3D computation of surface perturbations evolution in plasma cloud during its expansion in magnetic field.

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Previously, refs. [1-3] considered a 2D problem of initially spherical plasma cloud expansion in axial magnetic field. The papers indicated, in particular, that the cloud surface was nonresistant to “chute” type instability evolution and estimated the instability growth increments.

The objective of the paper is tracking the evolution of the above instabilities with account for their actual, i.e. three-dimensional, nature.

Two approaches are used for this purpose:

- the initial stage of the perturbation growth is considered analytically under the assumption of the perturbation smallness,
- the nonlinear stage is computed with 3D numerical code TREK [4].

1. Unperturbed plasma cloud dynamics in magnetic field.

Recall the features of solving the problem of dynamics of a plasma cloud expanding in the external magnetic field.

Consider a cloud of energy E, mass M and assume the magnetic field to be axial, homogeneous, of strength $\vec{H} = (0,0,H_0)$ with r→∞. Also, assume that the initial shape cloud is spherical of radius r_0.

A detailed pattern of the cloud expansion and deformation is obtained by numerical computations and discussed in ref. [3]. To find out the qualitative pattern, an approximate model can be used, which implies that the motion of each “sector” of the cloud depends on magnetic pressure on its surface. If the pressure is given (by relations presented in ref. [1]), then we can obtain the equation for the cloud surface radius:

$$R(\Theta,t) = \left[r_0^3 + R_0^3 \cdot \frac{\sin^2 \omega t}{\sin^2 \Theta} \right]^{\frac{1}{3}} \tag{1.1}$$

where $u_0 = \left(\frac{E}{M}\right)^{\frac{1}{2}}$ is initial expansion rate, $\omega(\Theta) = \frac{\omega_0}{\sin\Theta}$, $\omega_0 \cong \frac{u_0}{R_0}$, $R_0 = \left(\frac{E}{\frac{4}{3}\pi \cdot p_m^0}\right)^{\frac{1}{3}}$,

$p_m^0 = \frac{H_0^2}{8\pi}$, Θ is an angle with respect to the axis of symmetry.

Thus, from the solution it follows that the expanding cloud is decelerated by the magnetic field, its radius periodically changes depending on time t, with the deceleration being most noticeable at the “equator” ($\Theta = \frac{\pi}{2}$) and missing at the poles ($\Theta = 0$).

As the comparison to the data of ref. [3] suggests, this simple cloud dynamics model is valid at $\omega_0 t \le \pi$.

The plasma cloud deceleration by the magnetic field leads to the perturbation evolution on the cloud surface (by analogy with the R-T instability in hydrodynamics: the role of the “heavy” fluid is played by the cloud plasma, that of the “light” by the magnetic field).

2. Linear stage of instability growth

When considering this stage of the perturbation growth on the plasma cloud surface, of interest to us will be most hazardous of them, that is such, for which increment $\gamma >> \omega_0$. In this case the unperturbed surface can be considered as spherically symmetric and the unperturbed magnetic field near the surface as having only one, tangential, component $H_\Theta = \frac{3}{2} H_0 \sin\Theta$ [1].

If the surface perturbation form is given as

$$R(\Theta,\varphi,t) = R_0(t) + \sum_{lm} \xi_{lm}(t) \cdot Y_{lm}(\Theta,\varphi) \tag{2.1}$$

where $Y_{lm}(\Theta,\varphi)$ are spherical harmonics, then, by extending the well-known conclusion [5] for plane plasma-magnetic field interface to the spherical case under consideration, the following equation system can be obtained for harmonics $\xi_{lm}(t)$:

$$\frac{\partial}{\partial t}\left(R^2 \frac{\partial \sigma_{lm}}{\partial t} \right) = \sum_{k=l,l\pm 2} a_{km}^{lm} \cdot \sigma_{km} \tag{2.2}$$

where $a_{km}^{lm} = -\left[\ddot{R}_0 \cdot R_0 \cdot \lambda_{km}^{lm} + c_A^2 \cdot \mu_{km}^{lm} \right] \cdot l$ (2.3)

where $c_A^2 = \frac{\left(\frac{3}{2} H_0\right)^2}{4 \cdot \pi \cdot \rho_0(t)}$, λ_{km}^{lm} and μ_{km}^{lm} are algebraic functions l and m. The expressions for the functions prove quite cumbersome, here it is sufficient to note that for $l >> 1$, $\lambda_{km}^{lm} \sim \delta_{kl} + \delta\lambda_{km}^{lm}$, where $\delta\lambda_{km}^{lm} \sim \frac{1}{m+1}$, $\mu_{km}^{lm} \sim (l-m)$.

