{avail}}$

Lorenz (1955), Andrews (1981), Shepherd (1993)

In unforced, decaying flow

$$\frac{d}{dt}\left(E_{back} + E_{avail}\right) = 0$$ E is conserved

$$\frac{d}{dt}\left(IE_{back}\right) = \varepsilon$$ gain in IE_{back} due to turbulent dissipation and molecular mixing

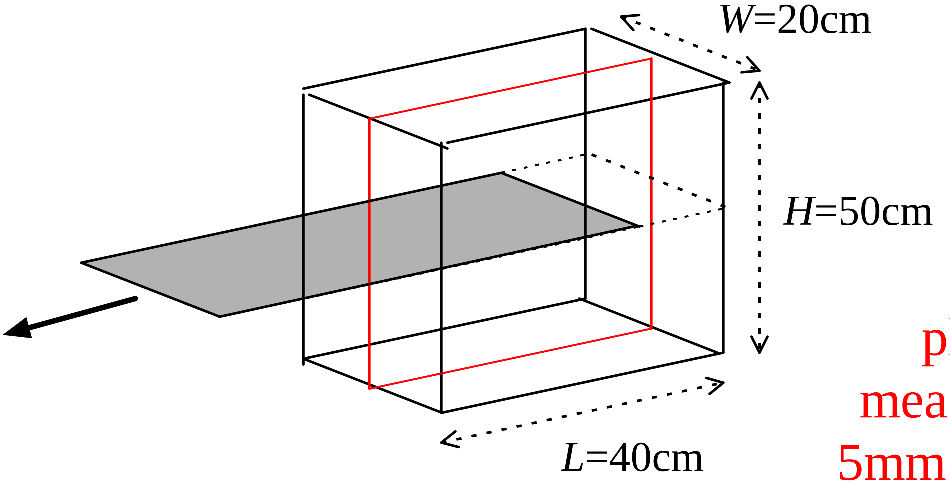
$$\frac{d}{dt}\left(PE_{back}\right) = q$$ gain in PE_{back} due to molecular mixing

Same definitions of mixing efficiency apply, so η is still

fraction of energy lost to fluid motion (reduction in E_{avail})
doing work against gravity (gain in E_{back})

Laboratory Experiments

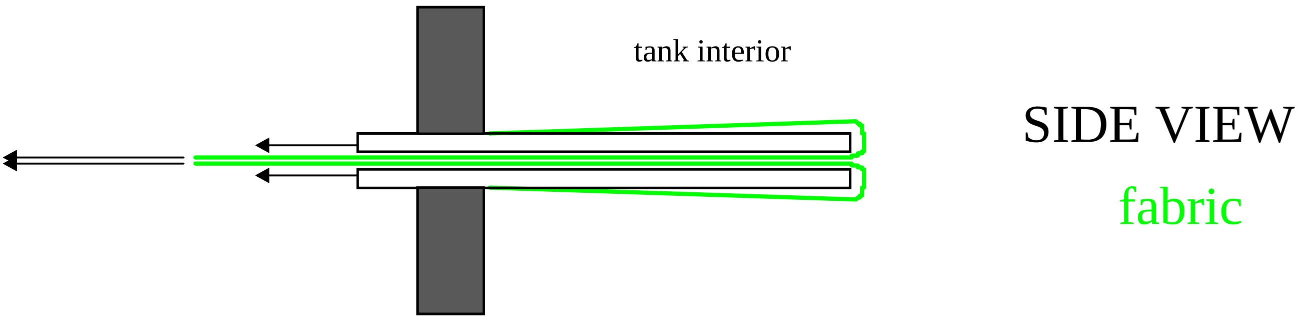
- Configuration



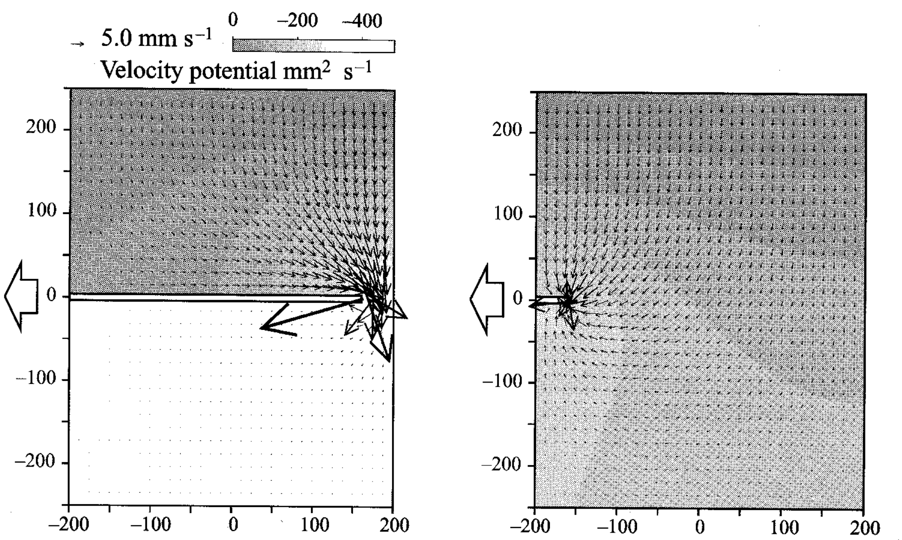
- Initial conditions

A solid barrier introduces significant shear

Reduced shear barrier: *Dalziel, Linden & Youngs (1999)*



Removal of finite thickness barrier causes initial velocity field



- Diagnostic measurements

Image analysis: spatial resolution 1 pixel ≅ 0.1cm
temporal resolution 25Hz

Assume statistical homogeneity across tank

Add propanol to fresh water to match refractive index

Density measurement

Dense fluid dyed with fluorescent dye

Images corrected for divergence of light sheet and attenuation

Velocity measurement

Fluid seeded with 400μm neutrally-buoyant particles

Lagrangian tracks for particles from tracking a frame sequence
Interpolating onto a grid gives Eulerian velocities

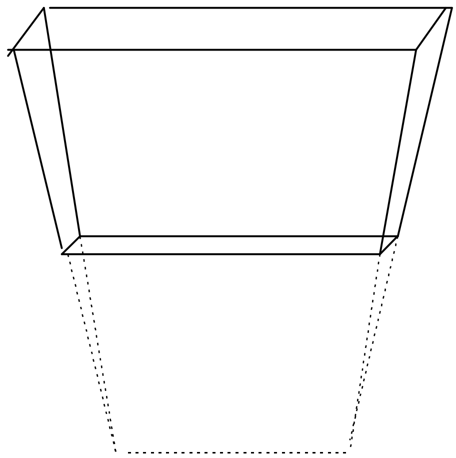
Gridded at two scales: 1cm - resolved velocity
3cm - mean velocity
(overcomes lack of similarity between experiments)

