8th IWPCTM: C20

LOCALIZATION AND SPREADING OF INTERFACES (CONTACT DISCONTINUITIES) IN PPM AND WENO SIMULATIONS OF THE INVISCID COMPRESSIBLE EULER EQUATIONS

VIZLAB

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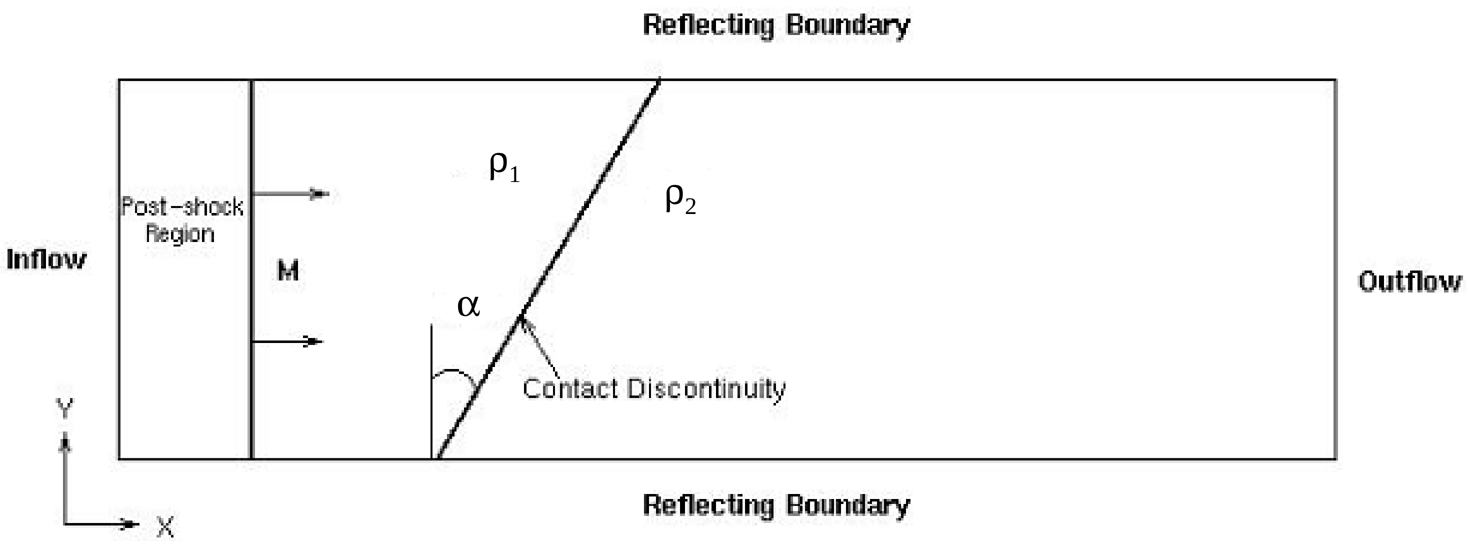
OBJECTIVE

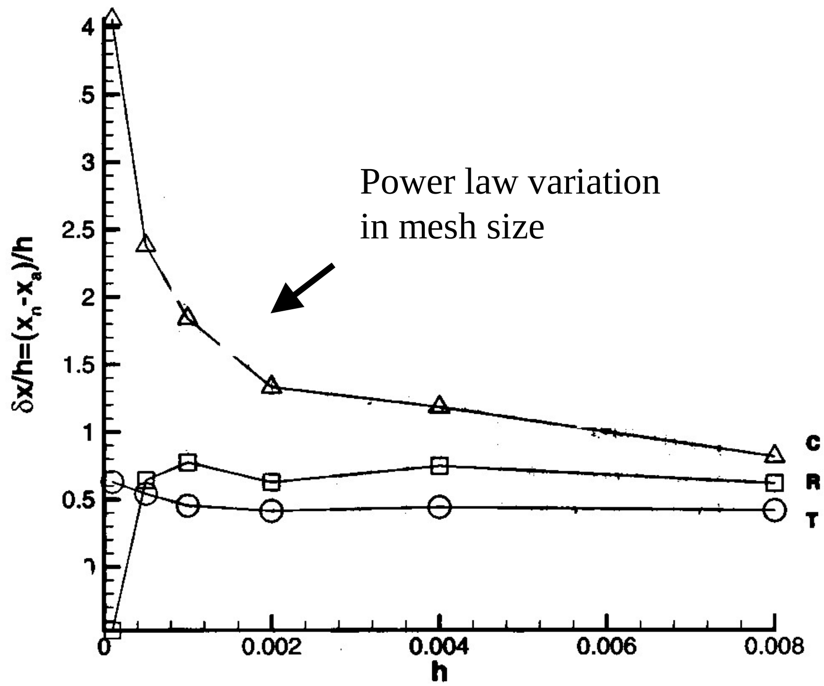
- Systematic approach to examine localization and temporal spreading of contact discontinuities(CDs) in 1D and 2D.
- Validity of near contact simulations of accelerated flows of high-gradient compressible media (RT and RM).
- Evolution of sinusoidal RM interface at late time and interfacial growth rate.



