Numerical Investigation of a Laser Induced Turbulent Mixing Zone

P. Seytor and M. Legrand
CEA/DAM Ile de France, BP12, 91680, Bruyères le Châtel, France

We have used high Mach number ($M \approx 20$) mix instability experiments which have been conducted using the NOVA laser system to investigate the growth of the Richtmyer-Meshkov instability resulting from a strong shock wave. The initial nonlinear single-mode two dimensional perturbation was machined into a brominated plastic ablator (density : 1.22 g/cc) adjacent to a low density carbon foam (0.10 g/cc). We compared the experimental results with LLNL simulations (CALE code) and with our own numerical simulations (FCI2 code). A non linear model (Ramshaw [1]) has been previously used to analyze the growth of a mixing zone in this experiment (Farley *et al.* [2]). We propose here to consider the possibility of a long turbulent phase in this type of experiment. We found mix width results to be in good agreement with the k-ε statistical model included in a 1D code when the transition time to turbulence is correctly estimated (high Mach number effects).

The first section of this talk describes the experimental setup. The instability phases of the flow are discussed in section II. The Ramshaw non linear model is described in section III and the necessary conditions for turbulence in section IV. High Mach number effects on the transition time to turbulence are discussed in section V (HESIONE 2D eulerian code) and k-ε computations are described in section VI. This is followed by a summary in section VII.

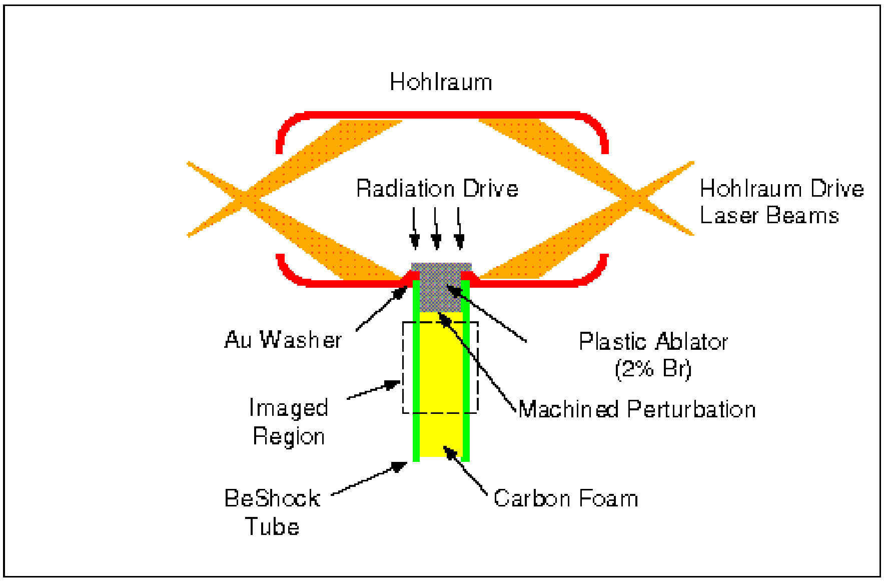
[1] J. D. Ramshaw
Simple model for linear and non linear mixing at unstable fluid interfaces with variable acceleration
Phys. Rev. E, **58**, N5, 1998

[2] D. R. Farley et al.
High Mach number mix instability experiments of an unstable density interface using a single mode, nonlinear initial perturbation
Physics of plasmas, **6**, N11, 1999

