Large Eddy Simulation of Strong-shock Richtmyer-Meshkov Instability

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Outline

- Objectives and physical problem setup
- Equations and numerical method
- Subgrid scale model description
  - *Stretched vortex SGS model for LES*
- Decaying isotropic turbulence test
  - *comparison between Pade and WENO*
    - *modified wave number behavior*
- RM Simulation results
  - *Plane averages and rms quantities*
  - *mixing width (with and w/o SGS models)*
- Conclusion

Strong-shock Richtmyer-Meshkov Instability (RMI)

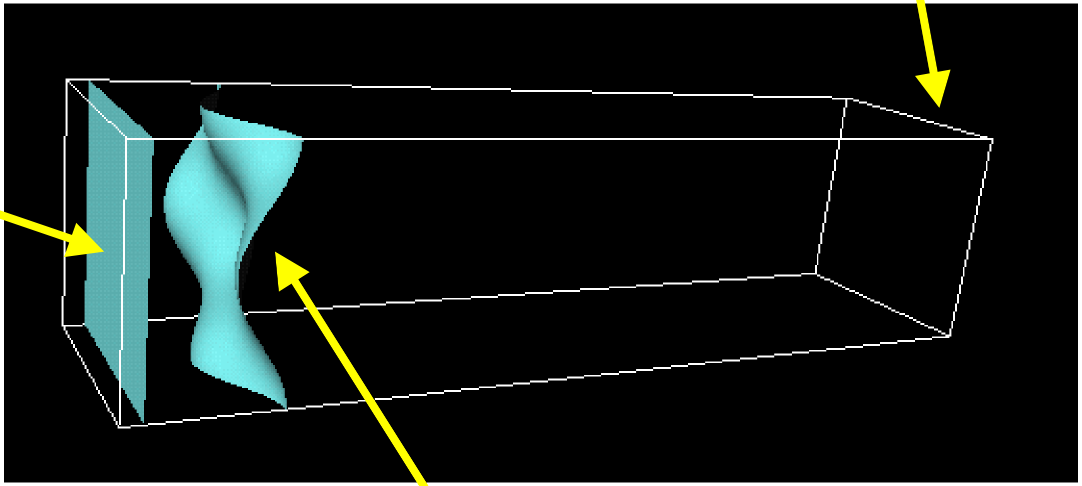
- Objectives:
  - *Pseudo-DNS of Richtmyer-Meshkov flow with strong shocks*
    - *shocks not resolved (requires shock-capturing method)*
    - *numerical method reverts to high-order in regions away from shocks*
  - *LES with the stretched-vortex model of same flow*
- Requirements:
  - *Shock-capturing method with good resolution characteristics in the high-wavenumber range (not only formally high-order)*
    - *WENO (Shu et al.)*
    - *Hybrid (Pade + WENO) (Adams and Shariff)*
    - *Spectral methods for compressible flows (Gottlieb et al.)*
  - *Numerical method compatible with AMR*
  - *SGS-Model applicable to flows with strong shocks*
    - *Stretched Vortex SGS (Pullin and co-workers)*

RM instability: Setup

- Strong shocks (M=10)
- Density ratios
  - *light to heavy (fast/slow) (5/1)*
  - *heavy to light (slow/fast)*
- Periodic boundary conditions in transverse directions
  - *homogeneous turbulence in cross-plane*

Shock reflects off end

Incident shock



Interface (multiple harmonic perturbation)

Favre filtered NS equations

- Favre average: $\widetilde{q} = \bar{\rho} q / \bar{q}$
- Continuity

$$\frac{\partial \bar{\rho}}{\partial t} + \frac{\partial \bar{\rho}\widetilde{u}_i}{\partial x_i} = 0$$

- Momemtum

$$\frac{\partial \bar{\rho}\widetilde{u}_i}{\partial t} + \frac{\partial(\bar{\rho}\widetilde{u}_i\widetilde{u}_j + \bar{p}/\gamma M^2 \delta_{ij})}{\partial x_j} = Re^{-1}\left(\frac{\partial \widetilde{\sigma}_{ij}}{\partial x_j}\right) - \frac{\partial T_{ij}}{\partial x_j}$$

where

$$T_{ij} = \bar{\rho}(\widetilde{u_i u_j} - \widetilde{u}_i\widetilde{u}_j) \equiv \bar{\rho}\,\tau_{ij}$$

Subgrid stress

Favre filtered NS equations

Energy

$$\frac{\partial \bar{E}}{\partial t} + \frac{\partial(\bar{E} + \bar{p}/\gamma M^2)\tilde{u}_i}{\partial x_i} = \frac{1}{Pr Re(\gamma - 1)M^2}\frac{\partial}{\partial x_i}\left(k\frac{\partial \tilde{T}}{\partial x_i}\right)$$

$$+ Re^{-1}\frac{\partial(\tilde{\sigma}_{ij}\tilde{u}_j)}{\partial x_i}$$

$$- \frac{1}{(\gamma - 1)M^2}\frac{\partial(\bar{\rho} q_i)}{\partial x_i}$$

$$- \frac{1}{2}\frac{\partial[\bar{\rho}(\widetilde{u_j u_j u_i} - \widetilde{u_j u_j}\tilde{u}_i)]}{\partial x_i}$$

$$+ Re^{-1}\frac{\partial(\widetilde{\sigma_{ij} u_j} - \tilde{\sigma}_{ij}\tilde{u}_j)}{\partial x_i}$$

$$\bar{E} = \frac{\bar{p}}{(\gamma - 1)\gamma M^2} + \frac{1}{2}\bar{\rho}(\tilde{u}_j\tilde{u}_j) + \frac{1}{2}\bar{\rho}(\widetilde{u_j u_j} - \tilde{u}_j\tilde{u}_j)$$

$$q_i = \widetilde{T u_i} - \tilde{T}\tilde{u}_i$$

$$\bar{p} = \bar{\rho}\tilde{T}$$

- Heat flux
- Triple correlation
- Viscous work
- Subgrid KE

Numerical method: WENO

- Finite difference formulation WENO (Jiang & Shu) for inviscid fluxes in the governing equations
- Conservative approximation of flux derivatives

$$\frac{1}{\Delta x_i}\left[\hat{f}_n(x_{i+1/2})-\hat{f}_n(x_{i-1/2})\right]=\partial f_n(x_i)/\partial x+O(\Delta x^k)$$

  - *Fluxes calculated in characteristic coordinates*
  - *Characteristics -eigenvalues and eigenvectors evaluated using Roe state*
- Runge-Kutta (TVD) time discretization

Prevention of Instabilities

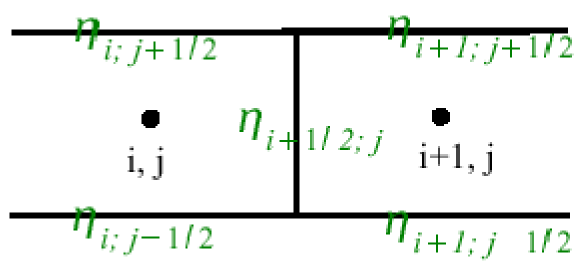
- "H-correction" by Sanders, Morano & Druguet adapted for FD-WENO

$$\hat{\lambda}^{*}_{p;i+1/2;j} = \max\left(\hat{\lambda}_{p;i+1/2;j}, \eta^{H}_{i+1/2;j}\right)$$

where

$$\eta^{H}_{i+1/2,;j} = \max\left(\eta_{i+1/2;j}, \eta_{i;j+1/2}, \eta_{i;j\ \ 1/2}, \eta_{i+1;j+1/2}, \eta_{i+1;j\ \ 1/2}\right)$$

