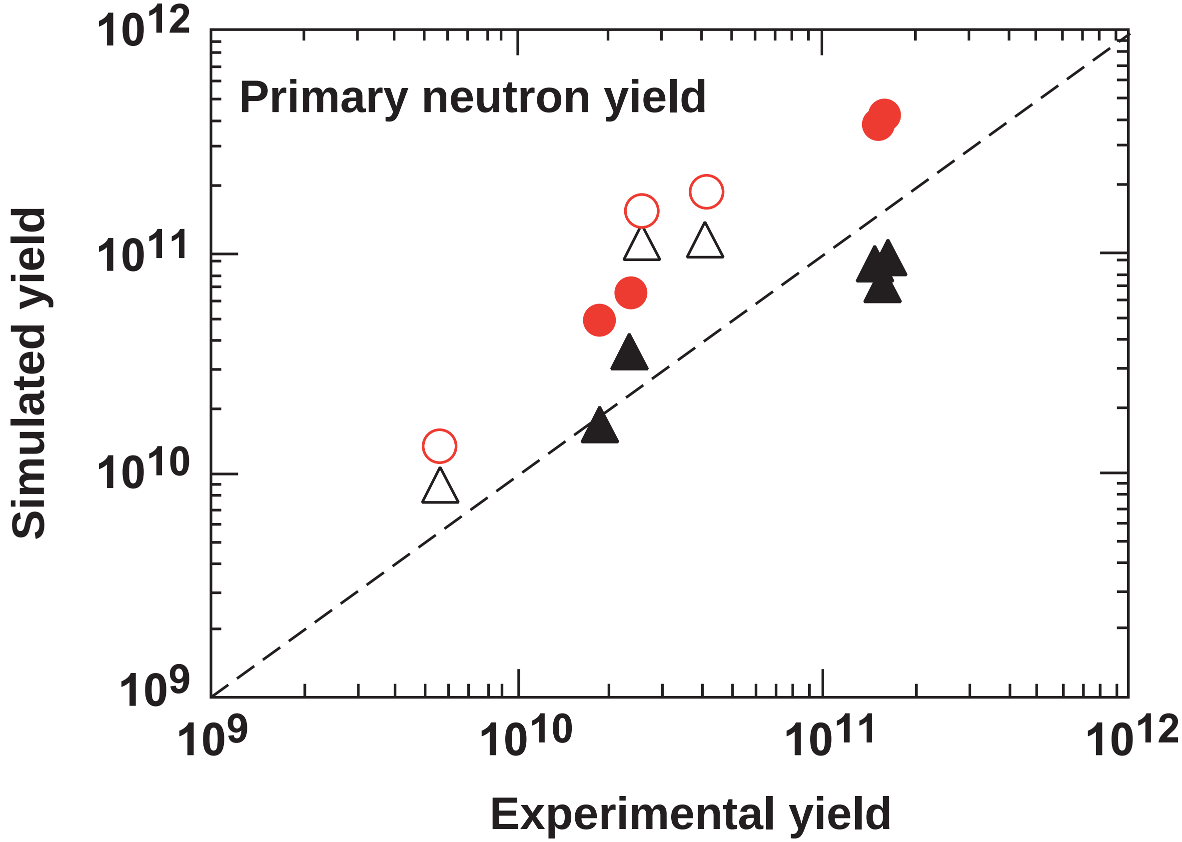
One-Dimensional Simulation of the Effects of Unstable Mix on Neutron and Charged-Particle Yield from Laser-Driven Implosions



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Collaborators

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Summary

Mix effects on particle yields can be described effectively by mix modeling in the 1-D hydrocode *LILAC*

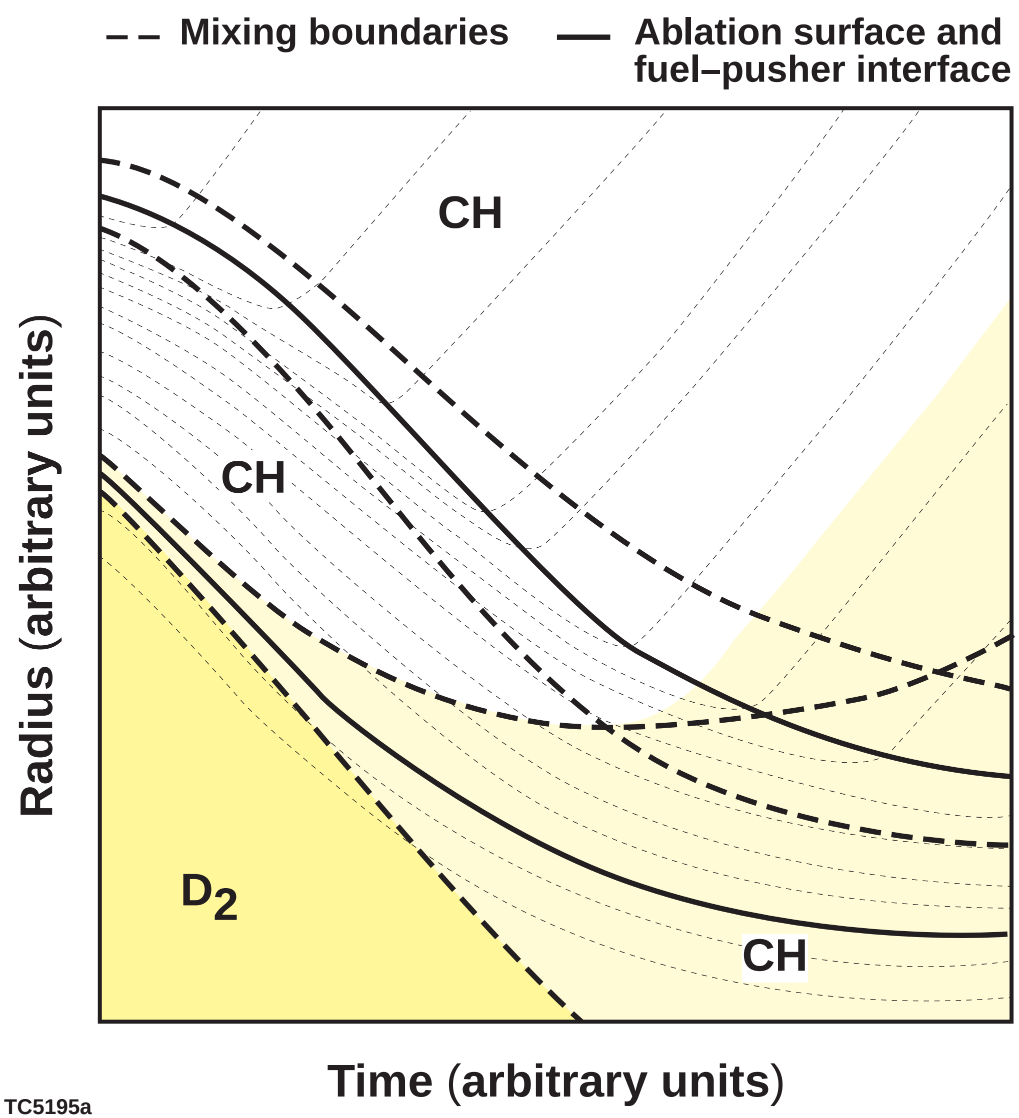
- The mix model includes the transport of target constituents, thermal energy, and turbulent energy due to both the acceleration and deceleration instabilities.

