APPLICATION OF Kε-MODEL FOR THE DESCRIPTION OF AN ATMOSPHERIC SURFACE LAYER

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The problem on determination of non-dimensional characteristics of turbulent flow in atmospheric surface layer is considered within kε-model. Kε-equations and their singular points are investigated. The mathematical program for calculations of characteristics of turbulent flow in surface atmospheric layer is developed. From the set of integral curves those curves are chosen which correspond to the solution of formulated task and ensure the satisfactory experiments description. Here the basic model constants are chosen according to the conventional criteria. At the same it is shown that the parameter C_θ responding to convection source term of an ε-equation should be chosen depending on stability conditions. The best agreement with experimental results is reached if $C_\theta = 0$ for steady stratification and $C_\theta \neq 0$ for unstable stratification. By a numerical choice of value C_θ and factor of turbulent diffusion $\hat{\alpha}$ the quite satisfactory description of experimental observations known as analytical interpolar dependencies is received.

Introduction

Semiempirical $K\varepsilon$ models of turbulence are widely used for the description of different classes of turbulent flows [1-3] (jet streams and channels streams, gravitational turbulent mixing of fluid layers with different densities and others).

$K\varepsilon$ model was used for solving of the micrometeorology problems as well. In [4,5] the calculations of universal profile functions of atmosphere surface layer were conducted. On the basis of $K\varepsilon$ model the technique of calculation reconstruction of vertical profiles of meteorological values in the atmosphere boundary layer was developed according to the data of standard surface meteorological measurements [6,7].

In the current paper we are also analyzing the problem about atmosphere surface layer (ASL) within the frameworks of $K\varepsilon$ model. ASL is an example of the stratified flow, which is interested by the fact that it includes simultaneously both shear and convective (gravitational) mechanisms of turbulence generation. Moreover, the profile characteristics of stationary ASL have been studied experimentally quite well up to present. These conditions make the ASL problem by a good test for any semiempirical turbulence model.

The conducted theoretical and numerical investigations of the problem under consideration allowed to perform the procedure of selection of the empirical model constants more accurately and to obtain more accurate description of the experimental profiles as compared with the previous papers [4,5]. As the analysis showed, the best agreement with the observation data is obtained in case we assume the empirical parameter C_θ (which regulates the convective source of turbulence in ε equation) depends on the ASL stability status: $C_\theta = 0$ in the stable stratification area and $C_\theta \neq 0$ in the unstable stratification area.

1. Atmosphere surface layer. Theory of similarity, dimensionless form of equations of *K*ε model.

To describe turbulent flow in stationary and horizontally homogeneous ASL the following system of equations is applicable

$$k\frac{\partial U}{\partial z}=u_*^2; \qquad (1.1)$$

$$-\alpha_\theta k\frac{\partial \theta}{\partial z}=q; \qquad (1.2)$$

$$k\left(\frac{\partial U}{\partial z}\right)^2-\frac{g}{T}\alpha_\theta k\frac{\partial \theta}{\partial z}-\varepsilon+\alpha_b\frac{\partial}{\partial z}k\frac{\partial b}{\partial z}=0; \qquad (1.3)$$

$$C_{\varepsilon 1}k\left(\frac{\partial U}{\partial z}\right)^2-C_\theta\alpha_\theta\frac{g}{T}k\frac{\partial \theta}{\partial z}-C_{\varepsilon 2}\varepsilon+C_\varepsilon\frac{b}{\varepsilon}\frac{\partial}{\partial z}\frac{b^2}{\varepsilon}\frac{\partial \varepsilon}{\partial z}=0; \qquad (1.4)$$

$$k=C\frac{b^2}{\varepsilon}.$$

Here the axis *oz* is up-directed, *T* is temperature of ground surface, θ is potential temperature, *U* is velocity, *g* is gravitational acceleration, u_* is dynamic friction velocity, which determines turbulent frictional force between horizontal layers, *q* is turbulent temperature flow on the surface, *b* is turbulent energy pulsation (turbulent kinetic energy), ε is turbulent energy dissipation, *k* is turbulent viscosity (diffusion) coefficient. $\alpha_b, \alpha_\theta, C_\varepsilon, C_{\varepsilon 1}, C_{\varepsilon 2}, C_\theta, C$ are empirical constants.

