# PRELIMINARY RESULTS OF LES SIMULATIONS OF SELF-SIMILAR VARIABLE ACCELERATION RT MIXING FLOWS

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**The importance of self-similar variable acceleration RT flows (SSVARTs) for the design and calibration of turbulent mixing models is shown in the presentation by Antoine Llor at this workshop.**

**Because experimental results on SSVARTs are not, and will probably not be available in the near future, we are currently investigating such flows by means of LES.**

**In these incompressible flows the acceleration has the form**

$$g = kt^n$$

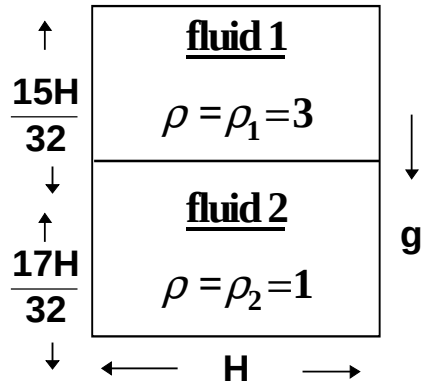
**and for self-similar mixing the mixing zone width grows in proportion to $t^{n+2}$.**

**Preliminary results, using low-resolution LES are given for n = 0, 1 and -1, using the TURMOIL3D code.**

**Results are compared with simple theoretical models.**

**THE TEST PROBLEM**

**This is based on the test problem proposed by Guy Dimonte (see presentation at this workshop).**




| | | |
|---|---|---|
| **Computational domain** | **:** | **H x H x 2H** |
| **Zoning** | **:** | **128 x 128 x 256** |
| **Dimension** | **:** | **H =1** |
| **Acceleration** | **:** | **(a) g = 2**<br>**(b) g = t**<br>**(c) g = 9/(4t)** |

**Run to t = 4.5, when for each case**

$\left(\int \sqrt{g}\, dt\right)^2 = 40.5$

**Compressible calculation with Mach number < 0.2**

**The major calculational problem is the treatment of variable g within a compressible calculation.**

**For incompressible flow, the pressure distribution adjusts at each instant of time to maintain** $\mathrm{div}\,\underline{u} = 0$. **If no mixing occurs this implies hydrostatic equilibrium.**

$$\frac{\partial p}{\partial z} = \rho g$$

**In the compressible simulations an appropriate pressure gradient is maintained by adding an internal energy source.**

**Initially adiabatic hydrostatic equilibrium (uniform entropy/neutral stability within each fluid) is assumed:-**

$$\frac{\partial p_0}{\partial z} = \rho_0 g_0$$

$p_0 = k\rho_0^{\gamma}$ , **in each region**

$$\rho_0 = \begin{cases} 3 & \text{just above the interface} \\ 1 & \text{just below the interface} \end{cases}$$

$\gamma = \frac{5}{3}$ **for both fluids**

If $g_{n+\frac{1}{2}}$ is the value of g for the n-th time step, at the start of the time step

the internal energy is scaled:-

$$\varepsilon' = \varepsilon . \frac{g_{n+\frac{1}{2}}}{g_{n-\frac{1}{2}}}$$

This maintains hydrostatic equilibrium outside the mixing zone and uniform entropy within each fluid.

The acceleration history needs to be modified slightly to give finite non-zero g at t = 0:-

case (b) $g = \max \{0.01, t\}$

case (c) $g = \min \{50, 9/(4t)\}$

The initial interface pressure is chosen high enough to ensure that the Mach no. of the flow remains small (M<0.2) at all times and also high enough to give small (<4%) variation in the initial density of each region.

## THE INITIAL PERTURBATION

**RT experiments with constant g give bubble penetration**

$$h_1 = \alpha \frac{\rho_1 - \rho_2}{\rho_1 + \rho_2} gt^2 \quad \text{, with } \alpha \sim 0.05 \text{ to } 0.06$$

**TURMOIL3D calculations with short wavelength initial perturbations (growth purely by mode coupling) give $\alpha \sim 0.03$.**

**Need to assume long wavelength initial perturbations with amplitude $\propto$ wavelength (as proposed by Inogamov [1]) to give self-similar growth with $\alpha \sim 0.05$.**

**Perturbation used** $\xi(x,y) = \xi_S + \xi_L$

$\xi_S$ : **wavelengths** **$4\Delta x$ to $8\Delta x$**
**s.d** **=** **$0.02\ \Delta x$**