- Including mix in 1-D simulations of experiments provides improved predictions of primary and secondary particle yields over a broad range of target performance.

Outline

- Modeling of mix in 1-D
- Comparison of simulated and experimental yields
- Secondary neutron and proton production
- Conclusions

“Bubble and spike” mixing thickness is obtained from a multimode Rayleigh–Taylor perturbation model*



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- $\frac{d^2}{dt^2} A_\ell = \gamma^2(t) A_\ell$ including Bell-Plesset effects
- Takabe/Betti form for $\gamma^2(t)$
- Haan saturation procedure for

$$A_\ell(t) > \frac{2R(t)^*}{\ell^2}$$

- Initial perturbation spectrum $A_\ell(t = t_0)$ specified at ablation surface and fed through to fuel–pusher interface over time.
- Mix is modeled as a diffusive transport process.

*S. W. Haan, Phys. Rev. A 39, 5812 (1989).

The mix model is based on carefully formulated phenomenology

- Perturbations due to single-beam imprint were obtained from *ORCHID* calculations based on measured single-beam nonuniformity.

- Beam-imbalance effects are based on power-imbalance measurements from each shot and the geometrical superposition of the acceleration distributions of 60 beams.

- The formulation of the perturbation growth using fully time-dependent perturbation equations allows secular nonuniform irradiation effects and “feedthrough” from the outer to the inner instabilities to be treated as driving terms, rather than as instantaneous effects.

- Plausible flux limitation of the diffusive mix transport is obtained by allowing that the mixed constituent profiles can remain self-similar under expansion.

Perturbation equations are best written in terms of a mass amplitude

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Incompressible planar approximation

$$\frac{d^2}{dt^2} A_\ell = \gamma_0^2 \, A_\ell \qquad \gamma_\pm = \pm\gamma_0$$

$$A_{\ell\pm} = A_{\ell 0} \, e^{\gamma_\pm t} \qquad \gamma_0^2 = \frac{\ell}{R}\left(\frac{\rho_2 - \rho_1}{\rho_2 + \rho_1}\right)\ddot{R}$$

Compressible spherical solution (i.e., Bell–Plesset*)

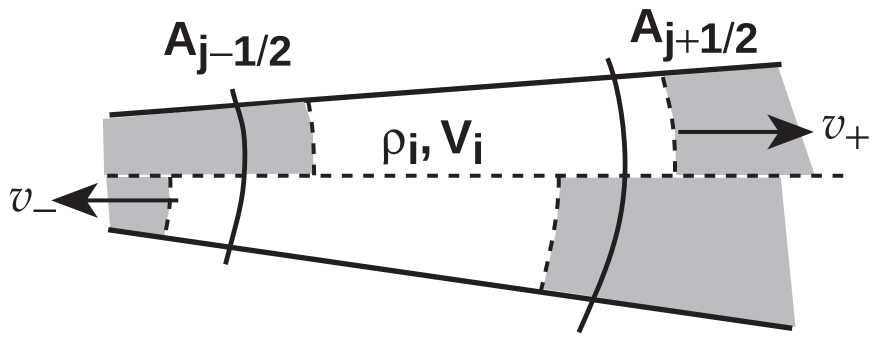
$$\left(-\gamma_\rho - \gamma_R + \frac{d}{dt}\right)\frac{d}{dt}\left(A_\ell \rho R^2\right) = \gamma_0^2 \left(A_\ell \rho R^2\right) \qquad \gamma_R = \dot{R}/R, \; \gamma_\rho = \dot{\rho}/\rho$$

$$\gamma_0^2 = \frac{\ell(\ell+1)}{R} \frac{(\rho_2 - \rho_1)\,\ddot{R}}{\left[\ell\rho_2 + (\ell+1)\rho_1\right]} \qquad \gamma_\pm = \frac{1}{2}\left(\gamma_\rho + \gamma_R\right) \pm \sqrt{\gamma_0^2 + \frac{1}{4}\left(\gamma_\rho + \gamma_R\right)^2}$$

*G. I. Bell, Los Alamos National Laboratory, Report No. LA-1321 (1951).
M. S. Plesset, J. Appl. Phys. 25 (1), 96-98 (1954).

