Turbulent Mixing Nuclear Burning in Type Ia Supernova Explosion based on Bubble Statistical Mechanics

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(This viewgraph was prepared for "the 8th International Workshop on the Physics of Compressible Turbulent Mixing (8th IWPCTM) " held at the California Institute of Technology (Caltech) in Pasadena, California, USA, from December 9-14, 2001. The Workshop will be held on campus in the Beckman Institute Auditorium, and is hosted by Lawrence Livermore National Laboratory (LLNL).)

It is well known that Type Ia supernovae explode when the masses of white dwarfs become close to the Chandrasechar's limiting mass. This is the reason why the Type Ia explosion is used as a standard candle in the universe to determine, for example, the Hubble constant and dark energy. The scale of explosion has been well studied with one-dimensional code with some mixing model; however, the physical mechanism has not determined from the first principle, yet. There are many works to understand the physics with large scale computing based on hydrodynamics in two-dimension or mostly three-dimension in these days[1]. It seems, however, that the smaller scale fluctuation appears, the smaller the grid size, and it is still open question how the instability grows and evolves into nonlinear stage and enhance the energy release by nuclear reactions.

In the present report, we would like to model the growth of the Rayleigh-Taylor instability coupled with the Landau-Darrieus instability. In the nonlinear stage, we consider the statistical mechanics of the bubbles following the way developed by Don Shvart's group[2] and estimate the increase in the nuclear burning rate due to the increase in the surface area of the burning wave in the form of fractal structure. This model is coupled with the multi-dimensional explosion code to predict the scale of explosion. Such work is expected to be used to identify the physical mechanism of the time evolution of the burning wave, which may change from deflagration wave to detonation wave.

Reference:

[1] W. Hillebrandt and J. Niemeyer, Ann. Rev. Astron. Astrophys. **38**, 191-230 (2000)

[2] D. Shvarts et al, Physics of Plasmas **2**. 2465 (1995).

1. Introduction

- We are promoting a new scientific field of "Laser Astrophysics".

- Many issues regarding hydrodynamics instabilities and turbulent mixings.

- Recent review of LA.
  H. Takabe, "*Astrophysics with intense and ultra-intense lasers –Laser Astrophysics-* ", Progress of Theoretical Physics Suppl. No. **143**, 202-265 (2001).

- Many works on hydrodynamic instabilities and mixings have been done in the case of Type-II supernova explosion, especially SN1987A, in the framework of LA. (B. Remington, P. Drake et al.)

- More sophisticated physics scenario should be studied in the case of Type-Ia SN explosion.

2. What is Type-Ia Supernova ?

- Type II
  - Massive star (~ 20 M_{Sun} for SN1987A), Single
- H_α line emission.

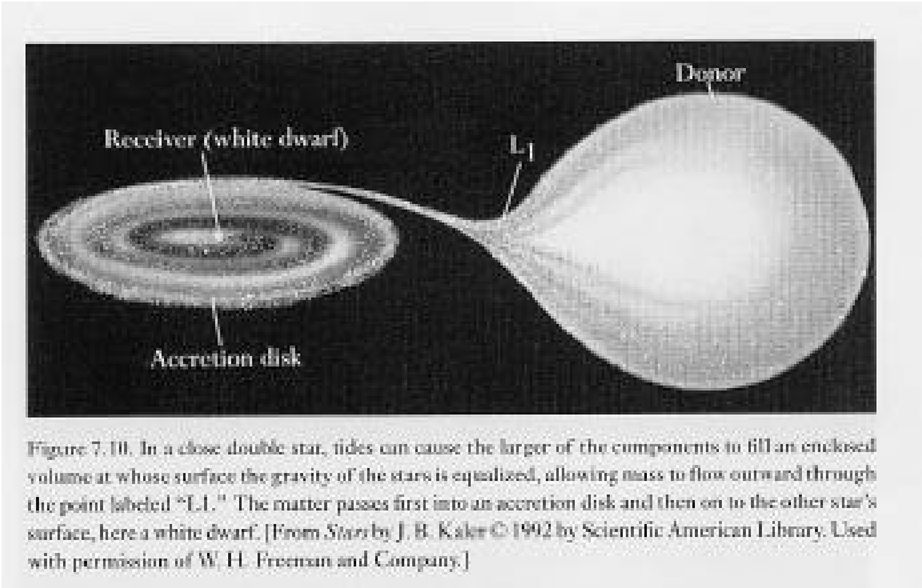

Figure 7.10. In a close double star, tides can cause the larger of the components to fill an enclosed volume at whose surface the gravity of the stars is equalized, allowing mass to flow outward through the point labeled "L1." The matter passes first into an accretion disk and then on to the other star's surface, here a white dwarf. [From *Stars* by J. B. Kaler © 1992 by Scientific American Library. Used with permission of W. H. Freeman and Company.]