Schematic of Shock Interaction with an Inclined Discontinuity. M is the Mach number, α is the angle between shock and contact discontinuity, ρ_1 and ρ_2 are the densities of two gases.

MOTIVATION

- Study by Samtaney & Zabusky: Visualization and quantification of compressible flows in *Flow Visualization(1999)*.
- Non-convergence of position of contact discontinuity(x_{num}-x_{anal})/h to exact analytical solution for 1D.



Convergence study using difference in the numerical and analytical locations of high gradient regions (shocks and CDs) vs **mesh size** h. M = 3.0 shock interacts with a density discontinuity (CD, ρ_2/ρ_1= 3.0) and yields a moving CD (C), upstream reflected shock (R), and downstream transmitted shock (T).

CONTINUUM LIMITS & DIFFERENTIAL APPROXIMATION

Consider a 1D Riemann problem for Euler System

$$\frac{\partial \mathrm{U}}{\partial t}+\frac{\partial F(\mathrm{U})}{\partial x}=0.$$

with initial conditions,

$$u(x,0)=u_0,\ \ p(x,0)=p_0,\ \rho(x,0) = \begin{cases}\rho_1, x < x_0 \\ \rho_2, x > x_0\end{cases}$$

Using **Differential Approximation** (Vorozhtsov and Yanenko, Springer1990) for a numerical method of **r-th** order spatial accuracy, system reduces to,

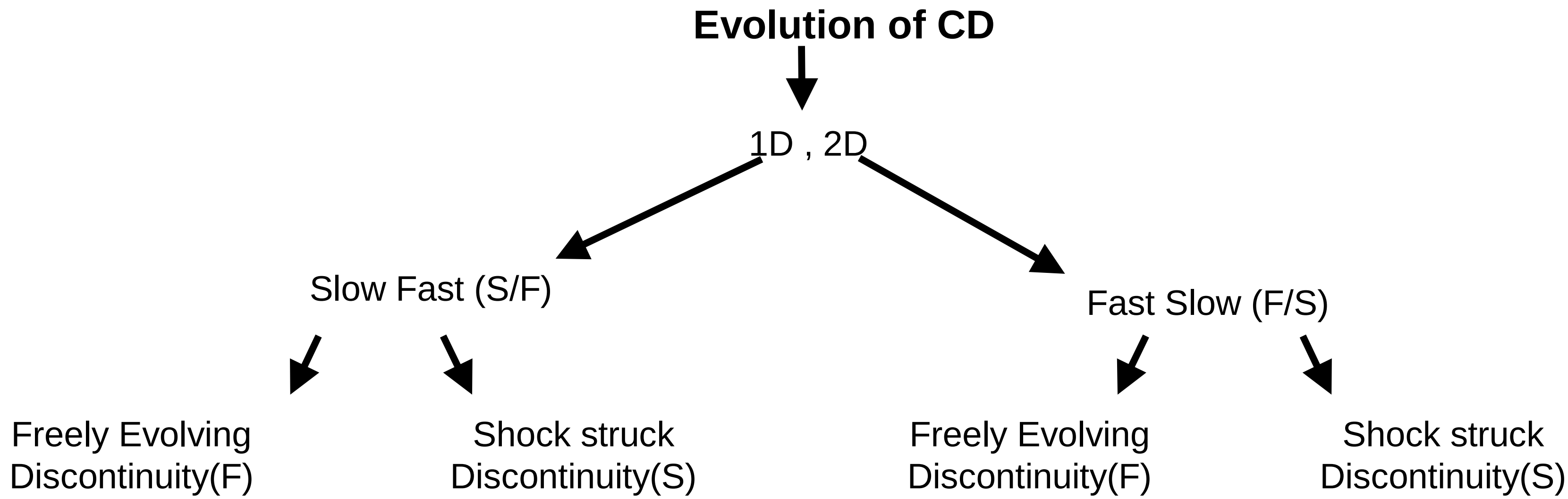
$$\frac{\partial \rho}{\partial t}+u_0\frac{\partial \rho}{\partial x}=(-1)^{r+1}\mu_{r+1}\frac{\partial^{r+1}\rho}{\partial x^{r+1}}$$