TC4513

Mix is modeled in 1-D as a diffusive transport process



Advection to and from nearest-neighbor zones is expressed as diffusion in 1-D.

$$\frac{\partial}{\partial t}\begin{bmatrix}\rho \\ n_e \\ n_H \\ C_v T_e \\ \vdots\end{bmatrix} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\sigma}{\beta_m}\frac{\partial}{\partial r}\begin{bmatrix}\rho \\ n_e \\ n_H \\ C_v T_e \\ \vdots\end{bmatrix}\right), \quad \text{where } \sigma = v_{mix}\lambda * \frac{4(r_b - r)(r - r_s)}{(r_b - r_s)^2}$$

v_{mix}: **obtained from trajectories of mix-region boundaries**

λ: **scale length of turbulence structure from rms perturbation wavelength**

f: "flux limit" parameter $\frac{\sigma}{\beta_m}\left|\frac{\partial \rho}{\partial r}\right| \Rightarrow \text{Min}\left[\frac{\sigma}{\beta_m}\left|\frac{\partial \rho}{\partial r}\right|, f\rho v_{mix}\right]$

TC3713

The mix computation is done as a separate step within the 1-D hydrocode

- **Diffusive transport of constituent densities $\{\phi\}$ keeps zone masses constant:**

$$\frac{d}{dt}(V\phi)_j = \left[A\left(u + \frac{\sigma}{\beta_m}\frac{\partial}{\partial r}\right)\phi\right]_{j+1/2} - \left[A\left(u + \frac{\sigma}{\beta_m}\frac{\partial}{\partial r}\right)\phi\right]_{j-1/2}$$

$$(\rho u)_{j+1/2} = -\left[\frac{\sigma}{\beta_m}\frac{\partial\rho}{\partial r}\right]_{j+1/2}$$

$$\frac{d}{dt}M_j = \frac{d}{dt}(V\rho)_j = 0$$

- **Hydrodynamics in terms of total mass velocity*** $v_{j+1/2} = \langle v_{j+1/2}\rangle + u_{j+1/2}$

$$\frac{d\rho}{dt} = \frac{-\rho}{r^2}\frac{\partial}{\partial r}\left(r^2 v\right), \qquad \rho\frac{dv}{dt} = -\frac{\partial}{\partial r}\left(P + Q + P_T + Q_T\right)$$

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*Leith, UCRL-96036 (1986).

TC3964

Mix-motion energy is computed as turbulent energy in a "k–λ" model

- **Turbulent energy density k:**

$$P_T = \frac{2}{3}k, \quad Q_T = -\frac{4}{3}\frac{\sigma}{\beta_q}\frac{\partial v}{\partial r}, \quad \sigma = v_{mix}\lambda \qquad (\beta_q = 1.0)$$

- **Buoyant force as source of k:**

$$S = \max\left(\rho u \frac{dv}{dt}, 0\right), \quad \rho u = -\frac{\sigma}{\beta_m}\frac{\partial \rho}{\partial r} \qquad (\beta_m = 0.7)$$

- **Dissipation rate:**

$$\varepsilon = C_\varepsilon \frac{k^{3/2}}{\rho^{1/2}\lambda} \qquad (c_\varepsilon = 0.09)$$

- **Evolution:**

$$\frac{dk}{dt} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\sigma}{\beta_k}\frac{\partial k}{\partial r}\right) - \left(P_T + Q_T\right)\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 v\right) + S - \varepsilon \qquad (\beta_k = 0.715)$$

$$\frac{dE_i}{dt} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\sigma}{\beta_m}\frac{\partial E_i}{\partial r}\right) + \varepsilon - S + \ldots \text{etc.}$$

TC3962

Growth rates of perturbations of arbitrary density profiles are estimated using Sturm–Liouville theory

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- The Rayleigh perturbation equation for an arbitrary density profile is a Sturm–Liouville eigenvalue equation:

$$\frac{1}{r^2}\frac{d}{dr}\left[r^2\rho\frac{dw_\ell}{dr}\right]-\rho\frac{\ell(\ell+1)}{r^2}w_\ell+\frac{1}{\gamma_\ell^2}\frac{\ell(\ell+1)}{r^2}g\frac{d\rho}{dr}w_\ell=0$$

- The perturbation growth rate is given by a variational expression:

$$\frac{\gamma_\ell^2}{g}=\mathrm{Max}\left\{\int_0^\infty\frac{\ell(\ell+1)}{r^2}\frac{d\rho}{dr}w_\ell^2r^2dr\Big/\int_0^\infty\rho\left[\left(\frac{dw_\ell}{dr}\right)^2+\frac{\ell(\ell+1)}{r^2}w_\ell^2\right]r^2dr\right\}$$

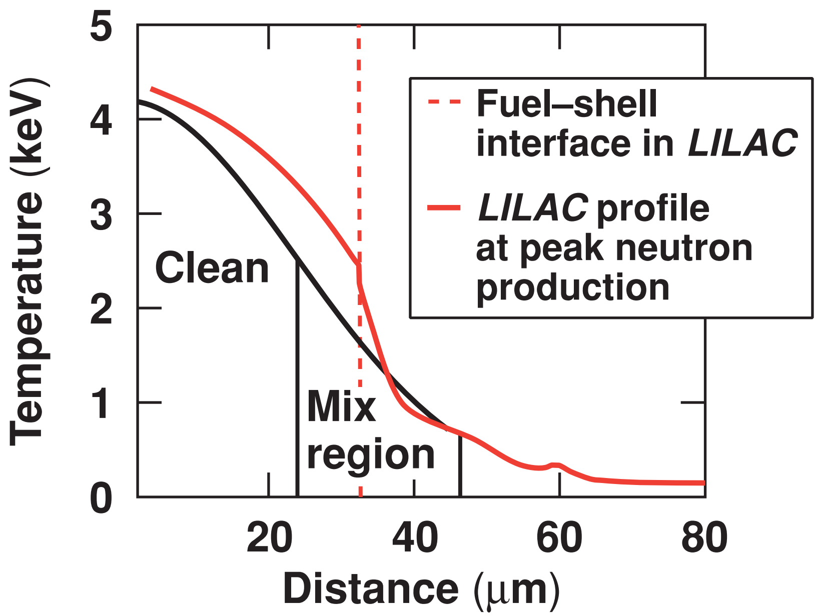
- Estimates obtained using only moderately accurate eigenfunctions are accurate to second order in variations of the postulated eigenfunction:

