Numerical Methods for Determination of RT and RM Mixing

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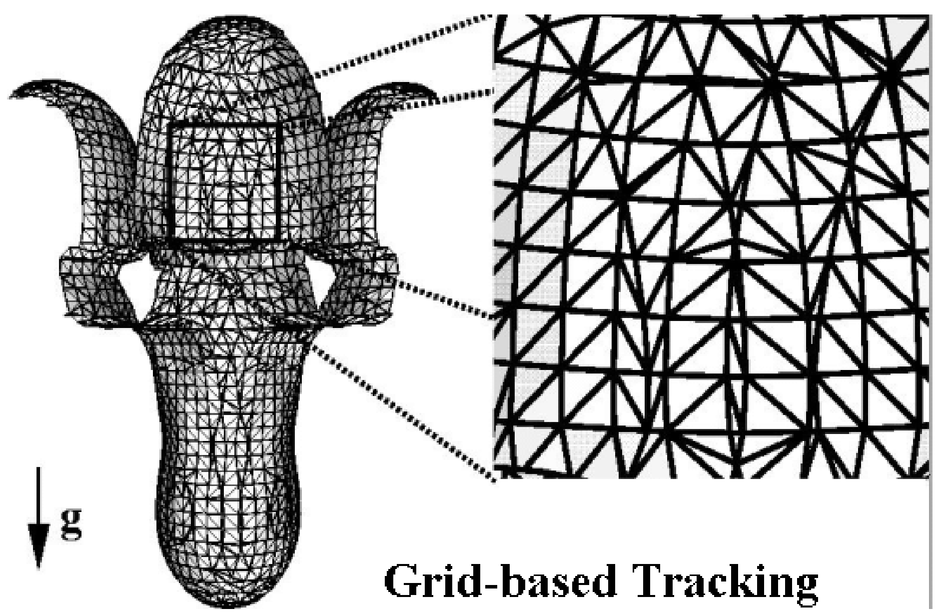
Outline of the Talk

- Tracking the contact interface
- Simulations of Random RT Mixing
- Comparison with untracked code
- Analysis of buoyancy acceleration
- Simulation of axisymmetric RM mixing
- Conclusion

The Front Tracking Method

- Interface divides space into subdomains
- Riemann solution to propagate interface
- Finite difference with ghost cells
- Coupling interface-interior solutions
- Dynamic resolution of interface topology
- Clear separation of discontinuous states
- Clear separation of material (EOS)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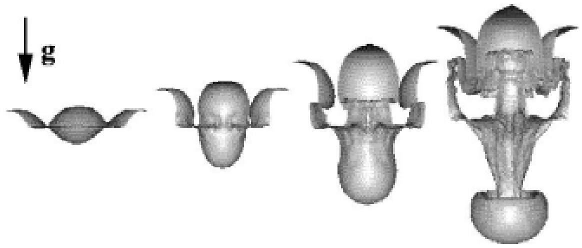
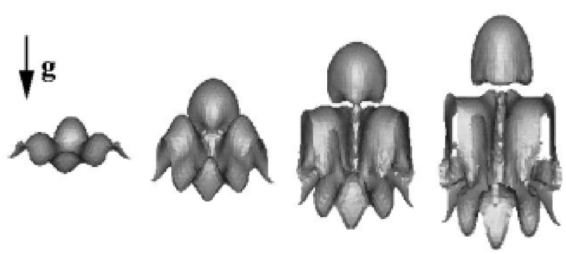
Grid-based Tracking