$\xi_L$ : **power spectrum** **P(k)**

$$\sigma_\lambda = \left\{ \int_{2\pi/\lambda}^{\infty} P(k)dk \right\}^{\frac{1}{2}} = \varepsilon\lambda$$

$$\Rightarrow P(k) \propto 1/k^3$$

$\varepsilon = 0.0005$

**wavelengths in the range** $4\Delta x$ **to** $\frac{H}{2}$

## RESULTS SHOWN

$f_1$ , $f_2$ : volume fractions of fluids 1 and 2

$\overline{f_1}$ , $\overline{f_2}$ : plane averaged values

$W = \int \overline{f_1}\ \overline{f_2}\ dz$ integral mix width

$h_1$ : bubble penetration - measured to point where $\overline{f_1} = 0.99$.
Approximation used here : $h_1 = 3.3W$

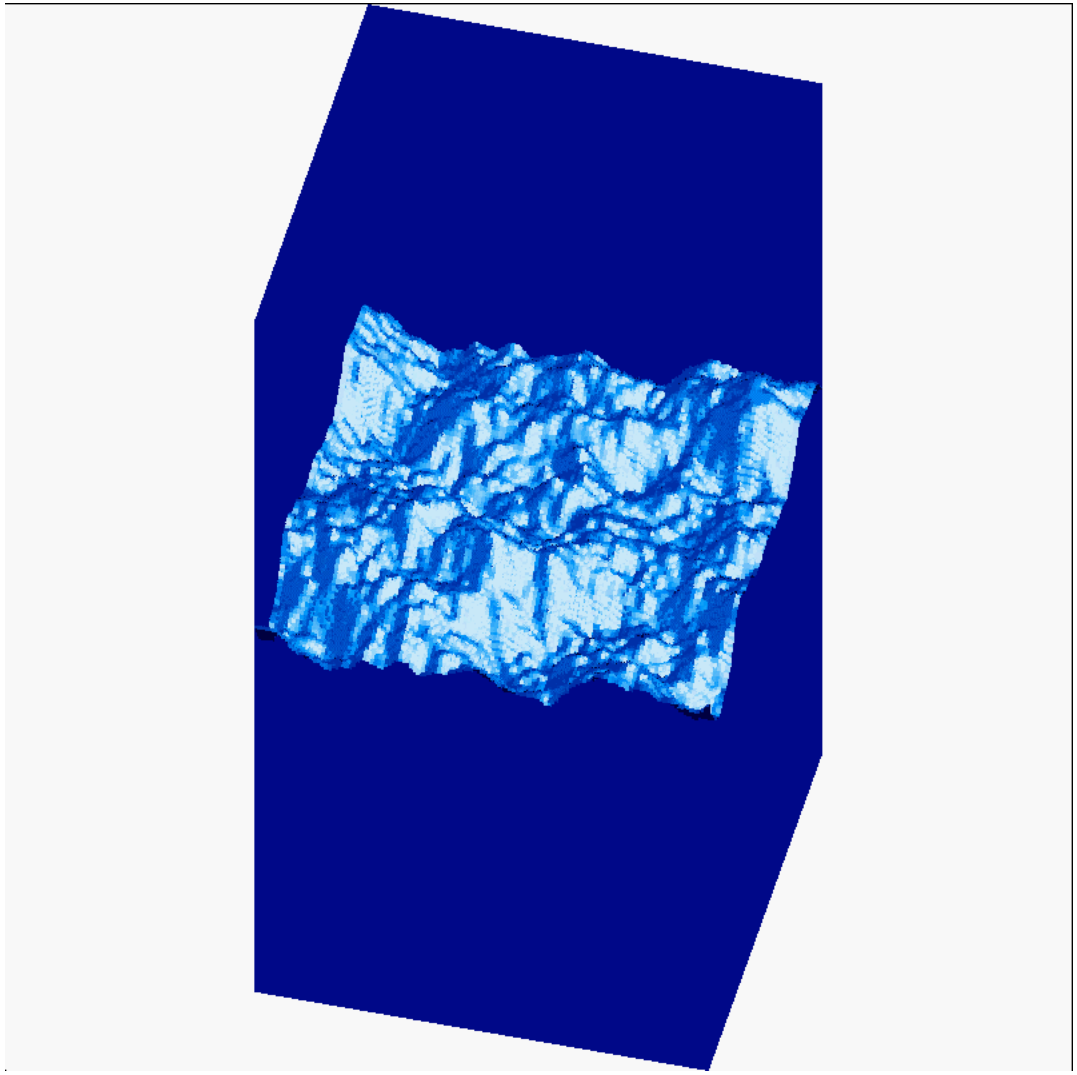
$\theta$ = $\dfrac{\int \overline{f_1 f_2}\ dz}{\int \overline{f_1}.\overline{f_2}\ dz}$ , molecular mixing fraction