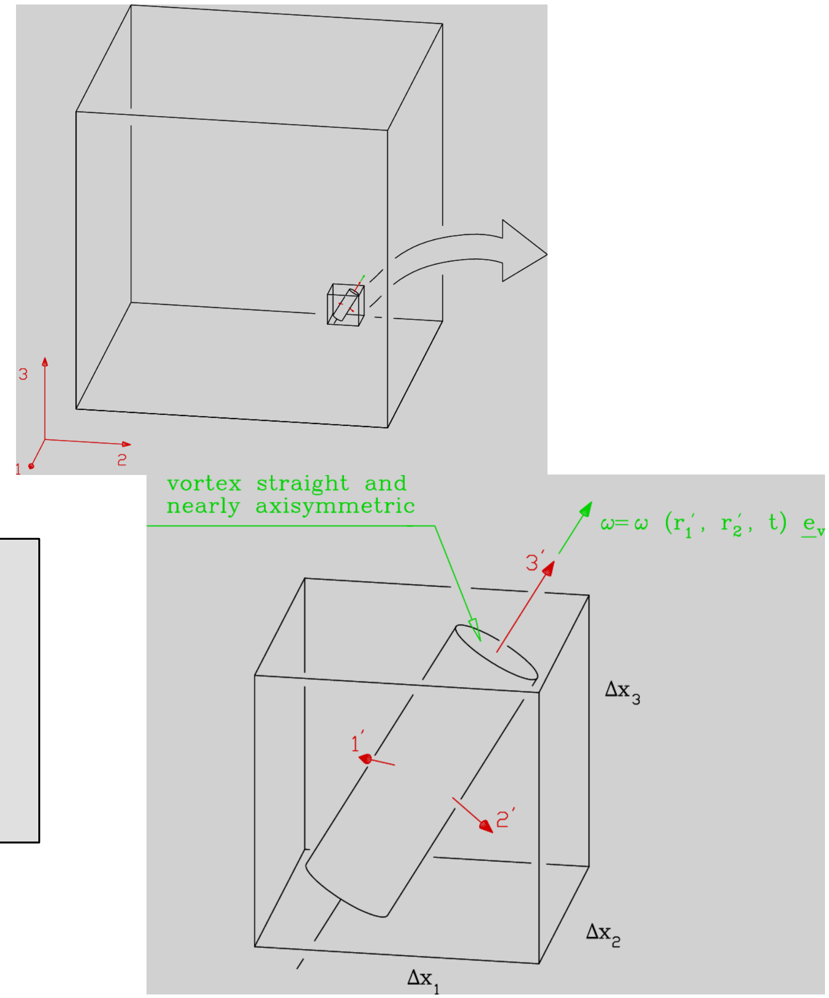
$$\eta_{i+1/2;j} = \frac{1}{2}\max_{p}\left(\left|\lambda_{p}(\boldsymbol{U}_{i+1;j}, \boldsymbol{e}_{x}) - \lambda_{p}(\boldsymbol{U}_{i;j}, \boldsymbol{e}_{x})\right|\right)$$



LES Model - Pullin SGS vortex model

- Extension of stretched vortex sub-grid stress model (Misra & Pullin 1997) to compressible turbulence
- Structure-based approach
  - *Subgrid motion represented by nearly axisymmetric vortex within each cell.*
- Subgrid stresses are:

$$T_{ij} = K\,(\delta_{ij} - e_i\,e_j)\,,$$
$$K = \int_{k_c}^{\infty} E(k)\,dk.$$



Pullin SGS vortex model

- Lundgren form assumed for subgrid energy spectrum:

$$K_0\,\epsilon^{2/3}\,k^{-5/3}\,\exp(-2\,k^2\,\nu/(3\,|a|))$$

$$a = \tilde{S}_{ij}\,e_i^v\,e_j^v$$

- PDF for vortex orientation in each cell

$$P(\mathbf{e}) = \mu\,\wp\,(\mathbf{e}^{\mathbf{v}}\mid\tilde{\mathbf{e}}_3) + (\mathbf{1}-\mu)\,\wp\,(\mathbf{e}^{\mathbf{v}}\mid\tilde{\mathbf{e}}_\omega)$$

$$\mu = \frac{\lambda_3}{\lambda_3 + \|\,\tilde{\boldsymbol{\omega}}\,\|}$$

- Subgrid temperature flux (analytical solution for the winding of the local resolved temperature by the elemental vortex)

$$[q_i]_{mod} = \frac{1}{2}\,\Delta\,K^{1/2}\,\left(\delta_{ij} - \mathrm{e}_i^v\,\mathrm{e}_j^v\right)\frac{\partial\tilde{T}}{\partial x_j}$$

Pullin SGS vortex model

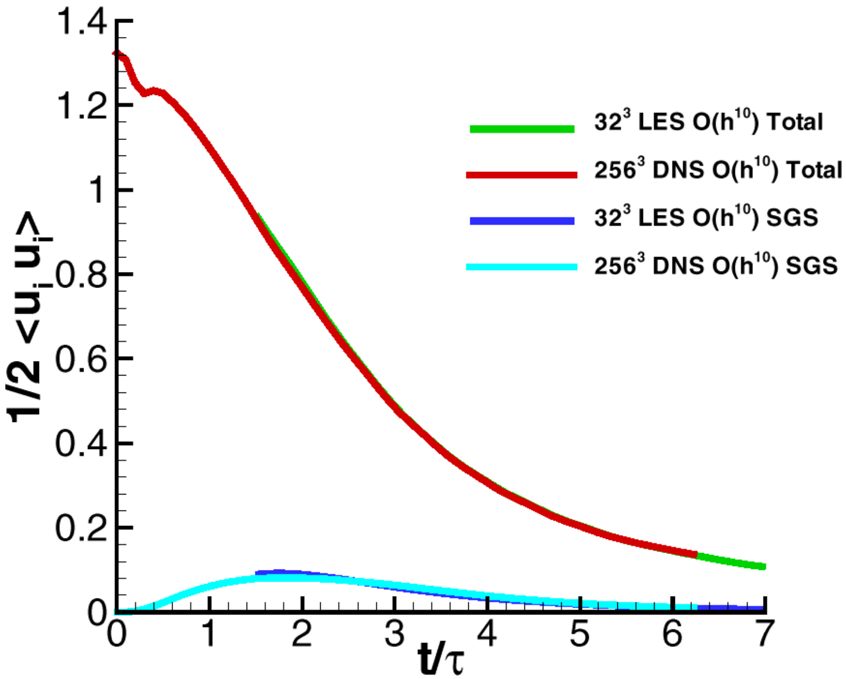
- $\mathcal{K}_0 \, \epsilon^{2/3}$ estimated locally by matching local resolved flow 2'nd-order velocity structure function to local subgrid estimate
- Stretched-vortex model is *not* an eddy-viscosity model
  - *allows "back scatter"*
- Elements of subgrid stress tensor and subgrid energy calculated directly
  - *Important for scalar and other subgrid quantities*
- No explicit filtering
- No explicit treatment for shock
  - *verified using aposteriori tests with DNS of decaying isotropic turbulence in the presence of shocklets at modest turbulent Mach numbers (0.3-0.5)*
- Plug-in model: ease of implementation

