Turbulent diffusion in solar type star

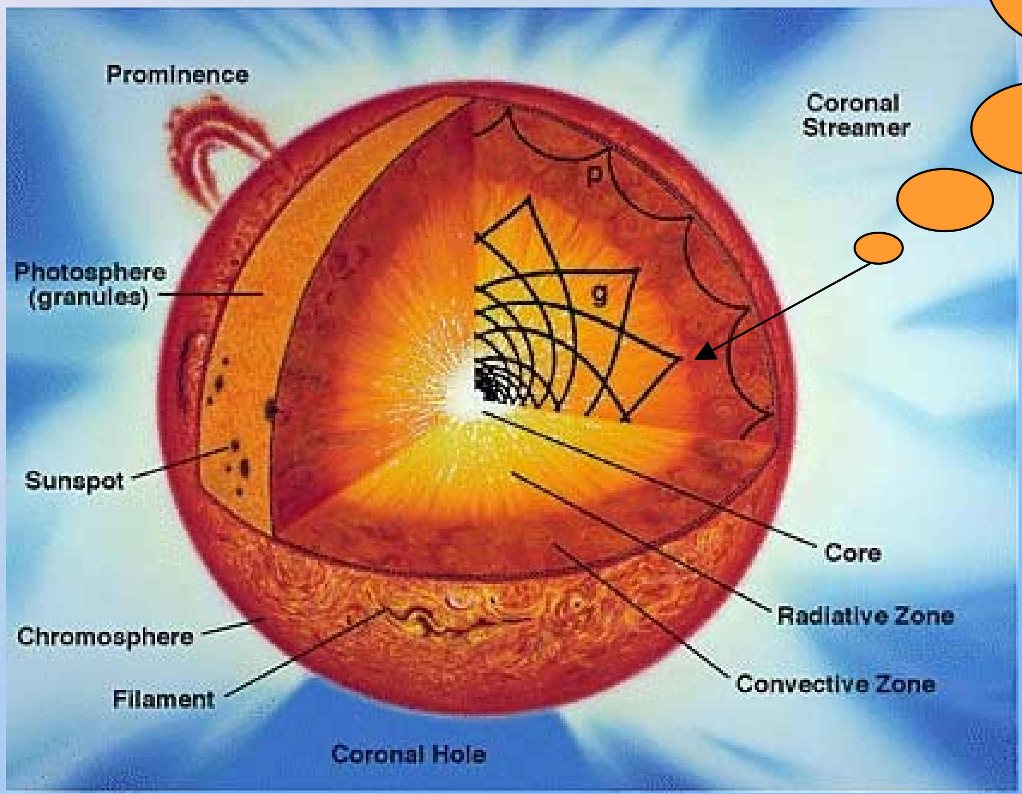
How may be trapped light elements as lithium in differential rotation zones of solar type star

Nathalie Toqué

Departement of Physics of the University of Montreal, Canada

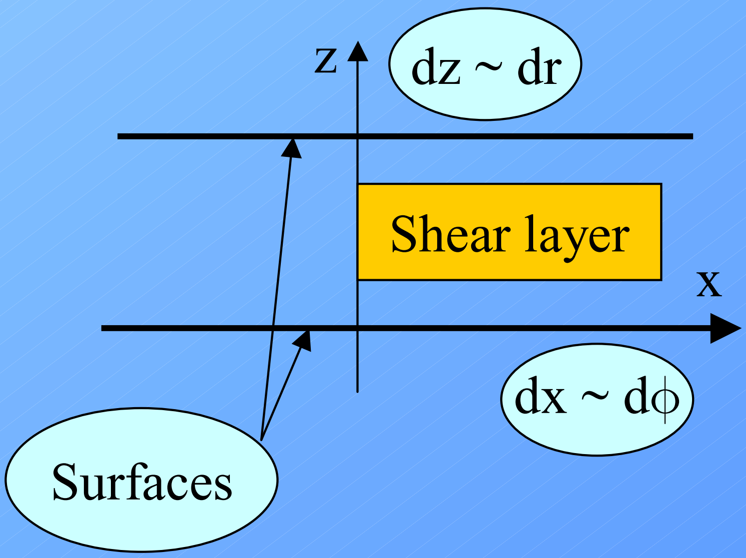
PhD Supervisor : Prof. Alain Vincent

Setup of the study



Tachocline: Differential rotation zone of about 35000 km thickness between the convective and radiative parts