Fig. 1 (J. Kaler, "Extreme Stars" (Cambridge, 2001) p.158)

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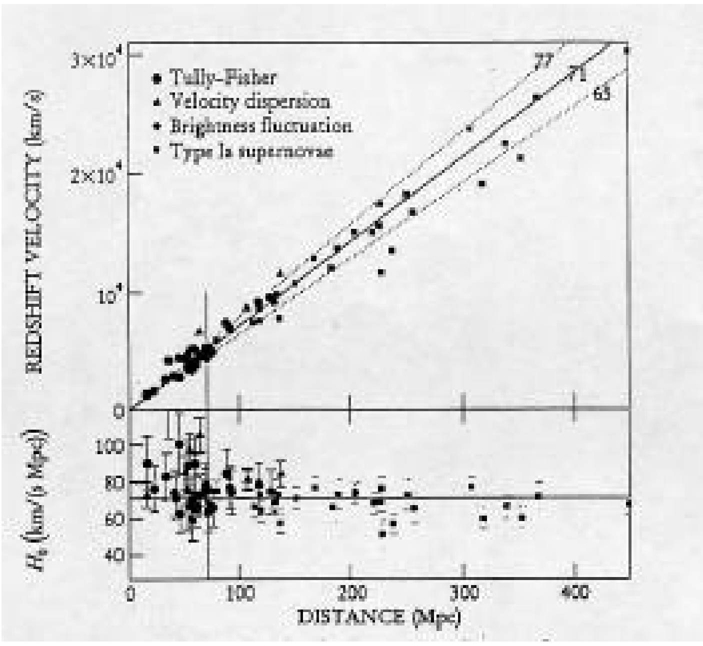

Fig. 2 (Physics Today, August (1999) p.20)

- Type Ia
  - White Dwarf, Binary system, Chandrasekhar-mass (~ 1.4 M_{Sun}) See Fig. 1.
  - No H_α line emission.

- Light curve of Type Ia SNe.

- Why important?

- Precise determination of Hubble constant
  H_0 = (velocity)/(distance)   for Z <0.1
  See Fig. 2

- Einstein constant is finite?   Dark energy.   For Z<1.0

  (cf: $Z = (\lambda - \lambda_0)/\lambda_0$,   L = 1 B-ly for Z = 0.1)

3. Hydrodynamic Instability in Laser Implosion

- Mixing reduces neutron yields --- Degrades thermonuclear reaction and burn.   [cf: H. Takabe et al, Phys. Fluids **31**, 2884 (1988)]

- Multi-dimensional simulation with surface-tracking is necessary.   See Fig. 3

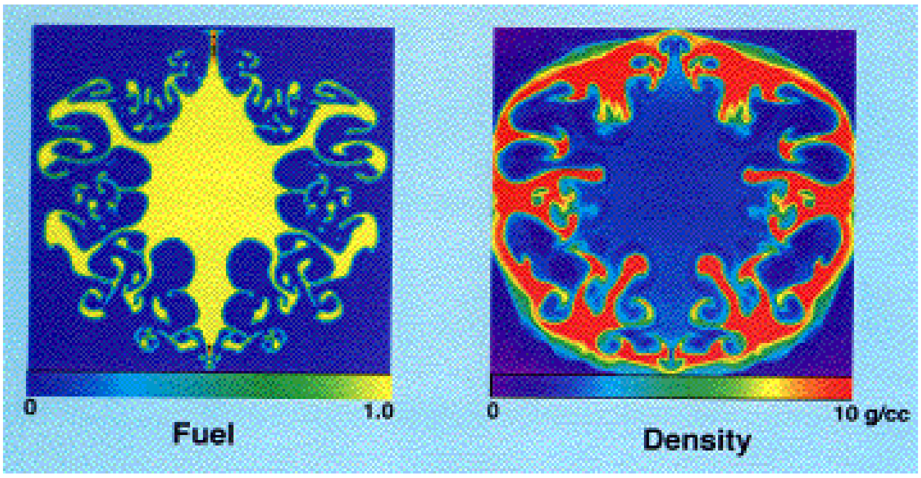

Fig. 3 (2D simulation of laser implosion. by H. Takeuchi and H. Takabe)