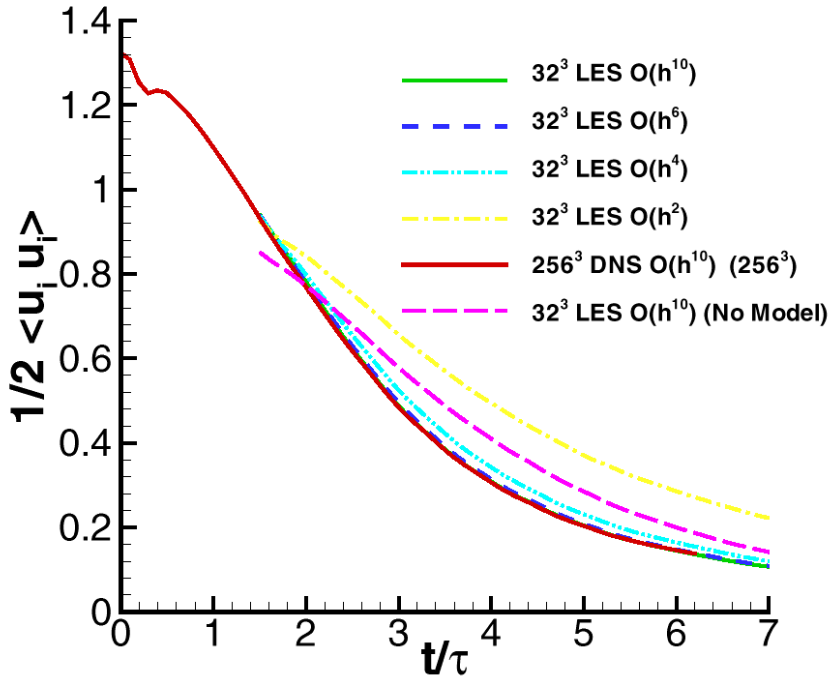
Comparison of DNS with LES + SGS

Decay of turbulent kinetic energy using Pullin stretched-vortex SGS model

("SGS modeling for LES of compressible turbulence" Kosovic, Pullin and Samtaney. To appear in Phys. Fluids)

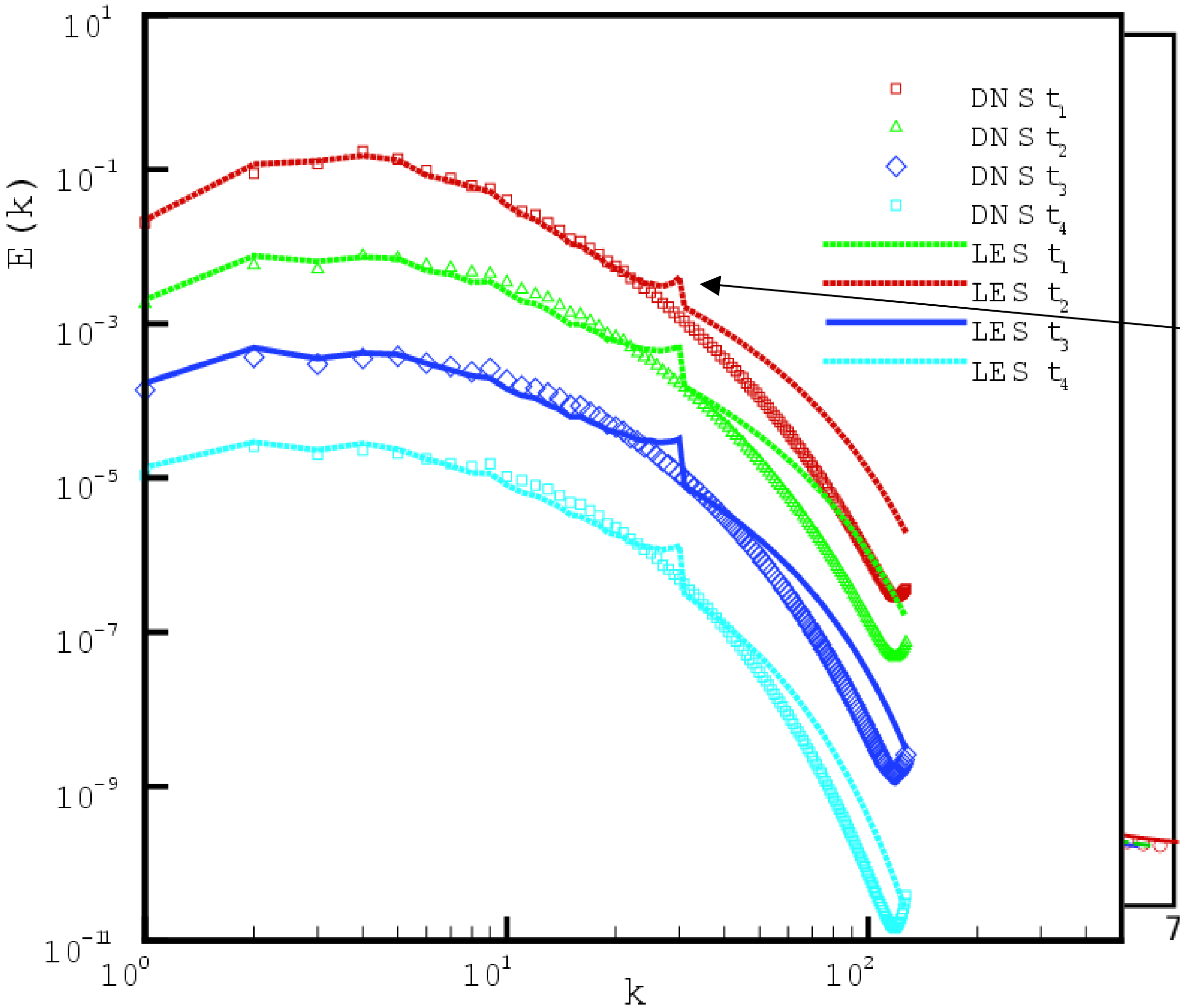
$$M_t = 0.488 \quad Re_l = 175 \quad O(h^{10}) \quad 256^3 \quad \text{IC4}$$





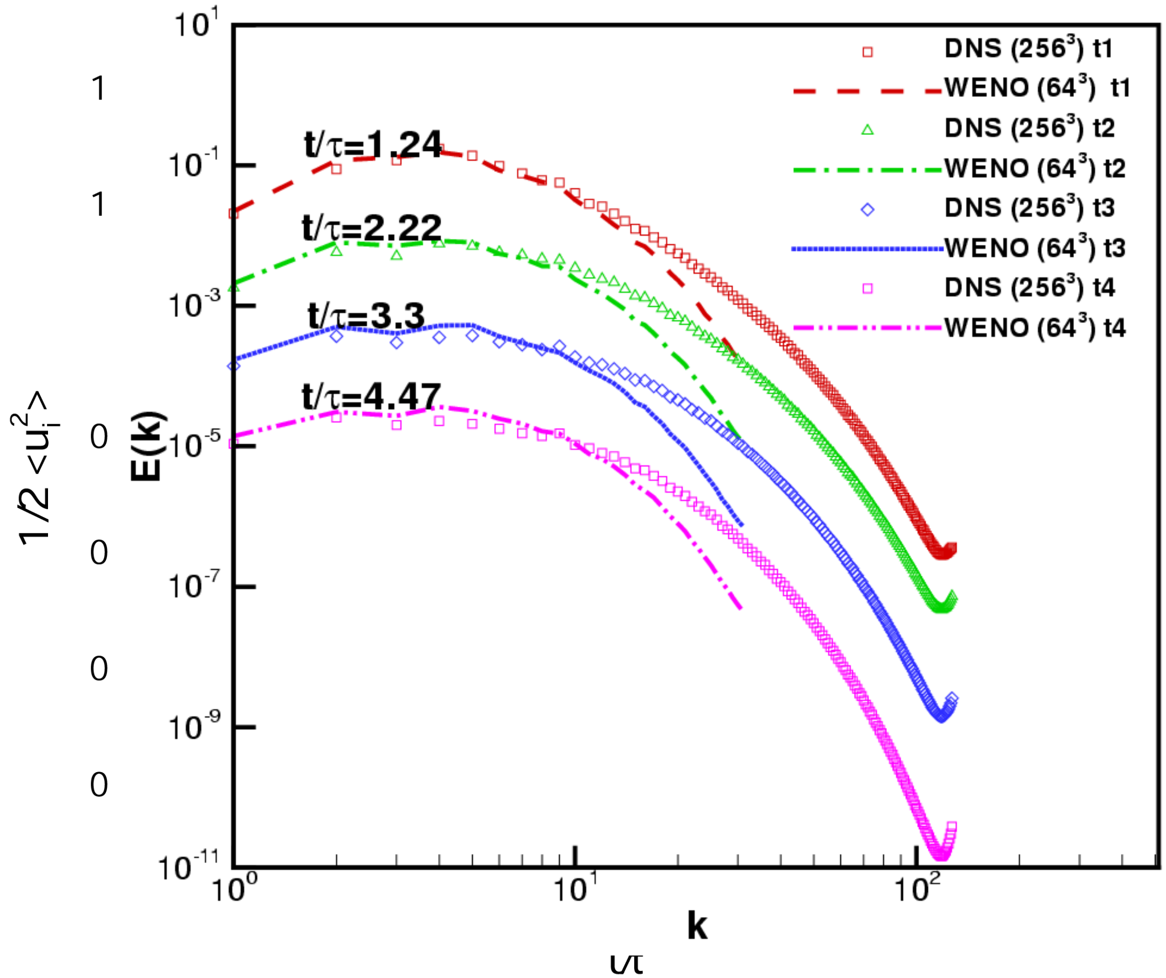
"DNS of decaying isotropic turbulence" - Samtaney, Pullin, Kosovic in Phys. Fluids, May 2001