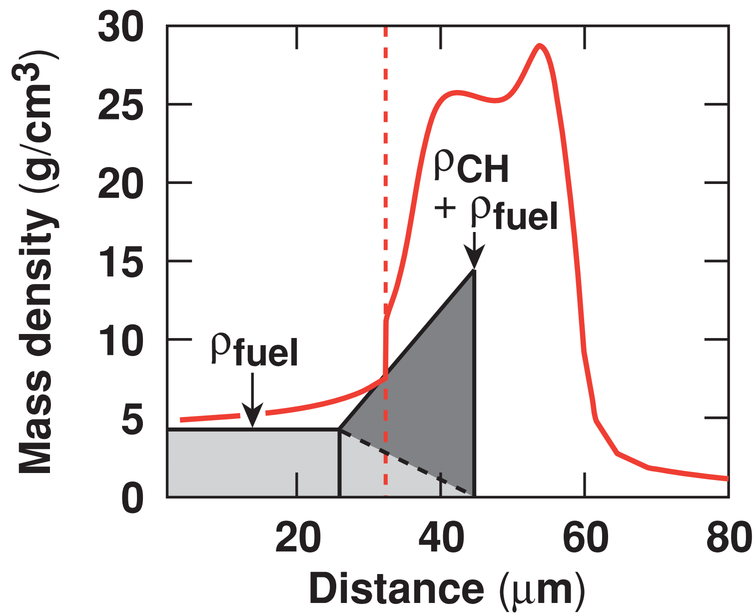
$$w_\ell\approx\left[(r/L)^\ell,\ r<L/2;\ a+b(r-r_0)+c(r-r_0)^2,-L/2<r<L/2;\ (r/L)^{-(\ell+1)},\ r>L/2\right]$$

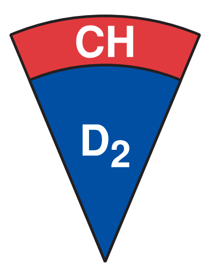
TC5857

A constrained set of static model core properties reproduces most experimental observables

- 1-ns square, 23 kJ, 20-μm-CH shells, 15 atm fill, CR ~ 15



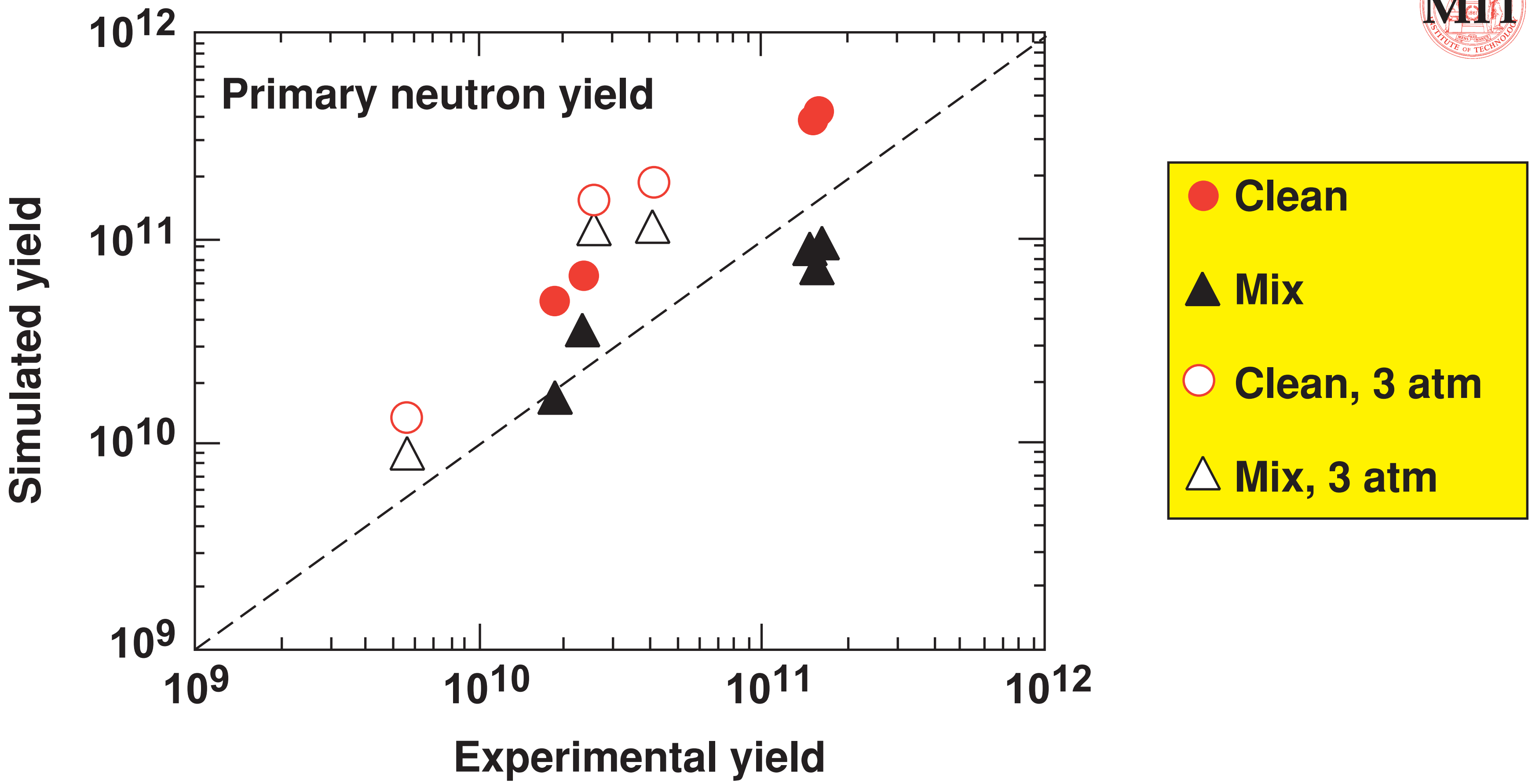




	Measured	% of model
Max: neutron rate	$(9\pm2) \times 10^{20}$	120
Burn width (ps)	170±20	94
T_{ion} (D_2) (keV)	3.7±0.5	90
Secondary neutron ratio	$(2.1\pm0.4) \times 10^{-3}$	90
Secondary proton ratio	$(1.8\pm0.3) \times 10^{-3}$	100

TC5855

Mix modeling improves the agreement of simulated primary neutron yield with implosion data