4. Spherical Symmetry Scenario of Nuclear Burning Wave Propagation in Chandrasekhar-mass WD.

- Very sharp wave front [(thickness) < mm] compared to the size of WD (~ 10000 km). See Fig. 4

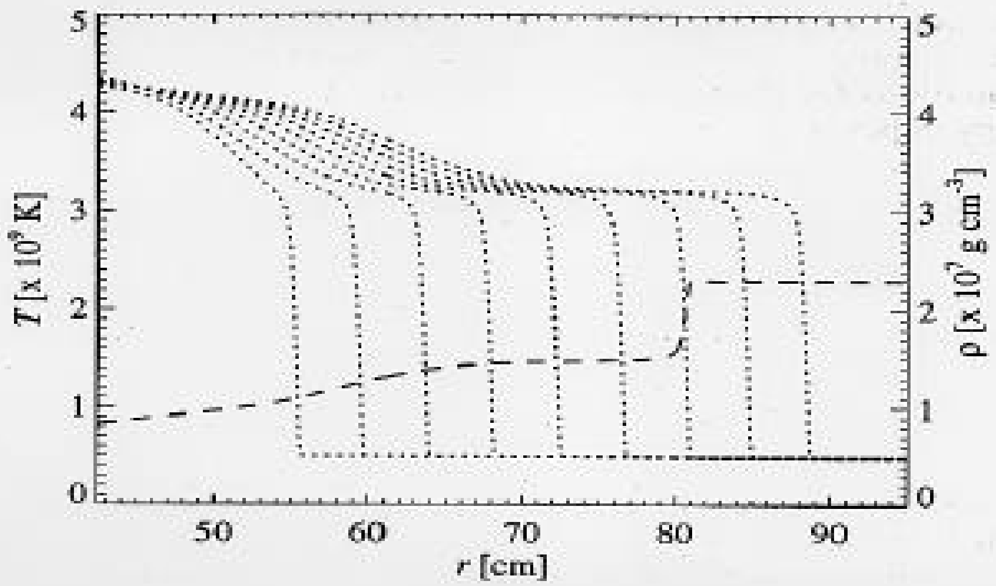

FIG. 2.—Propagation of an $X(^{12}C) = X(^{16}O) = 0.5$ conductive flame. The sequential temperature profiles (*dotted lines*) are shown at times $t = 1.3 \times 10^{-4}$, 3.0×10^{-4}, 6.5×10^{-4}, 8.3×10^{-4}, 1.01×10^{-3}, 1.19×10^{-3}, 1.36×10^{-3}, and 1.51×10^{-3} s. The dashed line is the density profile at $t = 1.19 \times 10^{-3}$ s. The carbon flame already moves in a steady state, while the temperature rise at the left-hand side comes from oxygen burning as a result of the energy stored at the initial state of the simulation.

Fig. 4 [A. M. Lisewski et al., ApJ **537**, 405 (2000)]

- Very week deflagration wave or combustion wave (M ~0.01).

- No explosion:
  (Released fusion energy) < -(Gravitational binding energy).

- Multi-dimensional physics scenario is necessary.

- Unstable to R-T instability.
- Unstable to Landau-Darius instability
- K-H instability is also important.

5. Fractal Dimension of the Nuclear Burning Wave Front.

- Effective area and velocity of the burning front (F. X. Timmes, ApJ, 1994)

$$A_{eff} = (l_{max}/l_{min})^{D-2} A_0$$

$$V_{eff} = (l_{max}/l_{min})^{D-2} V_{conduction}$$

D: Fractal dimension (D>2)

- Fractal dimension (S. I. Blinnikov, PRE, 1996).

$(l_{max}/l_{min})^{D-2} \sim 7$ Compare: we need ~ 100 !