Experimental Setup

Farley et *al.*

Front View

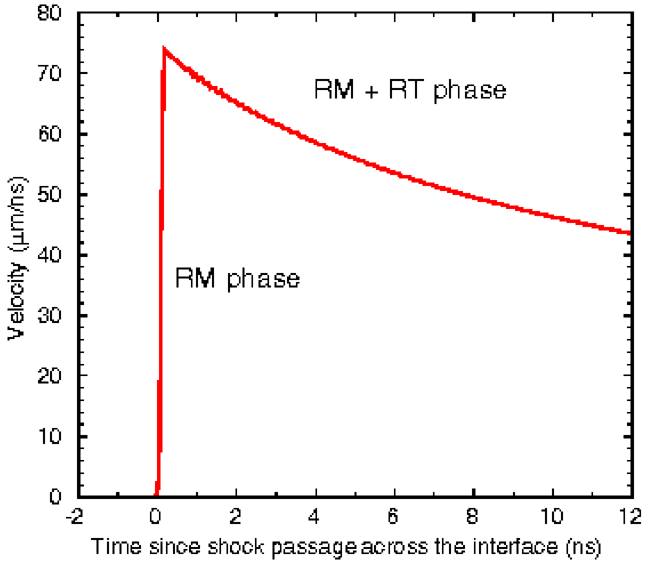


Hohlraum : 3080 µm long and 1600 µm diameter hollow cylinder

Shock tube : 2200 µm long **beryllium cylinder**
outer diameter : 700 µm
inner diameter : 500 µm
ablator : 300 µm long (ρ = 1.22 g/cm3)
carbon foam : 1900 µm long (ρ = 0.1 g/cm3)

Hohlraum drive : 1ns square pulse ($\approx$ 20kJ)
Radiation drive : peak flux temperature = 230 eV ($\pm$10 eV)

Instability phases



❒ A Richtmyer-Meshkov Instability (RMI) is induced by the shock passage across the interface

❒ This RM phase is followed by a slow decay of velocity which generates a Rayleigh-Taylor Instability (RTI)

❒ These results obtained with our 1D Radiation Hydrodynamics code are very close to Farley's (CALE 1D, [1])

[1] D. R. Farley et al.
High Mach number mix instability experiments of an unstable density interface using a single mode, nonlinear initial perturbation
Physics of Plasmas, **6**, N11, 1999

Nonlinear Model

- Ramshaw [1] uses a Lagrangian formulation to derive an ODE for the growth of a perturbed density interface subjected to an arbitrary acceleration g(t) in uncompressible fluids :

$$b|a|\ddot{a} + \frac{ba}{2|a|}\dot{a}^2 + 2\pi c|\dot{a}|\dot{a} - 2\pi A a g(t) = 0$$

a is the amplitude of the perturbation, A is the Atwood number, b is a nondimensional constant related to the RT α parameter and to the RM power law exponent θ {b = $\pi\theta/\alpha(2-\theta)$} and c is a dissipation factor of order unity.

- Farley *et al.* [2] use this model to fit the experimental results of the laser experiment.

[1] J. D. Ramshaw
Simple model for linear and non linear mixing at unstable fluid interfaces with variable acceleration.
Phys. Rev. E, **58**, N5, 1998

[2] D. R. Farley *et al.*
High Mach number mix instability experiments of an unstable density interface using a single mode, nonlinear initial perturbation.
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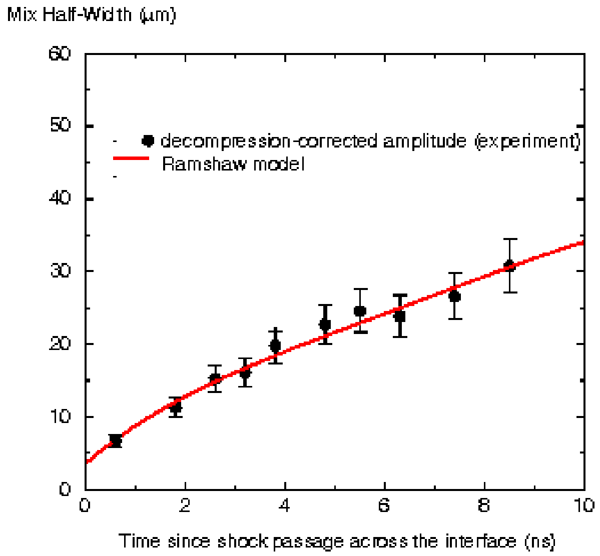
Nonlinear model

Farley *et al.*

❒ The amplitude (experiment) is equal to mix half-width

❒ The Ramshaw model is uncompressible and the experimental results have to be decompression-corrected in order to be compared to modelization.