Assume isotropy at small scales ⇒ estimate of total KE

- Parameters

Atwood number	$0.5\times10^{-3} < A < 2.5\times10^{-3}$	⇒ Boussinesq
Timescale	$10s > \tau > 4.5s$	
RMS velocity	$0.8cms^{-1} < \lvert \boldsymbol{u} \rvert < 2cms^{-1}$	
Integral lengthscale	$1.8cm < l < 2.5cm$	
Reynolds number	$150 < Re < 500$	

divergence

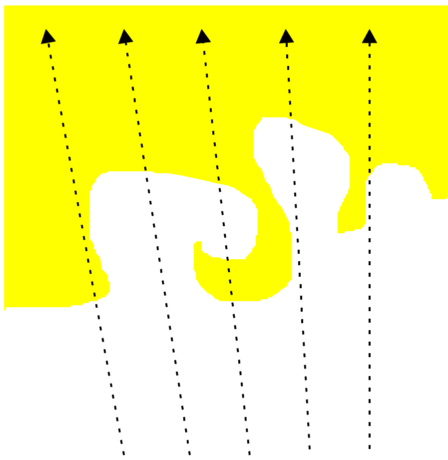


light source

corrected for
by comparing
fluorescence
pattern of image
with uniform dye
concentration

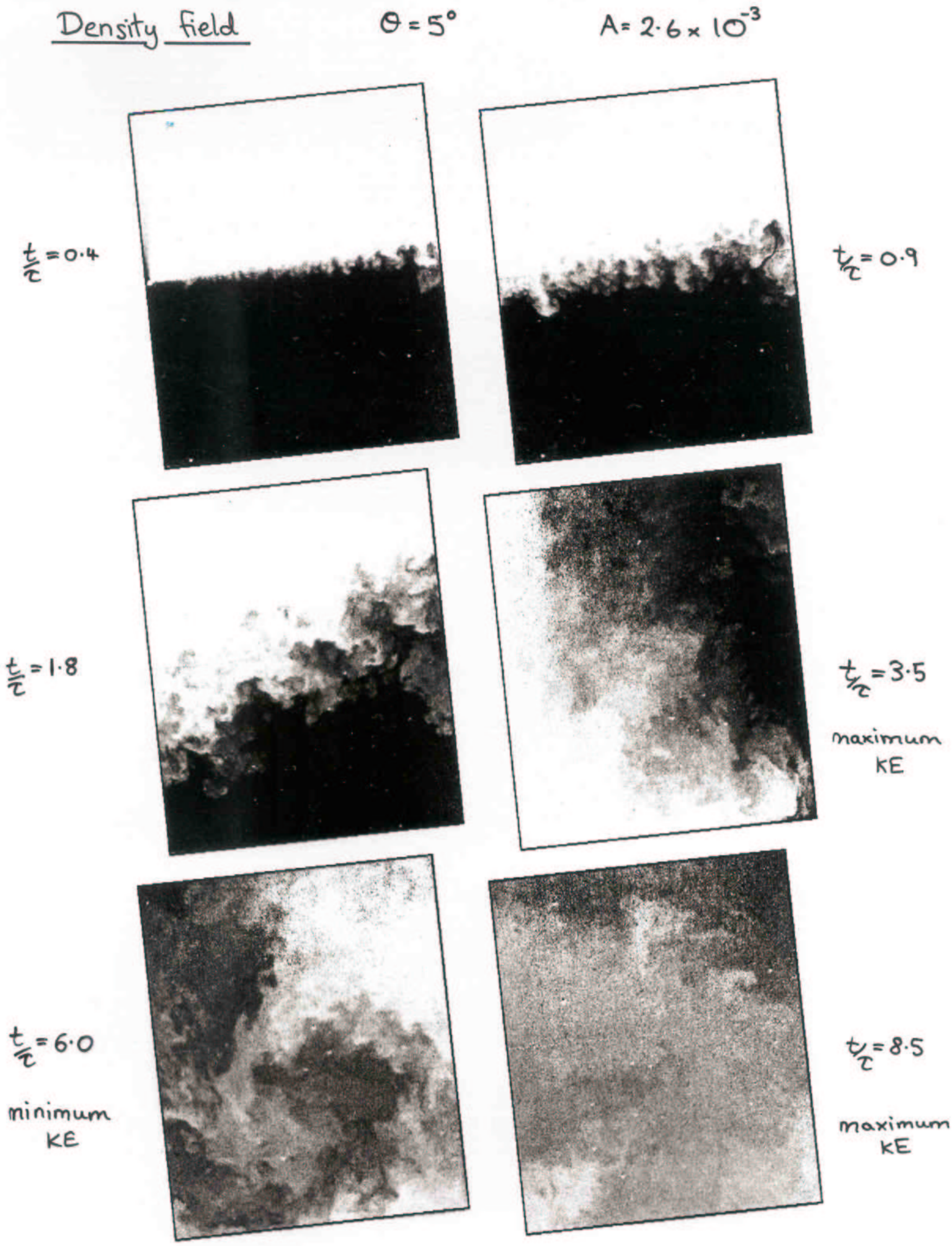
attenuation

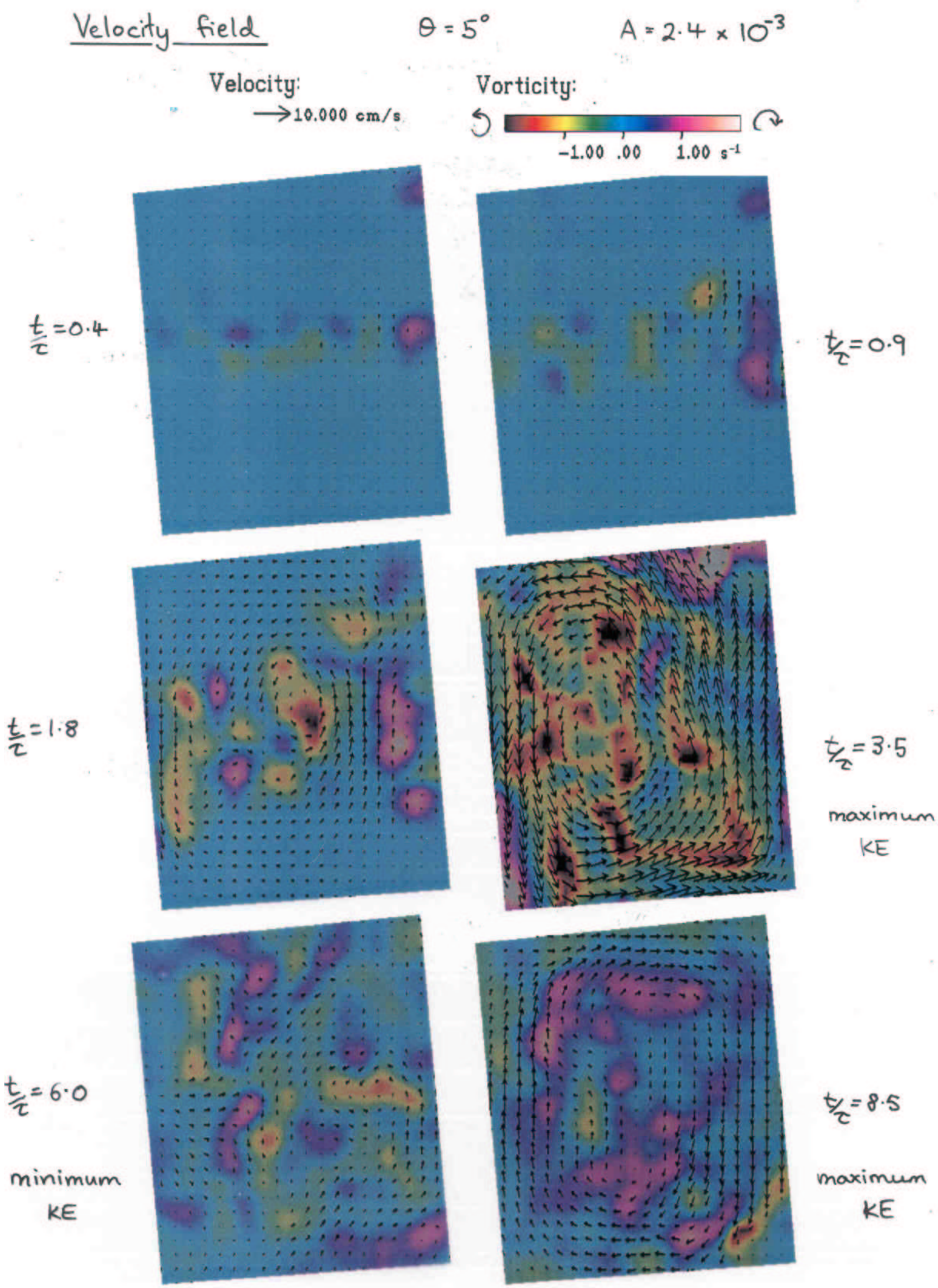
(of incident,
not fluoresced
light)