Note that the "engagement" of harmonics l and l±2 in equation system (2.2) is a consequence of the fact that the pressure of the unperturbed magnetic field on the surface depends on Θ ($p_m \sim \sin^2\Theta$). This "engagement" disappears, when the initial perturbation is localized at $\Theta = \Theta_0 \pm \Delta\Theta$, where ΔΘ<<1. Assuming m=0 and l such that $l\Delta\Theta >> 1$, equation (2.2) can be reduced to a simpler form:

$$\frac{\partial}{\partial t}\left(R^2 \frac{\partial \sigma_{lm}}{\partial t} \right) = -\sigma_{lm}\left[l \cdot \ddot{R}_0(\Theta_0) \cdot R_0 + c_A^2(\Theta_0) \cdot l^2 \right] \tag{2.4}$$

where $\ddot{R}_0(\Theta_0) \sim c_A^2(\Theta_0) \sim \sin^2\Theta_0$ are surface acceleration and Alfven velocity characteristic of a given angle.

In the quasi-static case ($\gamma >> \omega_0$) from (2.4) it follows that

$$\gamma_l^2 = -\left[k \cdot \ddot{R}(\Theta_0) + c_A^2(\Theta_0) \cdot k^2 \right] \tag{2.5}$$

where $k = \frac{l}{R_0}$. This expression coincides with that obtained previously [5] for the plane interface with parameters corresponding to angle Θ_0.

If we continue to consider l>>1, but m=l-Δ, where Δ~1, then from (2.2) it readily follows that

$$\frac{\partial}{\partial t}\left(R^2 \frac{\partial \sigma_{lm}}{\partial t} \right) = -\sigma_{lm}\left[l \cdot \ddot{R}_0 \cdot R_0 + c_A^2 \cdot l \cdot \Delta \right] \tag{2.6}$$

As seen from the comparison between (2.3) and (2.5), for such perturbations the stabilizing role of the magnetic field (addend ~c_A^2 in 2.5) is noticeably less than that for m=0. In particular, for $c_A^2 << (\ddot{R}_0 \cdot R_0)$ (2.5) yields an expression for increment γ_{lm} similar to that known [6] for a cylindrical problem with longitudinal magnetic field ($\gamma \sim \sqrt{-m\frac{\ddot{R}_0}{R_0}}$).

3. Numerical calculations

The computations were with code TREK [3]. The scaling was as follows: $p_1 = \frac{H_0^2}{8\pi}$ for pressure, $r_1 = \left(\frac{E}{\frac{4}{3}\pi \cdot p_1}\right)^{\frac{1}{3}}$ for length, $u_1 = \left(\frac{E}{M}\right)^{\frac{1}{2}}$ for velocity, $\frac{r_1}{u_1}$ for time, $\rho_1 = \frac{M}{\frac{4}{3}\pi \cdot r_1^3} = \frac{p_1}{u_1^2}$ for density.

The magnetic pressure on the cloud surface was calculated using two apporoaches:

- Vacuum was assumed outside the cloud; the magnetic field in it was calculated using quasi-stationary approximation [2]. In this case the problem is characterized with an only dimensionless parameter, $r_0' = \frac{r_0}{r_1}$, with the dependence on the parameter disappearing at $r_0' << 1$, $r' \sim 1$.
- "Background" plasma of quite a small density with a magnetic field frozen in it was assumed outside the cloud. In this case the magnetic field changes are calculated using MHD approximation; besides the parameter r_0, the problem is also characterized with parameter $M_A^2 = \frac{u_1^2}{c_A^2}$, where $c_A^2 = \frac{H_0^2}{4\pi \cdot \rho_1}$; options with $M_A^2 << 1$ were considered, in which the dependence on M_A^2 disappears.