- Data from eight shots (August 2000)
- Pure-CH shells, 20–27 μm, 900-μm diameter, D_2 fill, 3–25 atm

TC5667

Primary yield ratios indicate that implosion degradation is comparable to the predictions of mix modeling

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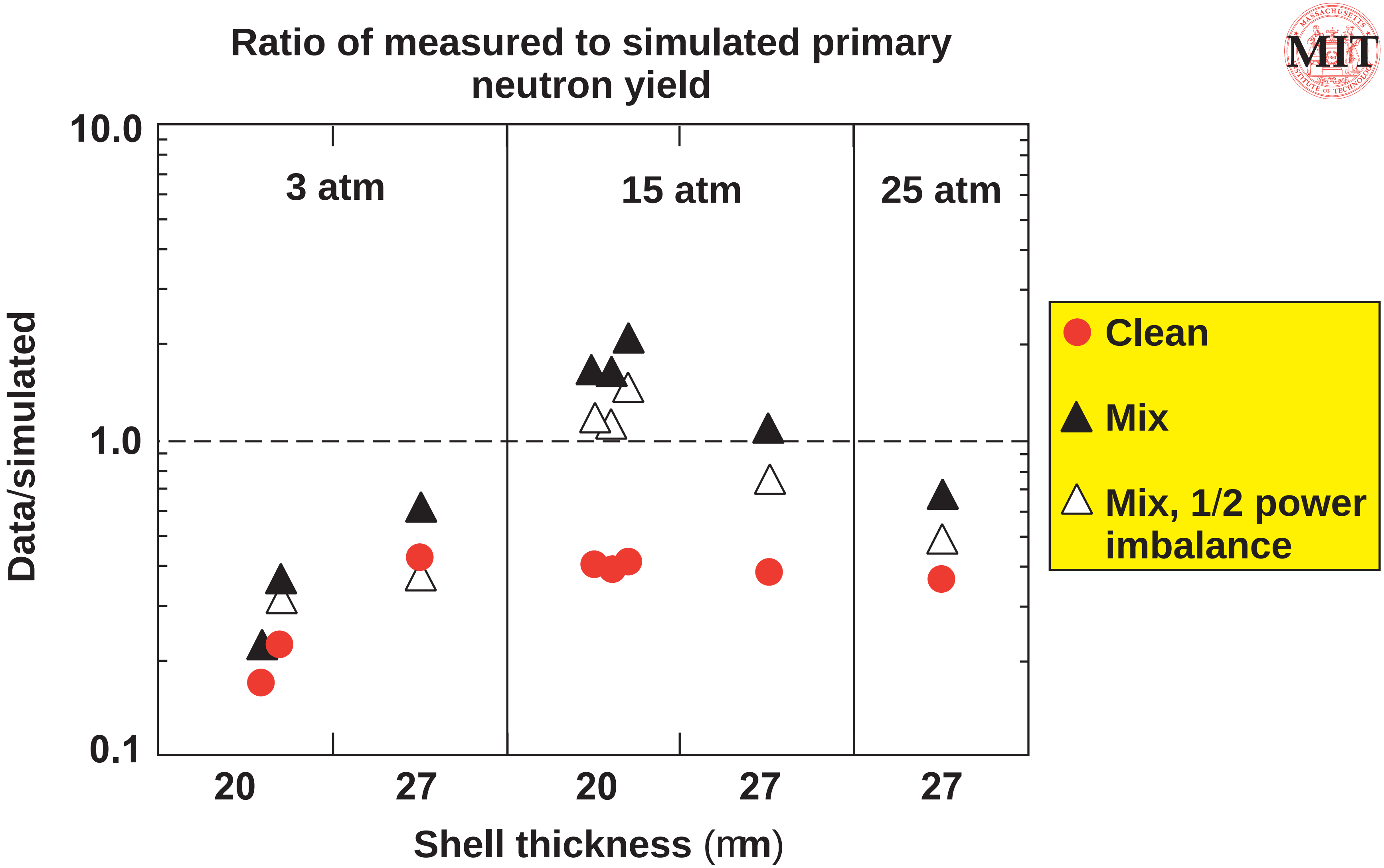
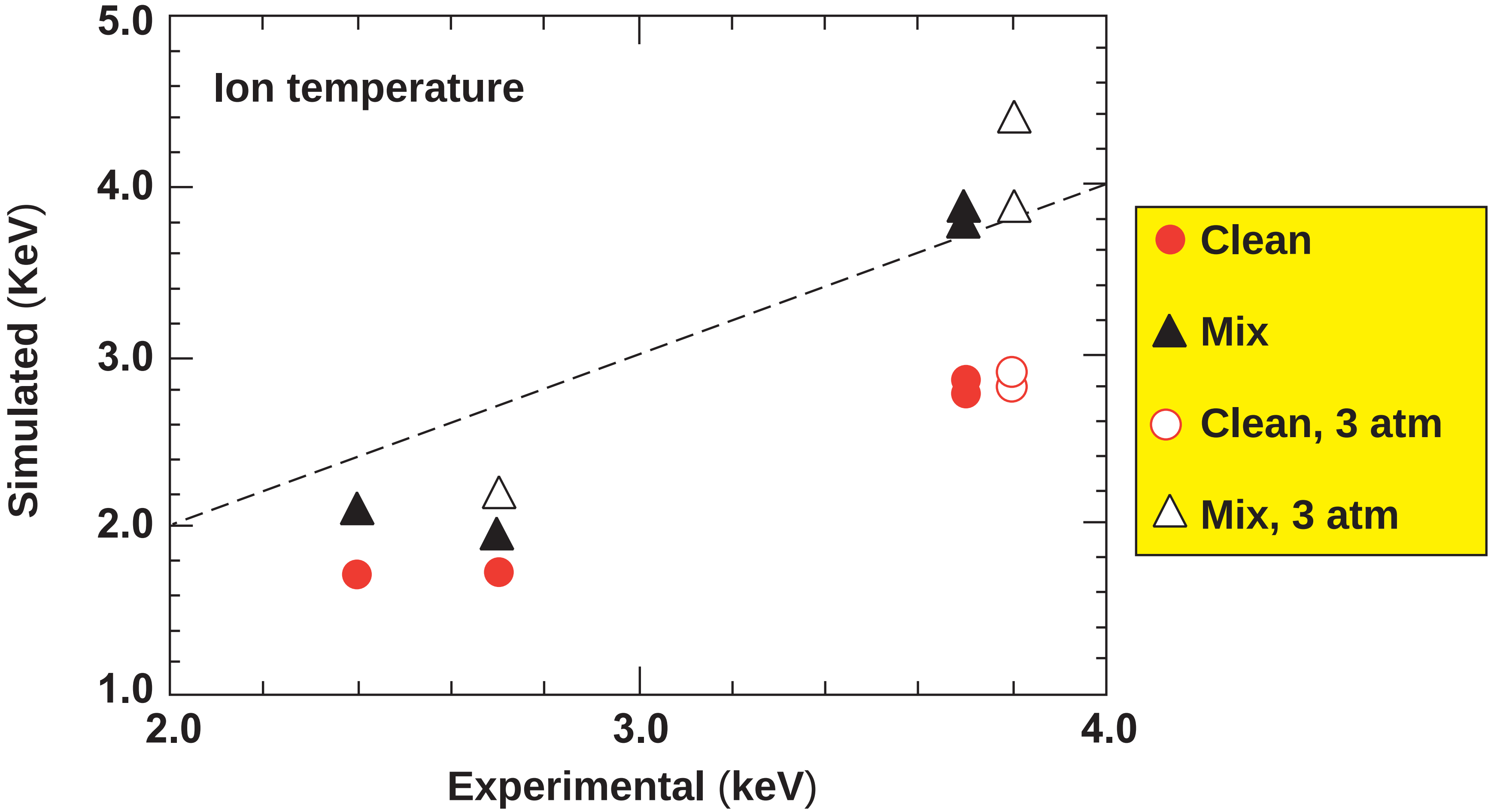