At the height much above the significant dimensions of surface roughness the dimensional parameters of the problem are: u_*, q, $\frac{g}{T}$. With them it is possible to construct the single length parameter $L=-\frac{u_*^3}{\kappa\frac{g}{T}q}=\frac{u_*^2}{\kappa\frac{g}{T}\theta_*}$ presenting the characteristic of stratification (relational influence of dynamic and heat factors). Here κ is Karman constant, $\theta_*=-\frac{q}{u_*}$ is temperature scale.

If $L>0$, turbulent heat flow is down-directed (air substratum is cooled with a colder ground surface), stratification is stable. If $L<0$, turbulent heat flow is up-directed (air is heated with a hotter surface), stratification is unstable. The limit $L\to\infty$ corresponds to neutral stratification (heat flow equals to zero, thermal convection is absent).

All ASL characteristics are the functions of dimensionless height $\xi=\frac{z}{L}$ and may be presented in the form

$$U(z)=\frac{u_*}{\kappa}u_n(\xi),\quad k(z)=\kappa u_* L k_n(\xi),\quad b(z)=\frac{u_*^2}{\sqrt{C}}b_n(\xi),\quad \varepsilon(z)=\frac{u_*^3}{\kappa L}\varepsilon_n(\xi),\quad \theta(z)=\theta_0+\frac{\theta_*}{\kappa\alpha_\theta}\theta_n(\xi).$$

Here the dimensionless functions are indicated with the index “*n*“, the empirical constants are traditionally introduced for convenience.

In the dimensionless variables instead of (1.1)-(1.4) after some transformations we finally have the following system of equations:

$$\frac{1}{k_n} - 1 - \varepsilon_n + \hat{\alpha} \frac{d}{d\xi} k_n \frac{db_n}{d\xi} = 0, \quad (1.5)$$

$$\frac{C_{\varepsilon 1}}{k_n} - C_\theta - C_{\varepsilon 2} \varepsilon_n + \hat{\sigma} \frac{b_n}{\varepsilon_n} \frac{d}{d\xi} k_n \frac{d\varepsilon_n}{d\xi} = 0, \quad (1.6)$$

$$k_n = \frac{b_n^2}{\varepsilon_n}.$$

Here $\hat{\alpha} = \alpha_b \frac{\kappa^2}{\sqrt{C}}$; $\hat{\sigma} = C_\varepsilon \frac{\kappa^2}{\sqrt{C}}$.

To interpret the experimental observations we use the dimensionless function $\varphi_u(\xi) = \frac{k\,z}{u_*} \frac{dU}{dz} = \xi \frac{\partial u_n}{\partial \xi} = \frac{\xi}{k_n}$ and Richardson flow number $Rf(\xi) = -\frac{\frac{g}{T} q}{u_*^2 \frac{\partial U}{\partial z}} = \frac{\xi}{\varphi_u} = k_n$, which is monotonous and single-valued function of ξ.

For limiting cases of neutral stratification ($z/L \to 0$), strong instability ($z/L \to -\infty$) and strong stability ($z/L \to +\infty$) the similarity considerations allow to make a number of specific conclusions without solving the system (1.5)-(1.6).

If $\xi = z/L \to 0$ ($L \to \infty$ or $q \to 0$ or $z \to 0$), the parameter q ceases to be the parameter of the problem, and dependence of the functions $\frac{\partial U}{\partial z}$, $\frac{\partial \theta}{\partial z}$, ε upon it should fall out. The length scale also disappears, i.e. that regime is self-similar. It is possible only if at $\xi \to 0$ $\frac{\partial u_n}{\partial \xi}, \frac{\partial \theta_n}{\partial \xi}, \varepsilon_n \to \frac{1}{\xi}$, hence it follows that the values of universal functions approach the constants: $\varphi_u(0) = 1$, $\varphi_\varepsilon(0) = 1$. Such constant values are provided by Karman constant κ previously introduced into the definition of universal functions.