❒ The initial values of a and da/dt are given by a 2D eulerian computation (CALE code)



The authors obtain a very reasonable fit of experimental data

Necessary conditions for turbulence

❒ The Reynolds number of the flow must be enough

- Reynolds number estimation

❒ The experiment duration must be greater than the transition time to turbulence

- Transition time model

Necessary conditions for turbulence

Reynolds number estimation

$$\mathrm{Re} = \frac{ul}{\nu}$$

ν = 0.001 μm^2/ns [1]

We propose [2] the following definitions for u and l :

$$u = \dot{a}(t_s) = 10 \mu m / ns$$

$$l = \lambda = 23 \mu m$$

$\mathrm{Re} \approx 10^5$ ($\approx$ Farley's result [1])

Grégoire [2] founds the following result for three classical shock tubes :

$5.\ 10^3 < \mathrm{Re} < 8.\ 10^4$

The laser induced instability can lead to turbulence

[1] D. R. Farley et al.
High Mach number mix instability experiments of an unstable density interface using a single mode, nonlinear initial perturbation
Physics of Plasmas, **6**, N11, 1999

[2] O. Grégoire
CEA/DIF internal report, 2000

Necessary conditions for turbulence
Transition time model [1]

$$\tau_{tr} = t_{nl} + t_{turb}$$

"filling" of the spectrum

end of the non linear phase

- t_{nl} $\quad a(t) = a_0^+ + \dot{a}_{IM}(t - t_0)$

linear phase, Impulsive model (IM)

$$t_{as}(a) = t - t_0 = \frac{a - a_0^+}{\dot{a}_{IM}} \qquad (d = ka < 1)$$

End of the linear phase :

Heuristic geometrical argument : $a(t_{nl}) = \lambda/2$ $(d = \pi)$

$$t_{nl} = \frac{\pi - d_0^+}{k\dot{a}_{IM}}$$

Grégoire [1] uses Vandenboomgaerde's impulsive model [2] valid for low Mach numbers to evaluate $\dot{a}_{IM}$

$$t_{nl} = \frac{2}{k}\left\{\frac{\pi - d_0^-\left(1 - \frac{\Delta U}{W}\right)}{\Delta U d_0^-\left[A^+\left(1 - \frac{\Delta U}{W}\right) + A^-\right]}\right\}$$

[1] O. Grégoire
CEA/DIF internal report, 2000

[2] M. Vandenboomgaerde et al.
Phys. Rev. E, 58 (2), 1874 – 1882, 1998

Necessary conditions for turbulence
Transition time model [1]

❑ t_{turb} : the initial spectrum is a $\delta(k\text{-}k_0)$ distribution. t_{turb} is the time necessary to establish the Kolmogorov cascade :

$$t_{turb} = \frac{\theta_T}{k\sqrt{K}}$$ (K is the fluctuating kinetic energy)

$$\theta_T = 2.1$$ (Clark *et al.* [2])

$$K \approx \frac{1}{2}\dot{a}^2_{t=t_{nl}}$$ (Grégoire [1])

$$t_{turb} = \frac{2}{k}\left\{\frac{\theta_T\sqrt{2}}{\Delta U d_0^- \left[A^+\left(1-\frac{\Delta U}{W}\right)+A^-\right]}\right\}$$

❑ We will use an alternate expression for $\dot{a}_{IM}$ (Meyer *et al.* [3]):

