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Richtmyer–Meshkov Instability in Inviscid and Viscous Flows

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Abstract. A new formula for the growth rate of the Richtmyer-Meshkov instability is proposed in the framework of the impulsive model. It allows us to predict the growth rate in heavy-light or light-heavy configurations. This expression reduces to the Richtmyer or the Meyer & Blewett formulas in specific cases. Extension to viscous flows has also been carried out. Comparisons are performed against numerical simulations.

1 Introduction

When two materials are impulsively accelerated into each other by a shock wave, small perturbations of the interface grow, first linearly and then into nonlinear structures. Under some circumstances, these nonlinear structures evolve towards a turbulent state. In a simplified picture of turbulence, the turbulent energy cascades up to a wavenumber small enough so that the dissipation acts. On the contrary, the behavior of the large scales is usually inviscid. The problem described above is a Rayleigh-Taylor (RT) type instability. It was discovered and analyzed by Richtmyer [1], and confirmed experimentally by Meshkov [2]. Richtmyer first established [1] a formula that gives the growth rate of the instability during its linear phase. It reads

$$\frac{da}{dt} = k\,[u]\,A_t^+ a(0^+), \tag{1}$$

where a is the amplitude of the perturbation, k its wavenumber, $[u]$ the velocity jump across the shock wave, $a(0^+)$ the amplitude immediately after the shock passage and A_t^+ the Atwood number after the interaction. The expression (1) gives relatively good results for light to heavy accelerations. For heavy to light acceleration, Meyer & Blewett found, on empirical grounds, that the term $a(0^+)$ in Eq.(1) has to be replaced by $(a(0^-) + a(0^+))/2$ where $a(0^-)$ is the preshock amplitude [3]. A more sophisticated approach has been proposed by Fraley [4] and used by Mikaelian [5]. Recently, Yang et al. compared Richtmyer's impulsive model to a small amplitude theory and noticed disagreement between the two [6].

In this paper, we present a new formula for the growth rate of the Richtmyer-Meshkov (RM) instability in the framework of the impulsive model. It allows us to predict the growth rate in heavy-light or light-heavy configurations, as long as compressibility effects are weak. This formula is first applied to inviscid flows and then generalized to viscous mixing flows with diffusion between species. Both formulas have been validated against numerical simulations performed with the code CADMEE [7].

2 Inviscid Richtmyer's model revisited

2.1 Derivation of the model

The linear analysis of the RT instability gives the differential equation for the amplitude of a perturbation in inviscid, incompressible fluids,

$$\frac{d^2a}{dt^2} = kgA_t a(t), \tag{2}$$

where g is the acceleration. The RM instability, which is characterized by a jump velocity $[u]$ of the interface imparted by the shock, is usually treated by the impulsive model [1]. In this approach, the shock is considered as an instantaneous acceleration of incompressible fluids, i.e., replacing g in Eq.(2) by $[u]\delta(t)$ where $\delta(t)$ is the Dirac function. However, it appears an ambiguity in the initial conditions. Richtmyer's recipe is to choose $a(0)$ and A_t after the shock passage. If we model the temporal evolutions of the amplitude and the Atwood number during the shock passage, we can obtain the following formula for the RM instability growth rate:

$$\frac{da(t)}{dt} = \frac{1}{2}\left(a(0^-)A_t^- + a(0^+)A_t^+\right) k\,[u]\,. \tag{3}$$

In situations where $a(0^-)A_t^- \approx a(0^+)A_t^+$, we find Richtmyer's classical result given by Eq.(1). In situations where the Atwood numbers, before and after the interaction, are very close to each other, i.e., $A_t^- \approx A_t^+$, Eq.(3) reduces to Meyer & Blewett's result.

2.2 Numerical results

Two configurations helium/air and air/helium, borrowed from Ref.[3], were simulated. It turns out that Richtmyer's formula and Meyer & Blewett's recipe are in good agreement with the simulations for the light-heavy and the heavy-light configurations, respectively. However, let us note that in the heavy-light configuration, the Atwood