$\dfrac{D}{P}$ = $\dfrac{\text{KE dissipated}}{\text{KE production}}$

$S = \dfrac{\rho_1 - \rho_2}{\rho_1 + \rho_2}\left(\int \sqrt{g}\ dt\right)^2$

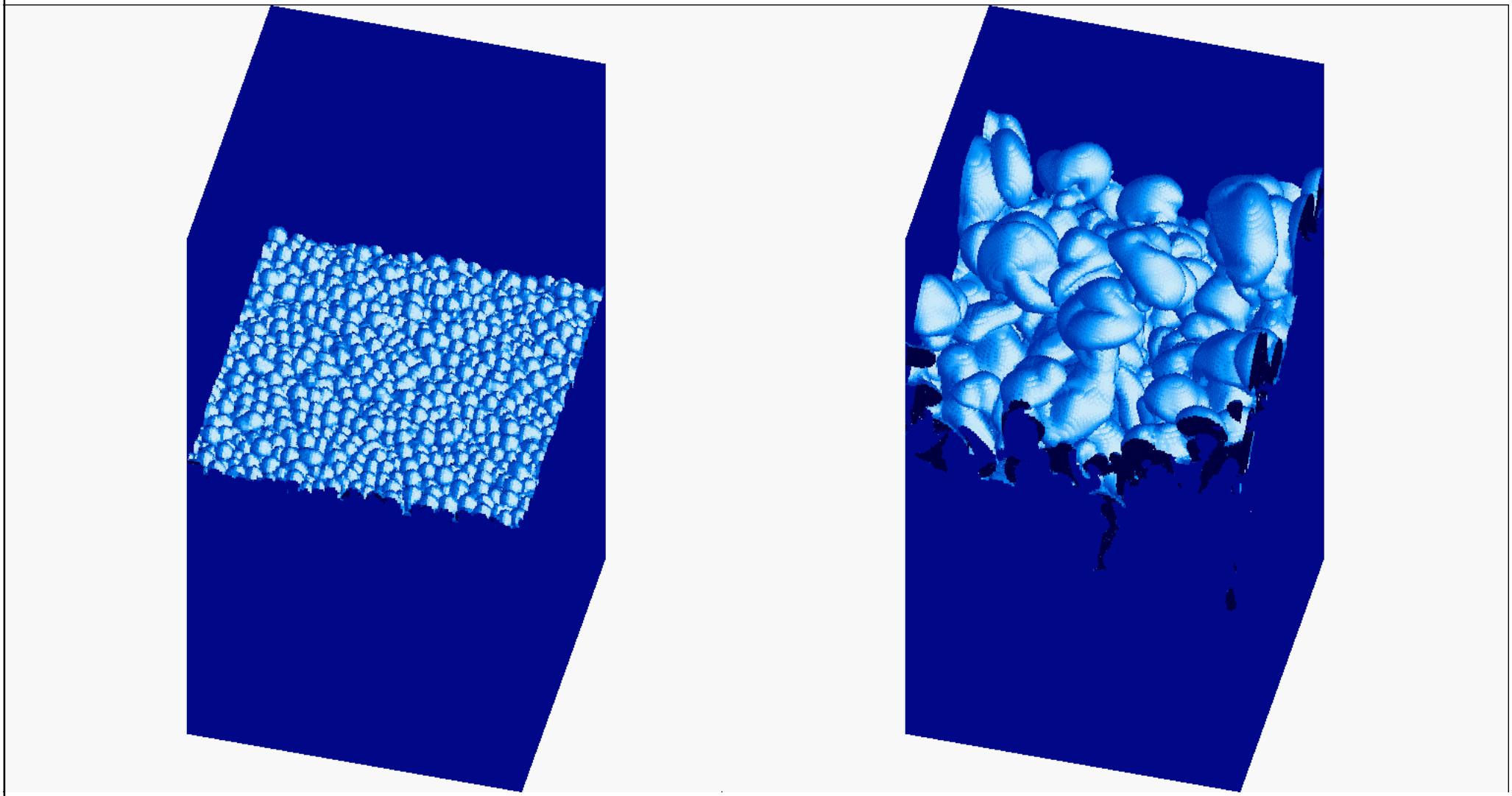
Fig 1 : initial long wavelength perturbation
Fig 2 : isosurfaces for the case g = k
Fig 3 : profiles of $\overline{f_1}$
Fig 4 : plots of $h_1$
Fig 5 : plots of $\theta$
Fig 6 : plots of D/P