Basic FronTier Test Simulations



Case Single-2



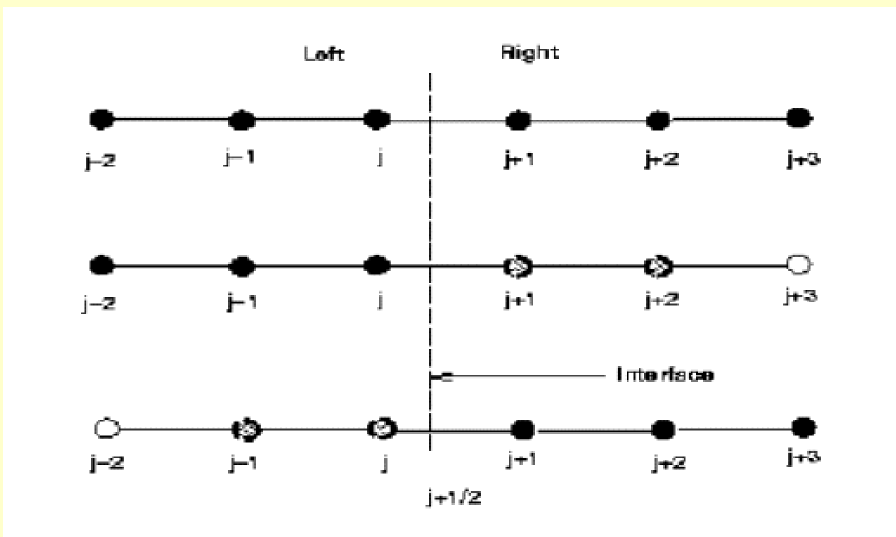
Case Bifure-1

Separating fluids via ghost-cell

1. Ghost-cell method

    Front Tracking (Glimm, McBryan, etc 1980)

2. Ghost cell has been the key design component of front tracking



The ghost-cell method

Numerical diffusion in 1st order upwinding

$$Du = u_t - au_x = 0$$

$$D_h u = \frac{u_j^{n+1} - u_j^n}{\Delta t} - a\frac{u_j^n - u_{j-1}^n}{\Delta x} = 0$$

$$D_h u - Du = -\frac{a\Delta x}{2}\left(1 - a\frac{\Delta t}{\Delta x}\right)u_{xx}$$

$$a\frac{\Delta t_{CFL}}{\Delta x} = 1$$

Numerical diffusion is large if $\Delta t << \Delta t_{CFL}$

Numerical dissipation at contact discontinuity

