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Quantitative Measurement Feasibility for 3D-Distributions of Hydrodynamic Quantities in the Turbulent Mixing Zone of Two Gases*

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1 Introduction

The study of turbulent mixing involves model experiments to understand the main conformities of this process, including those on hydrodynamic systems like shock tubes [1]. The important problem in the experimental investigation of instability evolution and turbulent mixing is not only the form of the desired turbulent motion of the media (gas-gas, gas-fluid, fluid-fluid), but also to extract quantitative data on the turbulent mixing zone (TMZ) structure and parameters. For study of physical properties of TMZ flows, it is preferable to use methods which do not introduce additional perturbations on the instability evolution zone. For this purpose it is desirable to use radiation to which the medium to be studied is sufficiently transparent, and visible light sources are commonly used in the cases of gaseous or liquid media. Experimental results obtaining the distribution of certain physical characteristic of the flow integrated along the light propagation trajectory. Using different experimental methods one can obtain shadow images, record phase shifts of the probing light wave moving through the TMZ with the interferential method or evaluate parameters from the absorption of probing radiation.

The integral distributions of flow characteristics themselves allow one to obtain certain quantitative information about the TMZ structure but local 3D-distributions (3DD) are more informative. It is possible to reconstruct 3DD if we can obtain integral ones from several directions of view (aspects). Such problems often appear in physics

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and medicine and are solved using the so called "reconstructive tomography" methods [2, 3, 4],

To extract needed physical values from experimental data, it is usually necessary to provide some mathematical procedures with them. From this point of view, tomography is a version of such procedures but much more complicated.

Although the mathematical basics of tomography appeared about 80 years ago [5], the use of reconstructive tomography became possible with the progress in computers. The numerous algorithms of 3DD reconstruction were developed for practical purposes. Some of them provide high accuracy at minimal number of aspects ($\leq 5 \div 10$) [6]. It means that it is possible to get 3DD data using several "identical" measurements without sophisticated complication of experimental techniques.

In this paper, we discuss the possibility of using few aspects laser interferometric tomography to reconstruct 3D density distributions of two gases in TMZ.

2 Optical Interferometry in the Problem of Turbulent Mixing of Gases

Choosing the method of measurements, we should be guided by those physical quantities which we want to receive as a result of reconstruction. If we are interested in obtaining the mixed gas density distribution, the most attractive as it seems to us, may be the method of optical interferometry. The fact is that the gas refraction indices differ slightly from unity ($n - 1 < 10^{-3}$), therefore it is possible to neglect the deflection of a light beam at a characteristic length of $L \approx 10$cm in the first approximation, and so the procedure of reconstruction may be appreciably simplified. Besides that, the refraction indices of gases and liquids are proportional to density. Thus the integrals of density along the light trajectories are directly obtained as a result of measurements. This means that we have to deal with reconstruction of a positive defined value which also simplifies this procedure. At last, the interference fringes system configuration does not depend on absorption of light in a gas, as the absorption influences only the contrast of a forming picture. Automatic formation of coordinate lines in interference patter may also be noted as a positive factor that simplifies spatial orientation.

The optical interferometric method for the evaluation of the material composition (density) (e.g. [7]) relies upon the following:

1. The phase of the light wave φ passing through the volume to be studied with length L along the light trajectory varies by

   $$\frac{2\pi}{\lambda} \int_L n \cdot dx,$$

   where λ is the light radiation wavelength and n is the material refraction index along L.

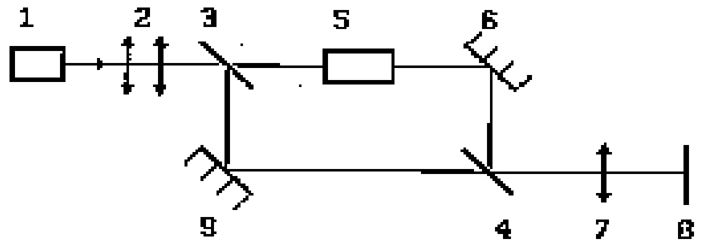


Figure 1: Schematic of Mach-Zehnder interferometer. 1. light source. 2. telescope. 3,4. semi-transparent mirrors. 5. volume to be studied. 6,9. rotation mirrors. 7. objective. 8. screen.