At t=0, sphere surface radial velocity perturbations were given, i.e.

$$u(0,\mu,\varphi) = u_0 + u_{lm} \cdot Y_{lm}(\mu,\varphi) \qquad (3.1)$$

was assumed. Here $u_0 = \sqrt{\frac{10}{3}}u_1$ is free expansion velocity of "cold" spherical cloud having a linear velocity profile and constant density. $u_{lm} = \alpha \cdot u_0$ was assumed.

Four problems were calculated.

In the first of them, α=0 (unperturbed surface) was assumed. The results of the calculation are plotted in Figs. 1 through 3 ($M_A^2 = 0.05$, $r_0' = 1$, t=0.25;2.0 and 3, respectively). The expansion pattern qualitatively agrees with relation (1.1): the plasma spreading in the longitudinal direction and oscillating motion (with period ~π) in the transversal direction occur.

In the second problem, "meridional" velocity perturbations were given at the initial time: α=0.1, l=12, m=0, the other parameters are the same as in problem 1. The calculation results are illustrated in Figs. 4 and 5.

In this case the ratio of the first addend to the second in (2.3) is $\sim \frac{u_0^2}{c_A^2 \cdot l} << 1$ (here c_A^2 is estimated by plasma cloud density $\rho_0 \sim 1$ at t≤1), hence, a significant perturbation growth stabilization can be expected thanks to the magnetic field.

This assumption is confirmed by comparison between the results of the calculation and similar calculation 3, which differs from the former in the fact that instead of "magnetic" pressure, "thermal" pressure of the same magnitude is given in the "external" plasma. In this case the cloud dynamics will remain about the same as in problem 2, however, the perturbation growth depends only on the first addend in (2.3), i.e. is significantly faster than in problem 2.

Problem 4 differs from problem 2 in given m=l=12, i.e. the initial perturbation was localized near “equator” and its amplitude depended on azimuthal angle φ. Its solution results appear in Figs. 8 and 9. In this case, according to (2.6), we can expect that increment $\gamma^2>0$ (unsteady conditions) and that the cloud surface will be perturbed, in the main, across azimuth. These assumptions agree with the calculated data presented in Figs. 8 and 9.

Conclusion

More extensive computational series with varying parameters r_0, M_A^2, l, m, as well as setting “random” surface perturbations are being planned.

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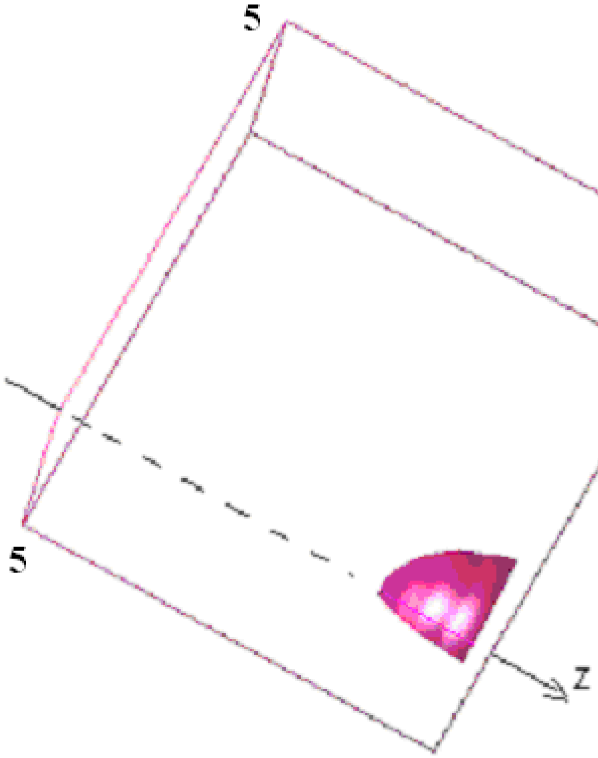

fig1. Plasma cloud expansion in the magnetic field (unperturbed surface), t=0.25, $M_A^2 = 0.05$