light rays

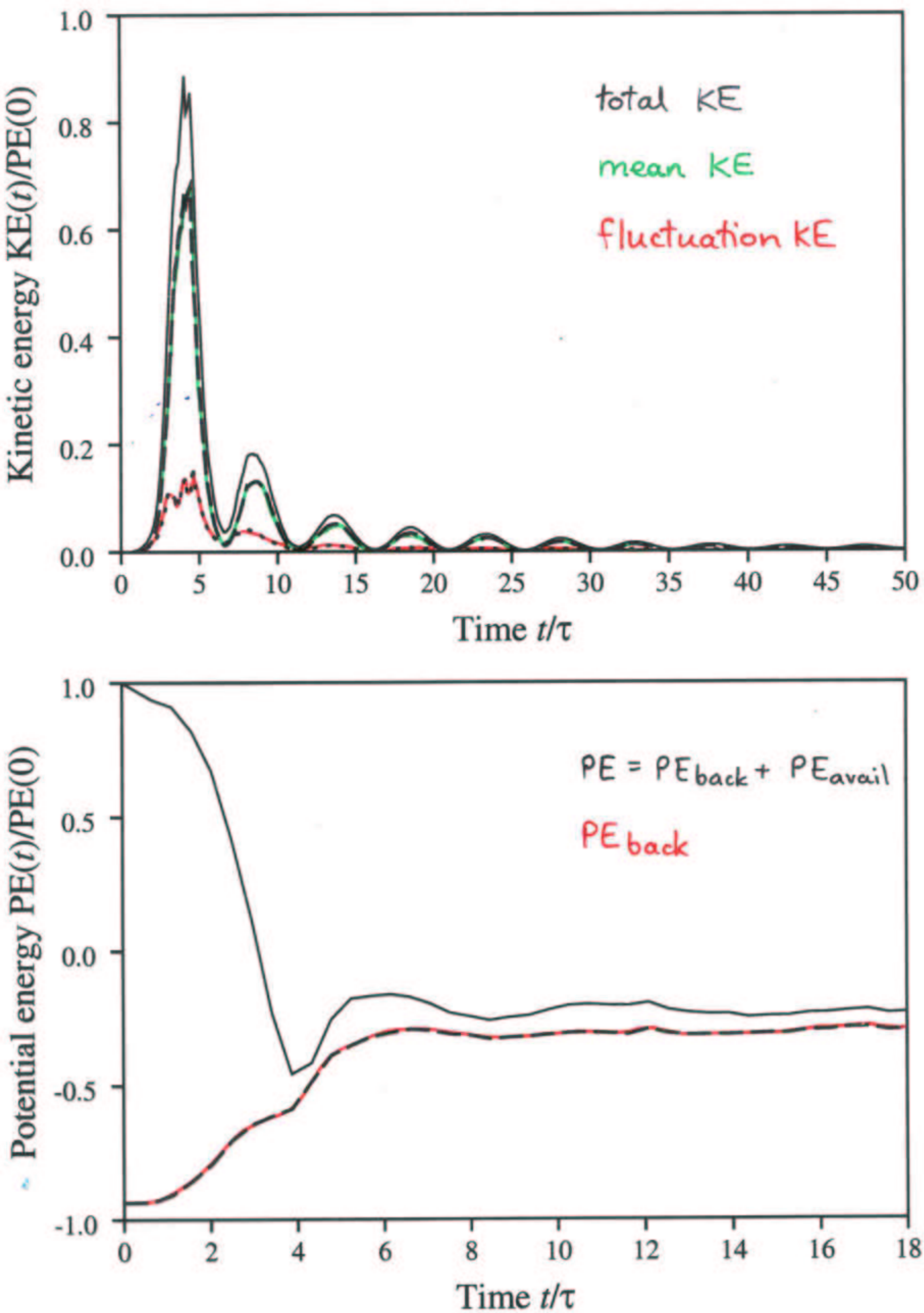
corrected for
by integrating
along rays to
determine the
actual illumination
and fluorescence
at each point


Density field
θ = 5°
A = 2·6 × 10⁻³
t/τ = 0·4
t/τ = 0·9
t/τ = 1·8
t/τ = 3·5
maximum KE
t/τ = 6·0
minimum KE
t/τ = 8·5
maximum KE

