Originally published in *Proceedings of the Fifth International Workshop on Compressible Turbulent Mixing*, ed. R. Young, J. Glimm & B. Boston. ISBN 9810229100, World Scientific (1996).

Modeling of Bubble and Spike Velocities for a Rayleigh-Taylor Instability

Mike Clover, William Powers, and Chan Choi

Los Alamos National Laboratory
MS: F663
Los Alamos, NM 87545

Abstract. We consider the two dimensional incompressible Rayleigh-Taylor unstable flow between parallel plates engendered by a single mode. We use LASNEX, a compressible, Lagrangian code, with various remapping strategies to simulate the flow and to compare the results to the analytical results of Layzer [1] and the numerical modifications of Ofer et al. [3]. Not surprisingly, we conclude that the best results are obtained with the minimum of remapping. The difficulty, and major concern of this work, is what is the optimum remapping strategy which, while keeping diffusion to a minimum, retains accuracy and permits completion of the simulation.

1 Introduction

For inertially confined fusion (ICF) the development of hydrodynamic instabilities can have significant effects upon the implosion and subsequent yield. For this reason it is important that the growth and development of these instabilities be adequately modeled by codes utilized in the design and simulation of ICF experiments. The instability environment encountered in such simulations is highly compressible, with large density gradients and Mach numbers. Lacking adequate analytic solutions in this regime, we seek guidance in the incompressible regime. It is hoped that the experience gleaned from the analysis of code performance in this regime will aid us in more adequately modeling instability growth in the former.

LASNEX, a finite volume, compressible, Lagrangian hydrodynamics code, will be used to model the growth and development of a Rayleigh-Taylor instability for an inviscid, incompressible medium. In addition, LASNEX provides various remapping schemes from full Eulerian to semi-Eulerian. The prevailing wisdom in the ICF community is that the Lagrangian formulation is most accurate, despite the difficulties associated with the significant distortion of the mesh which such flows induce. However, eventually the mesh becomes so entangled that all progress is halted by the diminishing timestep. At this point further progress can only be made by the implementation of various remapping strategies. Consequently, in practice, remapping is only used as a last resort to advance the calculation. It is well known that accuracy of the hydrodynamic algorithms, as well as other physics, would benefit from a more regular mesh. However, the diffusion engendered by remapping must necessarily affect the result; and less diffusive remapping schemes, unless flux limited, lead to spurious oscillations. The opportunity to investigate the accuracy and effectiveness of various remapping philosophies is offered by the comparison of simulation and analytic results.

Consider the two dimensional Rayleigh-Taylor unstable flow between parallel plates engendered by a single mode. By symmetry we only need consider the flow between the two parallel plates of half a wavelength. We will refer to the surface displacement opposite to the direction of gravity as the bubble, and that in the direction of gravity as the spike.

For a two dimensional flow between two parallel plates, the velocity potential ϕ during the linear phase is given by (Layzer [1])

$$\phi = v_0 e^{\sigma t} e^{-k|z|} cos(kx) \, ,$$

where t is time, the perturbed surface is centered about $z = 0$, and

$$\sigma = \sqrt{gkA_t} \, .$$

Here k is the wave number, g is the gravitational acceleration, and A_t is the Atwood number.

The two dimensional velocity field is given by $\vec{v} = -\nabla\phi$; and the perturbed surface by $\vec{r} = \vec{v}/\sigma$. Hence, when setting up the initial conditions both velocity and spatial perturbations must be established not only at the interface, but throughout the mesh. Violating this condition establishes unphysical initial conditions. However, apparently the hydro code will over a period of time establish the correct physical conditions.

For later times, the growth rate decreases when the amplitude becomes about 10% of the wavelength. The growth of higher order modes manifests itself in the development of a rising bubble and a descending, more narrow spike. The relative width of the bubble and spike is Atwood number dependent, with Atwood numbers near one exhibiting the greatest effect, while for Atwood numbers near one, there is little difference in the relative widths (Hoffman [2]). Eventually the flow reaches a regime which is nearly steady-state, where the bubble rises at a constant velocity. For Atwood numbers near one, Layzer [1] has derived a time-dependent velocity of the bubble from early to late times.