2. When one material is replaced with another, the light wave phase after passing through the volume shifts by

$$\Delta\varphi = \frac{2\pi}{\lambda} \int_L \Delta n \cdot dx,$$

where Δn is the difference of the refraction indices in the two materials.

3. $\Delta\varphi$ shifts are recorded in the interferential image occurring from the addition of the light wave passing through a medium whose properties may be changed and the non-perturbed light wave passing through a medium whose properties are constant (note that both waves must be coherent).

Figure 1 shows a schematic of a Mach-Zehnder interferometer which is commonly used for the study of gas flows. In [8] such an interferometer was used to find the gas density distributions and in [9, 10] to determine the density profile in experiments to study Rayleigh-Taylor instabilities, to visualize the mixing of flows, and to evaluate the boundary velocity for Richtmyer-Meshkov instabilities.

The light from the optical source 1, after the telescope lenses 2, goes to the mirror system. Semi-transparent mirrors 3 and 4 are designed to divide the source beam into the measurement beam passing through the volume 5 and reference beam and for further tracking. Mirrors 6 and 9 are rotational. The lens 7 forms the image of the observed region on the screen 8.

3 The Algebraic Image Reconstruction Technique

In experimental study of the turbulent mixing zone (TMZ) of heavy and light gases, the fringe phase shift would be dependent on the integrals of gas density distribution along the laser beam.

Tilt variations of the light beam traveling through the object of interest in any plane may provide a sufficient number of aspects to allow an attempt to reconstruct the 3D spatial gas density distribution of both gases with the desired accuracy. If only a limited number of aspects can be obtained experimentally, and the object symmetry is previously unknown, the primary consideration is what would be the smallest amount of experimental information to reconstruct the object image with the required accuracy. It is this situation we have to deal with when using optical interferometry, because some technical reasons make it almost impossible to obtain simultaneous interference patterns for angles in the range $0 \leq \theta \leq \pi$ with 10 or more aspects.

Reconstruction of the 3DD of an unknown function can often be reduced to reconstruction of a set of plane sections. Three dimensional distributions then would be obtained by arranging two dimensional ones. Hereinafter, we shall go after the image reconstruction technique based on the ideas stated in [11, 12, 13]. Basically, the concept of reconstruction as suggested in these papers is the following.

The reconstruction problem is in finding m values of the function in the spatial mesh $m \times m$. At the same time, what is known is only $m \times p$ data obtained experimentally, and often $p \ll m$. Thus, there is an indeterminate set of equations to calculate the values of the function to be reconstructed. In other words, the problem may have a great number of solutions, so that some additional conditions must be used to select the desired one. These conditions (criteria) should allow the selection of the most probable solution that incorporates the least amount of additional information out of the experimental data. The suggestion was that some norm must have *the maximum* at this solution and to take such a norm in the form of entropy of the non-equilibrium gas. As a rule, there is no rigorous reason to use concrete expressions for entropy functional, but good results justify this approach. In this paper we discuss the image reconstruction technique which is consistently based on minimization of the amount of information contained in the solution. This approach allows us to find the solution with maximal information entropy.

Let us use for reconstruction a mesh of $m \times m$ cells of the same size. Let us also assume that the *a priori* information available to us is the knowledge that each cell contains two gases with different but constant densities $\rho_1 > \rho_2$. These densities are constant across any section taken for reconstruction, and this is due to the fact that usually there is enough time for pressure and temperature relaxation. Let us split each cell into G parts, or sub-cells, so small as only one gas can exist in each of them. In other words, the sub-cell size is smaller that the minimum size of any non-uniformities in the TMZ. Let the number of sub-cells containing heavier gas be N_H, and that for lighter gas N_L, and $N_H + N_L = G$. This number of heavy-gas and light-gas sub-cells may be corresponded with the number of possible ways to achieve it (statistical weight of "state" having the numbers N_H and N_L). As is known, this statistical weight is [14]

$$\Gamma = \frac{G!}{N_L! N_H!}.$$

Based on the classic definition of the amount of information entropy I [15], contained in the set of G symbols with N_H of one kind and N_L of the other, we can put down

$$I = -G \cdot k \cdot \sum_{l=1}^{2} (n_l \cdot \ln(n_l))$$

where k is the normalization constant, $n_l = \frac{N_l}{G}$ the frequency of occurrence for a symbol with index l ($l = 1$ corresponds to the heavier gas and $l = 2$ to the lighter one).