Comparison of spectra (LES vs. DNS)



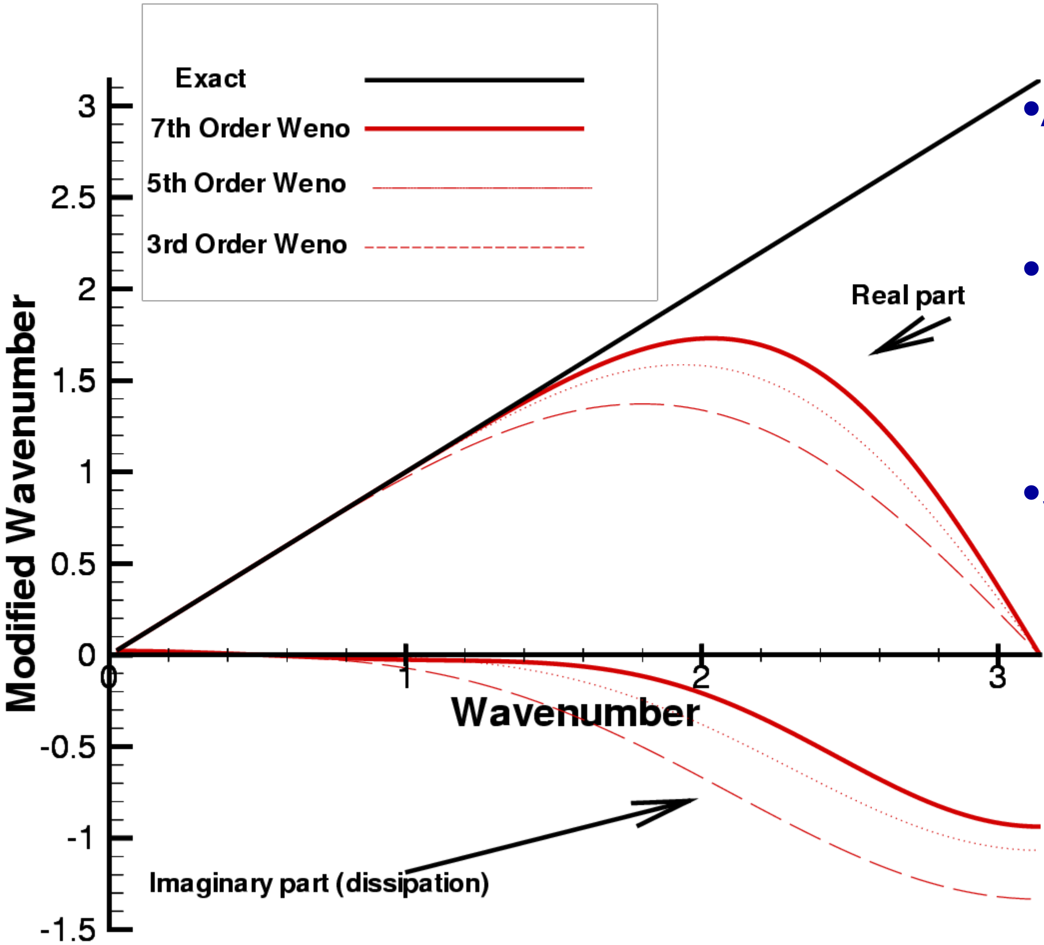
Pile-up is due to aliasing

Pade vs. WENO



DNS (256³) t1
WENO (64³) t1
DNS (256³) t2
WENO (64³) t2
DNS (256³) t3
WENO (64³) t3
DNS (256³) t4
WENO (64³) t4
t/τ=1.24
t/τ=2.22
t/τ=3.3
t/τ=4.47
E(k)
k
1/2 <u_i^2>
t/τ

Pade vs. WENO (Modified Wave number)