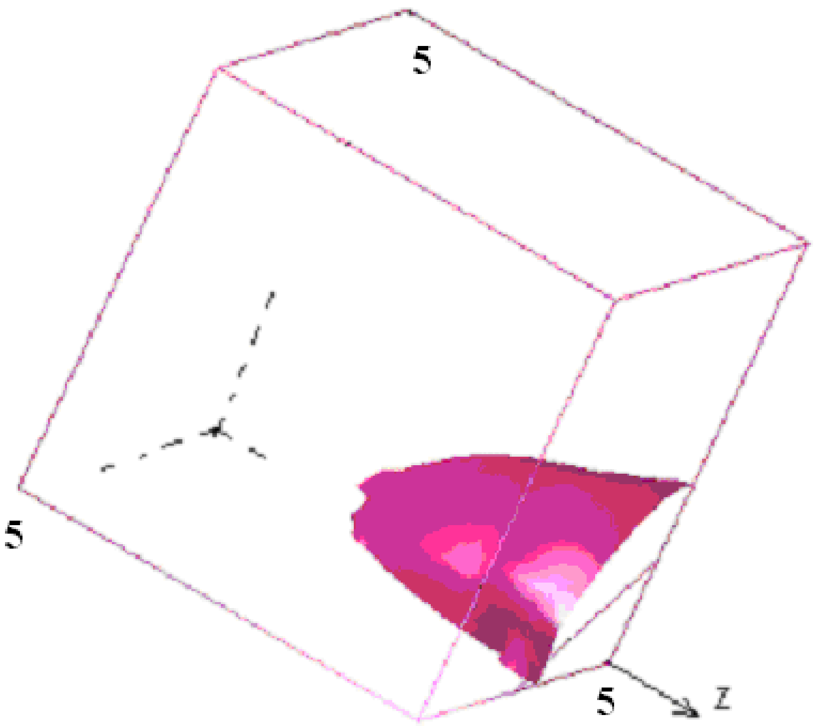

fig2. Plasma cloud expansion in the magnetic field (unperturbed surface), t=2.0, $M_A^2 = 0.05$

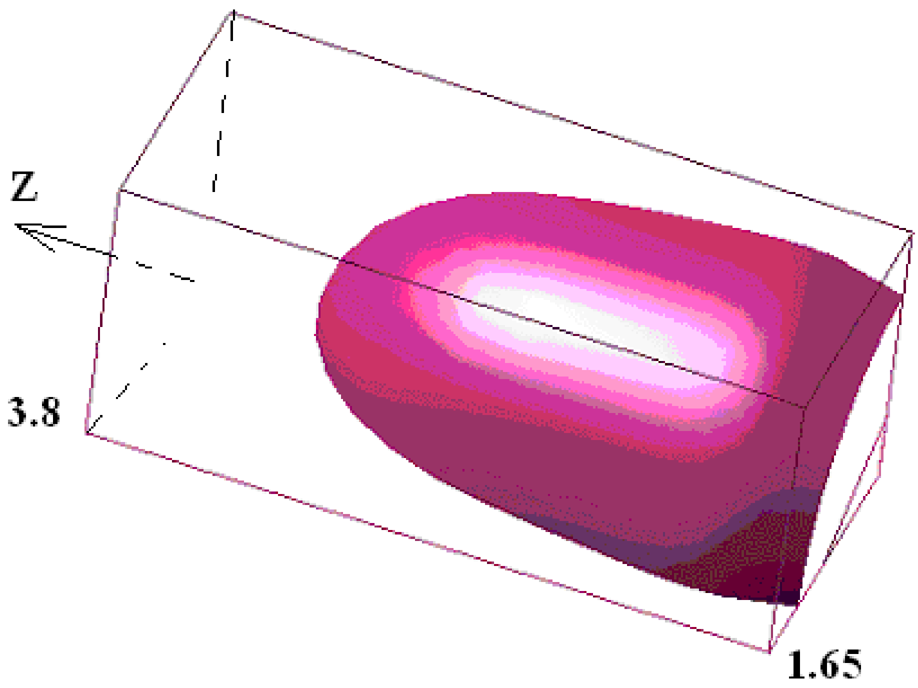

fig3. Plasma cloud expansion in the magnetic field (unperturbed surface), t=3.0, $M_A^2 = 0.05$