1. Most higher order schemes switch to 1st order at discontinuity to avoid oscillation, using either artificial viscosity or limiter.

2. For three characteristic wave speed $u \pm c,\ u$, the motion of contact is the inertial motion with speed u.

3. For first order scheme, the smaller is Δt away from the CFL condition, the more dissipative it becomes.

4. It is particularly dissipative at contact for low compressibility

$$c >> u \qquad \Delta t = CFL \times \frac{\Delta x}{|u| + c} << CFL \times \frac{\Delta x}{|u|}$$

The α Paradox

$$h_b = \alpha A g t^2$$

David Youngs and K.Read
(1984)

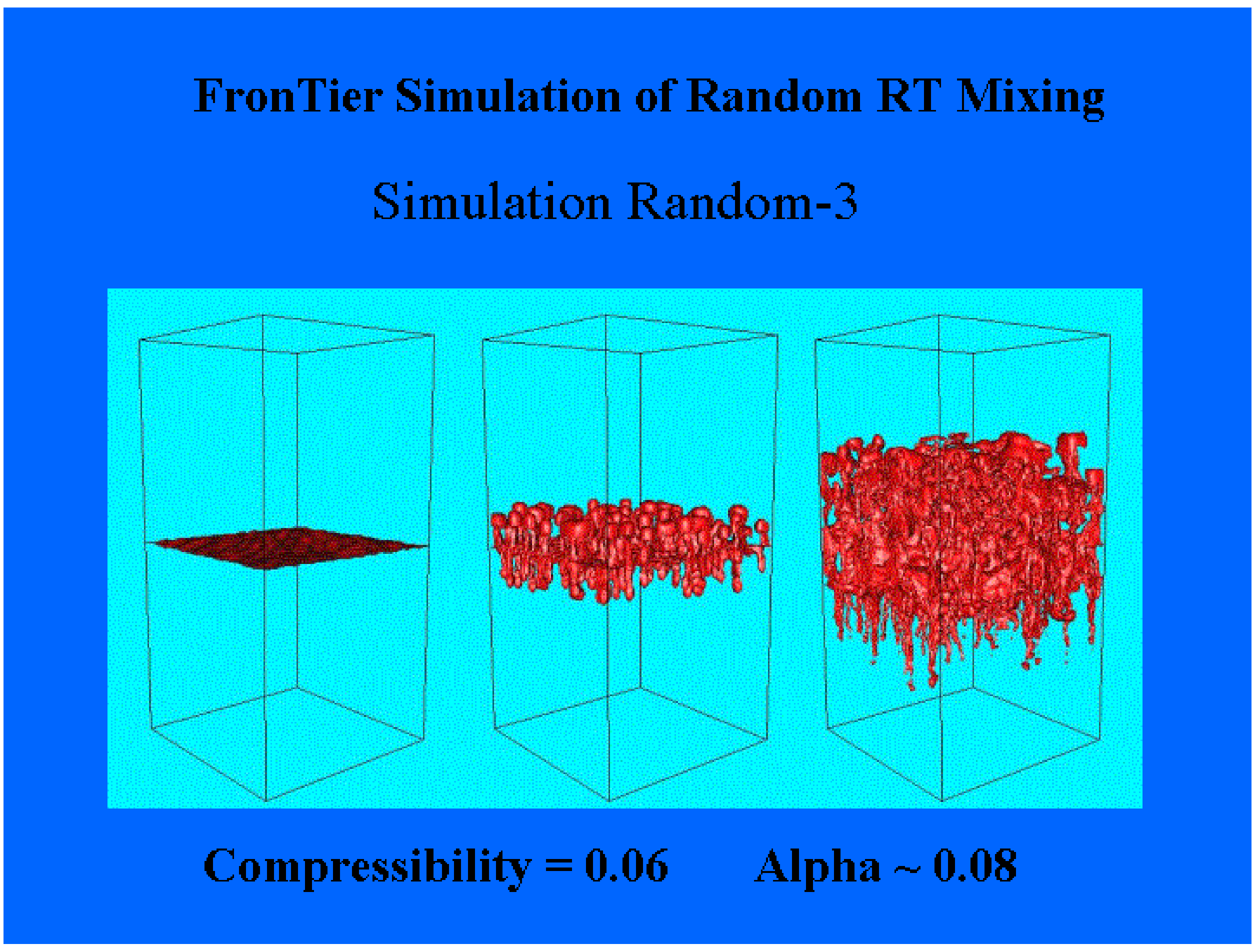
Read's Experiment (1984)