Thus, near to zero the following decompositions are true:

$$\left.\begin{aligned} u_n &= \ln\xi + \beta_1 \xi + const \\ \theta_n &= \ln\xi + \beta_1 \xi + const \\ b_n &= 1 + \gamma_1 \xi + \cdots \\ \varepsilon_n &= \frac{1}{\xi} + \delta_1 + \cdots \\ k_n &= \xi(1 - \beta_1 \xi) + \cdots \\ \varphi_u &= 1 + \beta_1 \xi + \cdots \end{aligned}\right\}, \quad (1.7)$$

It is verified by multiple atmosphere observations and laboratory experiments by studying surface layers in unstratified fluid.

According to the necessity of asymptotic (1.7) existence if $\xi \to 0$, the additional limitation for the values of Kε model empirical constants follows: $\hat{\sigma} = C_{\varepsilon 2} - C_{\varepsilon 1}$. At this for the decomposition coefficients it is possible to obtain:

$$\gamma_1 = \frac{1}{\hat{\alpha} - 2}; \qquad \delta_1 = \frac{3C_{\varepsilon 1} - C_{\varepsilon 2} - C_\theta (2 - \hat{\alpha})}{2(2 - \hat{\alpha})\hat{\sigma}}; \qquad \beta_1 = \delta_1 - 2\gamma_1 = \frac{3C_{\varepsilon 2} - C_{\varepsilon 1} - C_\theta (2 - \hat{\alpha})}{2(2 - \hat{\alpha})\hat{\sigma}} \quad (1.8)$$

The limiting case $z/L \to -\infty$ is the regime of purely convective turbulence, which may be obtained at $u_* \to 0$, i.e. u_* falls out from the determining parameters of the problem, hence it follows: $\varphi_u \approx C_u(-\xi)^{-1/3}$, $\varphi_\theta \approx C_\theta(-\xi)^{-1/3}$, $\varphi_\varepsilon \approx -C_\varepsilon \xi$. Then $Rf = k_n \approx -\frac{1}{C_u}(-\xi)^{4/3}$, $b_n \approx C_b(-\xi)^{2/3}$ (C_u, C_θ, C_ε, C_b are constants.)

The obtained asymptotic presents just qualitative information about behavior of universal functions and does not allow uniquely formulating the boundary conditions for b and ε for numerical solution of the system of equations (1.5), (1.6) at finite integrating interval.

The universal functions are considered rather well known, though different authors propose different formulas for experimental data interpolating. However, for the stratification, which is not very far from indifferent ($|\xi|<1$), different sources give close results especially for convective conditions. In [10] by processing of multiple experiments the formula adjusted for convective and stable conditions was obtained:

$$\varphi_u = \frac{\xi}{k_n} = \begin{cases} 1+\xi\left\{1+0.667\exp(-0.35\xi)\left[1+0.35(14.3-\xi)\right]\right\}, & \xi>0; \\ (1-19\xi)^{-1/4}, & \xi<0. \end{cases} \qquad (1.9)$$

Below we present the results of calculations of ASL characteristics within the frameworks of $K\varepsilon$ model.

2. Statement of boundary problem. Selection of values of kε model empirical constants.

As the basic sought functions we will use b_n and k_n. b_n corresponds to turbulent kinetic energy and is a positive and monotone decreasing function. Function k_n is also monotone and, according to the experimental observations, increase from $-\infty$ up to some finite value $k_n(\infty)\le 1$. Thus, the system of equations (1.5)-(1.6) is to be solved, which, if $\hat{\alpha}\ne 0$, with the variables b_n and k_n assumes the following form:

$$k_n b_n'' + k_n' b_n' = \frac{1}{\hat{\alpha} k_n}\left(b_n^2 + k_n - 1\right) \qquad (2.1)$$

$$b_n k_n'' + 4k_n' b_n' - 2\frac{k_n}{b_n} b_n'^2 - \frac{b_n k_n'^2}{k_n} = \frac{1}{\hat{\alpha}\hat{\sigma} k_n}\left[\left(2\hat{\sigma} - \hat{\alpha} C_{\varepsilon 2}\right) b_n^2 + \left(2\hat{\sigma} - \hat{\alpha} C_\theta\right) k_n + \hat{\alpha} C_{\varepsilon 1} - 2\hat{\sigma}\right] \qquad (2.2)$$

Here the stroke sign denotes ξ differentiation.