Layzer gives the late time asymptotic velocity as

$$V_\infty = \left(\frac{gR}{3\pi}\right)^{1/2} \, ,$$

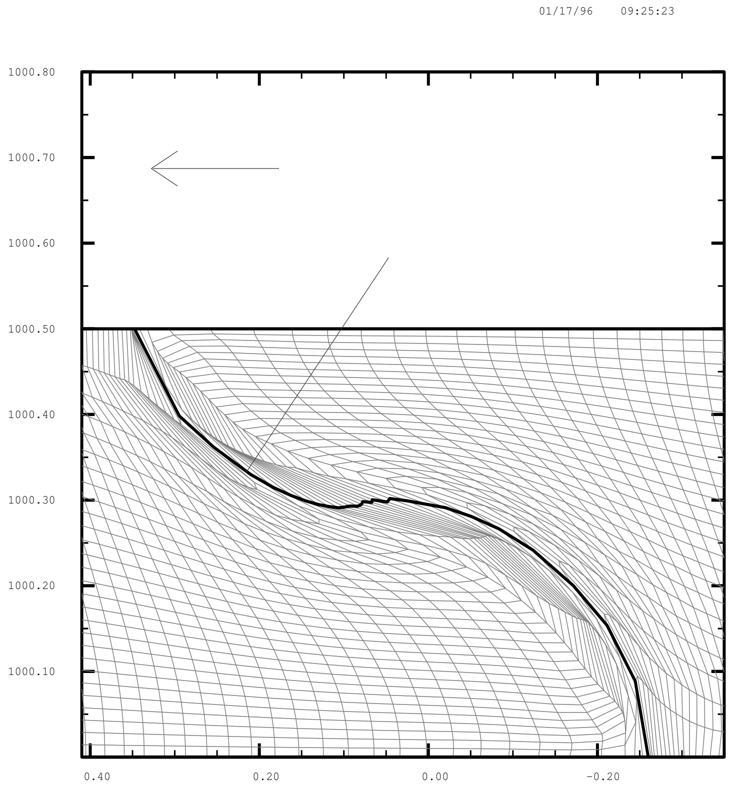

Figure 1:

where R is chosen to be half the wavelength. For Atwood numbers not near one, Ofer et al. [3] have found numerically that

$$V_\infty = \alpha \sqrt{\frac{2A_t}{1+A_t} g\lambda},$$

where $0.2 \geq \alpha \leq 0.3$. For a physical interpretation of this result see Hoffman [2].

For all calculations, we have chosen a value of $1cm/shake^2$ for the gravitational acceleration, a wavelength $\lambda = 1cm$, and, owing to the difficulties associated with Atwood numbers near one, an Atwood number of 1/3.

2 Lagrangian Simulations

The pure Lagrangian simulations all crashed for times greater than 8.5 shakes due to excessive distortion of the mesh. An examination of figure (1) indicates why. The shear generated at the interface between the two fluids produces severely elongated zones, resulting in small courant-limited timesteps.

Despite these difficulties, the Lagrangian mode runs through the linear growth stage. It is therefore possible to test the adequacy of various parameter settings by computing the growth rate σ from the resulting simulation. These results indicate that as the number of zones along the interface increases, and the perturbation is better resolved, correct linear growth rate is approached. Nevertheless, the approach to the theoretical value, although rapid for less than about 30 zones/wavelength, is very slow for larger resolutions. At 30 zones/wavelength, the growth rate is about 94% of the theoretical value, while at 80 zones/wavelength it's about 98%. Of course, it is also true that if the interface is poorly resolved, higher order modes are inadvertently introduced. There is also evidence that beginning with only a velocity perturbation yields inferior results to those obtained with only a surface perturbation. Indeed, these results indicate that the asymptotic fraction of the theoretical linear growth rate obtained with only a velocity perturbation is significantly lower than that for a surface (or surface plus velocity) perturbation. When the correct initial conditions are established, the results are marginally improved over that of a pure surface perturbation. The principal advantage being that no hydrodynamic adjustment is required; thus the flow is operating within the linear regime from the beginning of the run. Experience from these runs has also emphasized the importance of mass matching across the interface and correctly formulating the artificial viscosity.