For r = 1, $$\rho(x,t)=0.5(\rho_1+\rho_2)[1+\mathrm{erf}(\chi)] \qquad (1)$$

For r = 2, $$\rho(x,t)=(2\rho_2+\rho_1)/3+(\rho_2-\rho_1)\int_0^{\chi}\mathrm{Ai}(\chi')\,d\chi' \qquad (2)$$

where $$\chi(x,t)=(x-x_0-u_0t)/((r+1)\mu_{r+1}t)^{1/r+1} \qquad (3)$$

EXTRACTION OF CONTACT DISCONTINUITY



For 2D case, we examine a slice at y=YMAX/2

NUMERICAL METHODS

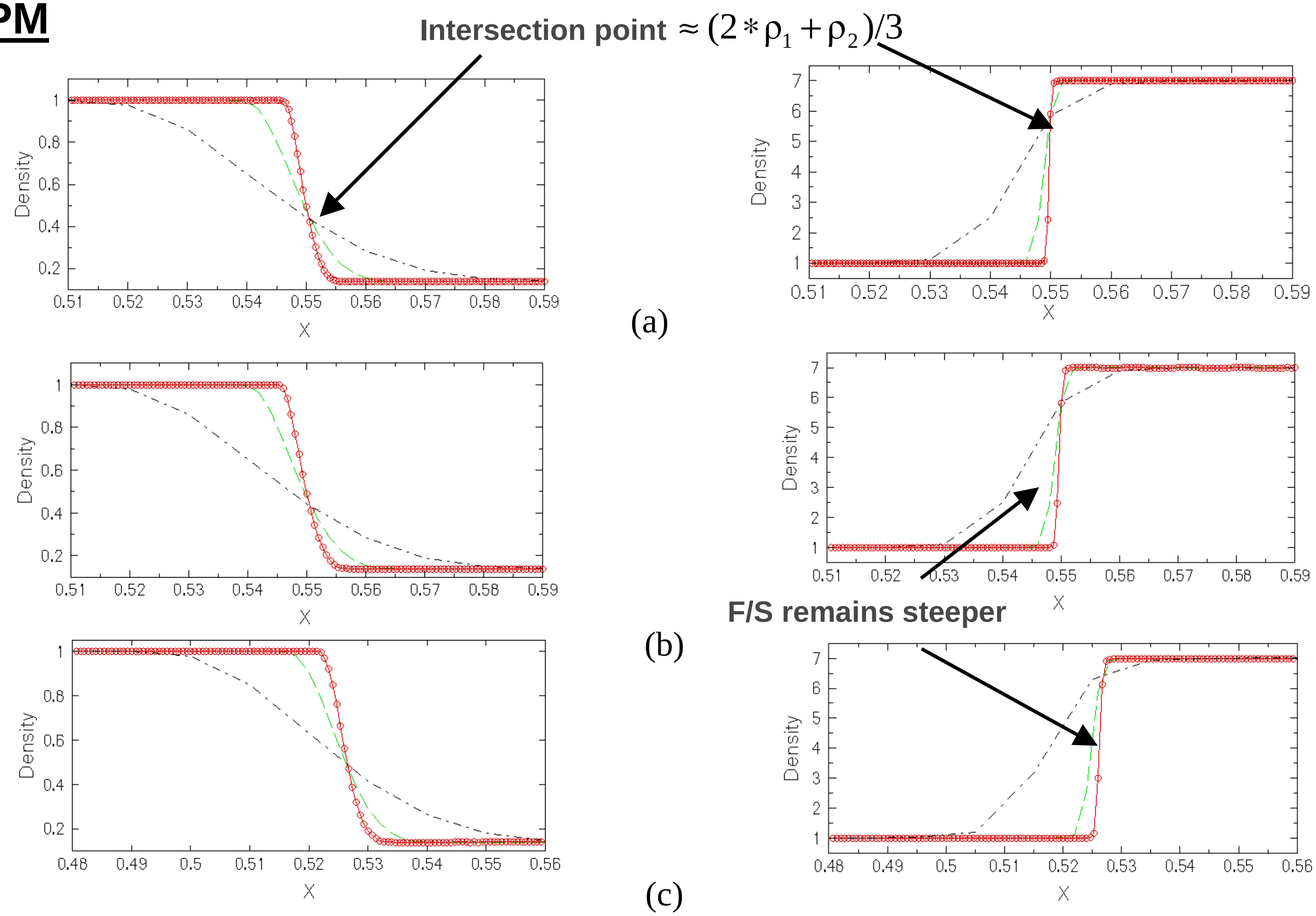
- Piecewise Parabolic Method (**PPM**)
- Weighted Essentially Non- Oscillatory (**WENO**,r=5)

EXTRACTION PROCEDURE FOR CD

- Point-wise Algorithm (A variation of edge detection technique)
- Width of CD = $X(d^2\rho_{max}) - X(d^2\rho_{min})$ where $d^2\rho$ is the second central difference
- Shock – Elimination using cost functions
  - Divergence of velocity $|\nabla.U| < |\nabla.U|_{thresh}$
  - Normalized pressure jump $dP < dP_{thresh}$