If we head for the dependencies (1.7) and (1.9) and attempt to approach them along the whole interval $-\infty<\xi<+\infty$, the boundary conditions for the system (2.1)-(2.2) will be as follows, taking into account the form of the functions k_n, b_n:

$$b_n = +\infty,\ k_n = -\infty \text{ at } \xi = -\infty \qquad (2.3)$$

$$b_n = 0,\ k_n = 1 \text{ at } \xi = +\infty. \qquad (2.4)$$

$$b_n = 1,\ k_n = 0 \text{ at } \xi = 0 \qquad (2.5)$$

Formally the model constants $C_{\varepsilon 2}, C_{\varepsilon 1}, C_\theta, \hat{\alpha}$ remain undefined. The constants $C_{\varepsilon 2}, C_{\varepsilon 1}, \hat{\sigma} = C_{\varepsilon 2} - C_{\varepsilon 1}, \hat{\alpha}$ are supposed to be uniform for the whole area $-\infty<\xi<+\infty$, where the solution is sought. If we consider the decomposition of the function (1.9), obtained from the experimental data, in series in the neighborhood $\xi=0$, we will obtain different decompositions to the right and to the left. Taking into account (1.7) the coefficient β_1 should be discontinuous that is C_θ should be taken as piecewise with discontinuity at $\xi=0$.

In meteorology the following constant values are considered to be conventional $C_{\varepsilon 2}=2; C_{\varepsilon 1}=1.45;\ \hat{\sigma}=C_{\varepsilon 2}-C_{\varepsilon 1}=0.55$; $\hat{\alpha}=0.54$; $C_{\theta}=1$.

However such selection of the constants is not unique [2,14]. For example, in [2] the description of self-similar profiles in experiments [13] is the basis for constant selection, and it is obtained that $C_{\varepsilon 2}=1.92; C_{\varepsilon 1}=1.43$; $\hat{\alpha}=1.7\cdot\hat{\sigma}=0.83$.

Values $C_{\varepsilon 2}, C_{\varepsilon 1}$ in these sets differ insignificantly. One can notice the difference in $\hat{\alpha}$ and the value C_{θ} remains virtually indefinite.

Below we describe the algorithms for selecting the values $\hat{\alpha},\ C_{\theta}$ for best description of the experimental observations at the finite interval $\xi\in[-a;+a]$ for the both nominal sets of constants $C_{\varepsilon 2}$ and $C_{\varepsilon 1}$ (which are assumed already known).

3. Behavior of solutions of the systems of equations of surface layer.

Let us investigate the behavior of the integral curves of the system (2.1), (2.2). We equate the right-hand sides to zero and obtain:

$$b_{n1}^2=\frac{C_{\varepsilon 1}-C_{\theta}}{C_{\varepsilon 2}-C_{\theta}};\qquad k_{n1}=\frac{C_{\varepsilon 2}-C_{\varepsilon 1}}{C_{\varepsilon 2}-C_{\theta}}. \tag{3.1}$$

The obtained flock of points is the solution of the system. Decomposition of the sought solution in the neighborhood $\xi=0$ is presented with the formulas (1.7), and the behavior of the integral curves near this point will be considered below.