3 Remapped Simulations

There were a number of remapping options attempted. The best remapping philosophy keeps the mesh fairly regular, thereby maintaining satisfactory timesteps, while not altering the essential physics of the simulation. It is for this reason that, especially when there is a large density gradient across the interface, that we would prefer not mixing material across the interface, thereby, rapidly diffusing material. However, under the present remapping schemes available in LASNEX, prohibiting remapping across the interface implies that along a curved interface the remapping of points along the interface is also prohibited, resulting in little improvement over a strictly Lagrangian simulation.

Since the best remapping option available appears to constrain the simulation to mixing across the interface, we would prefer to minimize that mixing. For this reason, we attempted to preserve the Lagrangian shape of the interface by constraining the remap along the interface to a cubic spline interpolation of the initial Lagrangian interface.

We will principally discuss two remapping strategies. The first, referred to as mrv39, utilizes a spline fit of the interface but is heavily and equally remapped throughout the problem. At 8.5 shakes, when the pure Lagrangian run has crashed due to timestep controls, the mesh for mrv39 looks regular and nearly orthogonal throughout the simulation.

The second remapping strategy is exemplified by the run referred to as mrv42. In this simulation there is less remapping being performed throughout the problem than for mrv39, additionally, considerably more remapping is enforced near the material interface than for the remainder of the mesh, although less than for mrv39. At the early time of 8.5 shakes, the meshes for mrv39 and mrv42 are virtually identical. However, run mrv42 crashes due to timestep controls at about 10.2 shakes, whereas mrv39 could apparently run indefinitely.

In order to address the question as to how to define the location of the bubble and spike under remapped simulations, the number fraction of the lighter gas was plotted at 8.5 shakes for both mrv39 and mrv42. Comparing these results with the pure Lagrangian simulation mrv35, indicates that the material interface is not associated with any particular number fraction. Near the spike, the 99% contour appears to best reflect the location of the spike if we are to trust the results from the Lagrangian simulation; while near the bubble vertex, the location of the bubble is best reflected by the 1% contour.

The asymptotic bubble velocities derived from the 50%, 95%, and 99% contours were all within the expected limits, with the higher percentage contours being suspiciously close to the upper bound. In all cases, remapping was turned on at 4 shakes. Contrasting the linear growth rates obtained before remapping was turned on with that afterwards, indicated that the results obtained with the 50% contours was marginally superior. Additionally, as expected, the spike velocities derived from the 50% contours exceeded the bubble velocities for all times, whereas this was not always true for the 99% contours. For these reasons we have chosen the 50% contour to describe the location of the bubble and spike.

As figure (2) indicates, the accuracy and efficacy of the different remapping strategies produces little distinction between the linear growth rates obtained, regardless of the fractional contour chosen. All results are about 95% of the theoretical value. As with the pure Lagrangian simulation mrv35, mrv39 and mrv42 were initiated with only a velocity perturbation. The linear growth rate derived from the mrv35 simulation is marginally above 95% of the theoretical value, implying that remapping has not adversely affected the linear growth rate. Consequently, on this basis alone, there is no reason to prefer full or partial remapping.

However, if we compare the shape of contours for the different remapping strategies, we find marked differences. Comparing the mrv39 and mrv42 simulations at 8.5 shakes, we find that the contours associated with mrv42, which was less heavily remapped, are more compact, indicating reduced diffusion of material across the material interface. Additionally, although it is difficult to judge, comparing these results with that of the pure Lagrangian run suggests that less remapping has more truthfully retained the