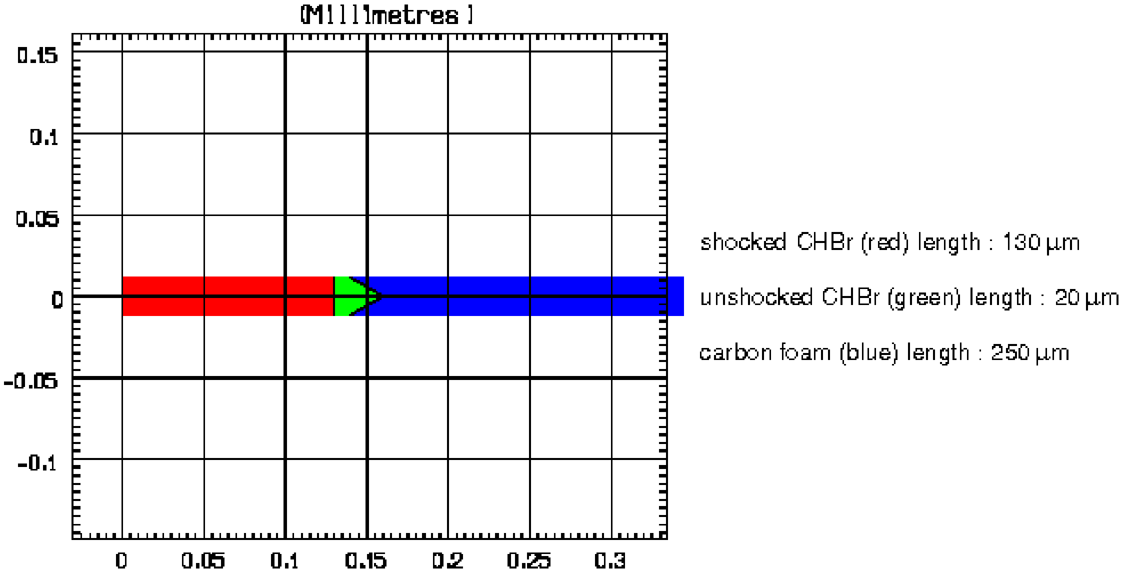
$$\dot{a}_{IM} = kA^+ a_0^* \Delta U \qquad \text{with} \qquad a_0^* = \frac{1}{2}\left(a_0^- + a_0^+\right)$$

τ_{tr} =0.44 ns after shock passage across the interface in the laser experiment (valid for low Mach number)

[1] O. Grégoire. CEA/DIF internal report, 2000
[2] T. T. Clark *et al.* Symmetries and the approach to statistical equilibrium in isotropic turbulence. Physics of Fluids, **10** (11),, 2846-2858,1998
[3] Meyer et al. Numerical investigation of the stability of a shock-accelerated interface between two fluids. Phys. Fluids 15, 753-75

2D eulerian computations (HESIONE code)

Initial setup



- ❒ Pure RMI computation (35 Mbar incident shock)
- ❒ Planar geometry
- ❒ Rigid walls computation box
- ❒ No radiation effects
- ❒ Standard EOS
- ❒ 100 zones/wavelength
- ❒ Computations valid till ≈ 2ns (non physical rarefaction waves effects)

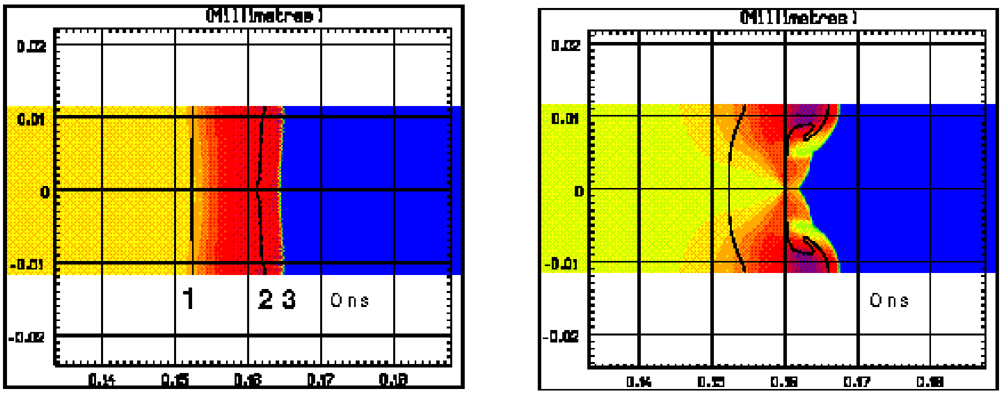
2D eulerian computations (HESIONE code)