The tachocline is approximated by a 2D shear layer as follows



Hypothesis of the model

Contribution from convective and radiative zones reduced to a **forcing in velocity** along the x direction and depending on the solar latitude coordinate

Perfect gas approximation for the fluid

Uncompressible and **isentropic** flow

No Boussinesq approximation, no thermic dissipation, kinetic and hydrostatic pressure gradients

Free-slip conditions on the surfaces, otherwise periodic

A concentration with an initial gaussian profile versus the z coordinate is convected by the flow. Its diffusion is studied for different values of the Reynolds number depending on the intensity of the forcing

Theory

$$\begin{cases} \partial_x V_x + \partial_z V_z = 0 \\ \partial_t V_x = -\partial_x P - R_x + \frac{1}{Re}\left\{\partial_x{}^2 V_x + \partial_z{}^2 V_x\right\} + \Phi \\ \partial_t V_z = -\partial_z P - R_z + \frac{1}{Re}\left\{\partial_x{}^2 V_z + \partial_z{}^2 V_z\right\} \\ \partial_t C = -\partial_x (V_x\, C) - \partial_z (V_z\, C) + \frac{1}{Pe}\left\{\partial_x{}^2 C + \partial_z{}^2 C\right\} \end{cases}$$

with

$$\begin{cases} R_x = V_z\,(\partial_z V_x - \partial_x V_z) \\ R_z = V_x\,(\partial_x V_z - \partial_z V_x) \end{cases}$$

Re is the Reynolds number

Pe is the Peclet number

Φ is the forcing along the horizontal direction

$(\partial_z V_x - \partial_x V_z)$: Vorticity component perpendicular to the domain of study

The theoretical equations are solved in spectral space

Written in spectral space with the following decompositions,

$$\begin{cases} C(t,x,z) = \sum_{\mathbf{k}} \hat{C}(t,\mathbf{k}) e^{ik_x x} \sin(k_z z) \\ V_z(t,x,z) = \sum_{\mathbf{k}} \hat{V}z(t,\mathbf{k}) e^{ik_x x} \sin(k_z z) \\ V_x(t,x,z) = \sum_{\mathbf{k}} \hat{V}x(t,\mathbf{k}) e^{ik_x x} \cos(k_z z) \end{cases}$$

The equations under study become

$$\begin{cases} \mathrm{i}k_x \hat{V}_x + \mathrm{i}k_z \hat{V}_z = 0 \\ \partial_t \hat{V}_x = -\mathrm{i}k_x \hat{P} - \hat{R}_x - \frac{(k_x{}^2 + k_z{}^2)}{Re} \hat{V}_x + \hat{\Phi} \\ \partial_t \hat{V}_z = -\mathrm{i}k_z \hat{P} - \hat{R}_z - \frac{(k_x{}^2 + k_z{}^2)}{Re} \hat{V}_z \\ \partial_t \hat{C} = -\mathrm{i}k_x (\widehat{V_x\, C}) - \mathrm{i}k_z (\widehat{V_z\, C}) - \frac{(k_x{}^2 + k_z{}^2)}{Pe} \hat{C} \end{cases}$$

The time iteration is made with

- ★ the **implicit** Cranck-Nicholson method for the **diffusion terms**
- ★ the **explicit** Leapfrog scheme for the **advection terms**

The time iteration is divided in two steps for the velocity components. After the first step, the pressure is calculated with the condition :

$$\widehat{\nabla . \mathbf{V}} = 0$$

The contribution of the small scales of turbulence is carried out by a subgrid model

Subgrid Model

(M.Lesieur&R.Rogallo, 1989)

$$\nu_t(k/k_c) = (A + Be^{C(k_c/k)})[E(k_c)/k_c]^{1/2}$$

A = 0.267, **B** = 9.21 and **C** = -3.03

An extra-diffusion term is added to the equations governing the velocity components and the concentration as follows :

$$\begin{cases} \hat{V}_x^{n+1} = \hat{V}_x^{n+1} - 2dt\ \nu_t\ k^2 \hat{V}_x^{n+1} \\ \hat{V}_z^{n+1} = \hat{V}_z^{n+1} - 2dt\ \nu_t\ k^2 \hat{V}_z^{n+1} \\ \hat{C}^{n+1} = \hat{C}^{n+1} - 2dt\ \nu_t\ k^2 \hat{C}^{n+1} \end{cases}$$

Simulation philosophy

Given a Reynolds number, a simulation is made in **two steps** with a forcing on several layers