3D alcohol/air	Exp # 29	Alpha = 0.073
	39	0.076
	58	0.077
3D NaI soln./Pentane	Exp # 33	Alpha = 0.066
	35	0.071
3D NaI soln./Hexane	Exp # 62	Alpha = 0.063
	60	0.073

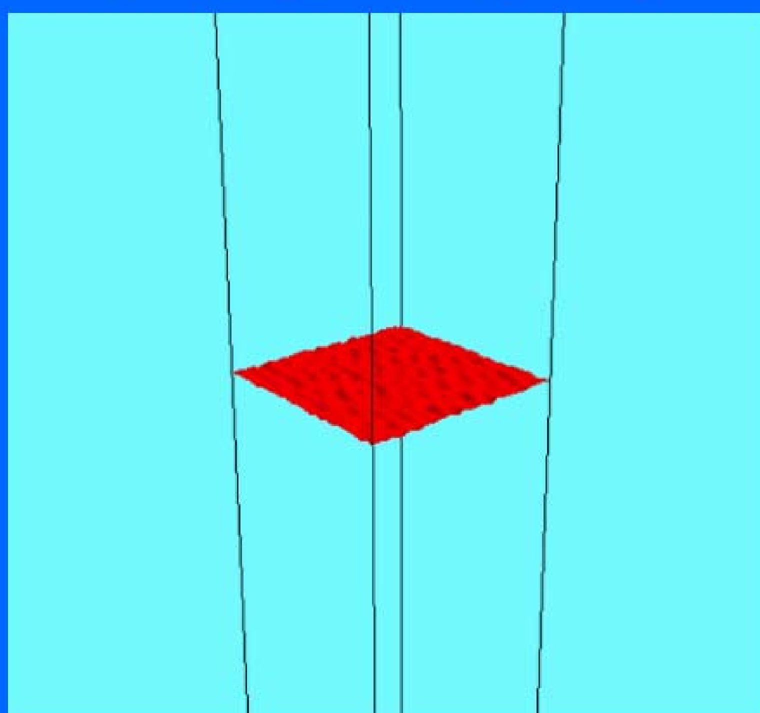
Summary of Experiments

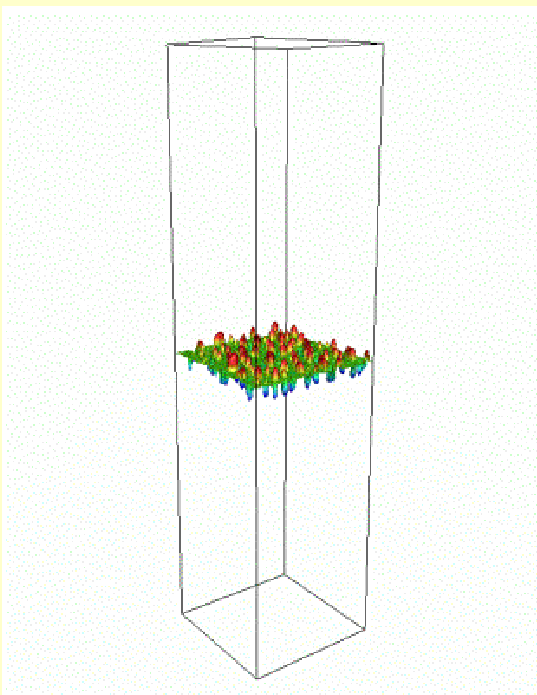
Experiments			
Read/Youngs	'84	$\alpha_b \sim 0.58 - 0.65$	2D
		$\alpha_b \sim 0.063 - 0.077$	3D
Kucherenko	'91	$\alpha_b \sim 0.07$	3D
Snider/Andrews	'94	$\alpha_b \sim 0.07 \pm 0.007$	3D
Schneider/Dimonte/Remington	'99	$\alpha_b \geq 0.054$	3D
Dimonte/Schneider	'99	$\alpha_b \sim 0.05 \pm 0.01$	3D

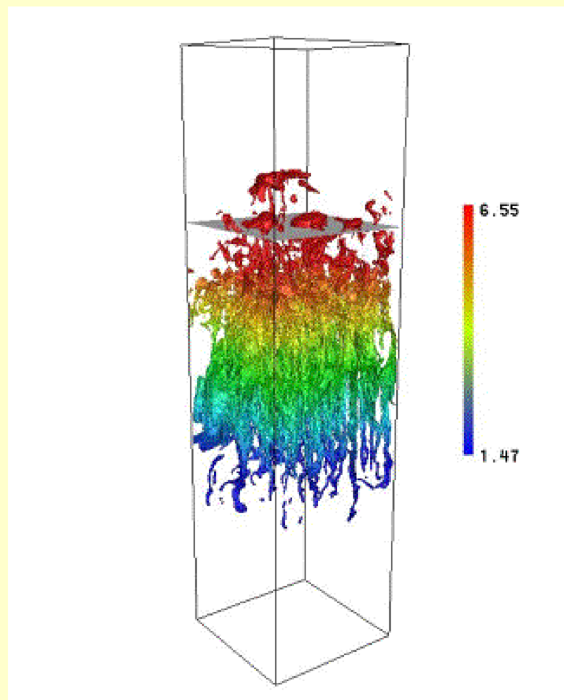
Bliznetsov	0.1
Shestachchenko	0.04


FronTier Simulation of Random RT Mixing
Simulation Random-3
Compressibility = 0.06
Alpha ~ 0.08

Movie Random-4



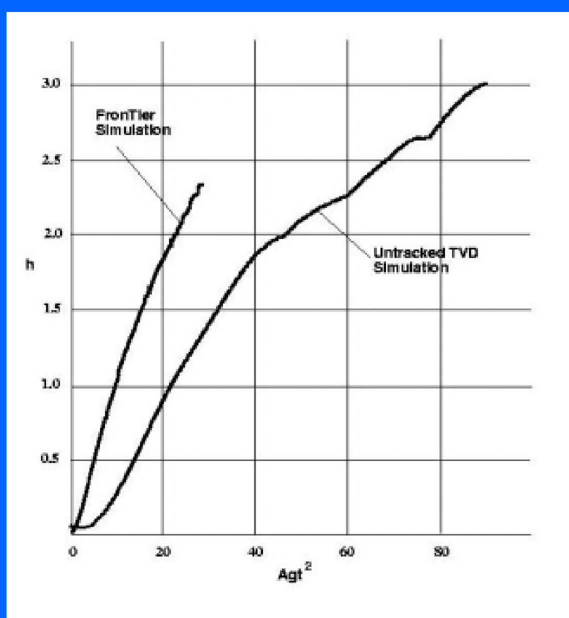




FronTier simulation of RT mixing, with A = 0.5.