**Figure 1**



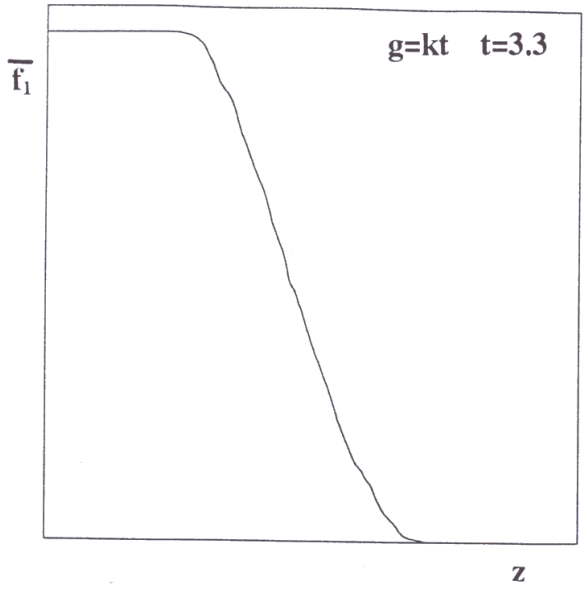
**ε = 0.005,  initial perturbation  x 200**

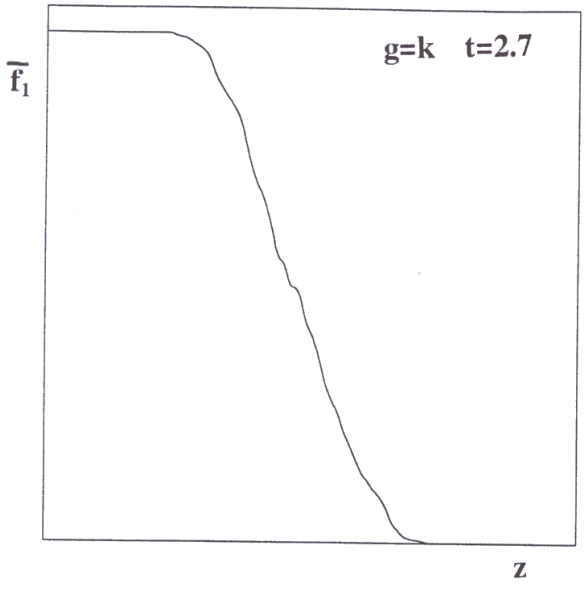
**Figure2: Isosurfaces ($f_1$ = 0.99) for g =k**