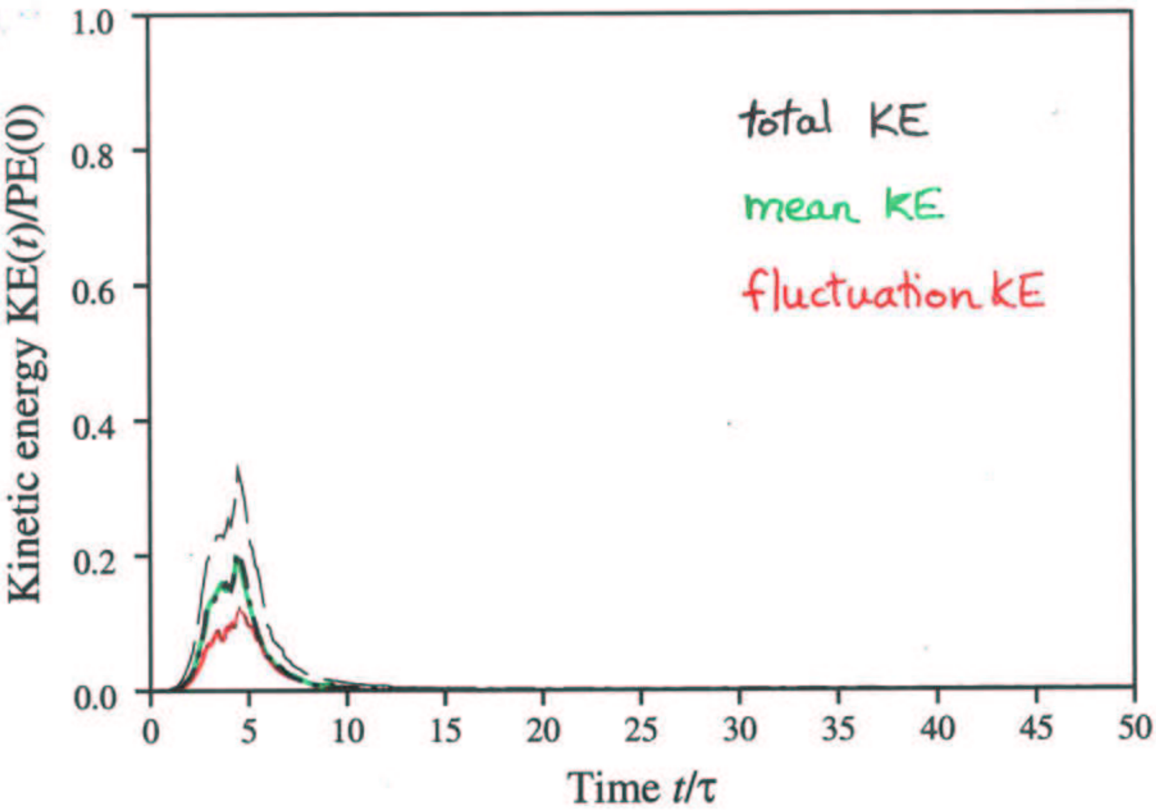


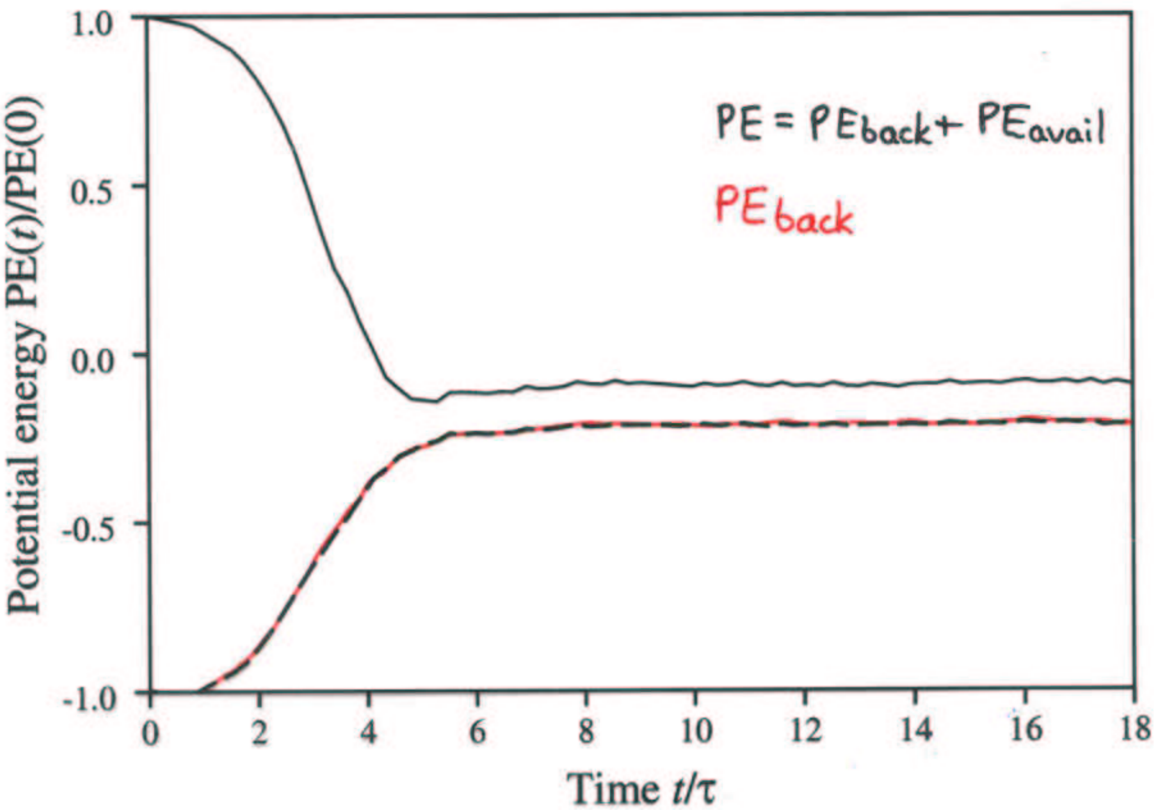
Velocity field
θ = 5°
A = 2·4 x 10⁻³
Velocity:
10.000 cm/s
Vorticity:
-1.00 .00 1.00 s⁻¹
t/τ = 0·4
t/τ = 0·9
t/τ = 1·8
t/τ = 3·5
maximum KE
t/τ = 6·0
minimum KE
t/τ = 8·5
maximum KE

Energy evolution $\theta = 5°$



Energy evolution $\theta = 0°$





Numerical Simulations

- Code type

Semi-Lagrangian finite volume code

(conservation of fluid masses and momentum)

Two ideal gases ($\gamma = 5/3$)

Typical simulation: 3D at resolution 200×160×80

- Viscosity and diffusion

Loss of resolution at grid scale ⇒ diffusion-like behaviour

for mass fractions, analogous to molecular mixing

for KE, analogous to dissipation, and added to IE

In some runs, an explicit viscosity was added

- Approximating an incompressible fluid

Normalisation: choose $H = 1$, $Ag = 1$, $\rho_1 = 1$

Non-dimensional parameters (ideally small):

Density ratio	$B = \frac{\Delta\rho}{\rho_0} = \frac{2}{g}$	≈ 0.18
Mach number	$M = \sqrt{\frac{AgH\rho}{p\gamma}} \approx \sqrt{\frac{3}{5p_0}}$	≈ 0.08
Incompressibility ratio	$I = \frac{gH\rho^2}{5p\Delta\rho} \approx \frac{g^2}{10p_0}$	≈ 0.12

Compromise $g = 11$, $p_0 = 100$

• Initial conditions - basic distribution

Away from interface:

Since $\boldsymbol{u} \approx 0$, require $\dfrac{\partial p}{\partial z} = -\rho g$.

Require neutral stability, buoyancy frequency

$N^2 = \dfrac{g}{T}\left(\dfrac{\partial T}{\partial z} + \dfrac{g}{c_p}\right) = 0 \Leftrightarrow$ isentropic fluid $p = k(s)\rho^\gamma$.