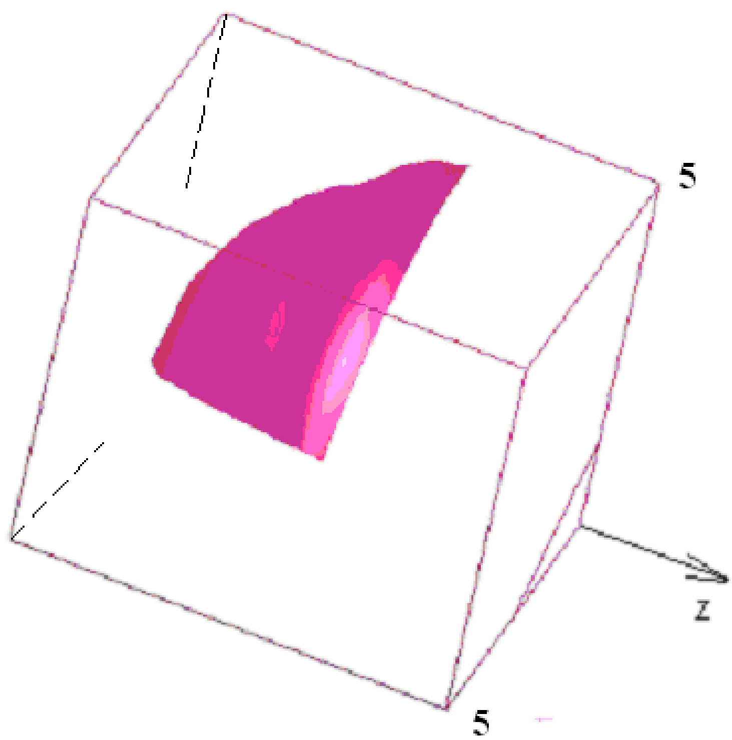

fig4. Plasma cloud expansion in the magnetic field (meridianal perturbations, l=12, m=0, $M_A^2 = 0.05$), t=0.25

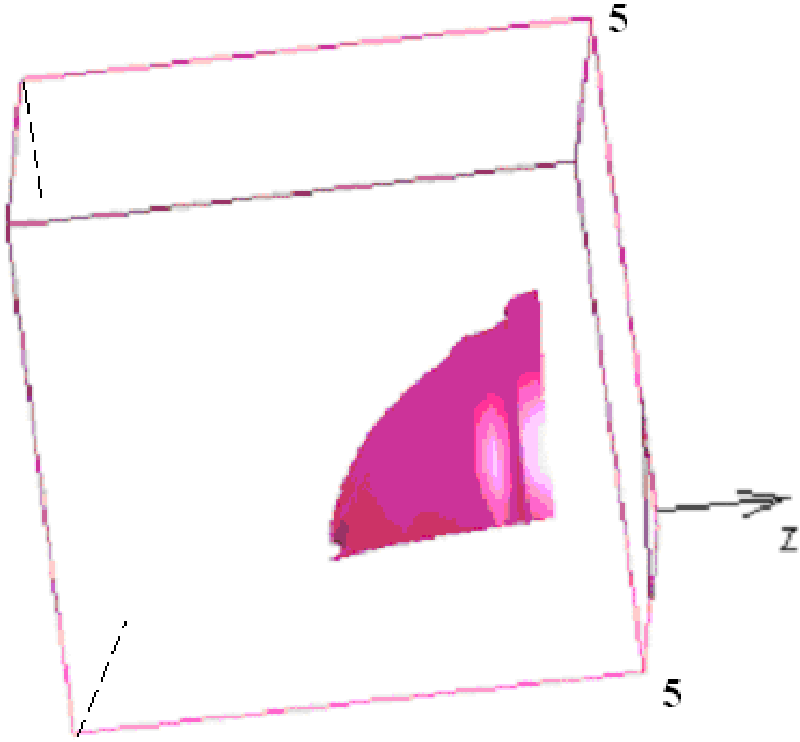

fig5. Plasma cloud expansion in the magnetic field (meridianal perturbations, l=12, m=0, $M_A^2 = 0.05$), t=0.5