**t = 0.9** **t = 3.5**

Figure3


g=kt t=3.3
$\overline{f_1}$
z


g=k t=2.7
$\overline{f_1}$
z

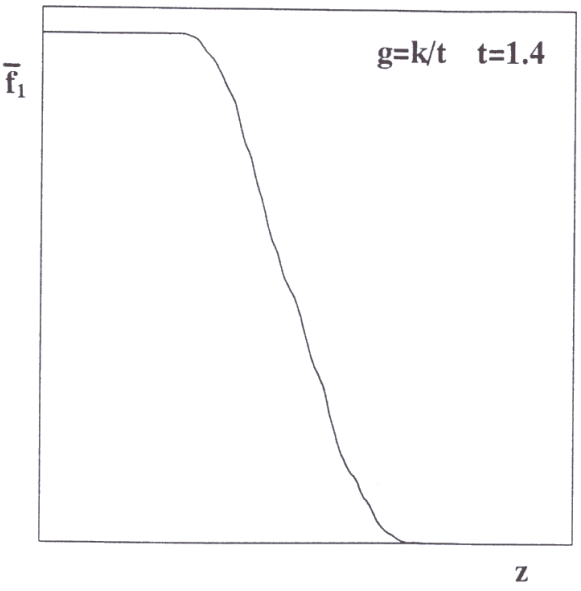

g=k/t t=1.4
$\overline{f_1}$
z

Figure4