It is known that $C_{\varepsilon 2}>C_{\varepsilon 1}$. We consider the behavior of the solution of the equations (2.1)-(2.2) at $\xi\in(0;+\infty)$. It would be natural to suppose that the solution should pass through the critical points $b_n\big|_{\xi=0}=1$, $k_n\big|_{\xi=0}=0$ и $b_n\big|_{\xi=+\infty}=b_{n1}$, $k_n\big|_{\xi=+\infty}=k_{n1}$. Since $0<k_{n1}\le 1$ and $b_{n1}^2\ge 0$ should be satisfied, then one should assume $0\le C_{\theta}\le C_{\varepsilon 1}$ at $\xi>0$.

When $\xi=-\infty$, the empirical formula (1.9) gives $k_n\big|_{\xi=-\infty}\approx c_k(-\xi)^{5/4}$. Let us find the decomposition of the solution of the equations (2.1)-(2.2) in the neighborhood $\xi=-\infty$ in the form $k_n\big|_{\xi=-\infty}\approx c_k(-\xi)^{\alpha}$, $b_n\big|_{\xi=-\infty}\approx c_b(-\xi)^{\beta}$. We obtain that the exponents α and β depend on the used values of the parameters $C_{\varepsilon 1}, C_{\varepsilon 2}, C_{\theta}, \hat{\alpha}$. From the allowed decompositions at $\xi=-\infty$ we select those, which correspond to the physical meaning of the problem, i.e. along the whole interval $\xi\in(-\infty;0)$ the conditions of monotone increase of the function k_n from $-\infty$ to 0 and monotone decrease of b_n from $+\infty$ to 1 should be satisfied. We obtain $3/2\le\alpha<2$ and $\beta=2-\alpha$ (that is $0<\beta\le 1/2$). Thus, at no values of the used parameters it is possible to obtain the solution of the equations (2.1)-(2.2), which would satisfy asymptotic of the empirical formula (1.9) at $\xi=-\infty$.

We consider the behavior of the solution at $\xi\approx 0$. Taking into account (1.7), at the fixed values $C_{\varepsilon 1}, C_{\varepsilon 2}$ it is always possible to select values $C_{\theta},\ \hat{\alpha}$, which allow to obtain coincidences for solution of the problem (2.1)-(2.2) with the values of the experimental curve slope at the right and at the left of $\xi=0$.

Thus, while solving the equations (2.1)-(2.2) for the selected fixed values $C_{\varepsilon 1}, C_{\varepsilon 2}$, at the expense of selecting $\hat{\alpha}$ and value C_{θ} discontinuous at $\xi=0$, one may accurately describe individually or the behavior of the experimental curve in the neighborhood $\xi=0$ or $\xi=+\infty$.

Though decompositions by $\xi=\pm\infty$, following from $K\varepsilon$ model, do not agree with the experimental formulas, which have different asymptotic at infinity with different authors, there may be no sense to try to satisfy them accurately. The experimental observations relate to the limited interval of change of dimensionless height, therefore the numerical integration of the equations (2.1)-(2.2) should be performed at the limited interval of height change. If we do not try to satisfy the empirical asymptotic at $\xi = \pm\infty$, but consider just the mean square deviation of the obtained solution from the experimental data at the limited interval, for example $-2 \le \xi \le 2$, then one may obtain good agreement with the experiment at the expense of parameter selection $C_\theta, \hat{\alpha}$.

4. Numerical integration of surface layer system of equations

We solve the system of equation (2.1)-(2.2) for the functions k_n, b_n. Since the coefficients included into the equations may be discontinuous at $\xi = 0$, integrating is performed by the two intervals: $[0.01; 2]$ and $[-2; -0.01]$. In the points $\xi_0 = \pm 0.01$ the values b_n and k_n are determined from the decomposition (1.7).

Let us determine the right boundary condition at the interval $[0.01; 2]$. The performed studying of the problem shows that at the right the values b_n and k_n quickly approach some constant positive values, which are formally dependent on the constants $C_{\varepsilon 1}, C_{\varepsilon 2}, C_\theta, \hat{\alpha}$. Therefore it is natural to take $\left.\frac{db_n}{d\xi}\right|_{\xi_R} = \left.\frac{dk_n}{d\xi}\right|_{\xi_R} = 0$ as a boundary condition.