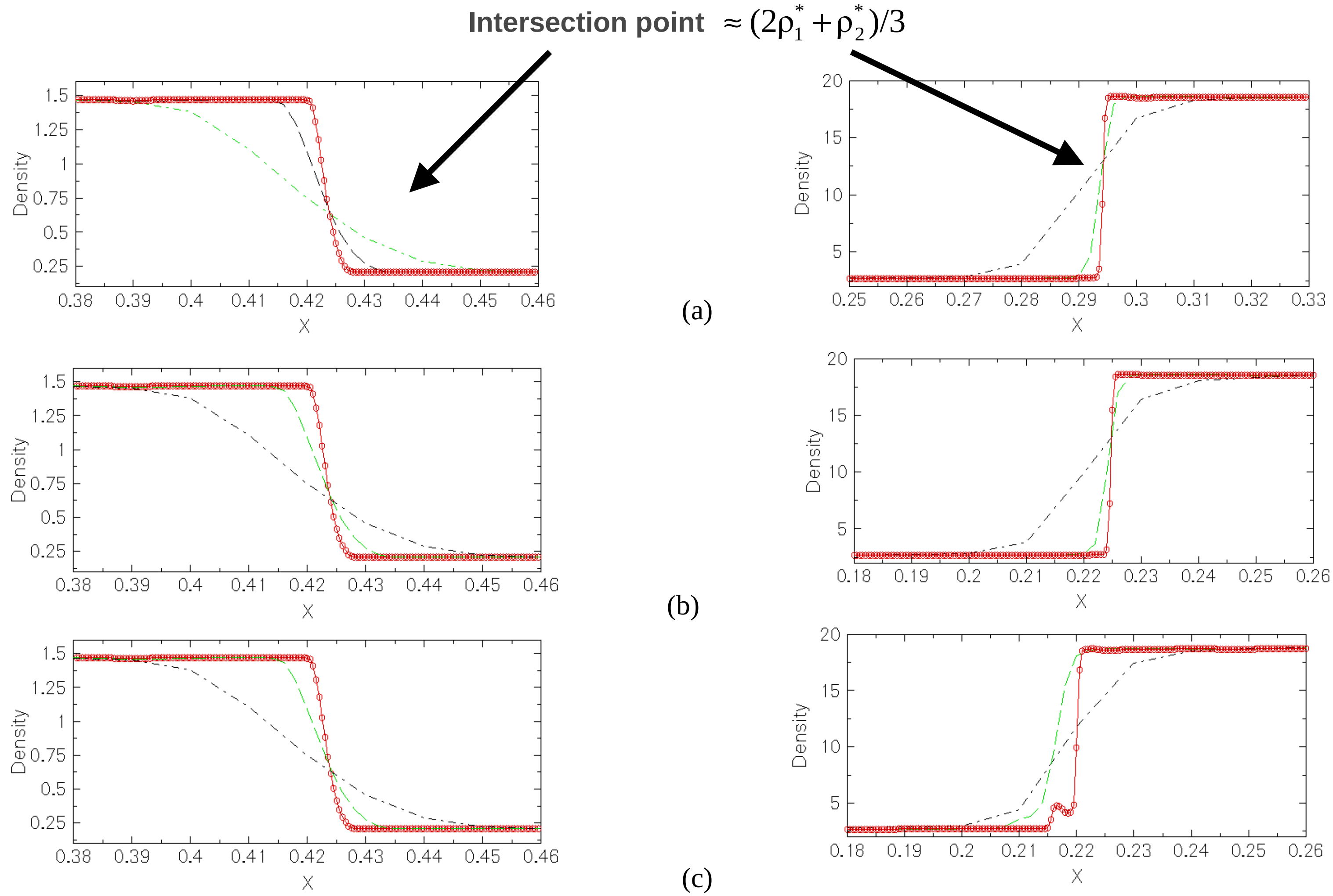
LOCALIZATION OF CD UNDER MESH REFINEMENT

PPM



S/F(density ratio=0.14) F/S (density ratio=7.0)

Density profiles for **Diffusing Contact Discontinuity** (u0 =1.5) at t=0.3. Top to Down (a) 1D (b) 2D, α =0 (c) 2D, α =30. The solid line with open circles is the highest resolution 0.0005 and - - - and - · - · - are 0.002 and 0.01 respectively.