Interface history

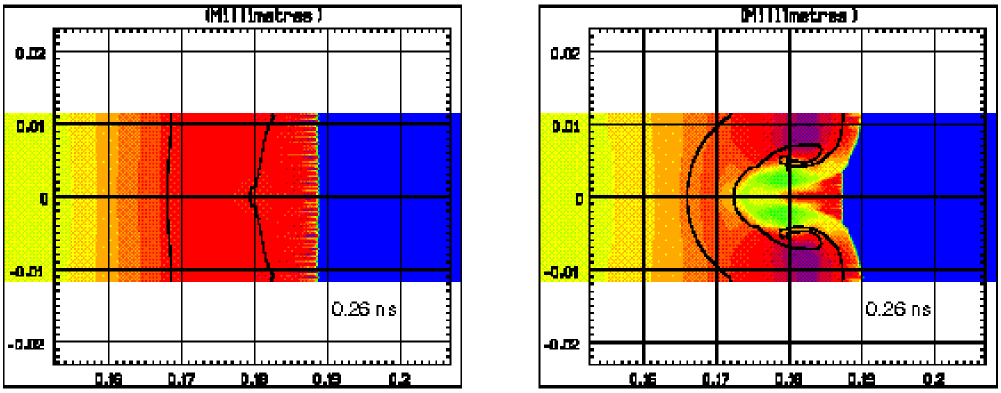
❒ Vx map + interface

❒ Time is referenced to when the shock completes the interface traversal for a0 = 10 µm

a0 = 1 µm   a0 = 10 µm



1 : artifact; **2** : interface; **3** : shock



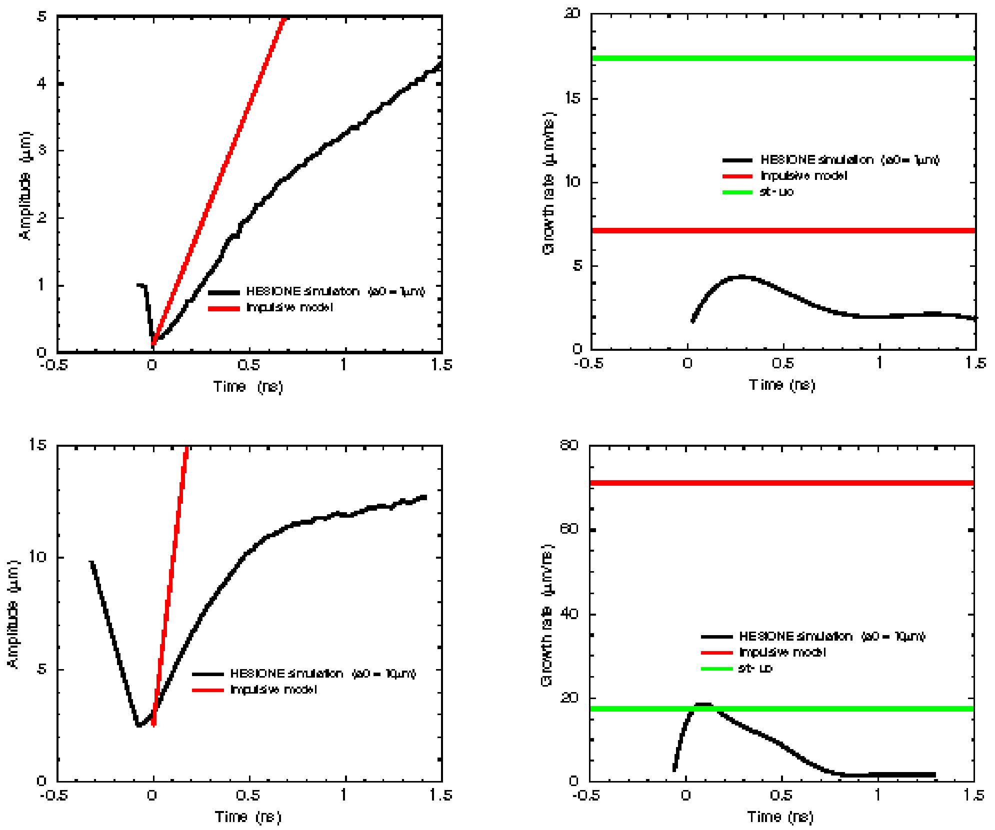
❒ The spike tip cannot overtake the transmitted shock (this penetration is energetically prohibitive)