At interface:

Choose specific heats at constant volume, c_{v1} and c_{v2}.

Require temperature continuous $\Leftrightarrow c_{v1}\rho_1 = c_{v2}\rho_2$.

Everywhere:

Pressure field cannot be entirely hydrostatic.

Require $\boxed{\dfrac{\partial}{\partial t}(\nabla.\boldsymbol{u}) = 0}$.

Ignoring terms of O(u^2), require

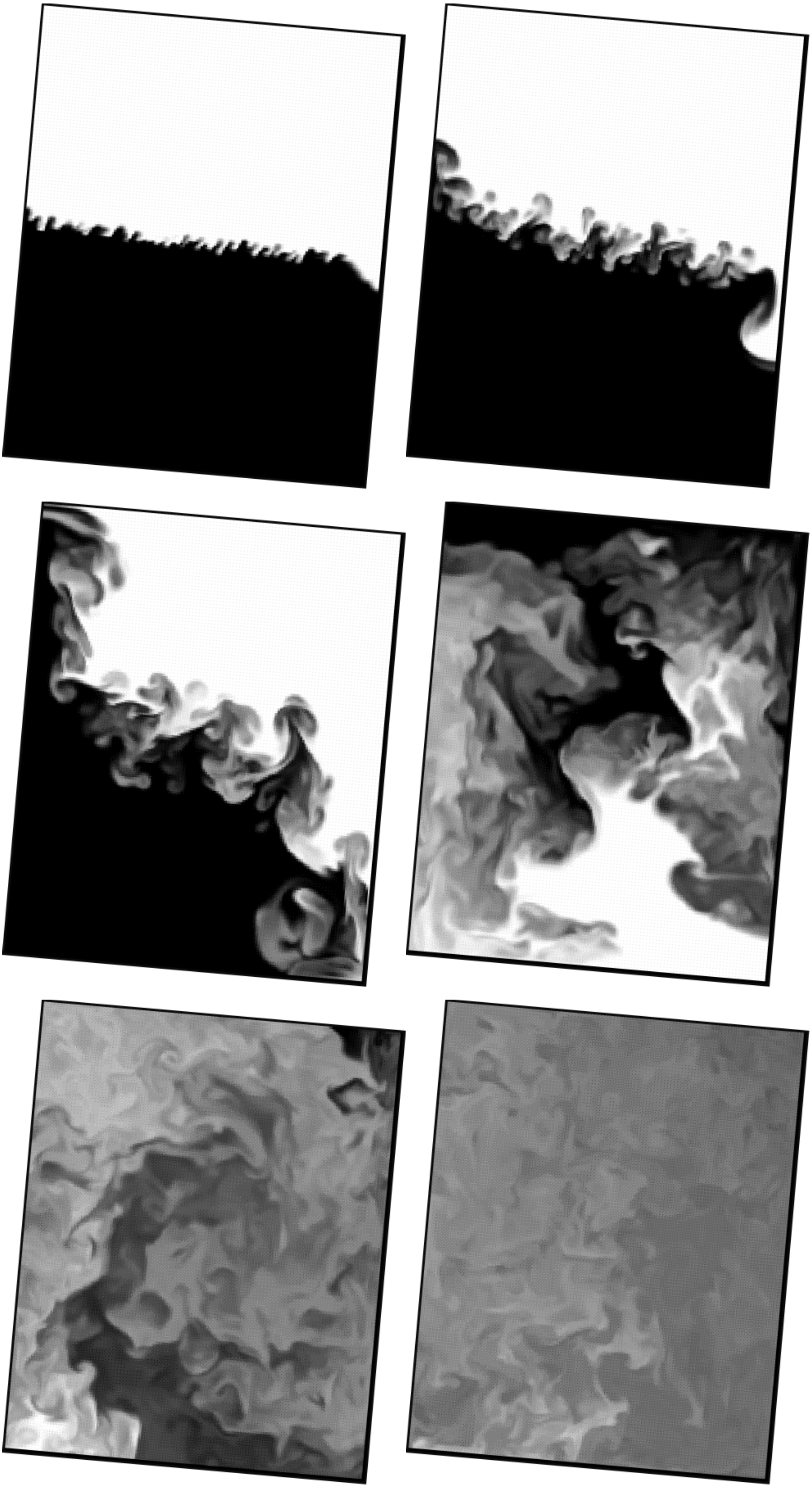
$$\nabla.\left(\frac{1}{\rho}\nabla p\right) \propto \nabla.\left(k^{1/\gamma}\nabla p^{(\gamma-1)/\gamma}\right) = 0,$$

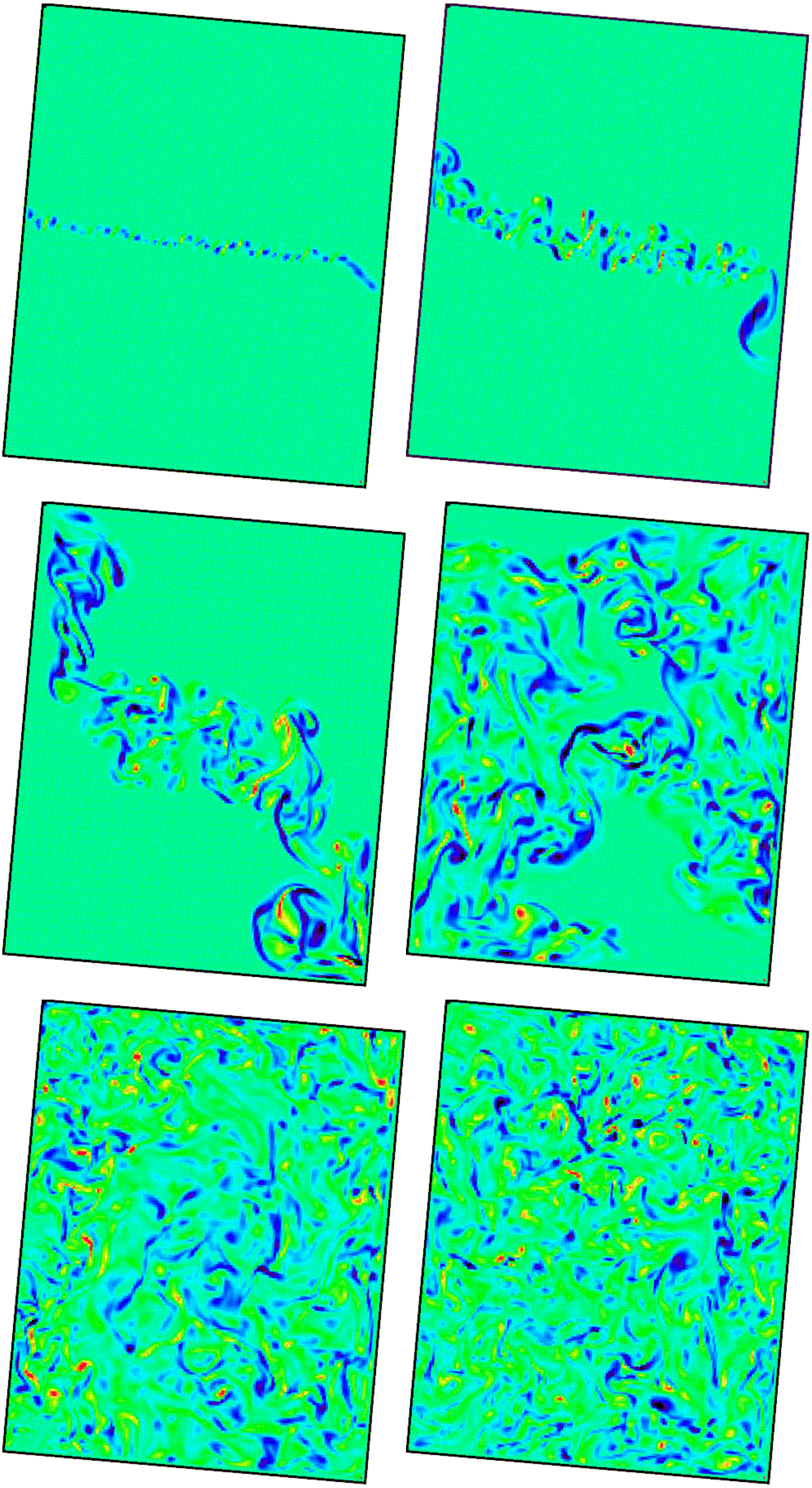
with $\dfrac{\partial p}{\partial n} = -\rho g \hat{\boldsymbol{n}}.\hat{\boldsymbol{z}}$ on boundaries with outward normal $\hat{\boldsymbol{n}}$.