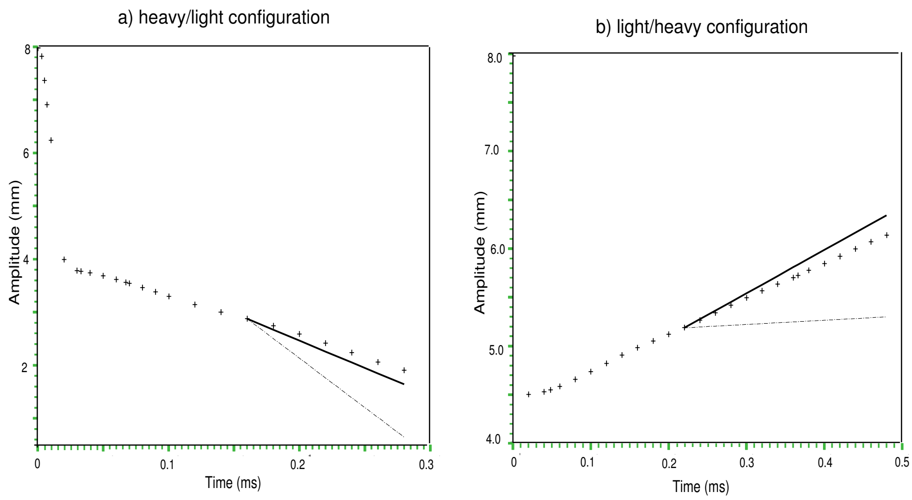

Figure 1: (a) Evolution of the perturbation amplitude $a(t)$ versus time for the heavy-light CO_2/argon configuration (run 1 in Table 1). Crosses correspond to values obtained from simulation. The formula proposed in this paper (bold line) agrees much better than Meyer & Blewett's result (line). (b) Same curves as in (a), for the light-heavy argon/XX configuration (run 2 in Table 1). As in (a), the formula proposed in this paper (bold line) agrees much better than Richtmyer's result (line).

number does not vary across the shock wave. For the light-heavy situation, this is the product aA_t that does not vary. In both cases, Eq.(3) gives good values of the growth rate. Other configurations have been designed to enhance differences between classical expressions and the new formula (3). Here, we present two of them. The incident shock Mach number is equal to 1.6. The wavelength and the initial amplitude of the perturbation are: $\lambda = 6.283\ cm$ and $a(0^-) = 0.8\ cm$. The first one is a CO_2/Ar case to compare with Meyer & Blewett's prescription (see Fig.(1a)). The other one is an Ar/XX case, where XX is a fictitious gas, ($\gamma = 1.9$, $\mathcal{M} = 44\ g/mol$) and Richtmyer's formula should apply (see Fig.(1b)). In Table 1, the different growth rates and the parameters $\left(A_t^-/A_t^+\right)$ and $\left(a(0^-)A_t^-/a(0^+)A_t^+\right)$ are given.

It can be verified that when the Atwood number is not constant, Meyer & Blewett's growth rate is far from the one given by the simulation, even for a heavy-light configuration. In the same way, when the product aA_t is not constant, Richtmyer's prescription fails to give the good result for the light-heavy case. On the other hand, Eq.(3) is in good agreement with CADMEE simulations, whatever the studied configuration (light-heavy or heavy-light) is. Comparisons with Yang et al.'s theory consolidate this conclusion: formula (3) gives good results provided that the shock strength is not too large and the

run	da/dt $CADMEE$	da/dt $M\&B$	da/dt $Richtmyer$	da/dt $Eq.(3)$	A_t^-/A_t^+	$\frac{a(0^-)A_t^-}{a(0^+)A_t^+}$
1	-9	-18.6	(-12.1)	-10.3	0.337	0.7
2	3.5	(0.59)	0.43	4.44	11.25	19.9

Table 1: Growth rates of the runs, given by the code CADMEE, Meyer & Blewett, Richtmyer and the formula (10). Brackets indicate situations where the corresponding formula does not apply.

two γ not too different, i.e in incompressible flows.

3 Extension to viscous flows

3.1 The linear Rayleigh-Taylor instability theory

The growth rate of the RT instability in presence of viscosity [8] and diffusion [9] is given by the expression:

$$n = (\frac{A_t g k}{\Psi} + \nu^2 k^4)^{1/2} - (\nu + D)k^2. \tag{4}$$

where $\Psi = \Psi(\alpha, A_t)$ with $\alpha = (kl)^{-1}$ and $l = 2(Dt)^{1/2}$. The kinematic viscosity ν is equal to $(\mu_2 + \mu_1)/(\rho_2 + \rho_1)$ and D is the diffusion coefficient.