First step : Stationarity of the anisotropy

The purpose of the forcing is to create a strong sheared flow with a constant mean value of the anisotropy defined as follows

$$A(t) = \sqrt{\frac{<V^2{}_x(x,z,t)> - <V_x(x,z,t)>^2}{<V^2{}_z(x,z,t)> - <V_z(x,z,t)>^2}}$$

Second step : Vertical diffusion of the concentration

The aim of the forcing is to produce the vertical diffusion of the concentration in the flow with a constant anisotropy.
The concentration is named after **C**

Quantities measuring the turbulent diffusion

Anisotropy

Turbulent diffusion coefficient

Flux

x averaged variancy

z averaged variancy

$$
\begin{cases}
A(t) = \sqrt{\frac{<V^2{}_x(x,z,t)> - <V_x(x,z,t)>^2}{<V^2{}_z(x,z,t)> - <V_z(x,z,t)>^2}} \\
D_T(z,t) = \frac{<V_z(x,z,t) \times C(x,z,t)>_x}{<\frac{dC(x,z,t)}{dz}>_x} \\
F(z,t) = < V_z(x,z,t) \times C(x,z,t)>_x \\
\sigma^2{}_x(z,t) = < (C(x,z,t) - < C(x,z,t)>_x)^2>_x \\
\sigma^2{}_z(x,t) = < (C(x,z,t) - < C(x,z,t)>_z)^2>_z
\end{cases}
$$

After averages on time and space coordinates,
it involves one value for **A**, $\mathbf{D_T}$, $\mathbf{F_{max}}$, $\sigma^{\mathbf{2}}{}_{\mathbf{x}}$ and $\sigma^{\mathbf{2}}{}_{\mathbf{z}}$

$\mathbf{D_T}$, $\mathbf{F_{max}}$ versus the anisotropy

$dx = dz = 1.2 \times 10^{-2}$
$dt = 10^{-5}$ and $Pe = 1$

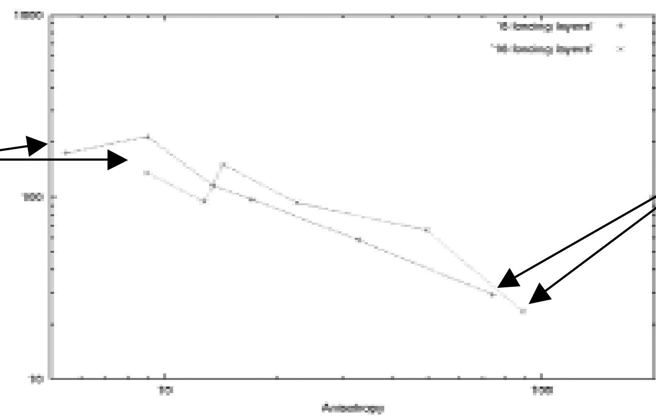


FIG. 1 – Dimensionless turbulent diffusion coefficient $D_T/(l_{0z}v_{0z})$ versus anisotropy