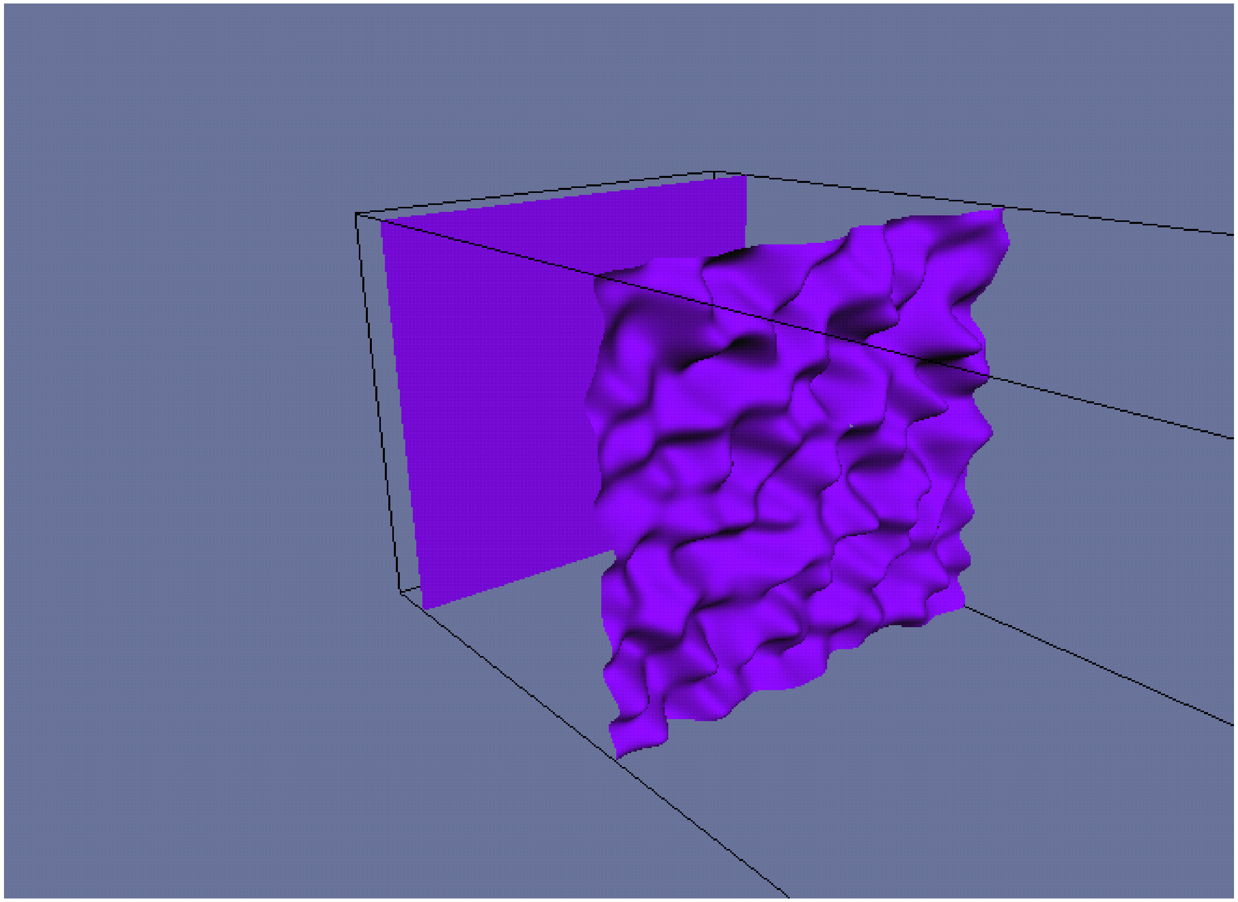


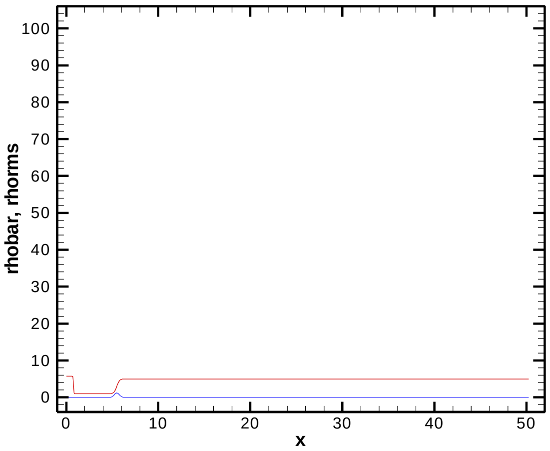
- Analysis assume periodic functions
- Modified wavenumber for WENO done for the optimal stencil
- See Lele (JCP 1992) for a discussion of modified wave number

WENO-RMI: Run Parameters

- Shock Mach number M=10
- Density ratio 1:5
- Interface Initial condition: Multi-mode perturbation with random phases and prescribed spectrum
- BC: Inflow (left), Reflecting (right), Periodic (transverse)
- Physical Domain $16\pi \times 2\pi \times 2\pi$
- 7th-order (formally) WENO with H-correction
- Three runs
  - *(A) 1024x128x128 (No SGS model)*
  - *(B) 512x64x64 (No SGS model)*
  - *(C) 512x64x64 (SV SGS model)*
- Simulations on ASCI Blue Mountain (nirvana)
  - *1024x128x128 on 128 procs., 18400 timesteps (40s/timestep)*
  - *512x64x64 on 64 procs., 10000 timesteps (20s/timestep)*