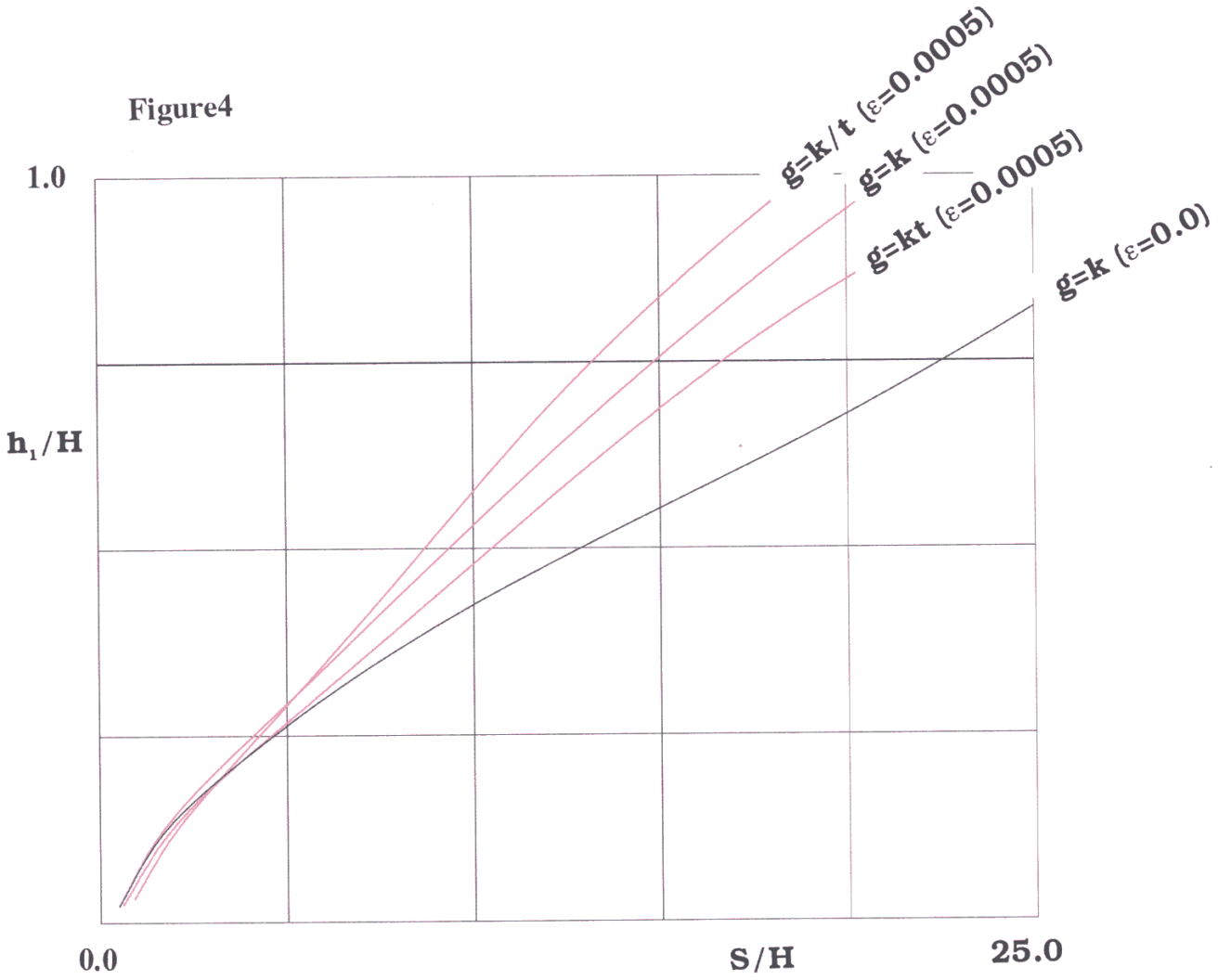


Figure5

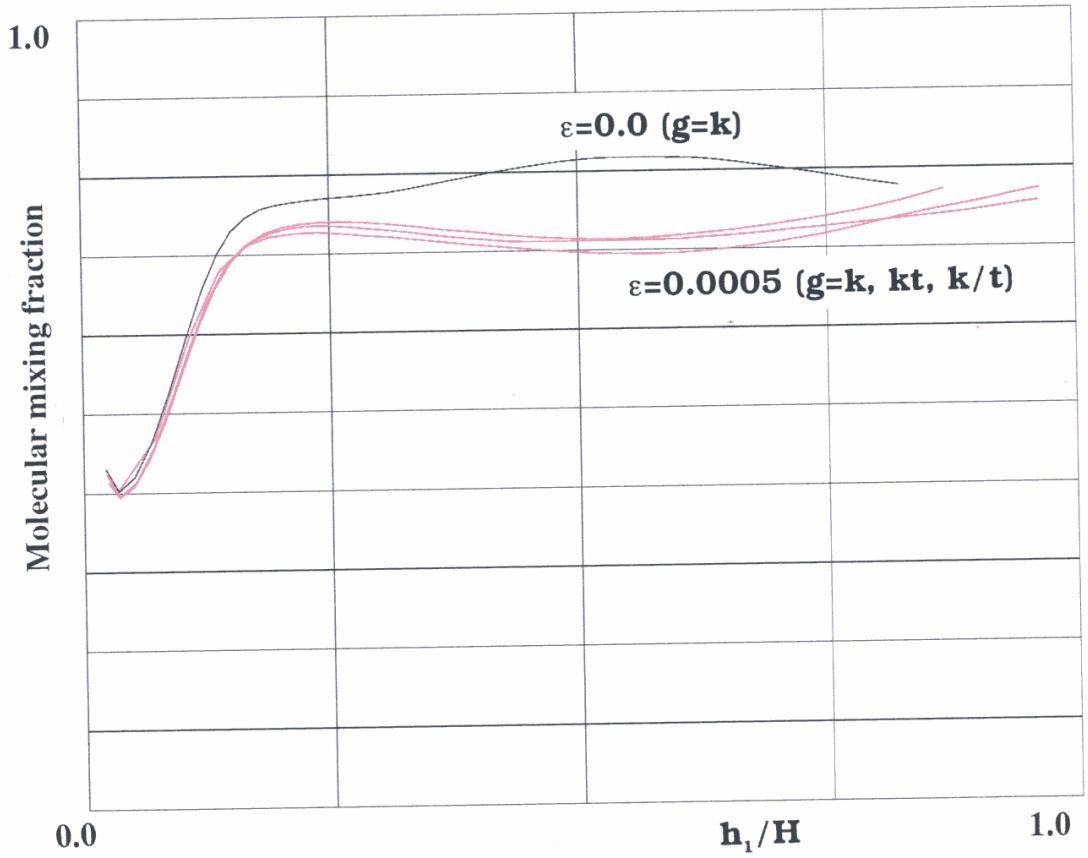


Figure6

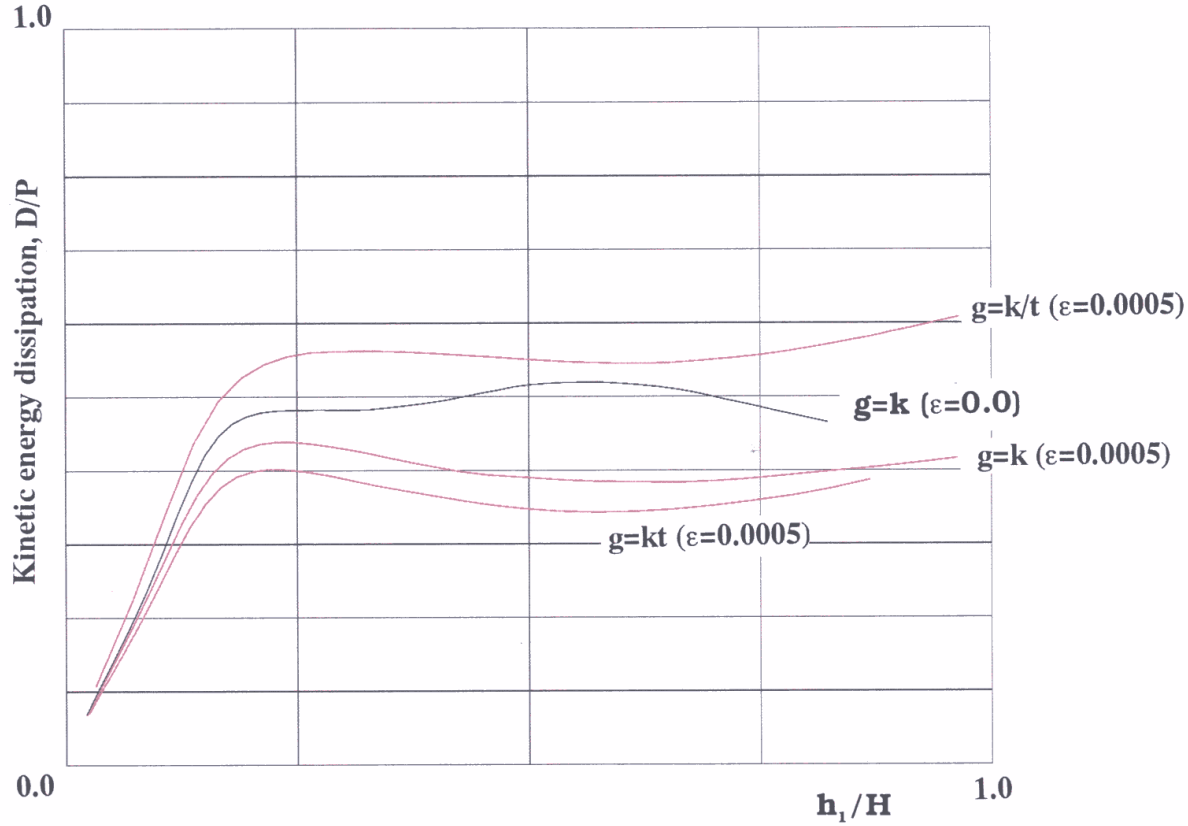
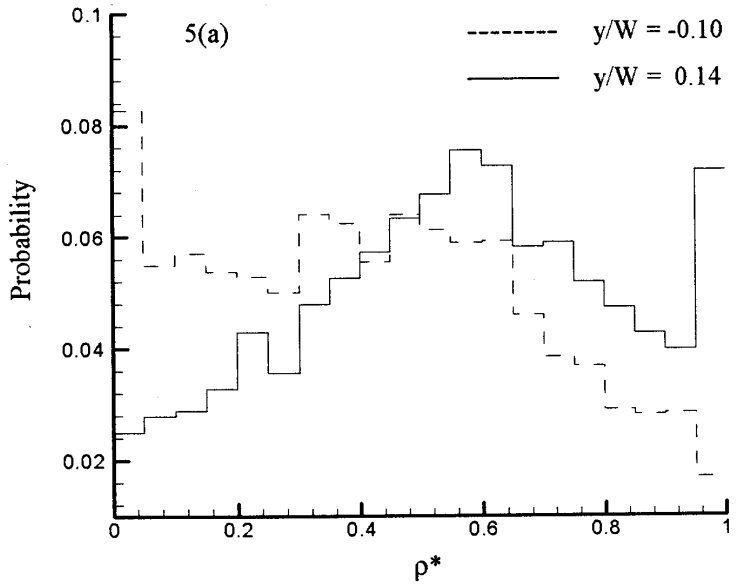