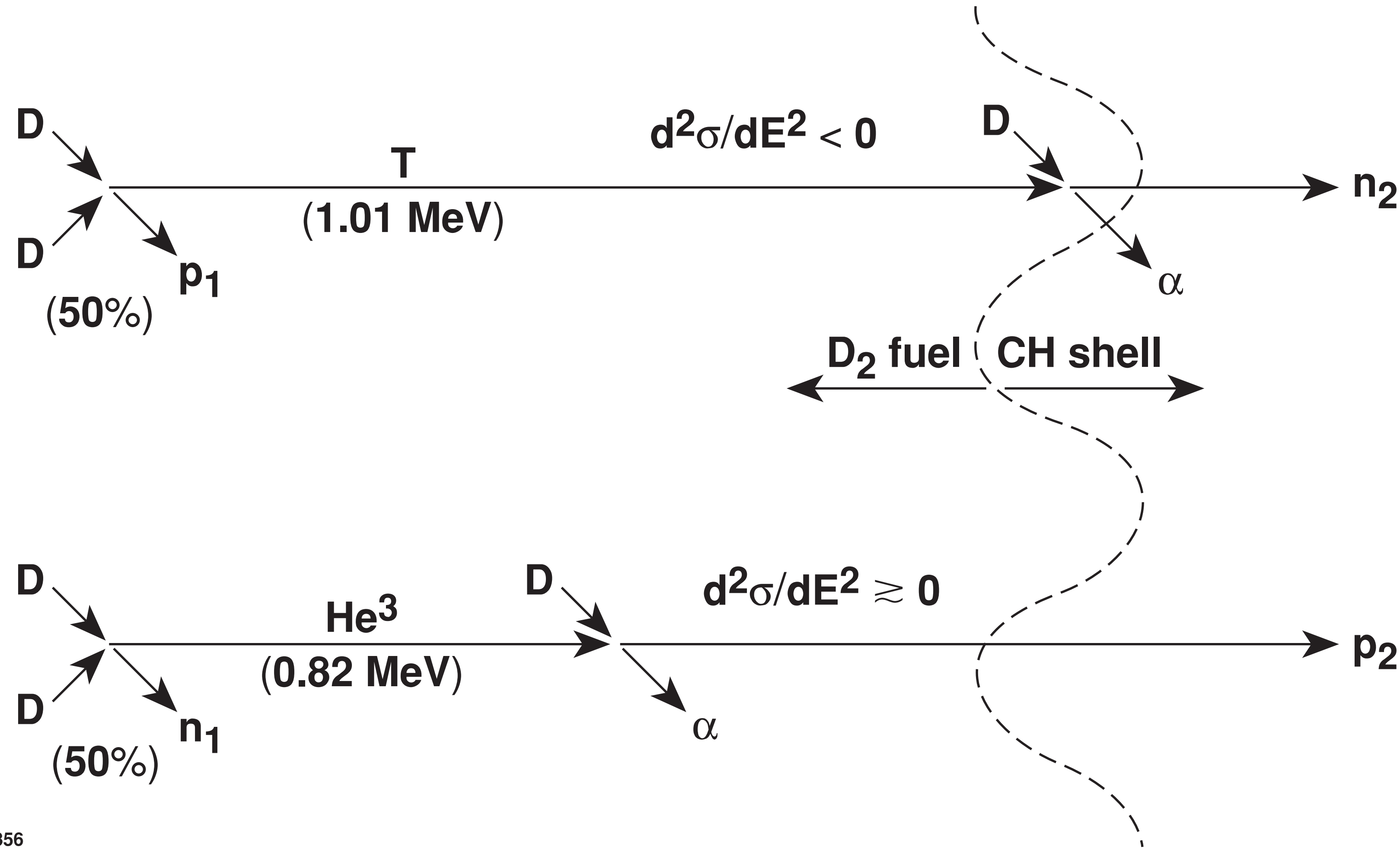
TC5668

Simulated and measured neutron-averaged temperatures show some improved agreement with mix modeling



TC5669

Secondary particle yields reflect different slowing rates and cross sections with contrasting energy dependence



TC5856

Comparison of simulated with measured secondary particle yield ratios suggests sensitivity to dynamics