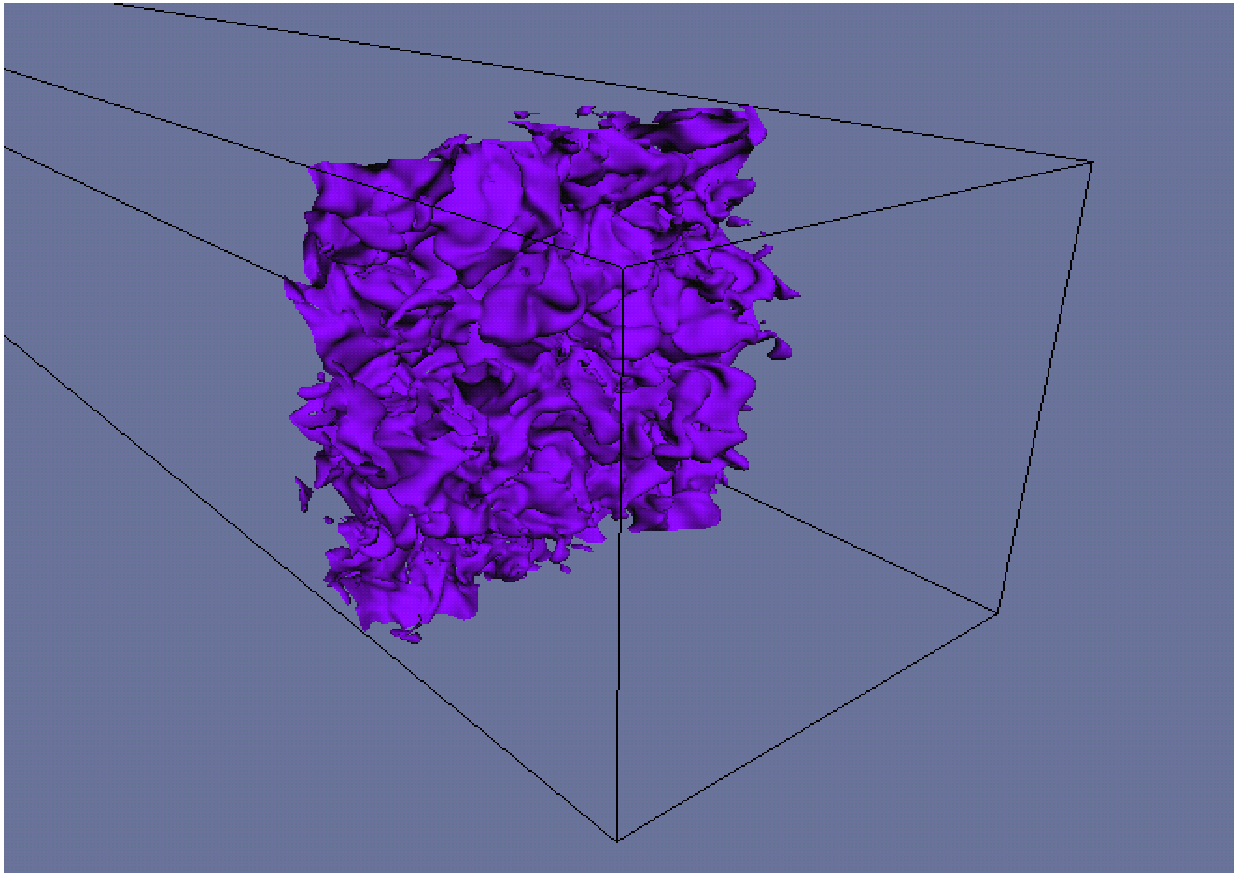
WENO- RMI simulation: Initial Condition

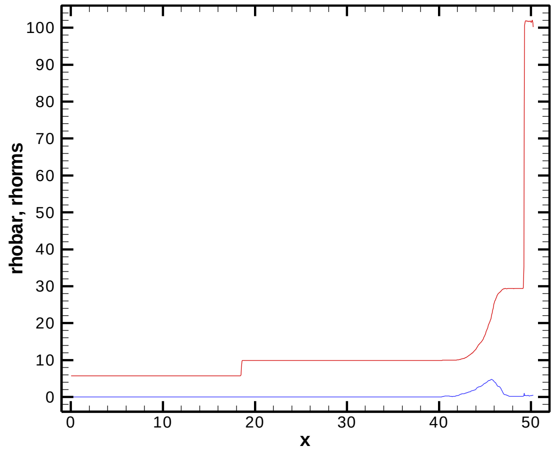




$$\rho = 3$$

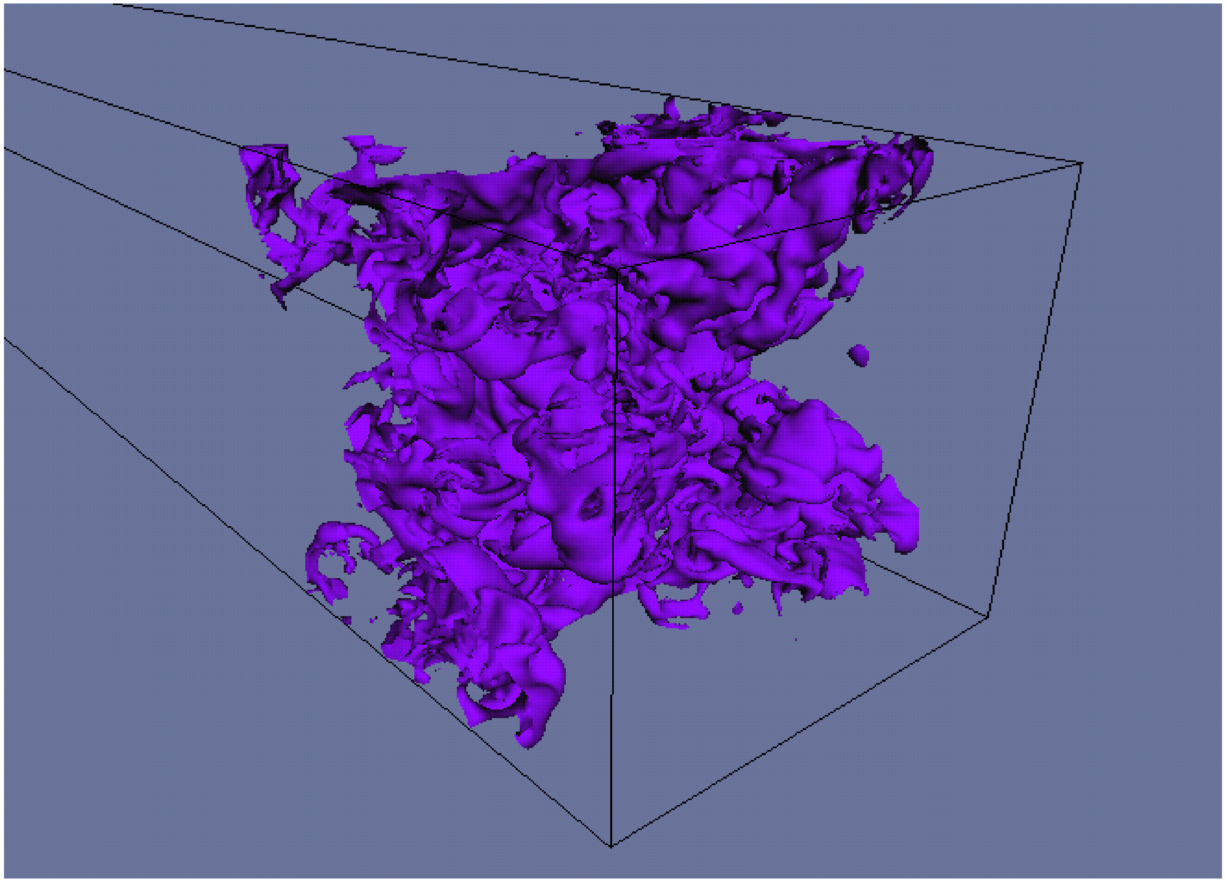
RMI: Before reshock (Run A)

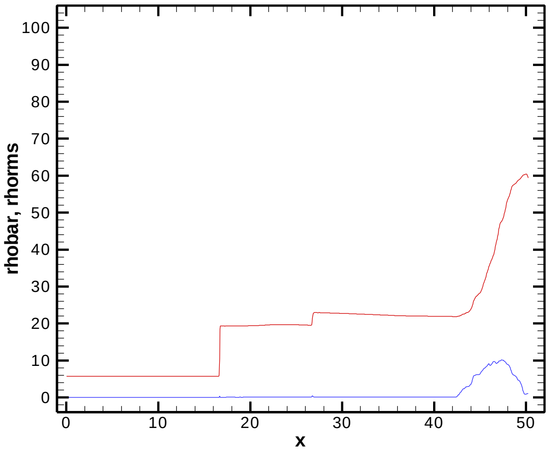




$$\rho = 20$$

RMI: After reshock (Run A)

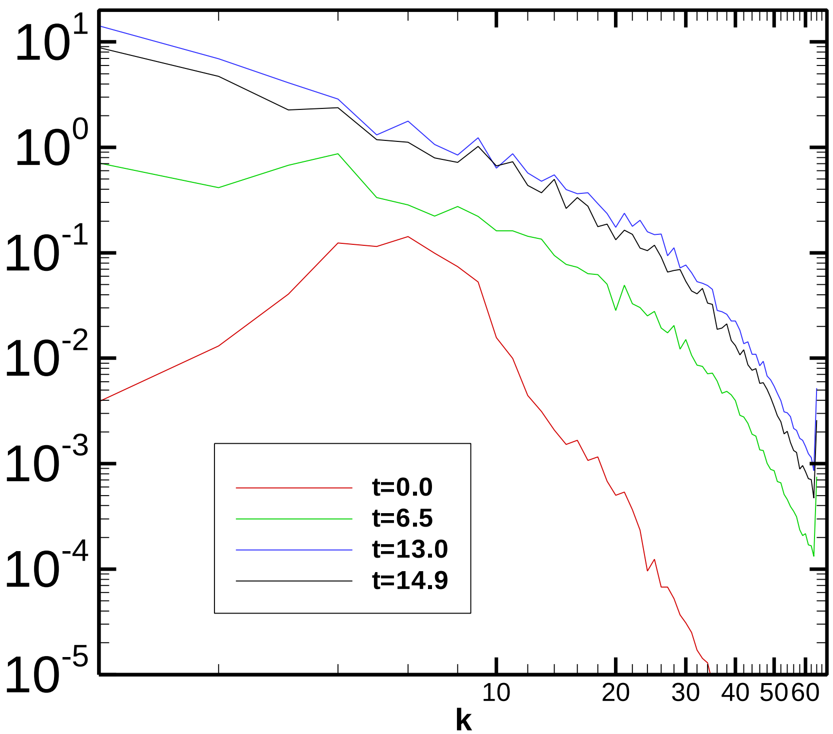




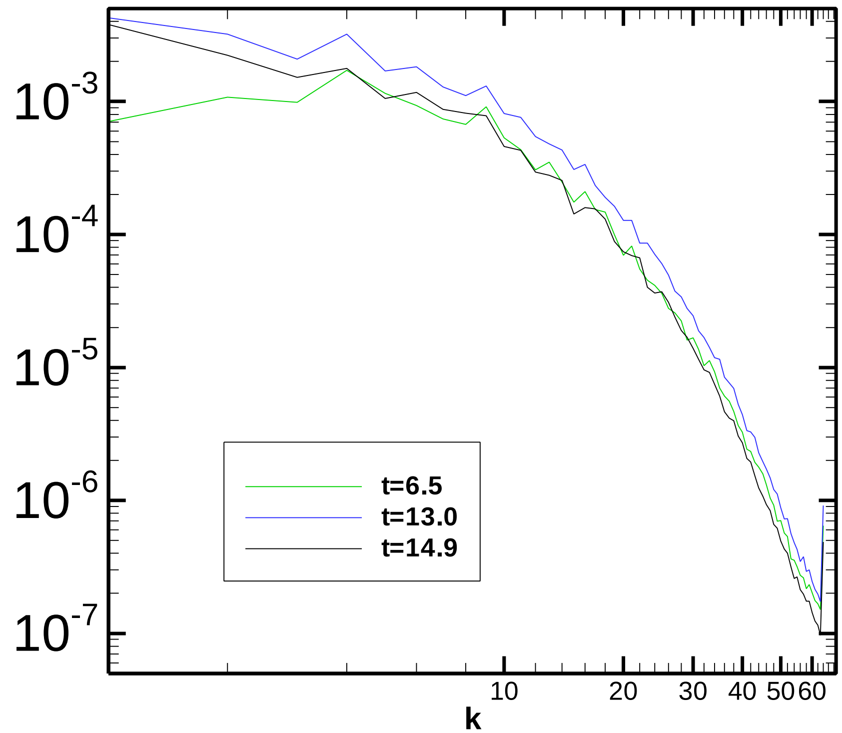
$\rho = 42$

RMI: Spectra (Run A)

- 2D spectra in planes normal to shock−propagation direction
- Location of plane determined by zero crossing of $\overline{\phi}$