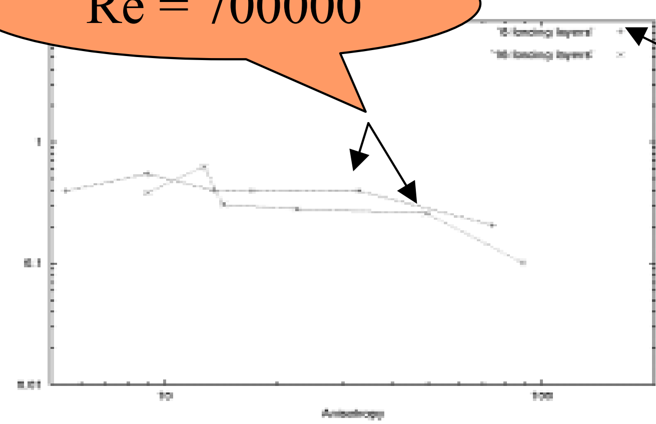


FIG. 2 – Maximum flux versus anisotropy

$\sigma^2{}_x$ and $\sigma^2{}_z$ versus the anisotropy

$dx = dz = 1.2 \times 10^{-2}$
$dt = 10^{-5}$ and $Pe = 1$

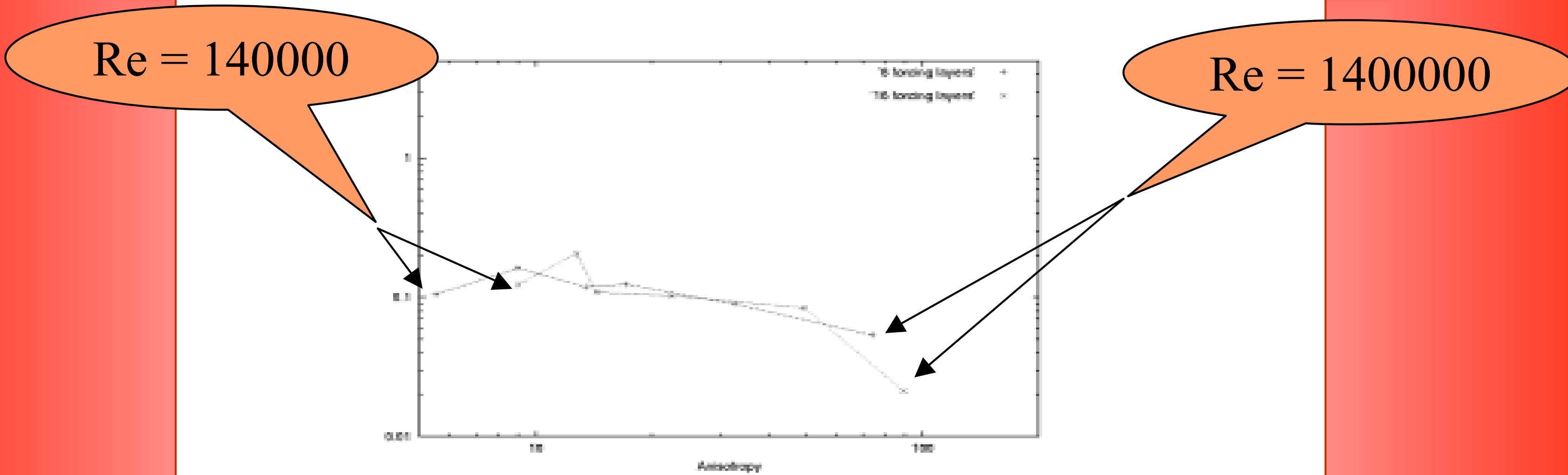


FIG. 3 – Variancy $\sigma^2{}_z$ versus anisotropy

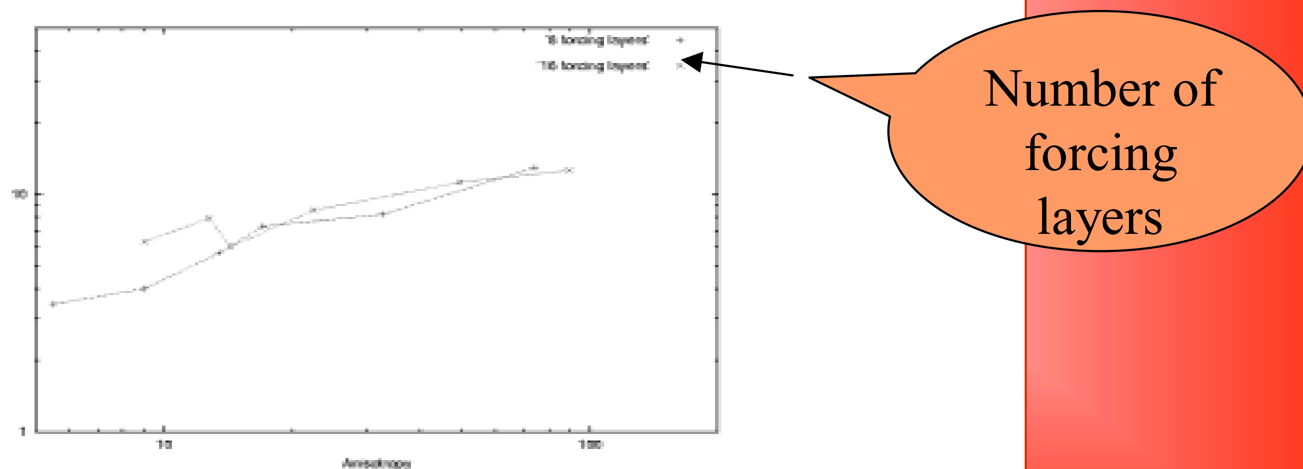


FIG. 4 – Variancy $\sigma^2{}_z$ versus anisotropy

Total kinetic energy, Re = 700000

16 forcing layers

$dx = dz = 1.2 \times 10^{-2}$
$dt = 10^{-5}$ and $Pe = 1$

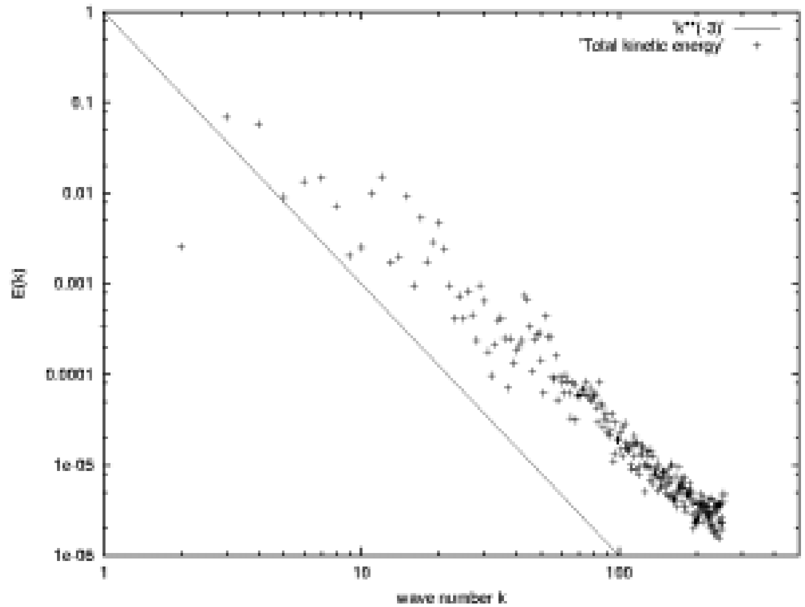