• Initial conditions - perturbations

2D velocity field with vorticity at interface models experimental barrier withdrawal.

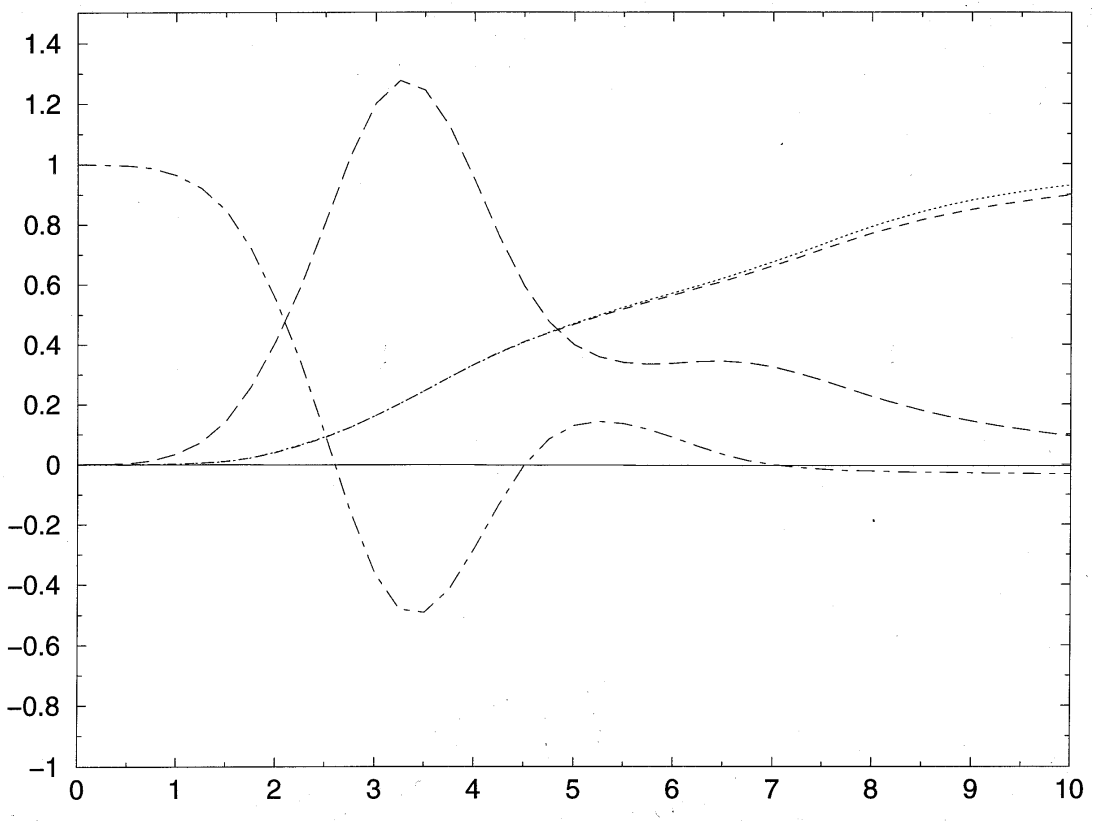
3D random perturbation to interface position, wavelengths $\dfrac{L}{40} < \lambda < \dfrac{L}{20}$, rms amplitude $\sigma = \dfrac{H}{2500}$.