See Fig. 5

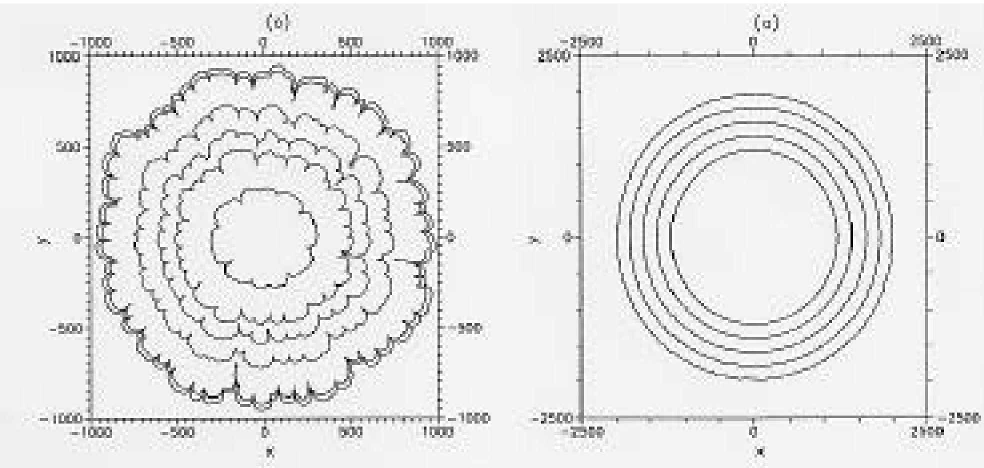

Figure 6.2: Flame front for several moments of time for $\eta = 0.8$

Figure 6.3: The front for several moments of time for $\eta = 0.4$

Fig. 5 (S. I. Blinnikov, Phy. Rev. E, 1996)

6. Multi-Dimensional Simulation (LES-like)

- Intensive works by W. Hillebrandt group (MPA).
  References:
  1. W. Hillebrandt and J. C. Niemeyer, Annu. Rev. Astron. Astrophys. **38**, 191 (2000).
  2. A. M. Lisewski et al., ApJ **537**, 405 (2000).
  3. M. Reinecke et al., A&A **347**, 739 (1999).
  4. M. Reinecke et al., astro-ph/0111473, 26 Nov (2001).
  5. M. Reinecke et al., astro-ph/0111475, 26 Nov (2001).

- Central ignition
- Off-center ignition
- Mesh-size dependence in 2-D simulation
- 3-D simulation   See Fig. 6

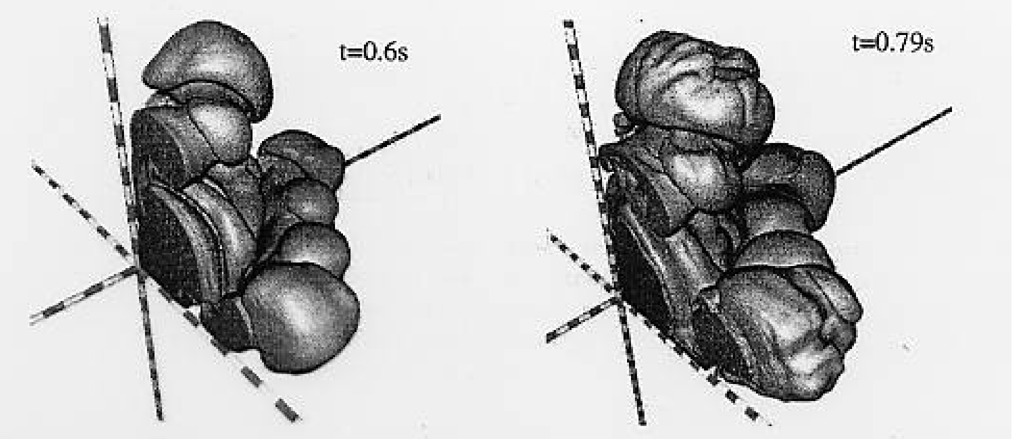

Fig. 3. Burning front geometry evolution of model c3_3d_256. One ring on the axes corresponds to 10^7cm.

Fig. 6 [M. Reinecke et al., astro-ph/0111473, 26 Nov (2001)]

7. Statistical Bubble Model is applicable to this case?

- Large-Eddy-type simulation based on the spectra by U. Alon et al. (PRL, 74, 534 (1995)). Possible ?

- Review by Don Shvarts in this morning session.

8. Summary

We have started numerical study to see more detail and complete explosion scenario of Type Ia supernovae. They are unstable to Rayleigh-Taylor instability and Landau-Darrius instability. The increase of the surface of the nuclear burning front determines and enhances the rate of thermonuclear energy release. It determines the fraction of the production of ^{56}Nn, which finally nuclear decays to ^{56}Fe. This is the origin of iron in the universe.

Type Ia is so bright, the peak luminosity is 10^{11} time the solar luminosity, and the luminosity is thought to be almost constant, so that it can be used to discuss not only to determine the Hubble constant precisely but also to know the existence of cosmological term in Einstein equation..

The project is not so simple. As summarized in Fig. 7, we have to start from one-dimensional code with realistic equation of state (EOS) in which Fermi pressure in relativistic regime should be modeled. The calculation of thermonuclear burn requires to calculate the equations of concentrations for original element (C/O) and burn product (Fe). Two dimensional simulations have been carried out by many authors, while the numerical resolution was a big issue. Prof. I. Hachisu, our collaborator at University of Tokyo, thinks that 2D is enough and no sub-grid model is necessary. He is think of usage of Earth simulator, the world-highest-speed vector-parallel computer at Yokohama.