FIG. 5 – Total kinetic energy at the time of maximum vertical flux of the concentration in the flow

The curves 1 to 4 show that for increasing Reynolds numbers, the turbulent diffusion coefficient, the maximum flux and the variancy σ^2_x are growing down for anisotropy values upper than 40, and the variancy σ^2_z is growing up

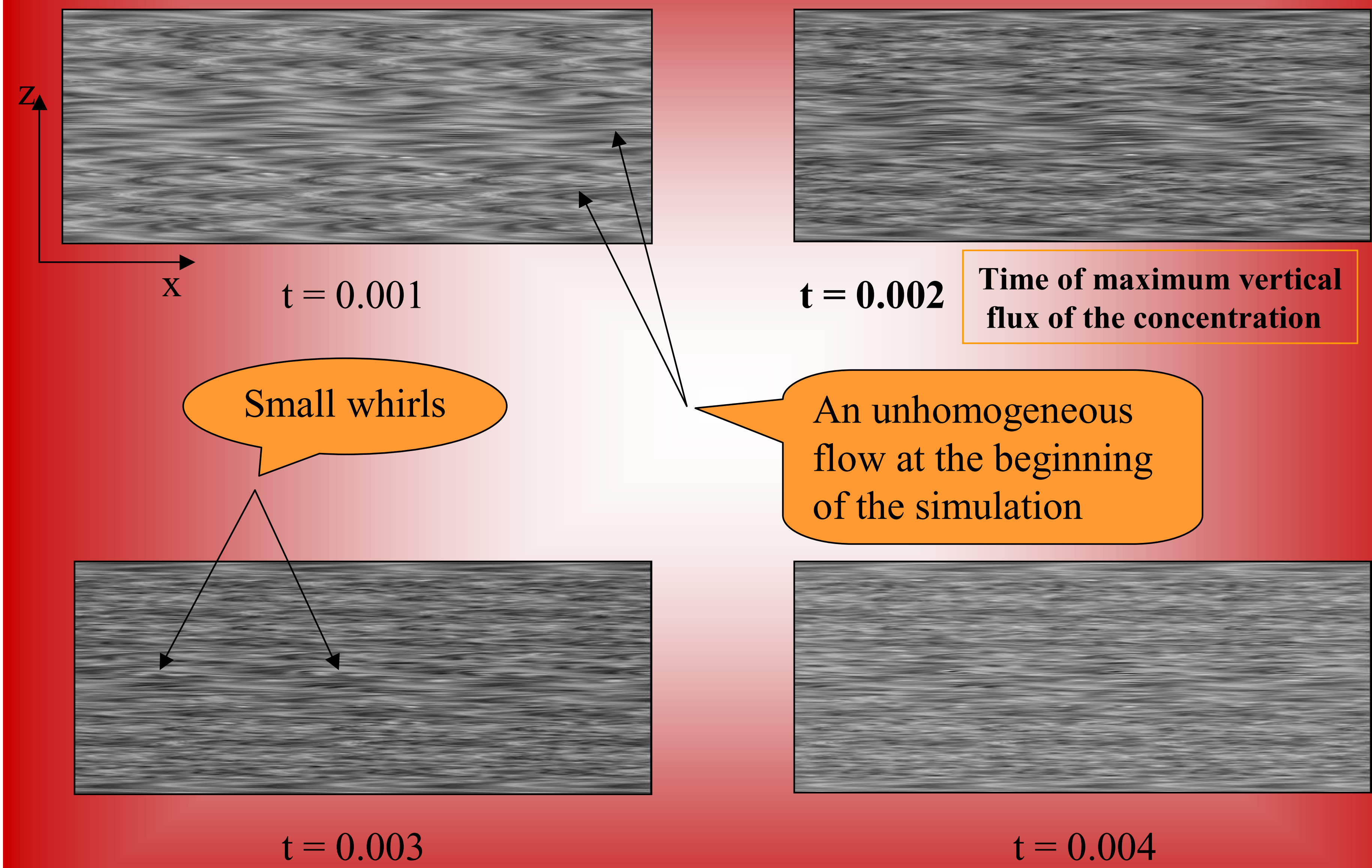
More forcing layers move down the maximum flux of the concentration in the flow because the integral scale and the quadratic fluctuation of vertical velocity are lower