⇒ **da/dt ≤ st - uc** (st : transmitted shock speed; uc : interface speed)

2D eulerian computations (HESIONE code)

Amplitude and growth rate history of the perturbation

The following analysis was proposed by Holmes et al. [1]



❒ Time is referenced to when the shock completes the interface traversal for each of the computations.

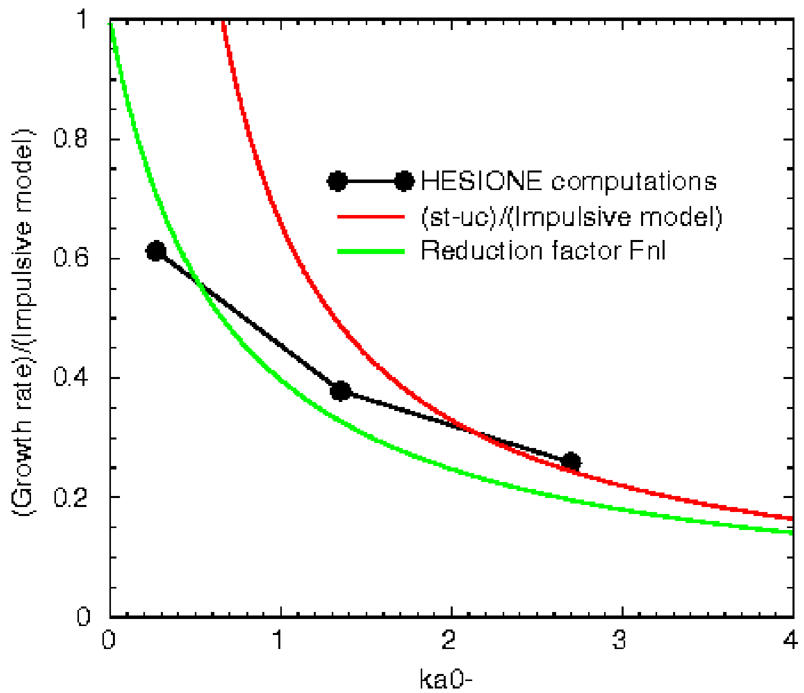
❒ For a0 = 1µm the impulsive model leads to a growth rate lower than st-uc.
⇒ The maximum growth rate given by HESIONE is of the order of 60% of the growth rate given by the impulsive model.

❒ For a0 = 10µm the growth rate given by the impulsive model is four times higher than st-uc which is the limiting factor (the maximum growth rate given by HESIONE is of the order of st-uc).