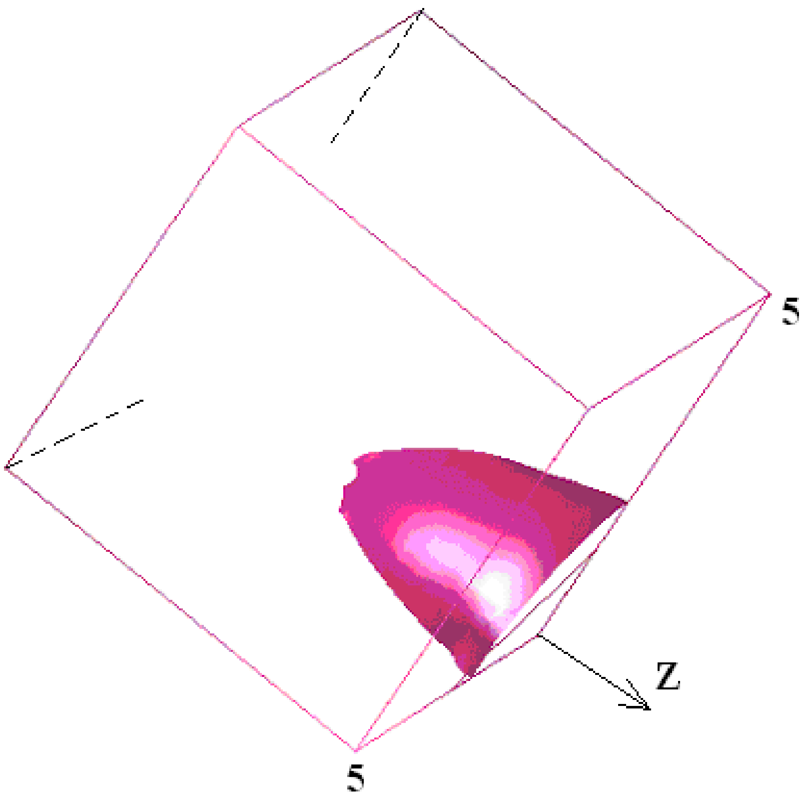

fig6. Plasma cloud expansion against termal pressure (meridianal perturbations), $M_{th}^2 = 0.05$, P_m=0, l=12, m=0, t=0.5

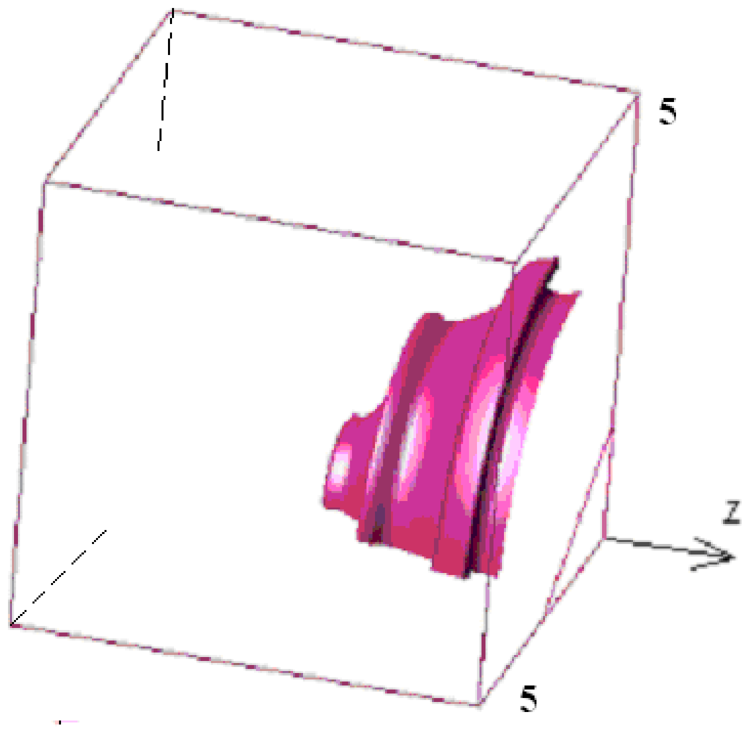

fig7. Plasma cloud expansion against termal pressure (meridianal perturbations), $M_{th}^2 = 0.05$, P_m=0, l=12, m=0, t=3.0