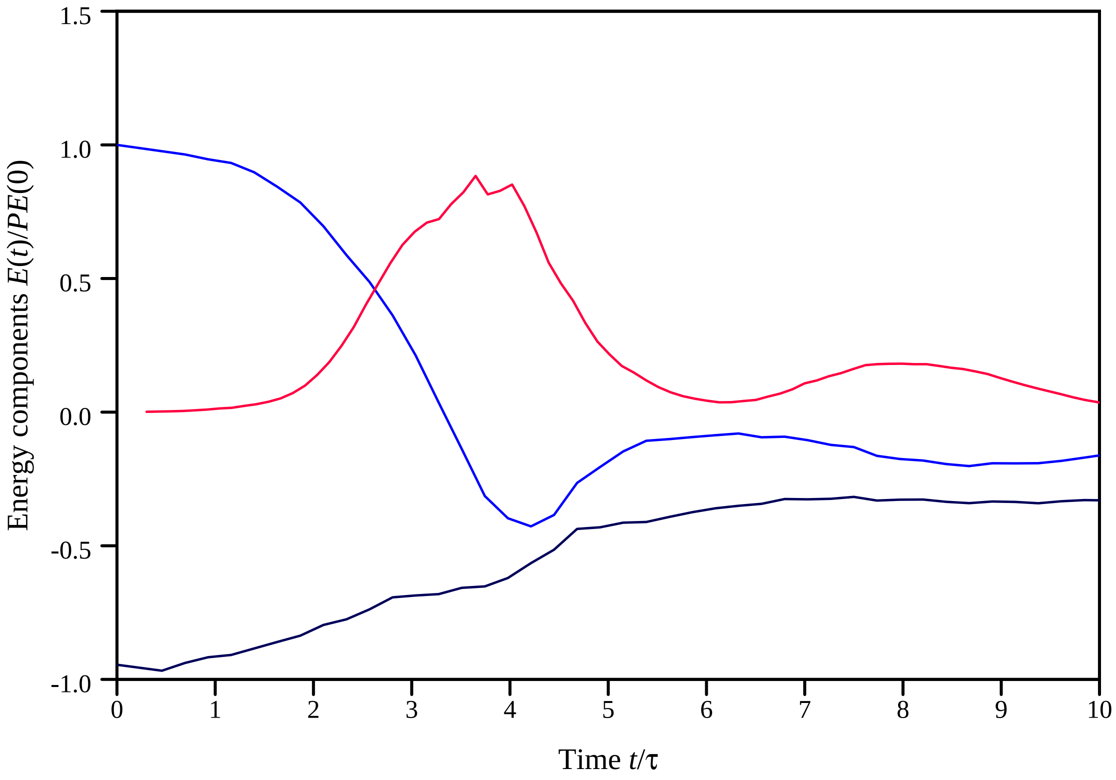


Energy budget

Numerical results $\eta \approx 0.48$



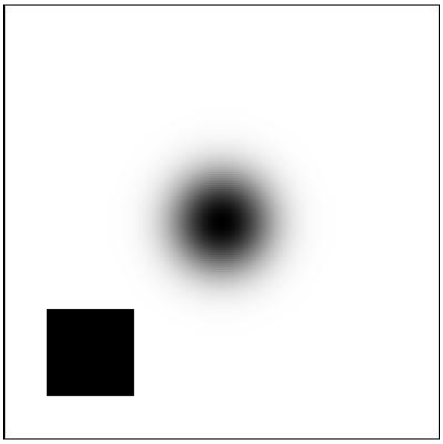
Typical experimental results $\eta \approx 0.38$



• Why the difference in energy budgets?

Numerical diffusion not ∇^2

2D advection test pattern



Numerical viscosity acts preferentially at small scales and is resolution and velocity-dependent

Total dissipation is unaffected by ratio of explicit/numerical viscosity until explicit viscosity dominates

Re of experiments is low

But experiments do not show *Re* dependence

Energy conservation

Small departures from energy conservation in stable waves

Sensitivity to initial conditions

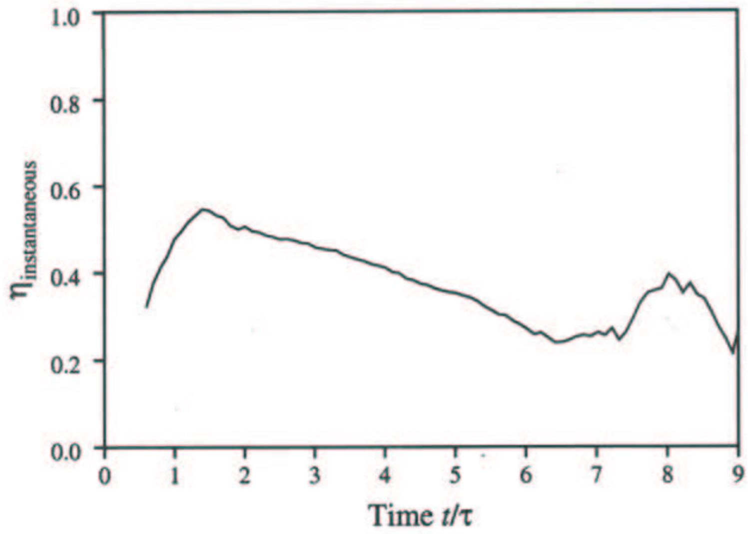
But there is no change when λ_{random} increased by 4

Different molecular Pr

Does small-scale dynamics adjust to forcing from larger scales?

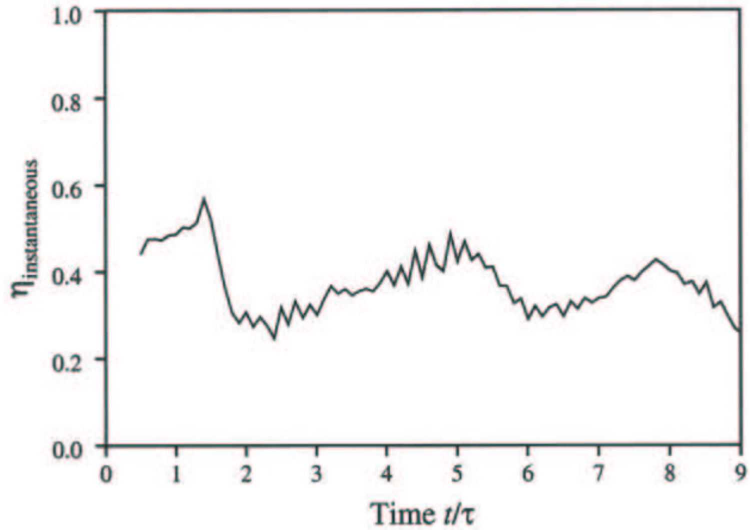
Instantaneous mixing efficiency

$\theta = 0°$



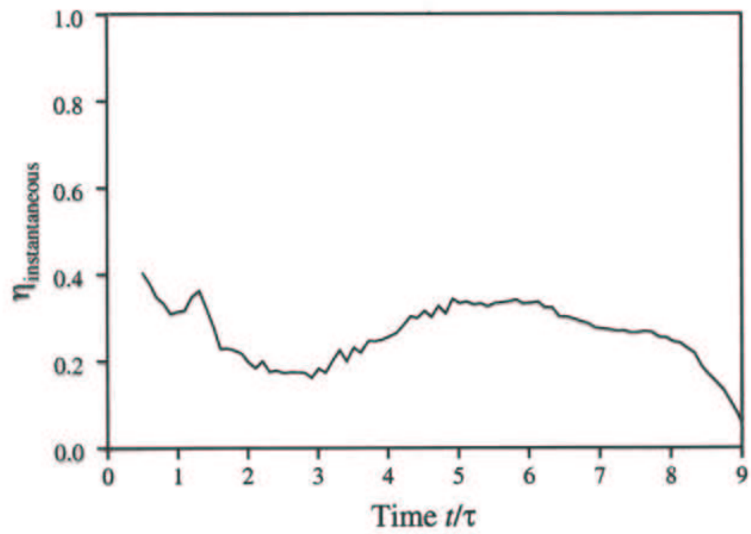
peak values $\eta_{instantaneous} \approx 0.5$ around $t/\tau = 1$ for $\theta = 0°, 5°$