The authors think of developing our original sub-grid model based on the accomplishment in relation to laser implosion physics. The simulation will be done in 2D and finally 3D. To check the sub-grid model we need to do a variety of local 3D simulations. We hope to relate such explosion simulation to the observable light curve.

Final point of this project is to find a new mixing model to be installed in 1D code so that many-parameter study is able to be done by supernova observation groups. Very tough job but very attractive, isn't it.

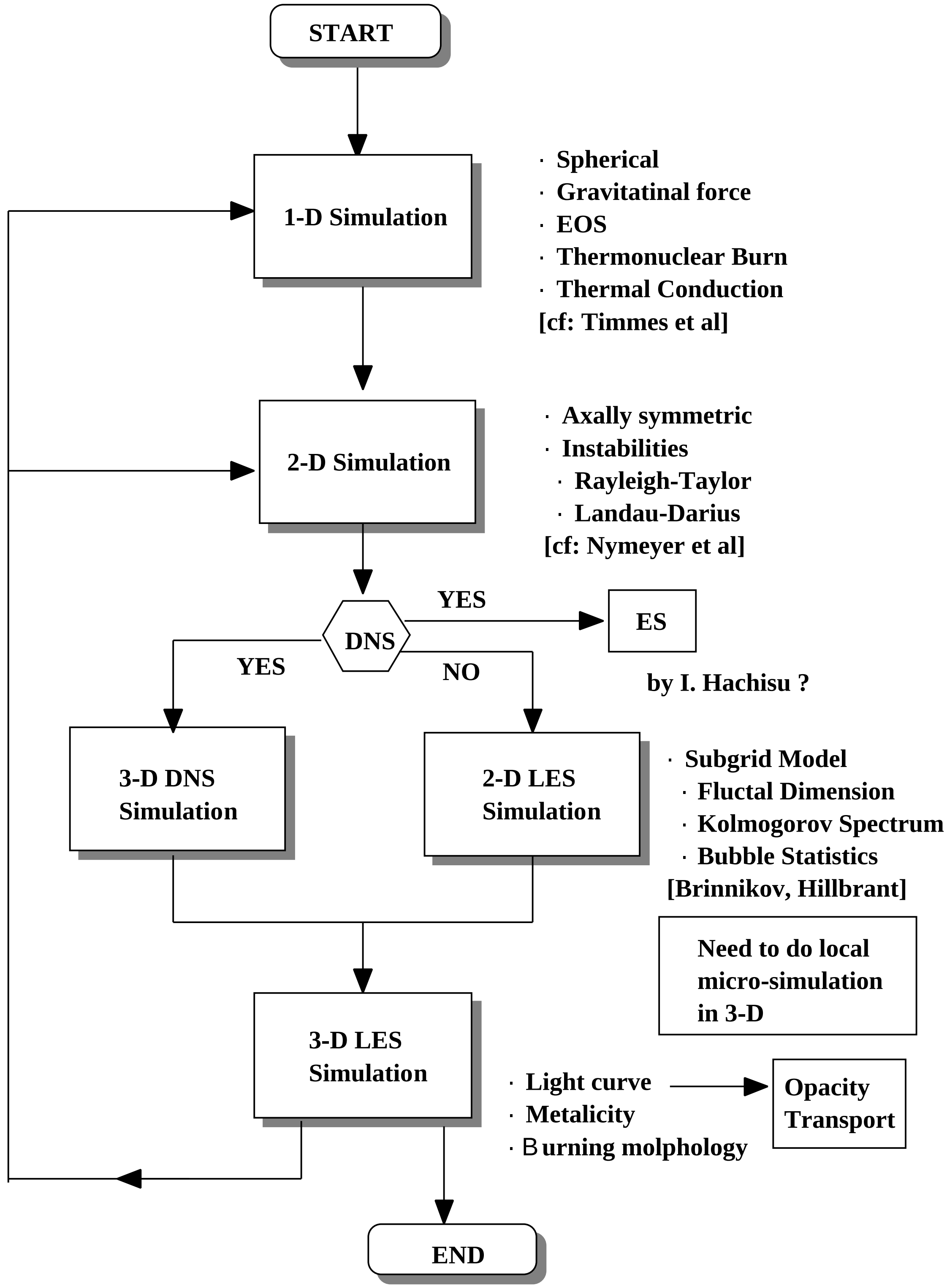

Fig. 7 Flow Diagram of the Present Project (H. Takabe)