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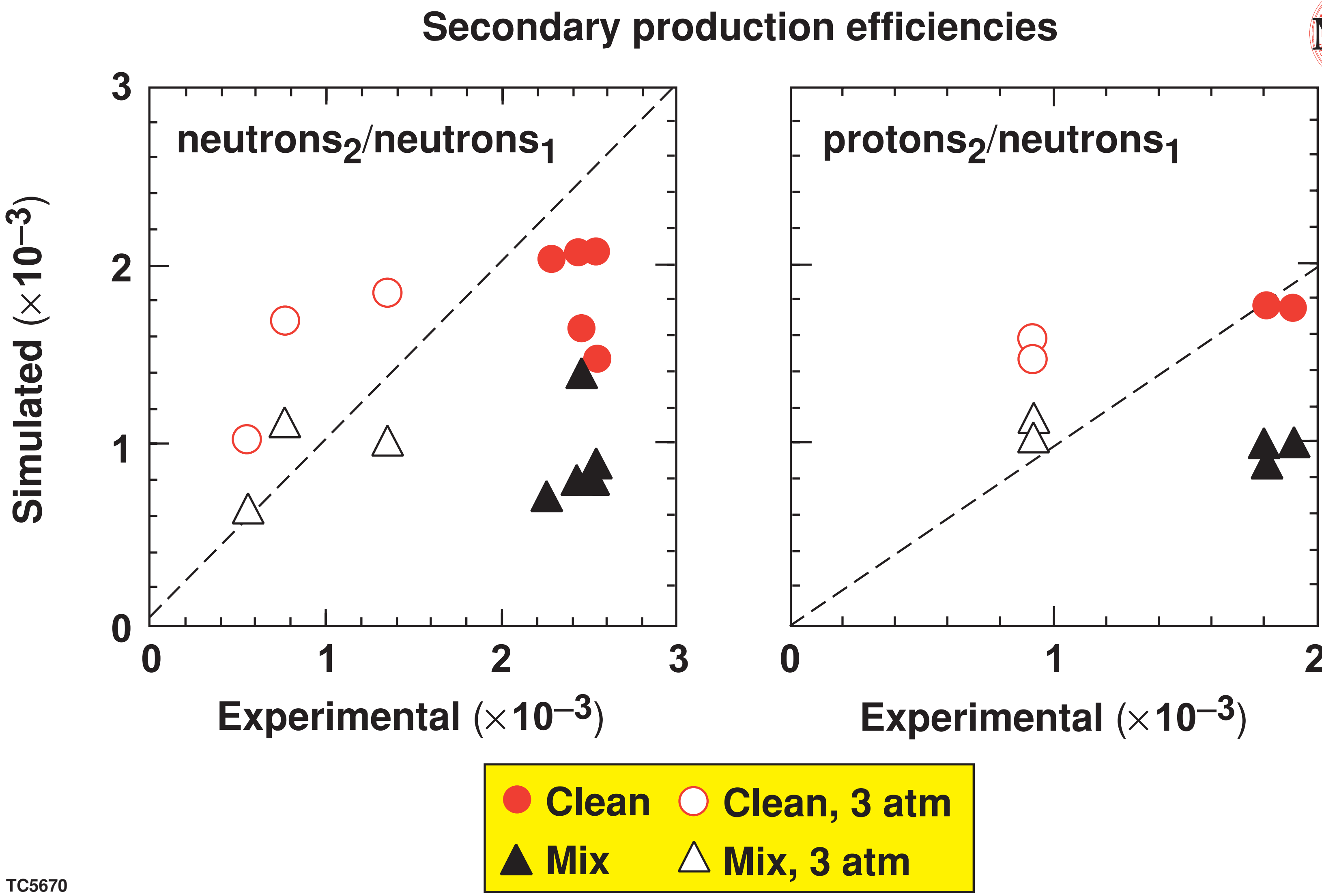

TC5670

The spatial distribution of secondary particle production depends on the extent of mix