Since we have $n_2 = 1 - n_1$ ($N_2 = G - N_1$), the mesh cell may have the value of information entropy in the "message" that the heavier gas fraction is n_1, expressed as follows:

$$I = -G \cdot k \cdot (n \cdot \ln(n) + (1 - n) \cdot \ln(1 - n)).$$

If n_{1ij} is the heavier gas fraction in the cell with indices ij ($i = 1 \ldots m, j = 1 \ldots m$) on the reconstruction mesh, then due to entropy additivity for statistically independent subsystems total entropy of the entire distribution of the lighter and heavier gases is:

$$I = -G \cdot k \cdot \sum_{i=1}^{n} \sum_{j=1}^{n} [n_{ij} \cdot \ln(n_{ij}) + (1 - n_{ij}) \cdot \ln(1 - n_{ij})] \qquad (1)$$

The resulting relationship coincides to within the constant normalization factor with the entropy equation for a non-equilibrium Fermi-gas.

Equation (1) for information entropy is a functional of the distribution of n_{ij} in the discrete mesh $m \times m$. The value of I will be maximal at some solution n_{ij} if the functional I will have zero variation. Experimental data serve as "external restrictions" and should be accounted for by Lagrangian factors. Consider these restrictions in more detail. Let us take interference patterns for p aspects. As already mentioned above, the shift of fringe or light signal phase is proportional to within the constant to

$$\Delta\varphi = c_1 \cdot \int_{l_{k_1}\theta} n_1 \cdot dl + c_2 \cdot \int_{l_{k_2}\theta} n_2 \cdot dl = c_2 \cdot l_{k_2} \cdot \theta + (c_1 - c_2) \cdot \int l_{k_1\theta} n_1 \cdot dl,$$

where $n_1 = 1 - n_2$ is the heavier gas concentration, and n_2 is the concentration of the lighter gas. Integration is made over the length of the light beam that crosses the given section at the angle of θ, k_1 is the number of the interference line. To incorporate this information into the reconstruction problem it would be suitable to have the experimental data preprocessed. A fringe pattern taken at a certain aspect angle defines the phase shifts at the sections as a function of the spatial coordinate normal to the laser beam direction. This function is determined at as many points as a number of interferometric lines crossing the given section. When we take the mesh $m \times m$ with m much smaller than the number of lines in the experiment, and we can find integrals of n over any band with the size of about the one cell in any section by adding together the line shifts within this band (see Figure 2).

The accuracy of the integral calculations over the fringe depends on interference line density, so the number of the interferometric lines across the cell must be adjusted to the needed accuracy.

Suppose that for each projection at the angle θ we know from the fringe pattern m integrals of n_j over the band of the number $k = 1 \dots m$:

$$R_{k\theta} = \sum_{i,j=1,m} n_{ij} \cdot S_{ijk\theta}, \tag{2}$$

where $S_{ijk\theta}$ is the fractional area of the cell of number i, j, which is common with the band k, θ. Thus the reconstruction problem tries to find maximum entropy 1 with restrictions 2.

For computer simulation of the image reconstruction procedure, we decided the distribution pattern of heavy and light gases in the turbulent mixing zone section to be the same as the black and white portrait of one of the authors of this paper. Here, the initial image has only two color intensity levels: zero or white, to simulate the distribution of light gas of constant density, and one or black, to simulate the distribution of heavy gas also of constant density. Thus the icon we selected for reconstruction corresponds both in symmetry and average grayness, as we think, to that expected in typical sections of the TMZ of two gases.

Numerical experiments were made for different numbers of mesh cells and aspects. The reconstruction results are illustrated in Figure 3.