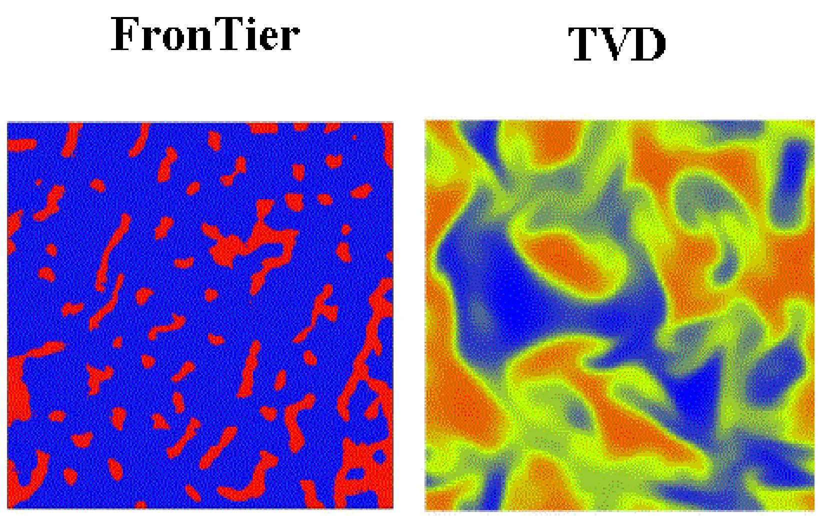
Left: early time. Right: late time. On 128*128*512 grid

The Alpha of Bubbles



FronTier:
Alpha = 0.08

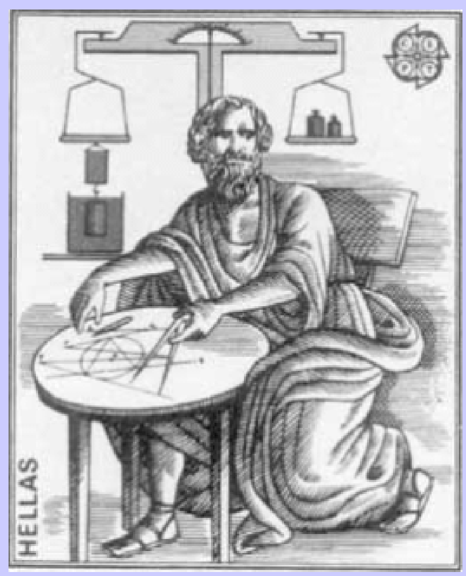
TVD:
Alpha = 0.025-0.045

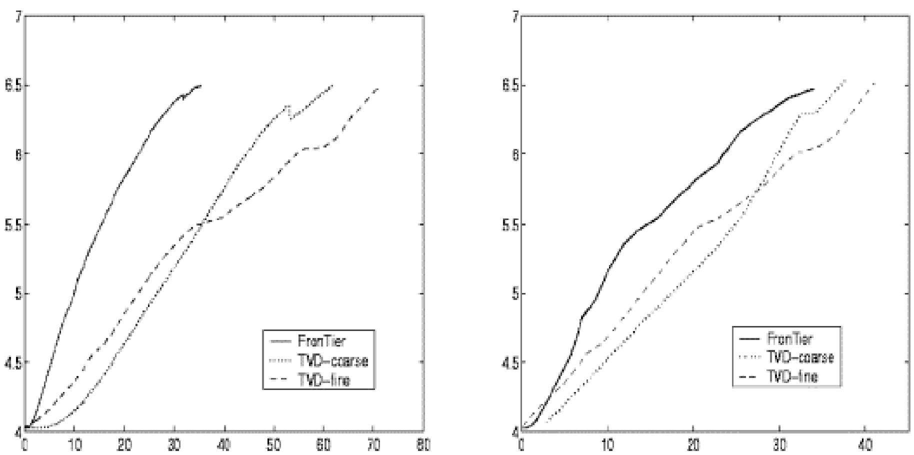


Agt^2 = 25.3 h = 4.16 Density plot

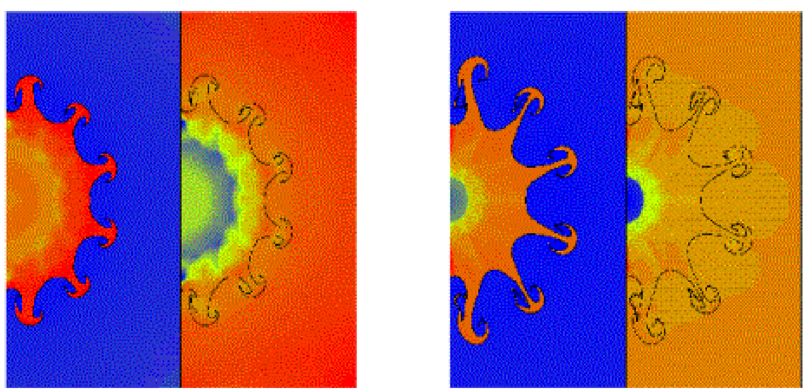
The Archimedes Principle

- The buoyancy force on an immersed body equals to the weight of the fluid the immersed body has repelled.