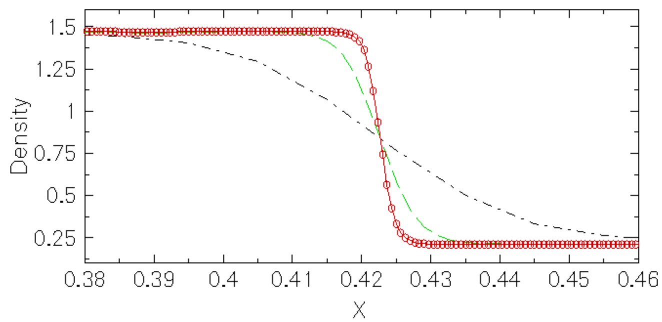
S/F(density ratio=0.14)

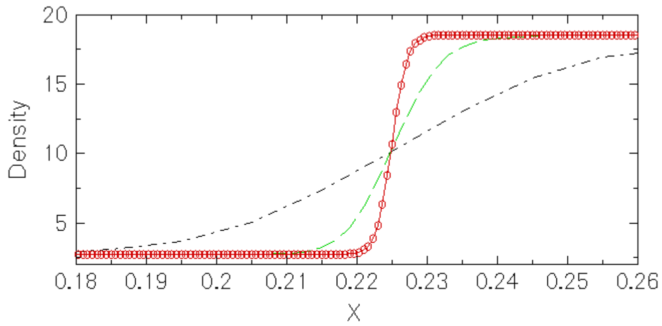
F/S (density ratio=7.0)

Density profiles for **Shock Contact Discontinuity Interaction** (M=1.5) at t=0.3. Top to Down (a) 1D (b) 2D, α =0 (c) 2D, α =10. The solid line with open circles is the highest resolution 0.0005 and - - - and - · · · - are 0.002 and 0.01 respectively. ρ_1^*, ρ_2^* are the post shock densities.

LOCALIZATION OF CD UNDER MESH REFINEMENT(CONT.)

WENO

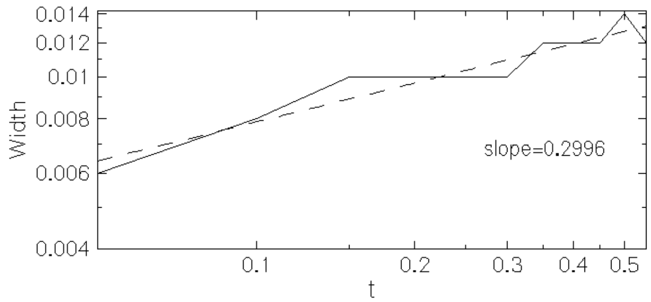




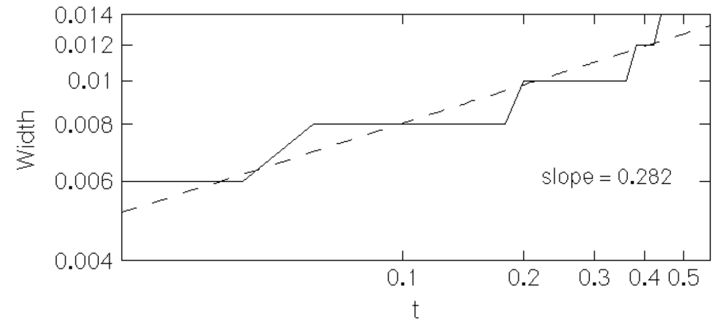
Density profiles for 1D **Shock Contact Discontinuity Interaction** (M=1.5) at t=0.3.The solid line with open circles is the highest resolution 0.000667 and - - - and - · - · - are 0.002 and 0.01 respectively.

SPREADING OF CD UNDER MESH REFINEMENT

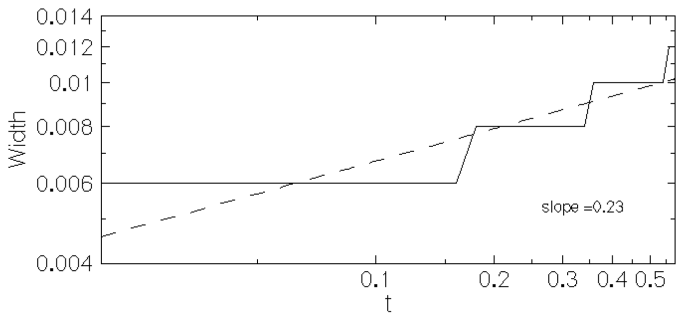
PPM (width $\propto t^{1/3}$)



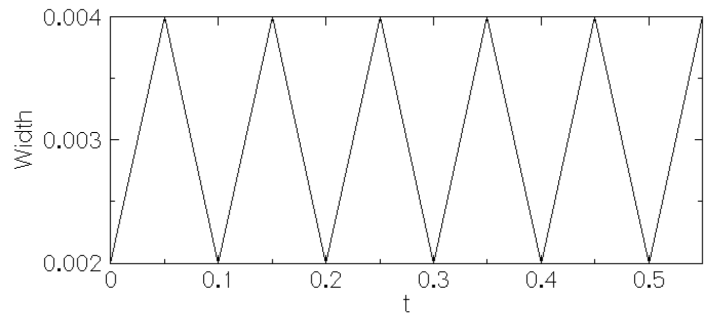
(a)



(b)