3.2 Generalization to the Richtmyer-Meshkov instability

As it was done in the inviscid case, the relation of dispersion (4) can be generalized to the RM instability. We have to solve the differential equation:

$$\frac{d^2 a}{dt^2} + 2(\nu + D)k^2 \frac{da}{dt} + \left[D(D+\nu)k^4 - \frac{A_t g k}{\Psi} \right] a = 0, \tag{5}$$

where Ψ is supposed to be constant and where g is replaced by $[u]\delta_\epsilon(t)$ as in the inviscid case. The solution of this equation is, for $t \geq \epsilon$:

$$\begin{aligned} a(t) &= \left[a_1 + (a(\epsilon) - a_1) e^{-2\nu k^2 (t-\epsilon)} \right] e^{-Dk^2(t-\epsilon)} \\ a_1 &= a(0^-) + a(0^+) - a(\epsilon) + \frac{[u]}{4\nu k \Psi} \left(a(0^-) A_t^- + a(0^+) A_t^+ \right) \\ &+ \frac{D}{2\nu} \left[2\left(a(0^-) + a(0^+) - a(\epsilon)\right) - a(\epsilon) - (2\nu + D) k^2 \epsilon \left(a(0^-) + a(0^+)\right) \right] \end{aligned} \tag{6}$$

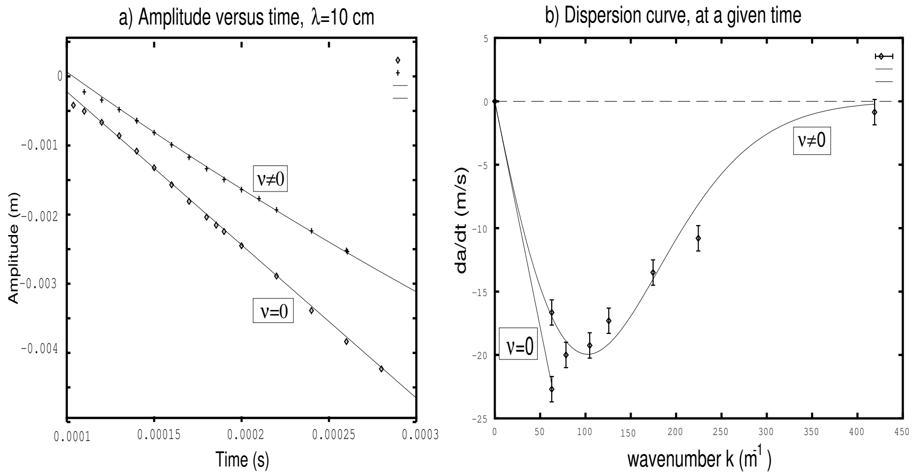

Figure 2: (a) Evolution of the amplitude of a perturbation for $\lambda = 10$ cm, with and without viscosity. The points correspond to the simulations and the full lines to the models. (b) Dispersion curve, at a given time. Each symbol is the result of a numerical simulation and the full lines correspond to the models.

3.3 Theory and numerical simulation comparisons

To assess the effects of molecular viscosity and diffusion, we compared the solutions given by formula (6) with the results of numerical simulations on air/helium shock tube flow at Mach 1.5. The initial amplitude of the interface perturbation is 2 mm peak to peak and we consider several values for the wavelength λ. To avoid too long runs, the values of the viscosity and diffusion coefficients have been multiplied by 10^4.

- Viscosity effects: Fig.(2a) presents the evolution of the perturbation amplitude $a(t)$ when $\lambda = 10$ cm, with and without viscosity. These curves show that there is a good agreement between the simulations (points) and the theory (lines), in both cases. Others simulations were performed with several values of the perturbation wavelength. Here again, we note that the expression (6) fits well the simulation results. Fig.(2b) presents the variation of the growth rate versus the wavelength, at a given time when all perturbations are still in linear phase. As we can see in Fig.(2b) there is a very satisfactory agreement between the model proposed in Eq.(6) and the numerical approach. In this figure, we also mentioned the straight line corresponding to the perturbation growth rate without viscosity. These curves clearly show that the main effect of viscosity is to attenuate the growth rate of the perturbations.

- Diffusion effects: we also compared the diffusive model given by Eq.(6) with numerical simulations. We verified that theoretical and numerical results agree. This work will be reported in a forthcoming paper.

4 RM Instability with a Multimode Interface

In order to understand physical phenomena involved in shock tube mix experiments, three preliminary simulations have been performed with the code CADMEE. A multimode interface is accelerated by a 1.4 Mach number shock wave moving from xenon to air, as in the experiments carried out in the shock tube laboratory at Vaujours. The characteristics of these three runs are summarized in Table 2.