1.0
Kinetic energy dissipation, D/P
0.0
1.0
h1/H
g=k/t (ε=0.0005)
g=k (ε=0.0)
g=k (ε=0.0005)
g=kt (ε=0.0005)

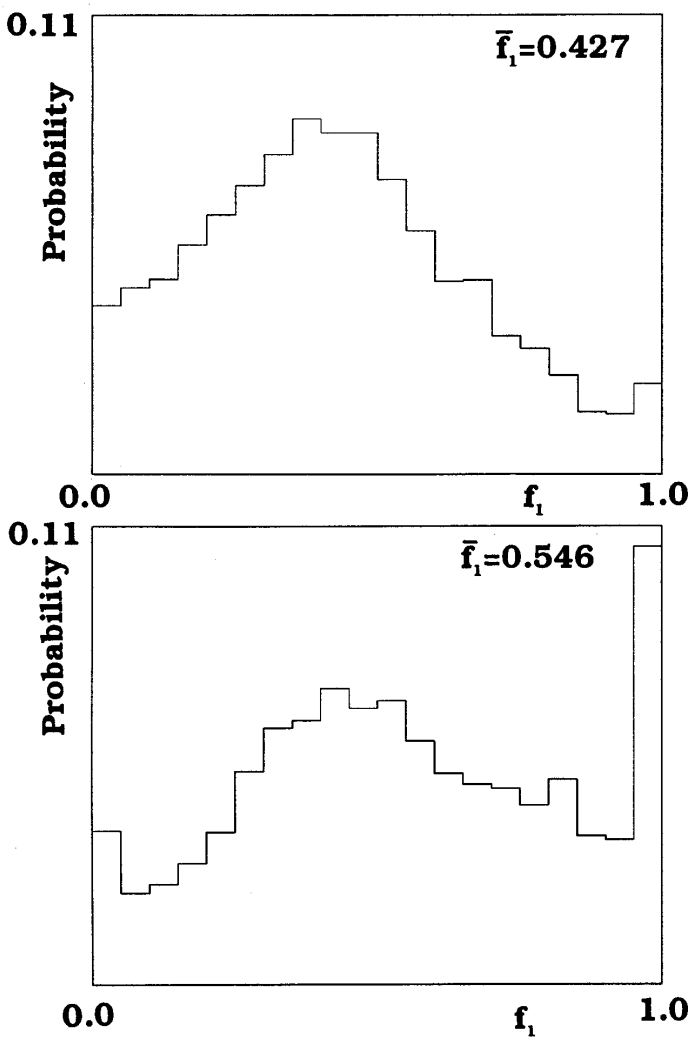
**Figure7 : Probability distributions for $f_1$ within a plane z=constant**

**Wilson and Andrews At =0.001**




**TURMOIL3D calculation**
**At=0.5**
**128x128x256 zones**

**g=constant**



## COMPARISON OF RESULTS WITH SIMPLE MODELS

### (A) The Simplest Model

Bubbles of radius R have a limiting velocity ~ $\sqrt{AgR}$. If it is assumed that $R \sim h_1$, then

$$\dot{h}_1 = c\sqrt{Agh_1}$$

ie $$h_1 = \alpha A\left\{\int_0^t \sqrt{g(t')}\, dt'\right\}^2 = \alpha S \qquad \ldots(1)$$