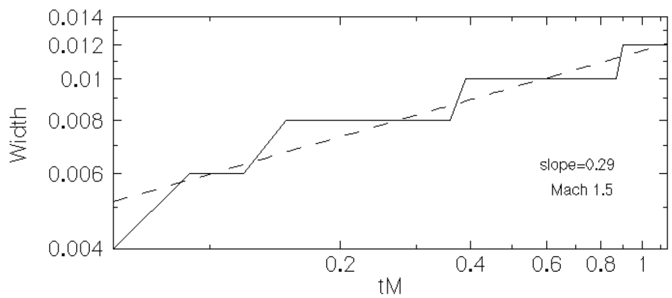


(c)

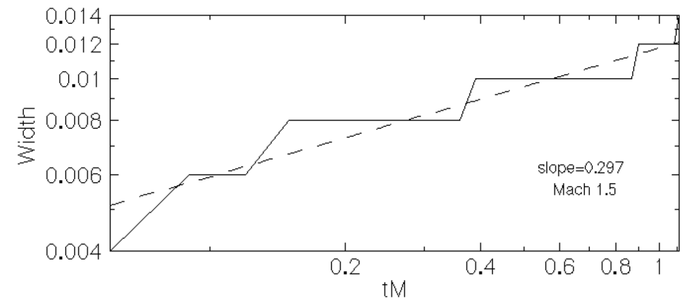


(d)

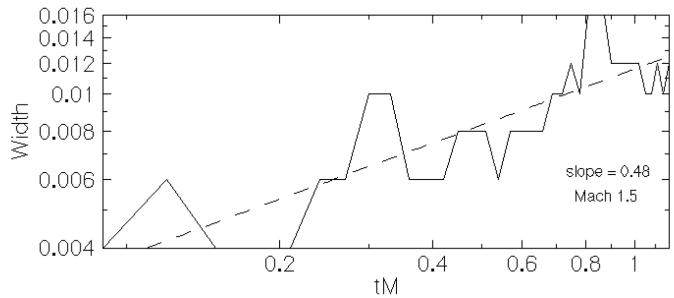
Growth of width of CD with time in **Diffusing Contact** for a resolution of 0.002 (a) S/F (1D) (b) S/F(2D, α=0) (c) S/F (2D, α =30). For (d)F/S, 1D **Width oscillates** between two values. Dashed line is the power law fit.



(a)



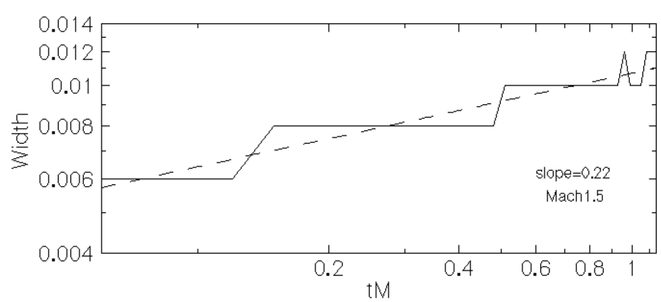
(b)



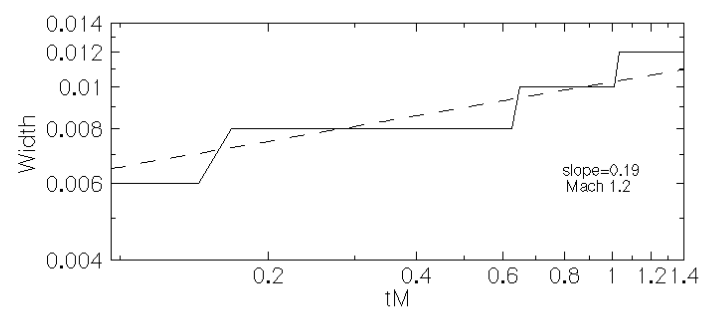
(c)

Growth of width of CD with time in **Shock Contact** interaction (Mach 1.5) for a resolution of 0.002 (a) S/F (1D) (b) S/F(2D, α =0) (c) S/F (2D, α =10). Dashed line is the power law fit.

WENO (width $\propto t^{1/4}$)



(a)



(b)

Growth of width of CD with time in **Shock Contact** interaction for a resolution of 0.002 (a)S/F (b) F/S Dashed line is the power law fit.

SPREADING RATES

Evolution	η or η*	Vel(U_0or M)	nD	α	C/r	Exponent(p)
F	0.14	1.5	1	N/A	PPM/2	0.2996
F	0.14	1.5	2	0	PPM/2	0.282
F	7.0	1.5	1	N/A	PPM/2	Oscillating
S	0.142	1.2	1	N/A	PPM/2	0.245
S	0.142	1.5	1	N/A	PPM/2	0.31
S	0.142	2.0	1	N/A	PPM/2	0.337
S	0.142	2.5	1	N/A	PPM/2	0.327
S	0.142	1.5	2	0	PPM/2	0.297
S	0.142	1.2	2	10	PPM/2	0.26
S	0.142	1.5	2	30	PPM/2	0.16
S	0.142	1.2	1	N/A	WENO/3	0.18
S	0.142	1.5	1	N/A	WENO/3	0.22
S	0.142	2.0	1	N/A	WENO/3	0.25
S	6.83	1.2	1	N/A	WENO/3	0.19
S	6.83	2.0	1	N/A	WENO/3	0.25