We consider the interval $[-2; -0.01]$. $k_n \to -\infty, b_n \to +\infty$ is to be satisfied within the limit $\xi \to -\infty$. If we take into account the kind of the solution decomposition at $\xi = -\infty$, $\left.\frac{db_n}{d\xi}\right|_{\xi=-\infty} = 0$ is to be satisfied as well. This condition is satisfied with adequate accuracy already for values ξ at small module, therefore one may take the derivative value close to zero: $\left.\frac{db_n}{d\xi}\right|_{\xi_L} = -0.001$ as the left boundary condition for b_n. The value of k_n in the point $\xi_L = -2$ we determine from the condition $k_n(\xi_L = -2) = \frac{\xi_L}{\varphi_u(\xi_L)} \approx -\frac{2}{0.4}$, where $\varphi_u(\xi_L = -2) = 0.4$ is the experimental value determined from (1.9).

The system of equations (2.1)-(2.2) with specified boundary conditions was solved numerically by chaser method at the intervals $[0.01; 2]$ and $[-2; -0.01]$. The program of numerical solution of the given system was created. Calculations were made with various number of points *N*. Convergence of the results was obtained at *N*>50 in positive and negative areas.

It was assumed that the values $C_{\varepsilon 1}, C_{\varepsilon 2}$ have been determined earlier and are constant for both intervals. Value $\hat{\alpha}$ is assumed to be constant and value C_θ is assumed to be discontinuous: $C_\theta = \begin{cases} C_{\theta+}: & \xi > 0 \\ C_{\theta-}: & \xi < 0 \end{cases}$. A great number of calculations with different values of $\hat{\alpha}, C_\theta$ was performed with the purpose to determine such their values, which allow to obtain numerical solution describing in the best way (that is mean–square deviation) the empirical functional dependence (1.9) at the interval $\xi \in [-2; 2]$.

5. Comparison of numerical solution with experimental observations

As we have noted above, the following set of constant values is typically used in meteorology: $C_{\varepsilon 2}=2; C_{\varepsilon 1}=1.45; \hat{\sigma}=C_{\varepsilon 2}-C_{\varepsilon 1}=0.55$; $\hat{\alpha}=0.54$; $C_\theta=1$.

It was obtained that the resulting solution at $\xi>0$ very weekly depends on $\hat{\alpha}$ and strongly depends on C_θ. At this the calculated values k_n, b_n approach very quickly (just at $\xi\approx 1$) the constant values, which coincide with the values in the critical points (3.1) (that verifies the correctness of the numerical solution of the problem).

Actually, having numerical solution at the finite right interval of height change and knowing the solution behavior beyond the interval at large values of ξ, we can construct the solution along the whole interval $\xi\in(0;+\infty)$. For k_n the best agreement with the empirical dependence is obtained at $C_\theta=0$.

Thus, it is necessary to assume $C_\theta=0$ at $\xi\in(0;+\infty)$ and it is required to match $\hat{\alpha}$ and C_θ at the left integrating interval.

Let us take the value $C_\theta=1$ at $\xi<0$. According to calculation results the optimal value of the left parameter $\hat{\alpha}=1$ was selected, which provides the best agreement between calculation and experimental data. The results of calculation of the case $\hat{\alpha}=1$, $C_\theta=\begin{cases}0: & \xi>0\\ 1: & \xi<0\end{cases}$ are presented in Figure 1 as compared with the empirical data.

Let $\hat{\alpha}=0.54$. According to the calculation results the optimal value $C_\theta=1.6$ at $\xi<0$. The results of calculation of the case $\hat{\alpha}=0.54$, $C_\theta=\begin{cases}0: & \xi>0\\ 1.6: & \xi<0\end{cases}$ are presented in Figure 1 as well and almost coincide with the previous case.

Above we presented another used set of constants of *ke* model $C_{\varepsilon 2}=1.92; C_{\varepsilon 1}=1.43$; $\hat{\alpha}=1.7\cdot\hat{\sigma}=0.83$. The results of calculation of this case provide the following optimal values of parameters $C_\theta=\begin{cases}0: & \xi>0\\ 1.2: & \xi<0\end{cases}$. The calculating results are presented in Figure 1 as well.