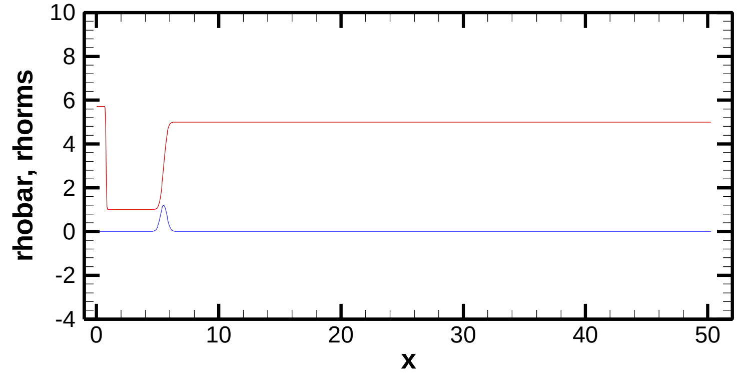
density spectra



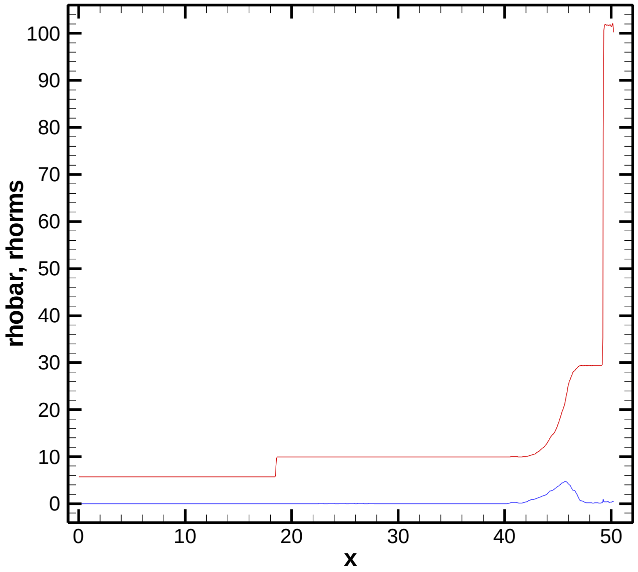
velocity spectra

RMI: Density plane averages and rms

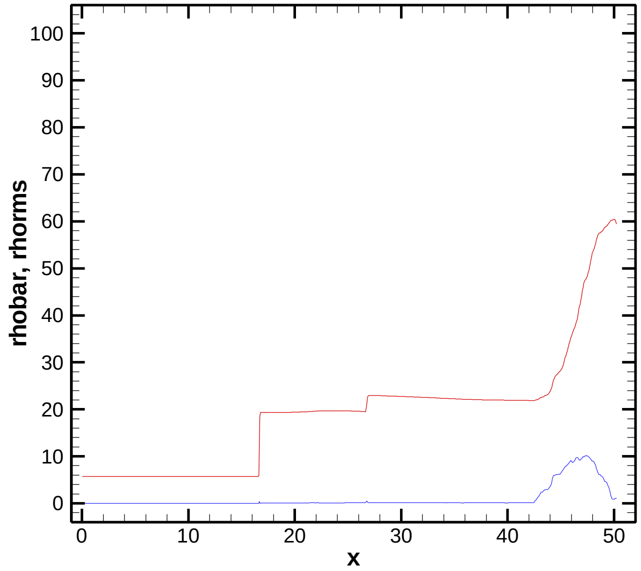
Run A



t=0.0

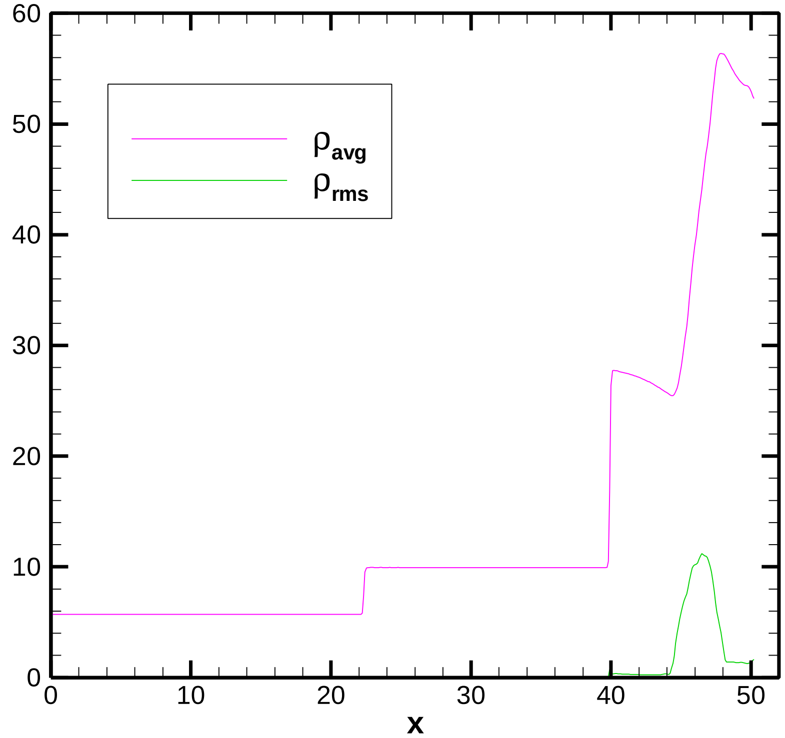


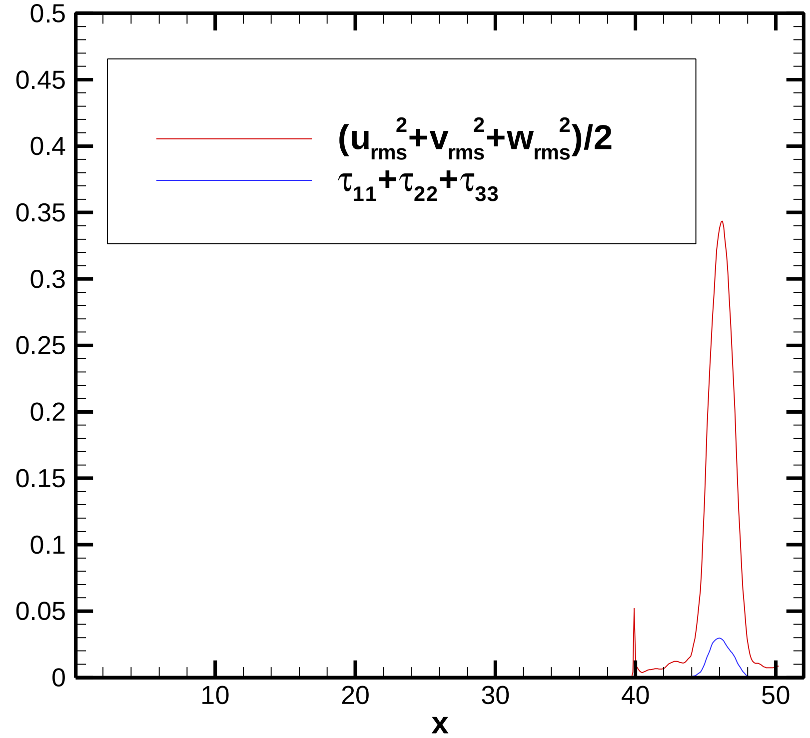
t=6.5



t=13.0

RMI: Plane averages and rms (Run C)