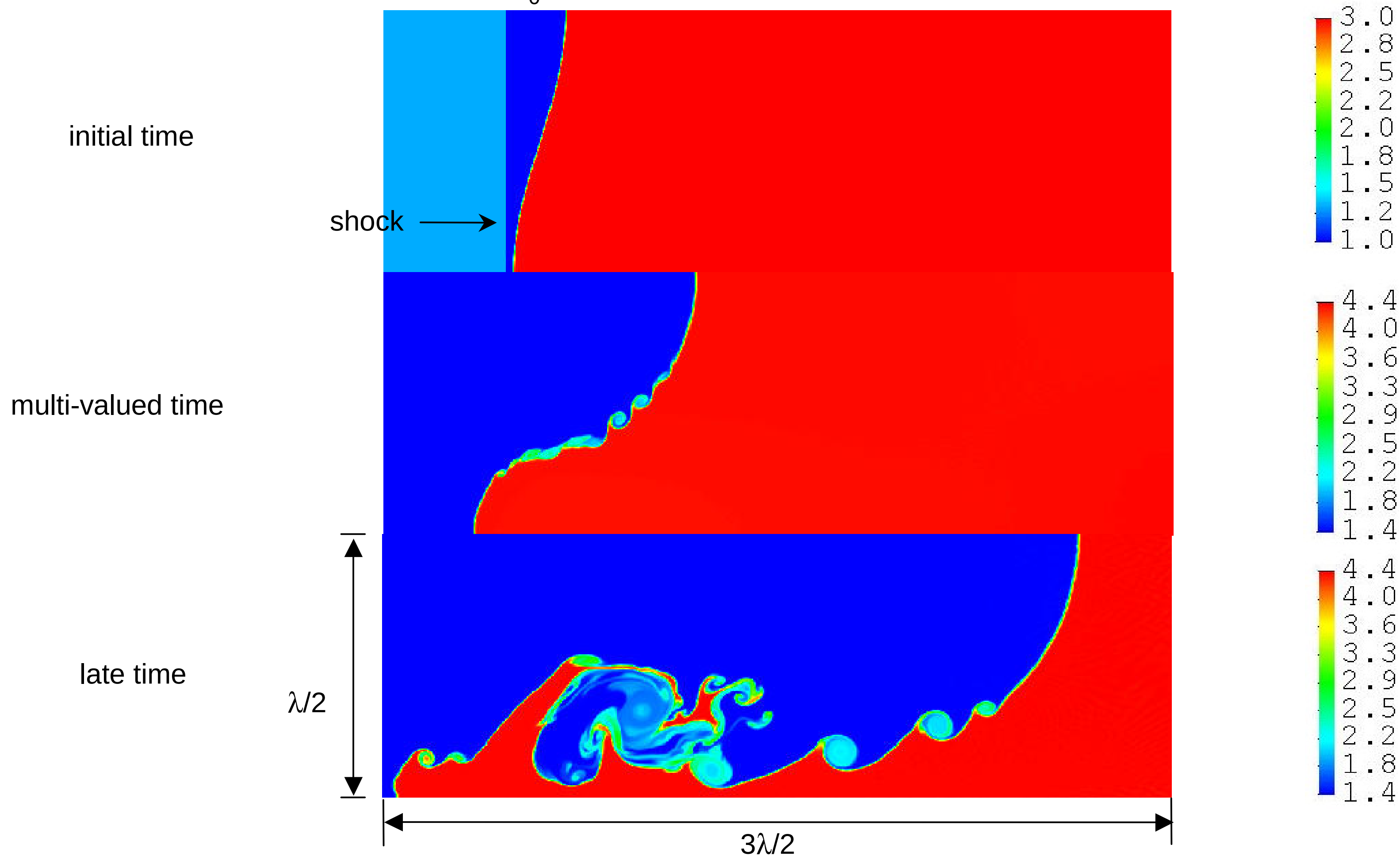
F or S – Freely evolving or Shock struck   η or η* - Density ratio before and after shock passage