[1] R. L. Holmes *et al.*
Richtmyer-Meshkov Instability growth : experiment, simulation and theory
J. Fluid Mech., vol. 389, pp. 55-79 (1999)

2D eulerian computations (HESIONE code)

Saturation of the growth rate



❒ The ratio of the actual growth rate to the IM growth rate (IMgr) is lower than 1 and lower than (st-uc)/IMgr

❒ Holmes *et al.* [1] propose a heuristic reducing factor Fnl which gives the IM growth rate in the linear limit :

$$Fnl = 1/\{1 + IMgr/(st - uc)\}$$

❒ Our results ($M \approx 20$) are similar to Holmes ($M = 15.3$)

[1] R. Holmes *et al.*
Richtmyer-Meshkov instability growth : experiment, simulation and theory
J. Fluid Mech., vol. 389, pp. 55-79 (1999)

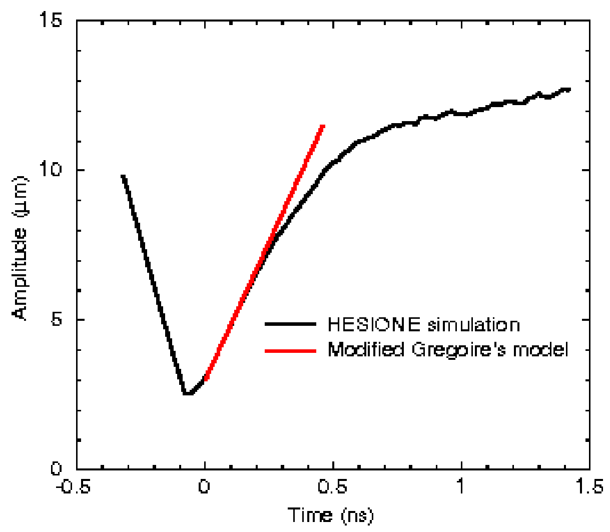
2D eulerian computations (HESIONE code)

Estimation of the transition time

Modifications of the Gregoire's model :

❒ The IM growth rate is replaced by the maximum growth rate given by the simulation to calculate tnl and tturb (low value of τtr)

❒ The growth rate at tnl can be used instead of the maximum growth rate to calculate tturb (high value of τtr)



1.2 ns < τtr < 1.9 ns

(after shock passage across the interface)

k-ε computations [1]
Initial conditions

- The k-ε model is a two equations statistical model (one equation for k, the turbulent kinetic energy and one equation for ε, its dissipation rate)

- k and ε have to be defined at τ_{tr}

- k and ε have the same extension L at τ_{tr} (the mixing zone width at τ_{tr}) and have a symmetric triangular shape

$$L = 2\alpha(1+\beta)A^{+}\Delta U\tau_{tr} \text{ (Mikaelian [2])}$$
$$k_{max} = \frac{4}{3}\alpha\left(A^{+}\Delta U\right)^{2}$$
$$\varepsilon_{max} = C\frac{k_{max}^{3/2}}{L}$$

This simple model has been widely used in classical shock tubes computations.

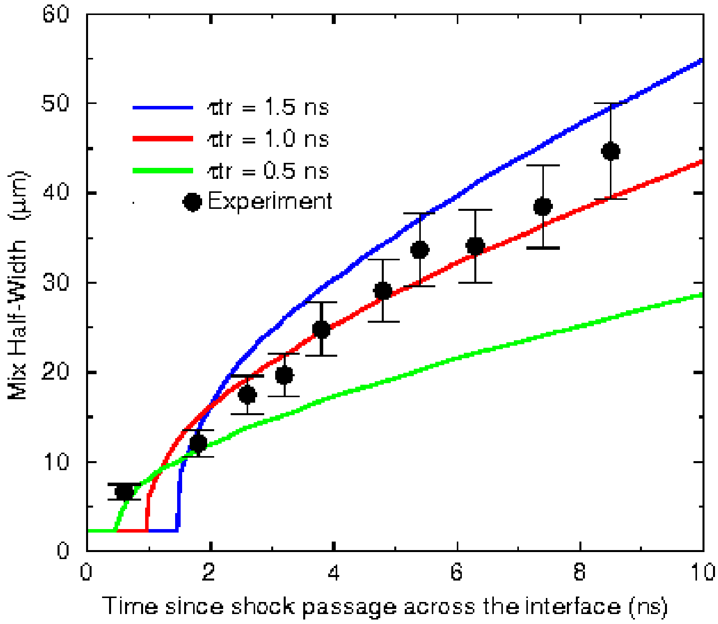
[1] S. Gauthier et al.
A k-ε model for turbulent mixing in shock tube flows induced by Rayleigh-Taylor instability
Phys. Fluids. A **2** (9) 1685-1694, 1990