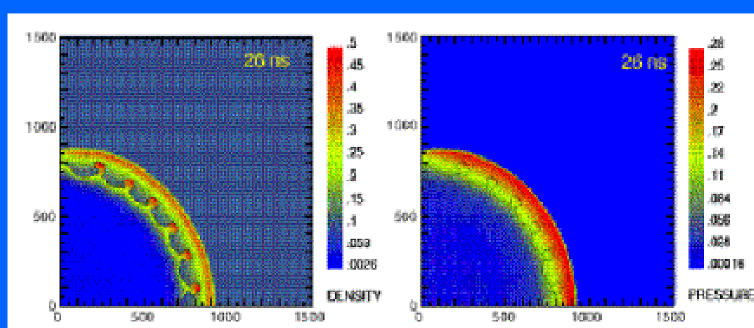


Normalization of alpha with effective Atwood number

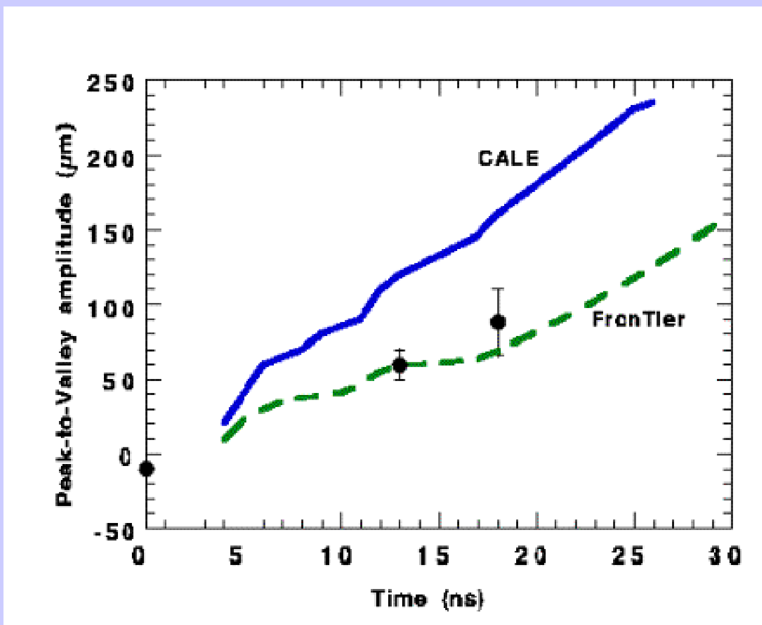


A 2D simulation of spherical RM instability with axisymmetry The Mach number of the imploding shock is 1.2 and the Atwood number is 2/3. The inner gas is SF_6 and the outer gas is air.

FronTier Simulation of NLUF 2 Experiment



CHGe capsule surrounded by CRF foam. The RM instability is driven by strong shock of Mach number 300 by the Omega laser



Comparison of *FronTier* amplitude and the CALE Simulation with the experiment

Conservative Interface-Interior Coupling

The conservation law:

$$u_t + f(u)_x = 0$$

The Rankine-Hugoniot condition:

$$f(u_L) - su_L = f(u_R) - su_R$$

Neglect higher order term and note that

$$\int_{\Delta V} u dV = \int_{S_M} u v_n dS \Delta t$$

We have the integral form of conservation

$$\frac{\partial}{\partial t}\int_V u dV - \int_{S_M} u v_n dS + \oint_S F_n(u) dS = 0$$

This can also be written as

$$\frac{\partial}{\partial t}\int_V u dV + \int_{S_F} F_n(u) + \int_{S_M} (F_n(u) - v_n u) dS = 0$$

Or simply

$$\frac{\partial}{\partial t}\int_V u dV + \oint_S (F_n(u) - v_n u) dS = 0$$

Conclusion

1. Numerical diffusion is a serious problem in simulation of contact surface in gas dynamics, especially when $c >> u$.

2. Front tracking prevents such diffusion through ghost-cell method, retains correct buoyancy force in RT simulation.

3. The introduction of local effective Atwood number bridges the gap between tracked and untracked simulations.

4. Front tracking shows agreement in axisymmetric spherical RM instability.

5. Front tracking is will replace ghost-cell by conservative tracking using dynamic flux at a cell with moving boundary.