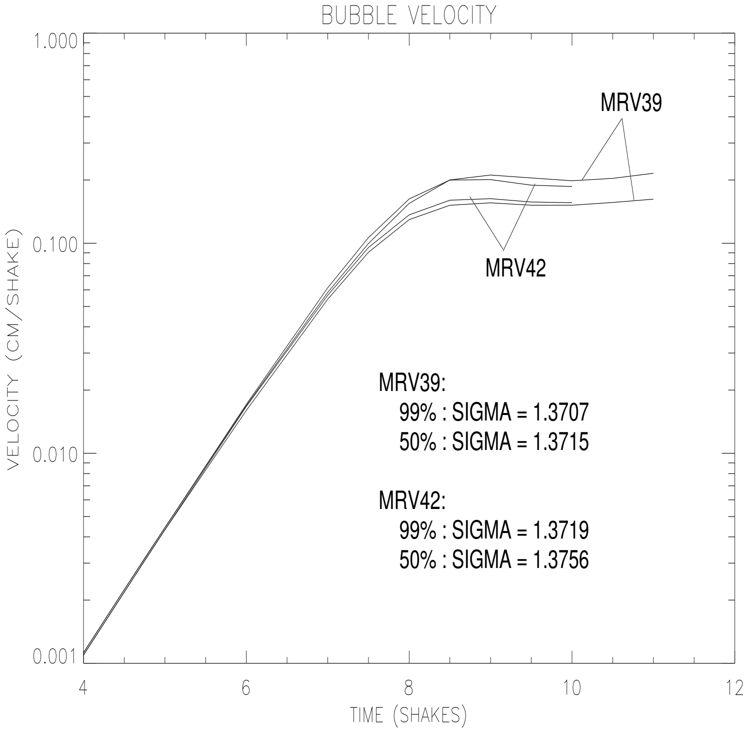

Figure 2:

shape of the bubble and spike.

Similarly, at 10 shakes, the less remapped simulation is less diffusive and appears to more fully capture the features of the flow. Figure (3) shows the contours near 50% for mrv42. The spike, bubble and spline interpolation of the interface are also indicated.

However, of greatest interest for future work is that the material interface defined by the spline interpolation much more closely tracks that of the contours for the less remapped simulation. Thereby, offering hope that with even less remapping or more clever remapping strategies that the material interface may be more truthfully represented with a minimum of diffusive remapping.

Authors Note: In order to conform to page limitations, most of the figures have been removed from this article. Should you desire a copy of the original document, contact Bill Powers.

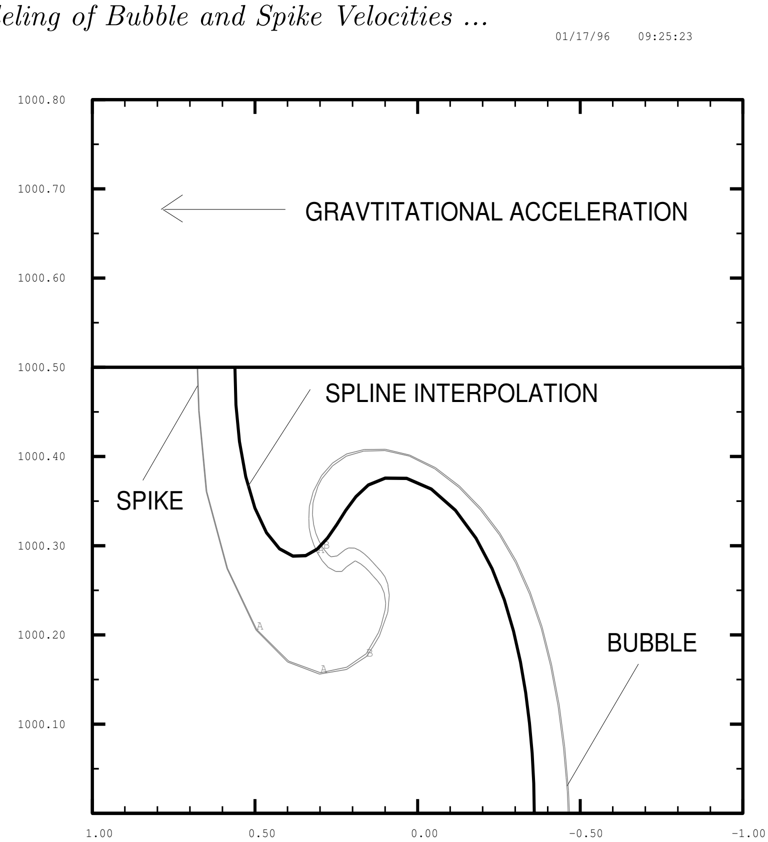

Figure 3:

References

[1] David Layzer, On the Instability of Superposed Fluids in a Gravitational Field, Astr. J, vol 122, pp 1-12, 1955.

[2] Nelson Hoffman, Hydrodynamic Instabilities in Inertial Confinement Fusion, Los Alamos Unclassified Report, LA-UR-94-3945, 1994.

[3] D. Ofer, D. Shvarts, Z. Zinamon, S.A. Orszag, Mode Coupling in Nonlinear Rayleigh-Taylor Instability, Phys Fluids B, vol 4, pp 3549, 1992.