As is seen, the solutions for all the obtained sets of constants almost coincide between each other and are close to the experimental data. The calculations at various values of parameters $\hat{\alpha}$ and C_θ were conducted, however, the available results would not be improved.

If we know the values of the used parameters, we may determine the character of the solution behavior at $\xi\approx 0$ and $\xi=\pm\infty$.

For example, the set of parameters $C_{\varepsilon 2}=2; C_{\varepsilon 1}=1.45; \hat{\sigma}=0.55$; $\hat{\alpha}=1$, $C_\theta=\begin{cases}0: & \xi>0\\ 1: & \xi<0\end{cases}$ provides the following behavior of the functions: $k_n\big|_{\xi=-\infty}\sim(-\xi)^{1.62}$, $b_n\big|_{\xi=-\infty}\sim(-\xi)^{0.38}$, and the decomposition coefficient for k_n in the neighborhood $\xi\approx -0$ (see (1.7)) $\beta_1=3.23$. At $\xi>0$ the values k_n, b_n quickly approach the constant values coincident with the values in the critical points (3.1), at this in the neighborhood $\xi\approx +0$ the value $\beta_1=4.17$.

The parameter set $C_{\varepsilon 2}=2; C_{\varepsilon 1}=1.45; \hat{\sigma}=0.55$; $\hat{\alpha}=0.54$, $C_\theta=\begin{cases}0: & \xi>0\\ 1.6: & \xi<0\end{cases}$ provides the following results: $k_n\big|_{\xi=-\infty}\sim(-\xi)^{1.54}$, $b_n\big|_{\xi=-\infty}\sim(-\xi)^{0.46}$, $\beta_1=1.38$ at $\xi\approx -0$ and $\beta_1=2.83$ at $\xi\approx +0$.

If $C_{\varepsilon 2} = 1.92; C_{\varepsilon 1} = 1.43;$; $\hat{\alpha} = 0.83$, $C_\theta = \begin{cases} 0: & \xi > 0 \\ 1.2: & \xi < 0 \end{cases}$, then $k_n\big|_{\xi=-\infty} \sim (-\xi)^{1.59}$, $b_n\big|_{\xi=-\infty} \sim (-\xi)^{0.41}$, $\beta_1 = 2.55$ at $\xi \approx -0$ and $\beta_1 = 3.78$ at $\xi \approx +0$.

Thus, the values of the model parameters are obtained, at which the numerical solutions of the system (2.1)-(2.2) describe well the experimental dependence (1.9) along the interval $\xi \in [-2; 2]$.

