

Implementation of a Turbulent Mix Model in a 2D ALE Code

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Abstract

The addition of a turbulent mix model to a two dimensional finite element ALE hydrocode, CORVUS, is discussed. Use is made of the existing mixed-cell data structure package to facilitate the inclusion of the model.

This first stage of the model is based on the multiphase flow equations, and is a simplified form of the model implemented by Youngs (see paper at this workshop) in a 2D eulerian code. This is applicable to simple Rayleigh-Taylor and Richtmyer-Meshkov instabilities and some results are presented.

A simple buoyancy-drag model is used to calculate the early stages of the instability growth at internal nodes, and this is used to initialise the turbulent mix model calculation.

2D ALE Code, CORVUS

Corvus is a 2D finite element arbitrary lagrangian eulerian (ALE) hydrocode. It uses a staggered quadrilateral grid which is explicitly integrated using bilinear finite elements; pressure and internal energy are cell centred and the velocity is node centred. Each region is logically quadrilateral and ALE works across adjacent regions. The time step is split into a lagrangian and rezone phase.

Corvus uses a predictor corrector scheme for time discretisation and the rezone is accomplished using a second order van leer scheme. This gives a nominal second order accuracy both spatially and temporally. Corvus also incorporates strength, slide, friction, void opening and void closure.

The purpose of this work is to add a mixing phase to the time step.

A. J. Barlow, "ALE in Corvus", proceedings of new models and numerical codes for shock wave processes in condensed media, oxford, 1997, 581-596

The Model

Equations for the transport of mass, momentum and internal energy.

$$\frac{\partial}{\partial t} (f_r \rho_r) + \frac{\partial}{\partial x_j} (f_r \rho_r u_{rj}) = 0$$

$$\begin{aligned} \frac{\partial}{\partial t} (f_r \rho_r u_{ri}) + \frac{\partial}{\partial x_j} (f_r \rho_r u_{ri} u_{rj}) = & -f_r \frac{\partial p}{\partial x_i} \\ & + \sum_s A_{rsi} + \sum_s D_{rsi} + f_r \rho_r g_i \end{aligned}$$

$$\frac{\partial}{\partial t} (f_r \rho_r e_r) + \frac{\partial}{\partial x_j} (f_r \rho_r e_r u_{rj}) = -h_r f_r \rho_r \frac{\partial \bar{u}_j}{\partial x_j} + f_r \varepsilon$$

f_r = volume fraction of material r	h_r = compressibility factor of material r
ρ_r = density of material r	u_{rj} = velocity of material r in direction j
e_r = internal energy of material r	A_{rsi} = Added mass of fluid r due to s
g_i = gravity	D_{rsi} = Drag on fluid r due to s

The Model - Mixing phase

Volume fluxes calculated

$$f_{rj}^1 V_j^1 = f_{rj}^0 V_j^0 + \delta V_{rj} - \delta V_{rj+1}$$

$$\delta V_{rj} = A_j f_{rj}^x (u_{rj}^0 - \bar{u}_j^0 + \delta u) \Delta t$$

A_j = area

f_{rj}^x = van leer limited advection value

u_{rj}^0 = fluid r velocity

δu = correction used to ensure $\sum \delta V = 0$

Density Fluxes

$$\rho_{rj}^1 f_{rj}^1 V_j^1 = \rho_{rj}^0 f_{rj}^0 V_j^0 + \delta M_{rj} - \delta M_{rj+1}$$

Internal Energy

$$e_r^1 \rho_{rj}^1 f_{rj}^1 V_j^1 = e_r^1 \rho_{rj}^0 f_{rj}^0 V_j^0 + \delta E_{rj} - \delta E_{rj+1}$$

The model - Van Leer

Van Leer limited advection flux calculated as follows
 if $(u_{rj}^0 - \bar{u}_j^0 + \delta u) > 0$ then $a=j-2, b=j-1, c=j$
 otherwise $a=j+1, b=j, c=j-1$

define $\varepsilon_{rj} = (|u_{rj}^0 - \bar{u}_j^0 + \delta u| A_j \Delta t) / V_b$ then

$$f_{rj}^x = f_{rb}^x + (1 - \varepsilon_{rj}) D_{rj} / 2$$

D_{rj} which represents the change in f across the upwind cell is obtained by Van Leer limiting. This is the minimum of:

$$\Delta_1 = f_{rb}^0 - f_{ra}^0$$

$$\Delta_2 = f_{rc}^0 - f_{rb}^0$$

$$D_{rj}^\diamond = V_b \left(p \frac{\Delta_1}{\frac{1}{2}(V_a + V_b)} + (1-p) \frac{\Delta_2}{\frac{1}{2}(V_b + V_c)} \right)$$

where $p = \frac{V_a + \varepsilon_{rj} V_b}{V_a + V_b + V_c}$

Similarly $\delta M_{rj} = \rho_{rj}^x \delta V_{rj}$ and $\delta E_{rj} = e_{rj}^x \delta M_{rj}$ where ρ_{rj}^x and e_{rj}^x are also Van Leer limited values

Implementation in Corvus

The mix model is initialised by specifying the interfaces across which the mixing is calculated. A list of cells is generated on each side of this interface.