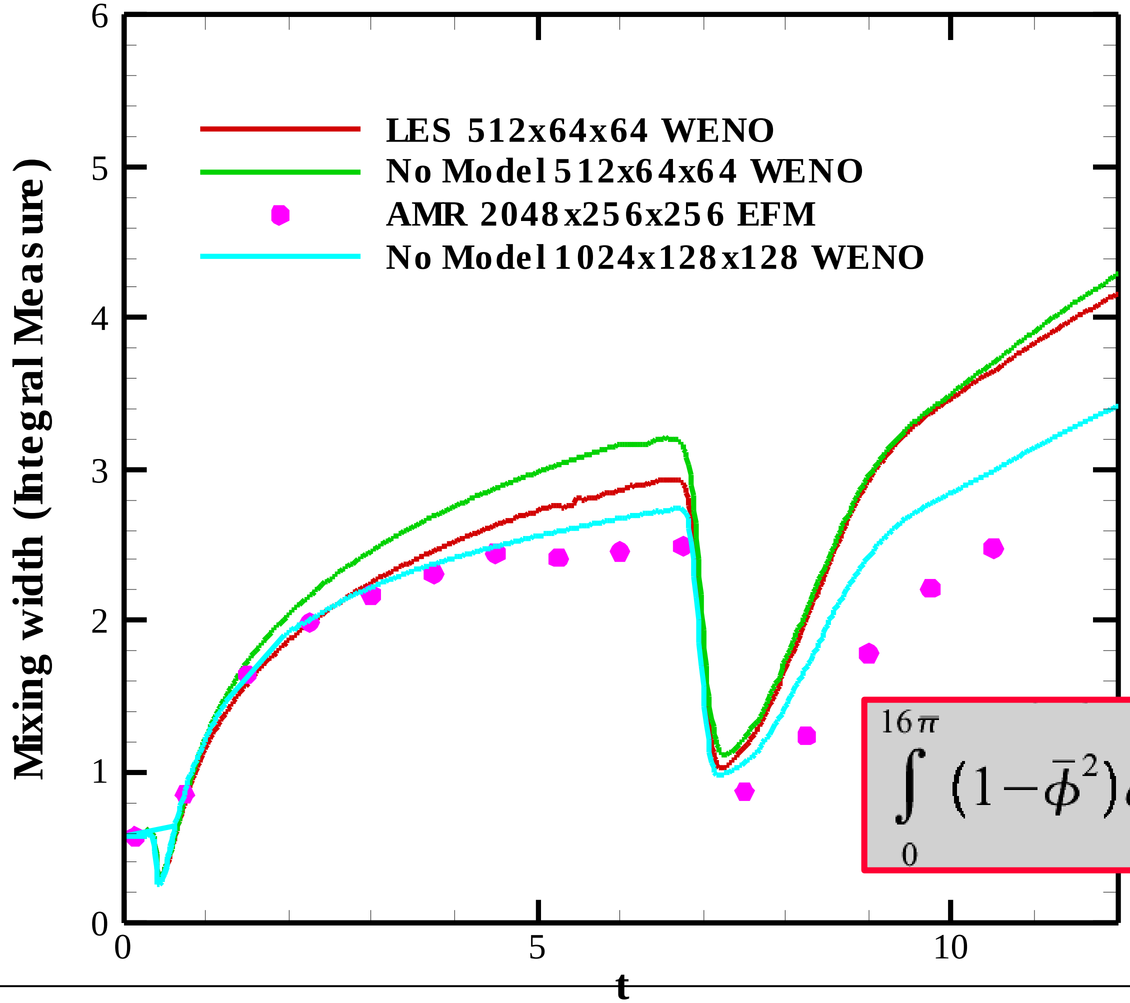




t=8.3 (after reshock)

Turbulent Mach number is approx. 0.13 (0.1-0.35 after reshock)

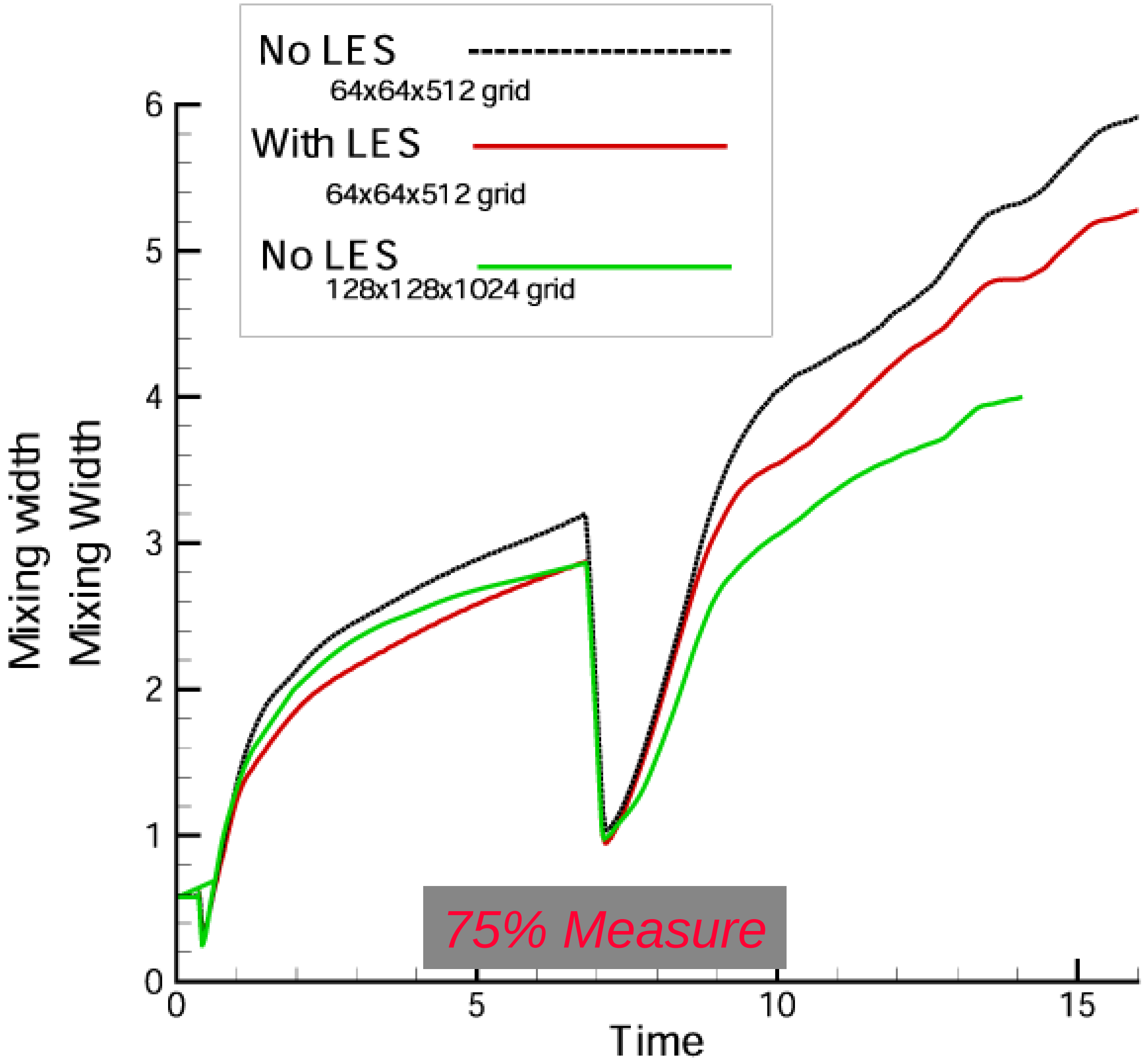
RMI: Mixing width (Integral Measure)



$$\int_0^{16\pi} (1-\bar{\phi}^2)\,dx \quad \text{with} \quad \phi \in [-1;1]$$

AMR done with EFM, no SGS model and with reflecting BC

RMI: Width of density interface



Conclusion

- Requirement of shock-capturing and higher-order is difficult to achieve in practice
  - *WENO schemes investigated*
    - *Compared with Pade schemes for decaying isotropic turbulence*
    - *High modified wavenumber behaviour not favourable*
    - *Require "Carbuncle fix" to stabilise the shock*
- LES of strong-shock RM performed using the stretched vortex SGS model
  - *SV - SGS model implemented in the WENO code*
    - *works as a plug-in*
    - *no explicit filtering*
    - *SGS model is robust (I.e., no. numerical stability issues)*
- Compared LES with SV model and LES with no model
  - *SGS model active but subgrid TKE is a small fraction of the totalTKE (~10%)*
  - *Small differences in the "mixing width" with and w/o SGS model*