The variances seem not influenced by the number of layers. They measure the turbulent homogenization in the two directions. For a conservative flow, less kinetic energy in the vertical direction involves more in the horizontal direction. So, the turbulent homogenization is better in the x direction than in the z direction. For increasing Reynolds numbers, the variancy σ^2_x decays and the variancy σ^2_z grows up.

The total kinetic energy evolves as k^{-3} as shown on the diagram 5

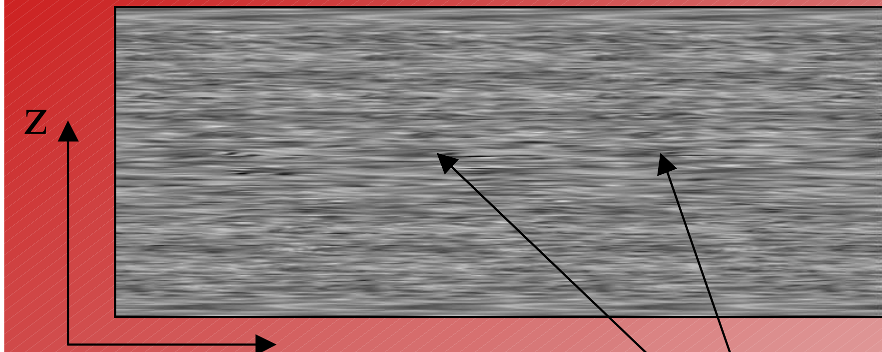
Vorticity component perpendicular to the domain of study, Re = 700000

16 forcing layers



Vorticity component perpendicular to the domain of study, Re = 700000

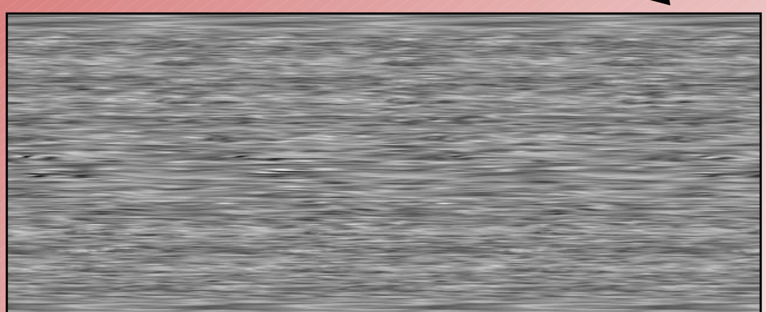
16 forcing layers



t = 0.005

t = 0.006

Whirls are getting thinner and thinner and flattened



t = 0.007

No more evidence of inhomogeneity between the center, the top and the bottom of the flow

Continuation of the study

- Simulations with different values of the Peclet number
- Boussinesq approximation to take into account a temperature stratification
- Forcing with internal waves to quantify their part on the turbulent diffusion in differential rotation zones
- 3D compressible simulations