The mixing routines are then called after the lagrangian and rezone phases of each time step. The equations are solved along “rows” and “columns” of cells. The cell numbers of the edges are stored and the line of cells between these edges is generated and passed to the mixing routines one at time. The volume fractions are stored using the ALE package which calculates the effect of mesh movement. Other variables are stored on a “mixed cell” basis, in the same way as the ALE package. This minimises storage and utilises the pre-existing code.

Results - buoyancy-drag model

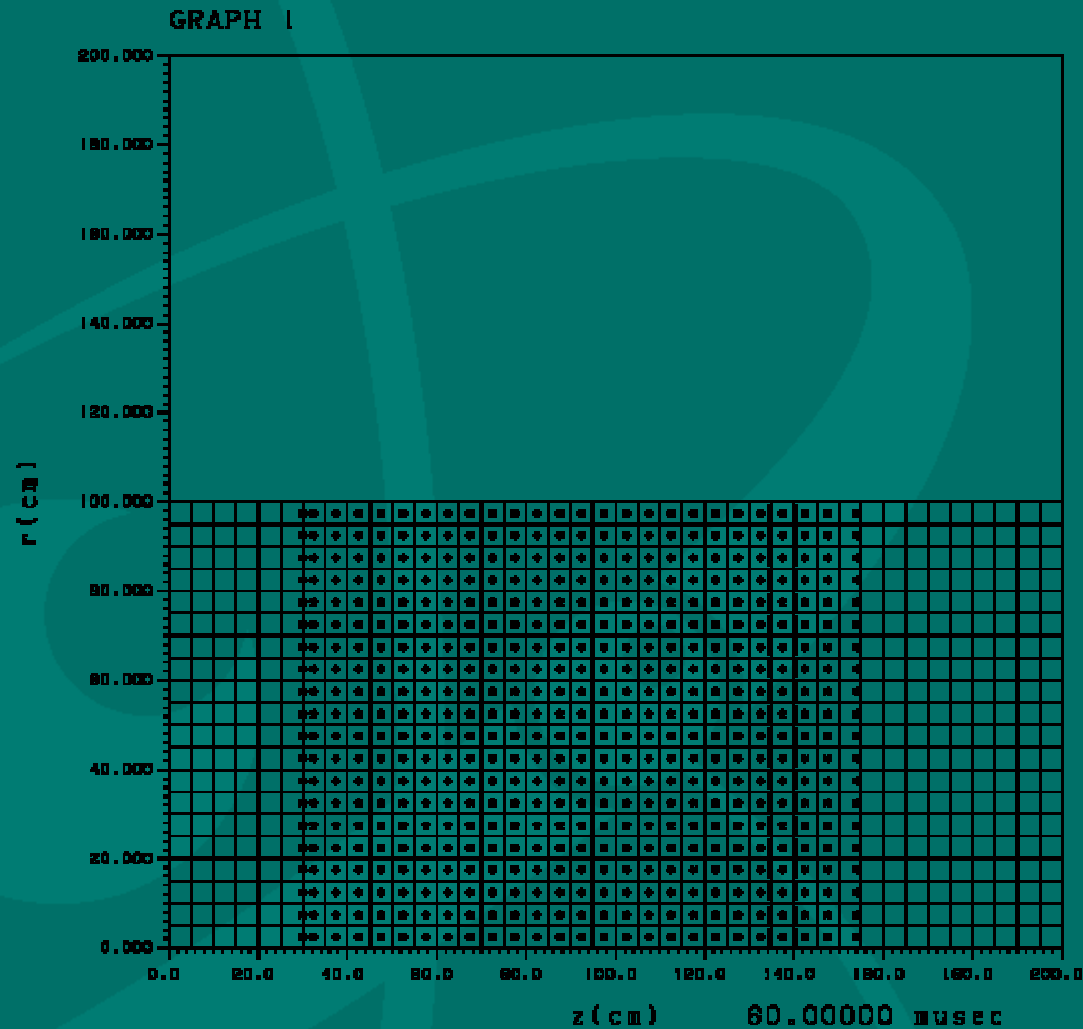
A simple problem is used to illustrate the model. This is two regions of density 1 and 3. The Equation of State of both fluids is $-\frac{1}{\rho} \frac{\partial p}{\partial x} = 0.5$ with boundary conditions $p=400$ and $p=200$. This gives mixing limits of 54.0 and 70.0 at $t=60$.