Figure 5 shows plots of $h_1$ vs S. The slopes of the curves (for the range $h_1/H$ = 0.25 to 0.75) give; for $\varepsilon = 0.0005$

| | | | | |
|---|---|---|---|---|
| n = 0 | , | $\alpha$ | = | 0.0464 |
| n = 1 | , | $\alpha$ | = | 0.0415 |
| n = -1 | , | $\alpha$ | = | 0.0559 |

The model works surprisingly well, but there is some variation of $\alpha$ with n:-

$$\frac{\alpha_1}{\alpha_0} = 0.89$$

$$\frac{\alpha_{-1}}{\alpha_0} = 1.20$$

**(B) A Buoyancy - Drag Model**

**A model of this type, based on a modified form of Layzer's equation for a bubble rising in a cylindrical tube, was used by Hansom et al [2].**

$$\ddot{h}_1 = Ag - \frac{C_D\left(\dot{h}_1\right)^2}{h_1} \qquad \text{. . . (2)}$$

**acceleration = buoyancy - drag**

**For constant g, $C_D = 4.5$ gives $\alpha = 0.05$. Then if $\alpha$ is defined as in equation (1):-**

$$\frac{\alpha_1}{\alpha_0} = 0.97$$

$$\frac{\alpha_{-1}}{\alpha_0} = 1.11$$

**This is closer to the TURMOIL3D results than taking $\alpha$ independent of n but the change is not large enough.**

**Dimonte and Schneider [3] include a factor $\beta < 1$ in front of the Ag term in equation (2). This improves agreement with the 3D simulations.**

## (C) An Energy Balance Model

This is version of the model proposed by Ramshaw [4], but with different settings for the model coefficients.

Let $K = \frac{1}{2}\left(\rho_1 + \rho_2\right) h_1 V^2$ = kinetic energy within the mixing layer.

$\dot{P}$ = KE production rate (loss of potential energy)

$\dot{D}$ = dissipation rate

Then the model equations used are

$$\dot{K} = \dot{P} - \dot{D}$$

$$\dot{P} = c_1\left(\rho_1 - \rho_2\right) g\, h_1 \dot{h}_1$$

$$\dot{D} = c_2 \frac{1}{2}\left(\rho_1 + \rho_2\right) V^3$$

$$\dot{h}_1 = c_3 V$$

The coefficient $c_1$ depends on the shape of the volume fraction profile. For a linear distribution $c_1 = 1/3$. For the TURMOIL3D profiles (figure 4) $c_1 = 0.30$, and this is the value used here.

If for constant g we assume, $\alpha$ = 0.05 and D/P = 0.4 (figure 7) then

$c_2 = 1.4055$

$c_3 = 1.0541$

The equation $\dot{K} = \dot{P} - \dot{D}$ gives

$$\ddot{h}_1 = c_1 c_3^2 Ag - \frac{c_2 + c_3}{2c_3} \frac{\left(\dot{h}_1\right)^2}{h_1}$$

$$= \frac{1}{3} Ag - \frac{7}{6} \frac{\left(\dot{h}_1\right)^2}{h_1} \qquad \ldots (3)$$

This has the same form as the buoyancy - drag model (2). The coefficient in front of the Ag term is less than unity as in Dimonte and Schneider [3].

This choice of the coefficients gives

$\frac{\alpha_1}{\alpha_0} = 0.91$

$\frac{\alpha_{-1}}{\alpha_0} = 1.43$

| | | | |
|---|---|---|---|
| also | D/P | = | 0.40 assumed for n = 0 |
| | D/P | = | 0.36 for n = 1 |
| | D/P | = | 0.57 for n = -1 |

$\alpha_1/\alpha_0$ agrees with the 3D simulations

$\alpha_{-1}/\alpha_0$ appears to be somewhat too high. However, the variation of D/P with n (see fig. 6) is represented very well.

## FUTURE PLANS

- Values of n outside the range [-1, 1]
- Higher resolution TURMOIL3D calculations
- Use of the SSVARTs results to differentiate between various types of RANS models.

## REFERENCES


1. N A Inogamov et al, Proceedings of the 3rd IWPCTM, Royaumont (1991)
2. J C V Hansom et al, Laser and Particle Beams, Vol 8, p 51 (1990)
3. G Dimonte and M Schneider, Phys Rev E54, p 3740 (1996)
4. J D Ramshaw, Phys Rev E58, p 5834 (1998)