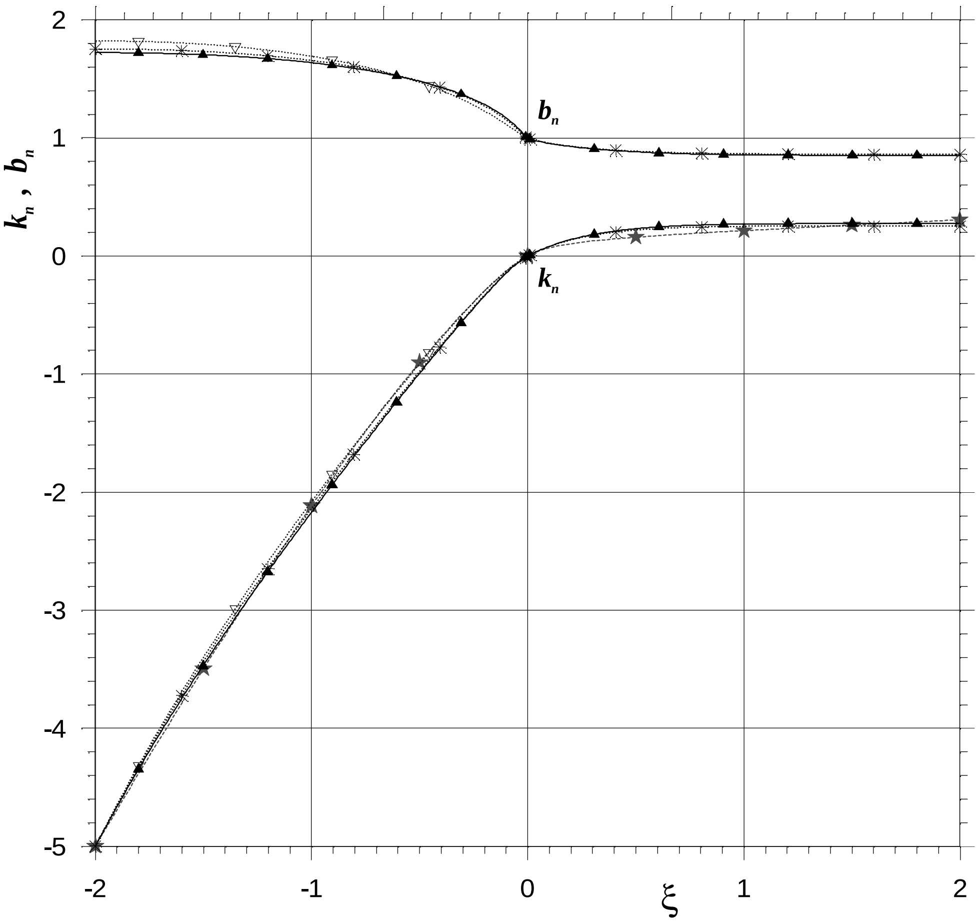

Figure 1. Dependence of functions k_n, b_n on dimensionless height ξ:

--▲-- calculation with $C_{\varepsilon 2} = 2; C_{\varepsilon 1} = 1.45; \hat{\sigma} = 0.55$; $\hat{\alpha} = 1$, $C_\theta = \begin{cases} 0: & \xi > 0 \\ 1: & \xi < 0 \end{cases}$;

- - - calculation with $C_{\varepsilon 2} = 2; C_{\varepsilon 1} = 1.45; \hat{\sigma} = 0.55$; $\hat{\alpha} = 0.54$, $C_\theta = \begin{cases} 0: & \xi > 0 \\ 1.6: & \xi < 0 \end{cases}$;

- - ⋇ - - calculation with $C_{\varepsilon 2} = 1.92; C_{\varepsilon 1} = 1.43;$; $\hat{\alpha} = 0.83$, $C_\theta = \begin{cases} 0: & \xi > 0 \\ 1.2: & \xi < 0 \end{cases}$;

- - ★ - - the empirical curve [10],

Conclusions

Within the frameworks of $K\varepsilon$-model the problem about determination of dimensionless characteristics of turbulent flow in atmosphere surface layer is considered.

The complete investigation of $K\varepsilon$-equations and their critical points are conducted. From the variety of the integral curves the ones are selected, which correspond to the solution of the stated problem and describe quite satisfactorily all experiments. At this the basic empirical constants of the model $(C_{\varepsilon1}, C_{\varepsilon2})$ are selected according to the conventional criteria. At the same time it is shown that parameter C_θ responsible for the convective source term in the equation for ε should be selected different depending on stability status of the surface layer. At this the best agreement with experimental observations is obtained, if $C_\theta = 0$ in the area of stable stratification and $C_\theta \neq 0$ in the area of unstable stratification.

The program, which allows solving the obtained model equations, is created. The calculations for various values C_θ and $\hat{\alpha}$ are conducted.

By the numerical matching of values C_θ and turbulent diffusion coefficient $\hat{\alpha}$ the quite satisfactory description of the experimental observations is obtained at the finite interval of dimensionless height change for any state of atmosphere stability.

Taking into account the obtained asymptotic of the model equation solutions for the values of dimensionless height $\xi = \pm\infty$, the solution is virtually constructed along the whole interval $\xi \in (-\infty; +\infty)$.

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