$\theta = 5°$



typical values 0.2–0.4

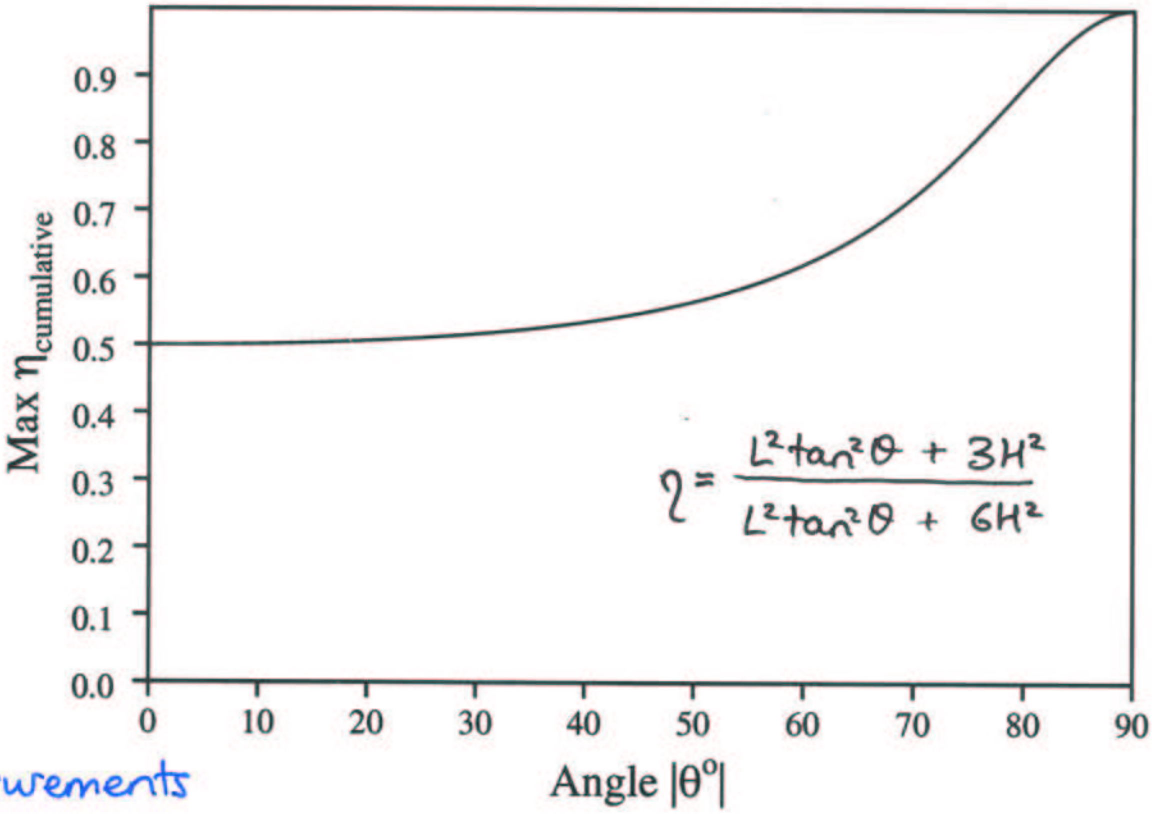
$\theta = 10°$



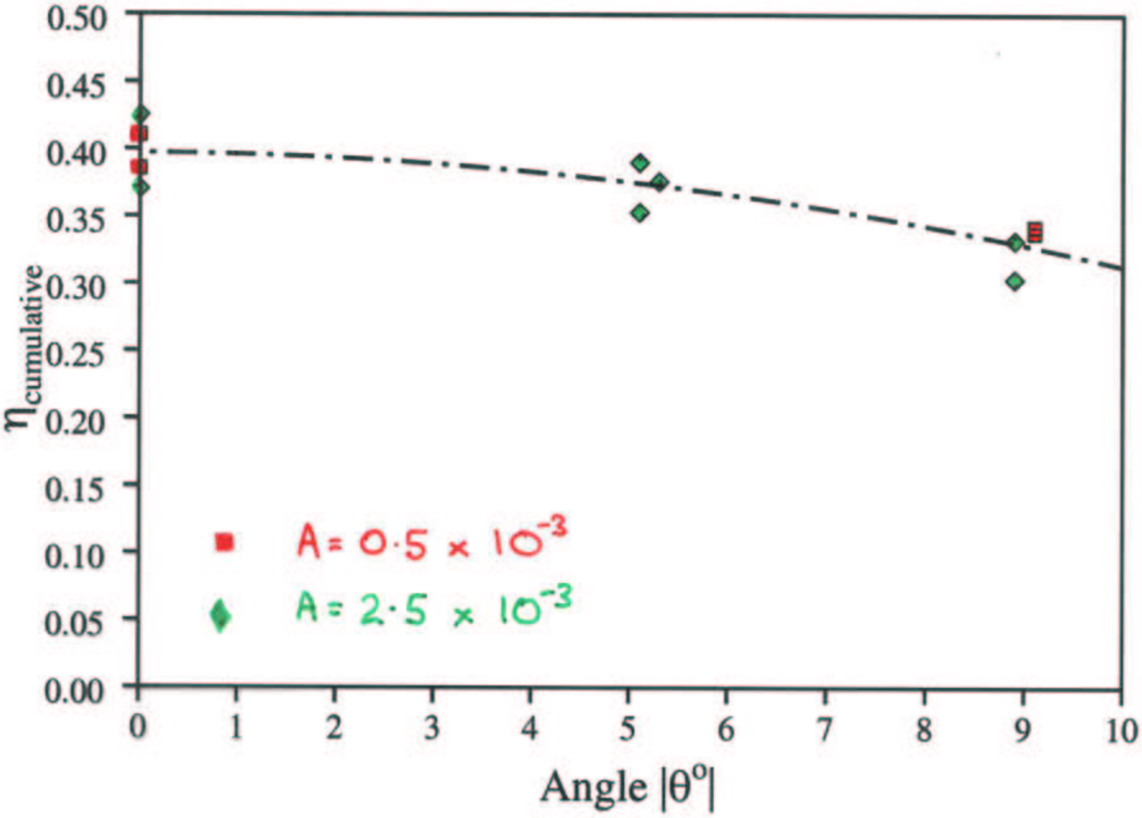
Dominant contribution to $\eta_{cumulative}$ when E_{avail} decreases most rapidly – during overturning

Cumulative mixing efficiency

theoretical maximum



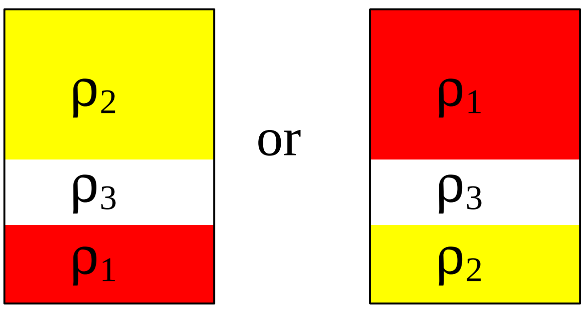
measurements



Conclusions and further work

- Laboratory experiments
  - $\theta = 0^\circ$ - $\eta_{cumulative} \approx 0.4$
  - As $\theta \uparrow$, $\eta_{cumulative} \downarrow$
  - For $\theta \leq 5^\circ$, $\eta_{instantaneous} \approx 0.5$
- Numerical simulations
  - Models experiments at suitable parameters
  - Good agreement in large-scale overturning
- Further work
  - Investigate sensitivity of mixing to various factors
  - Investigate instability at higher angles - up to limiting case:

  - Extend study of mixing efficiency to more complex stratifications



$\rho_1 > \rho_2 > \rho_3$

Dalziel & Jacobs (2000)