Fig.(3) shows the xenon concentration contour plots at three different times. These contour plots are used to calculate the mixing zone width. Fig.(4a) displays the evolution versus time of the mixing zone. The compression due to the interaction of the shock wave with the mixing zone can be seen. The mixing zone width obtained from the coarse run is at any time larger than those obtained from the two other simulations. This is principally due to the initial amplitudes of the perturbation ($8.5\,mm$ for the coarse run and only $2\,mm$ and $2.5\,mm$ for the medium and the fine runs respectively). The evolution of the average vorticity in the mixing zone is displayed in Fig.(4). We clearly see the strong generation of vorticity at each interaction with a reshock. The important result is that the medium and fine runs converge when the number of zones is increased. This point may be seen in the evolution of the mixing zone and in the average vorticity. The differences in the maxima of vorticity at each reshock may be partially due to different temporal resolutions.

5 Conclusion

A new formula for the growth rate of the RM instability is proposed within the incompressible inviscid impulsive model. It reconciles all the recipes available in the literature for inviscid flows. It has been validated against numerical simulations. This formula holds for shock strength not too large and γ not too different. Richtmyer's and Meyer & Blewett's prescriptions appear to be particular cases of this new formula. The success of these recipes is due to the fact that either the Atwood number or the product aA_t

run	*number of zones*	*zone sizes*(mm^2)	*initial amplitude*(mm)	*initial wavelengths*(mm)
coarse	144000	1×0.25	$a = 2.$	$3.5, 3, 2.5, 2$
medium	236800	0.5×0.25	$a = 0.5$	$3.5, 3, 2.5, 2, 1$
fine	551000	0.35×0.25	$0.3 \leq a \leq 0.4$	$3.5, 3, 2.5, 2, 1, 0.64$

Table 2: Characteristics of the three multimode calculations.

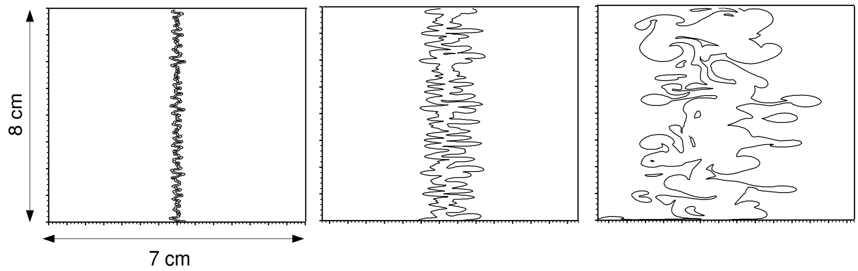

Figure 3: Contour plots of the concentration at: (a)$t = 0$, (b) $t = 1\,\mu s$ and (c) $t = 1.7\,\mu s$. Two isovalues are displayed. $c = 95\%$ on the left and $c = 5\%$ on the right.

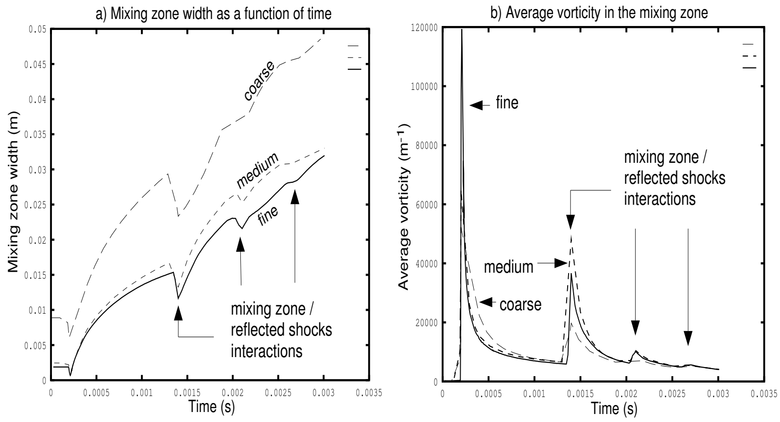

Figure 4: (a) Evolution of the mixing zone width versus time. The medium and fine runs are almost converged. Note the compression of the mixing zone at each reshock. (b) Evolution of the average vorticity in the mixing zone versus time. The quantity increases at each reshock on a very short time scale.

were fortunately time-independent. Extension to viscous flows has also been achieved. Comparisons with numerical simulations show a remarkable agreement. Preliminary high-resolution calculations of the RM instability with a multimode interface have also been presented.

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