U_0 or M – Constant initial velocity or Mach number

nD – Number of dimensions   C/r - Code/Order of code

VORTEX LOCALIZATION & NONLINEAR EVOLUTION SINGLE-MODE RM INTERFACE

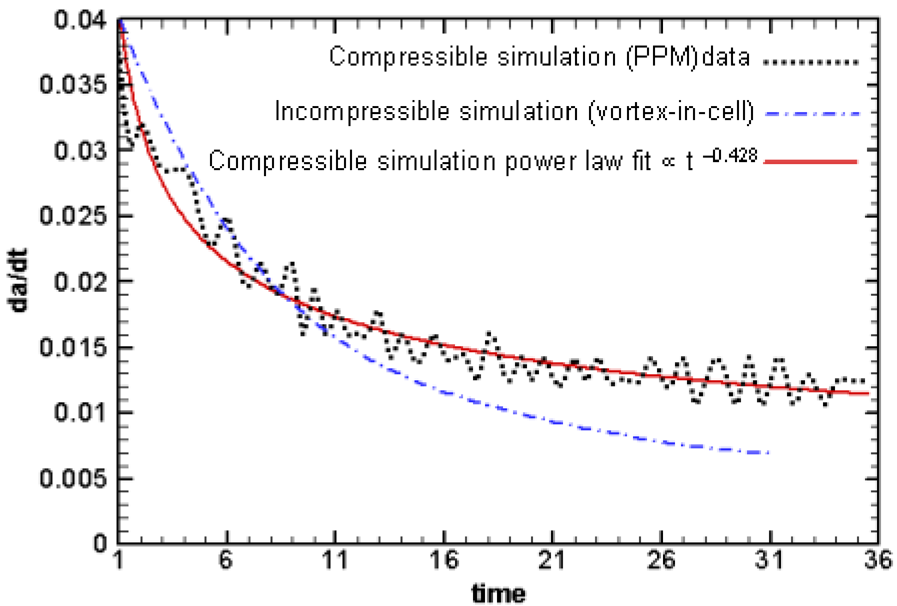
A = 0.5, a_0/λ = 0.05, M = 1.2, resolution 840 ×280

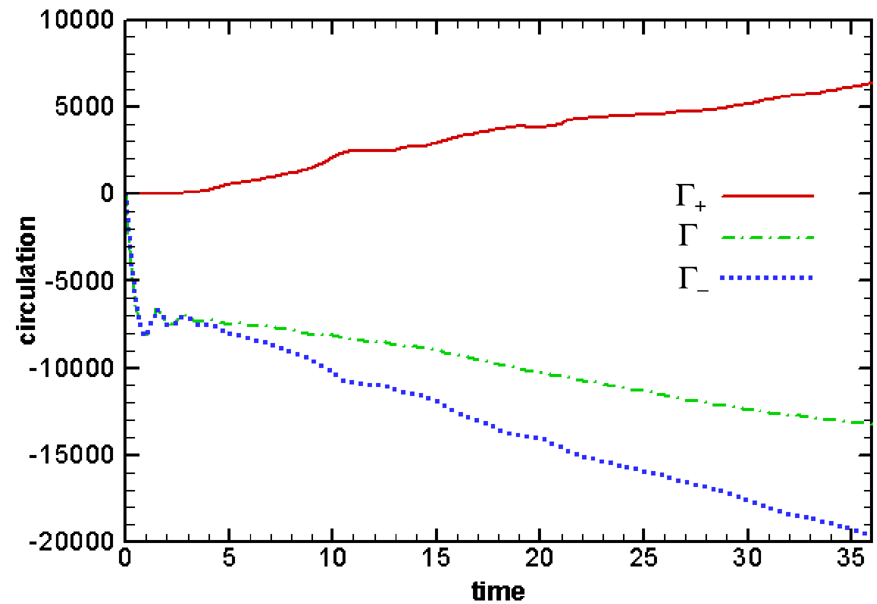


Three stages(*density*) of RM instability in shock-sinusoidal interaction : “initial time”, “multi-valued” and “late time.” The actual times are 0, 6.0 and 36; Atwood number is 0.5; initial density ratio is 3.0; Incident shock is M = 1.2; the perturbation is a_0/λ = 0.05; resolution 840 ×280 (PPM)

INTERFACIAL GROWTH RATE AND GLOBAL CIRCULATION

$A = 0.5$, $a_0/\lambda = 0.05$, $M = 1.2$





Interfacial growth rate obtained from compressible simulation(PPM), incompressible simulation (vortex-in-cell) and power law fitting for compressible simulation (PPM) data. For the power fitting: total RMS error ~ 8.7%, but for 16 < t < 36, da/dt = 0.013-0.15/ t and RMS error ~ 7%.

Positive (Γ_{+}), negative (Γ_{-}) and net (Γ) circulations obtained from compressible simulation (PPM). Secondary vorticity generation and associated instability contribute to interfacial growth rate.

CONCLUSIONS

- We present a systematic approach to quantify interfacial localization and temporal spreading in one dimension.
- We observe asymmetry in interfacial spreading rates for the one and two dimensional PPM simulations for F/S and S/F configurations. These are not present in a WENO simulations.
- Evolution of sinusoidal RM interface at late time exhibits large growth of positive and negative “secondary” baroclinic circulation. Interfacial growth rate is not (1/t) and depends on Atwood number.