The table below gives the results.

Time	Mix penetration	Mix penetration
0.000E+00	1.000E-05	1.000E-05
2.500E+00	1.243E-01	9.594E-02
5.000E+00	4.918E-01	3.794E-01
1.000E+01	1.955E+00	1.508E+00
1.500E+01	4.391E+00	3.387E+00
2.000E+01	7.798E+00	6.015E+00
2.500E+01	1.217E+01	9.394E+00
3.000E+01	1.753E+01	1.352E+01
3.500E+01	2.385E+01	1.840E+01
4.000E+01	3.115E+01	2.403E+01
4.250E+01	3.516E+01	2.712E+01
4.500E+01	3.942E+01	3.041E+01
4.750E+01	4.392E+01	3.388E+01
5.000E+01	4.866E+01	3.753E+01
5.250E+01	5.364E+01	4.138E+01
5.500E+01	5.887E+01	4.541E+01
5.750E+01	6.434E+01	4.963E+01
6.000E+01	7.006E+01	5.404E+01

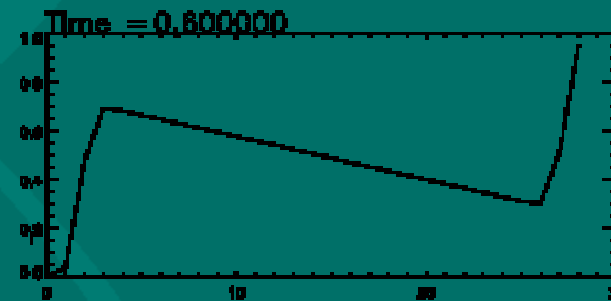
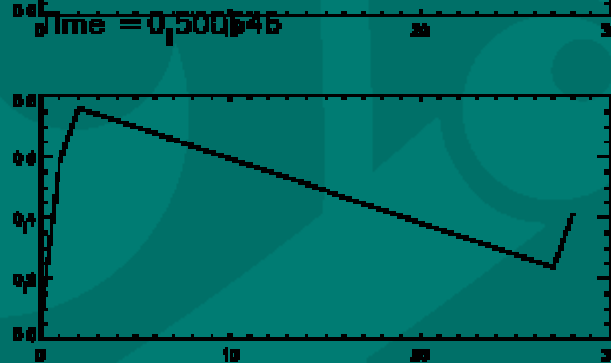
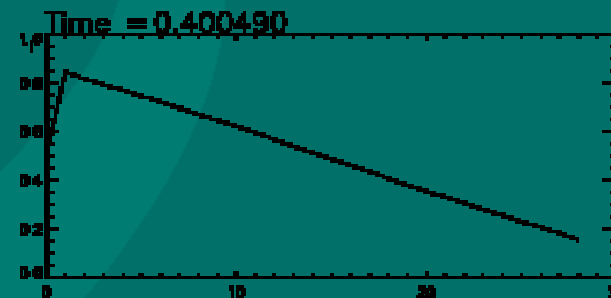
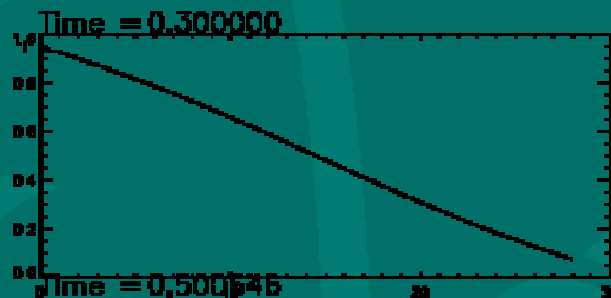
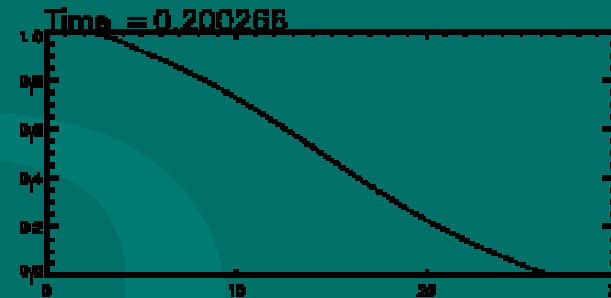
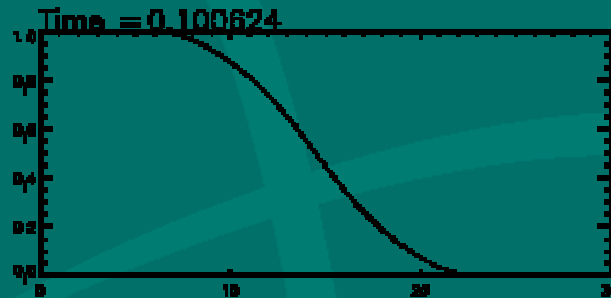
Results - buoyancy-drag model

This is a mesh plot, the stars show the mixed cells.