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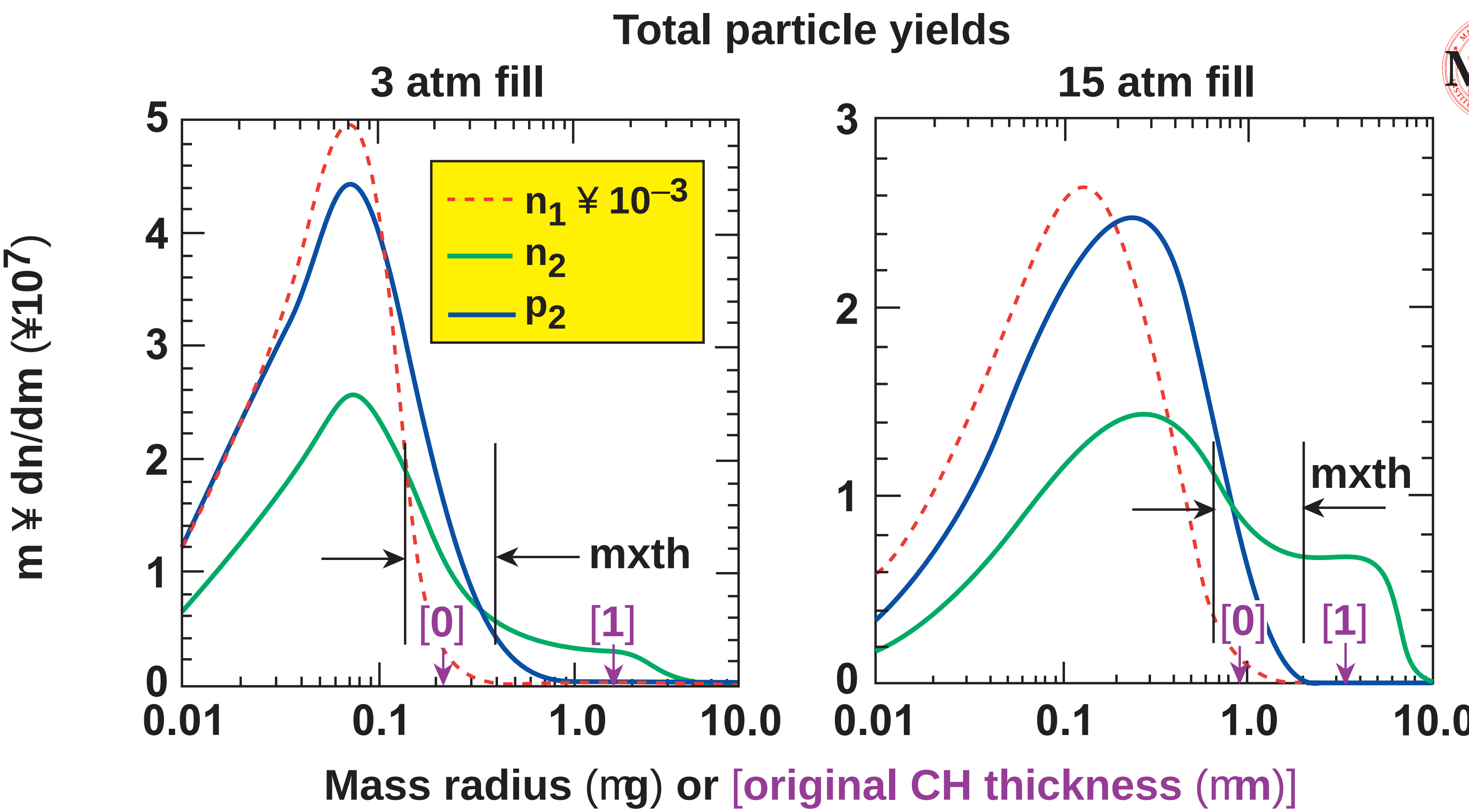
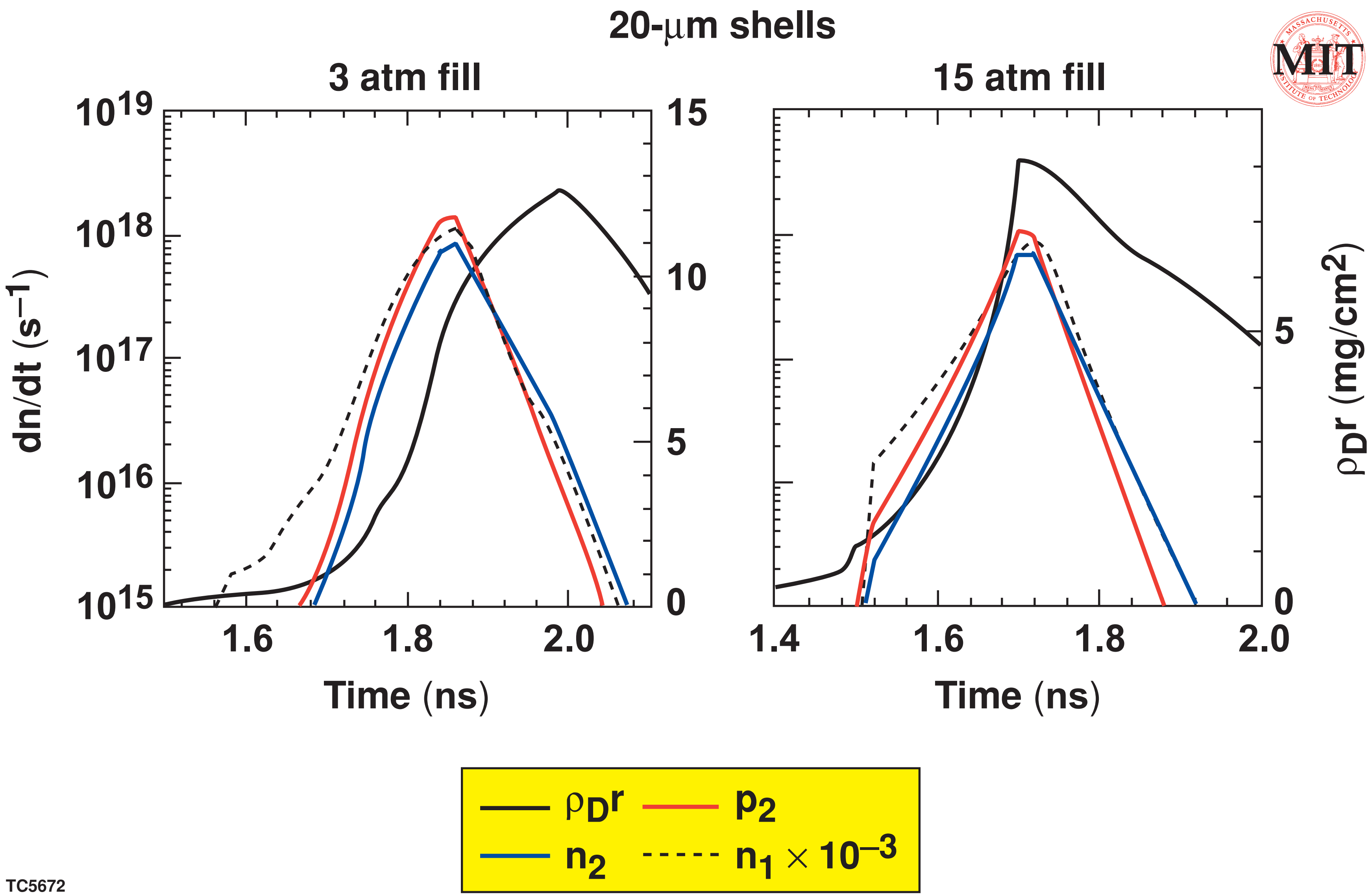


- Mix thickness (mxth) is from the 1:3 to 3:1 mix points at the time of peak n_1 production rate.
- With the mass-spatial distribution as plotted here, the area under the curve is preserved.

TC5671

The relative timing of peak neutron production and peak compression does not affect the coincidence of primary and secondary production times



TC5672

Summary/Conclusion

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- The mix model includes the transport of target constituents, thermal energy, and turbulent energy due to both the acceleration and deceleration instabilities.

- Including mix in 1-D simulations of experiments provides improved predictions of primary and secondary particle yields over a broad range of target performance.

- The validity of approximating multidimensional hydrodynamics with a spherically symmetric model remains an issue.