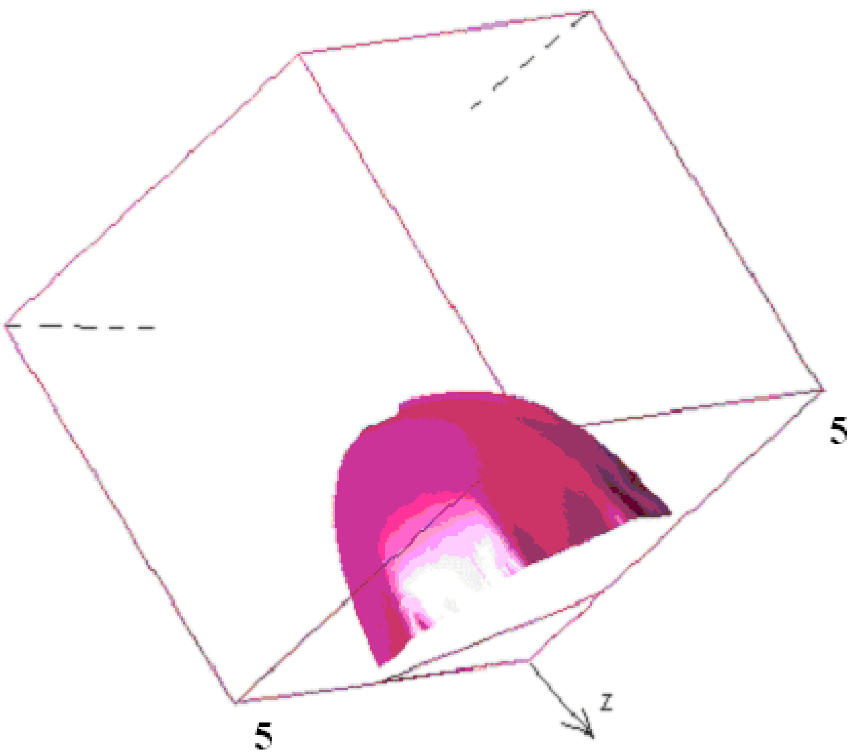

fig8. Plasma cloud expansion in the magnetic field (perturbation l=12, m=12, $M_A^2 = 0.05$), t=0.4

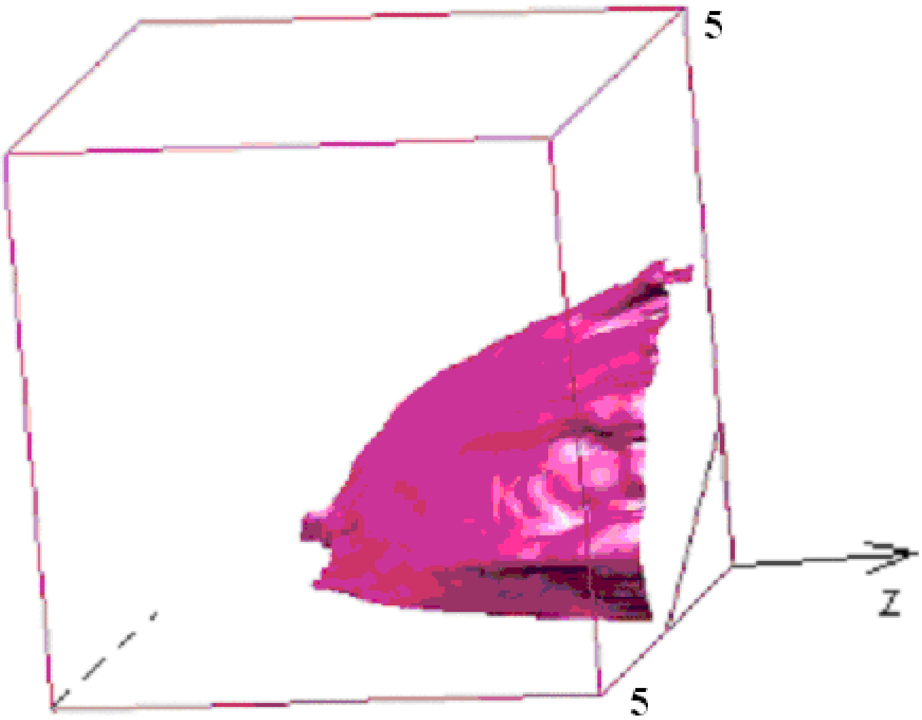

fig9. Plasma cloud expansion in the magnetic field (perturbation l=12, m=12, $M_A^2 = 0.05$), t=1.