Results

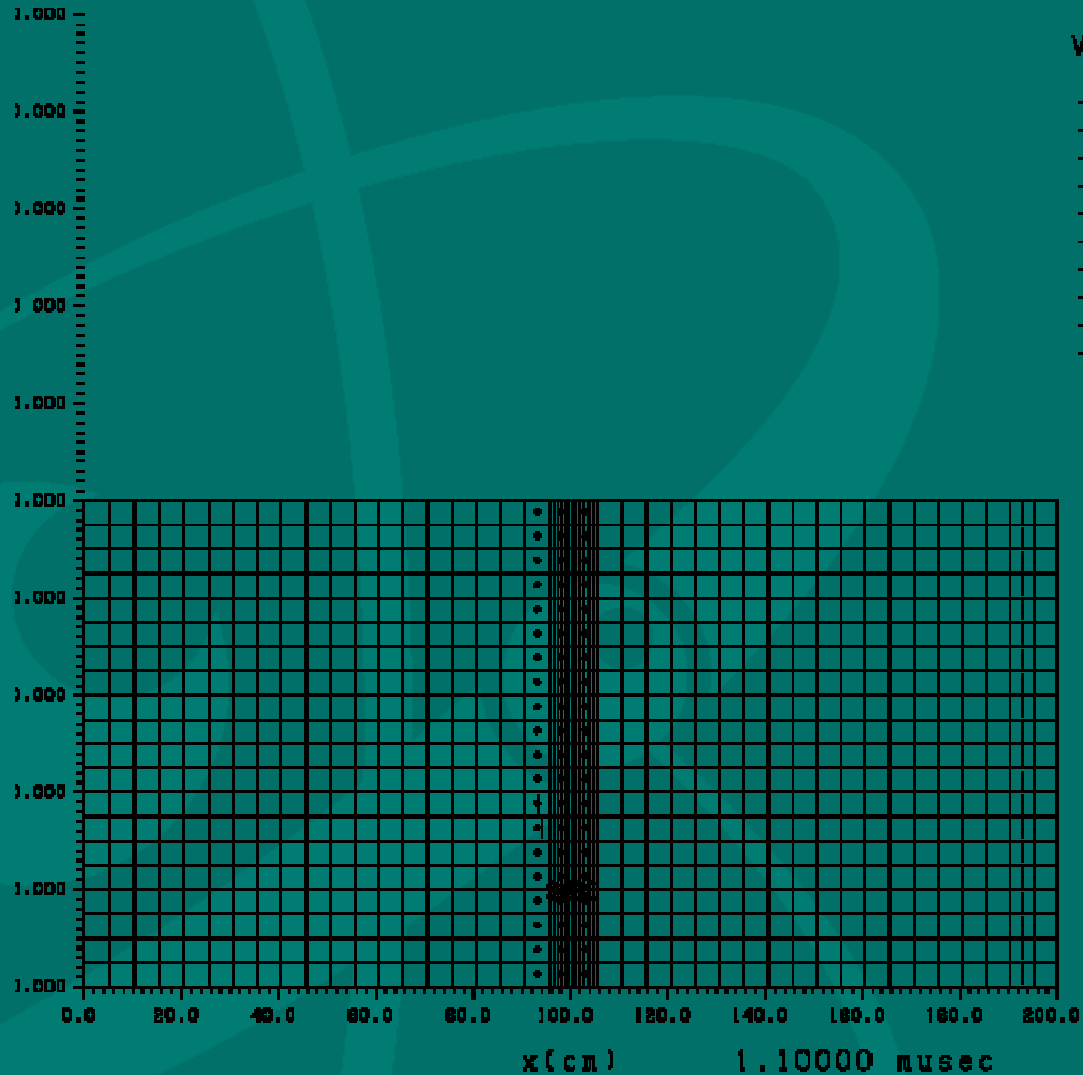
These are the volume fractions along a line.



Results

This is a meshplot showing volume fractions.

GRAPH ALE



Volume Fraction

0	0.9090909E-
1	0.1818182E1
2	0.2727273E1
3	0.3636364E1
4	0.4545455E1
5	0.5454545E1
6	0.6363638E1
7	0.7272727E1
8	0.8181818E1
9	0.9090909E1

Further work

Initialisation using a simple buoyancy-drag model to start the turbulent model, this should increase the accuracy of the early time behaviour on a coarse mesh.

Mass exchange in the cells. Currently each mix cell consists of separate fluids.

Conclusions

A turbulent mix model has been implemented in a 2D ALE code, Corvus. It has been used on a simple problem and the results shown.