[2] K. O. Mikaelian
Turbulent mixing generated by Rayleigh-Taylor and Richtmyer-Meshkov instabilities
Physica D, **36**, 343, 1989

k-ε computations

Transition time estimation

k-ε computations were carried out for three arbitrary values of τtr (0.5, 1.0 and 1.5 ns)



❒ The amplitude (Mix half-Width) history for τtr ≈ τtrIM (τtr = 0.5 ns) doesn't fit the experimental data

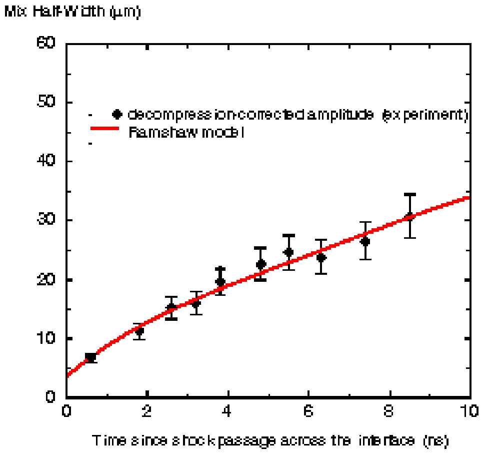
❒ A reasonable fit for the long term amplitude is obtained for 1.0 ns < τtr < 1.5 ns

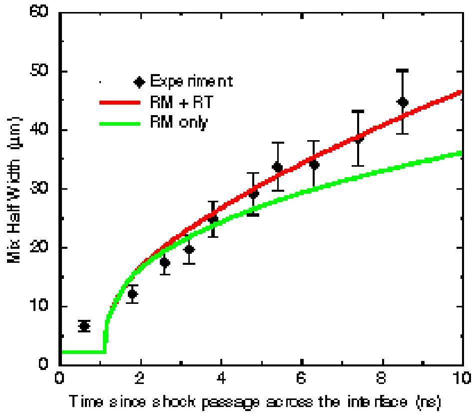
❒ This τtr range reasonably agrees with the Gregoire's model applied to HESIONE computations :

1.2 ns < τtr < 1.9 ns

k-ε computations

Ramshaw model [1] and k-ε model





❒ **A reasonable fit for the long term amplitude is obtained either with the Ramshaw model or with the k-ε model (τtr = 1.125 ns)**

❒ The k-ε transient phase doesn't permit a satisfactory description of the beginning of the perturbation growth

❒ The pure RM case has also been considered in [1] (aRM/aRM+RT = 0.81 at 10 ns) and the k-ε model leads to a similar result (aRM/aRM+RT = 0.78 at 10 ns)

[1] D. R. Farley et al.
High Mach number mix instability experiments of an unstable density interface using a single mode, nonlinear initial perturbation.
Physics of Plasmas, **6**, N11, 1999

Summary

- ❑ The Reynolds number of the flow induced by laser in the experiment analyzed here is of the order of magnitude of the Reynolds number obtained in classical shock tube experiments

- ❑ The transition time related to the Impulsive Model growth rate is very small as compared to the duration of the experiment

- ❑ The transition time related to the growth rate evaluated through 2D computations stays very shorter than the duration of the experiment

- ❑ The k-ε computations fit the experimental data in a range of τ_{tr} compatible with the former result