The first column is the original portrait, the second was obtained from two iterations, the third from eight, and the N^{th} from $4^{N-3/2}$ iterations. Thus, the final column presents images after 512 iterations. Each row shows results with the same number of aspects. The top with two, the second with three, the third with four, the fourth with six, and the fifth with eight. In all cases, aspect angles θ_p were uniformly distributed between zero and π. The angles were measured from the horizontal direction. Finally, the bottom row presents the reconstruction results for 10 aspects with five of these covering the angles $-\pi/6 \leq \theta \leq \pi/6$ against the horizontal axis and the other five the same angles against the vertical one. Figure 3 shows the convergence of the iterative process and also illustrates the dependence of the reconstruction results on the number of aspects selected. One may conclude from the pictures shown in the figure that quite satisfactory quality of reconstruction can be achieved with as few as $p = 4$ aspects once

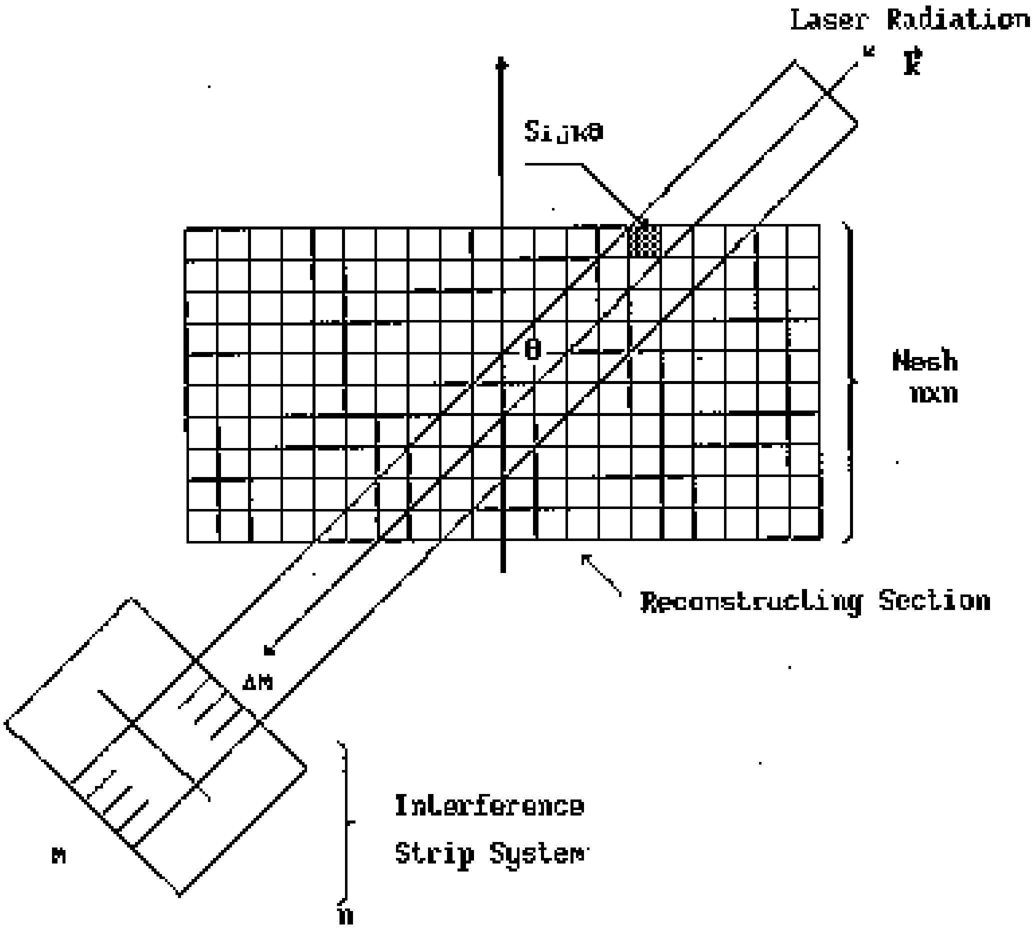


Figure 2: The experimental data preprocessing scheme.

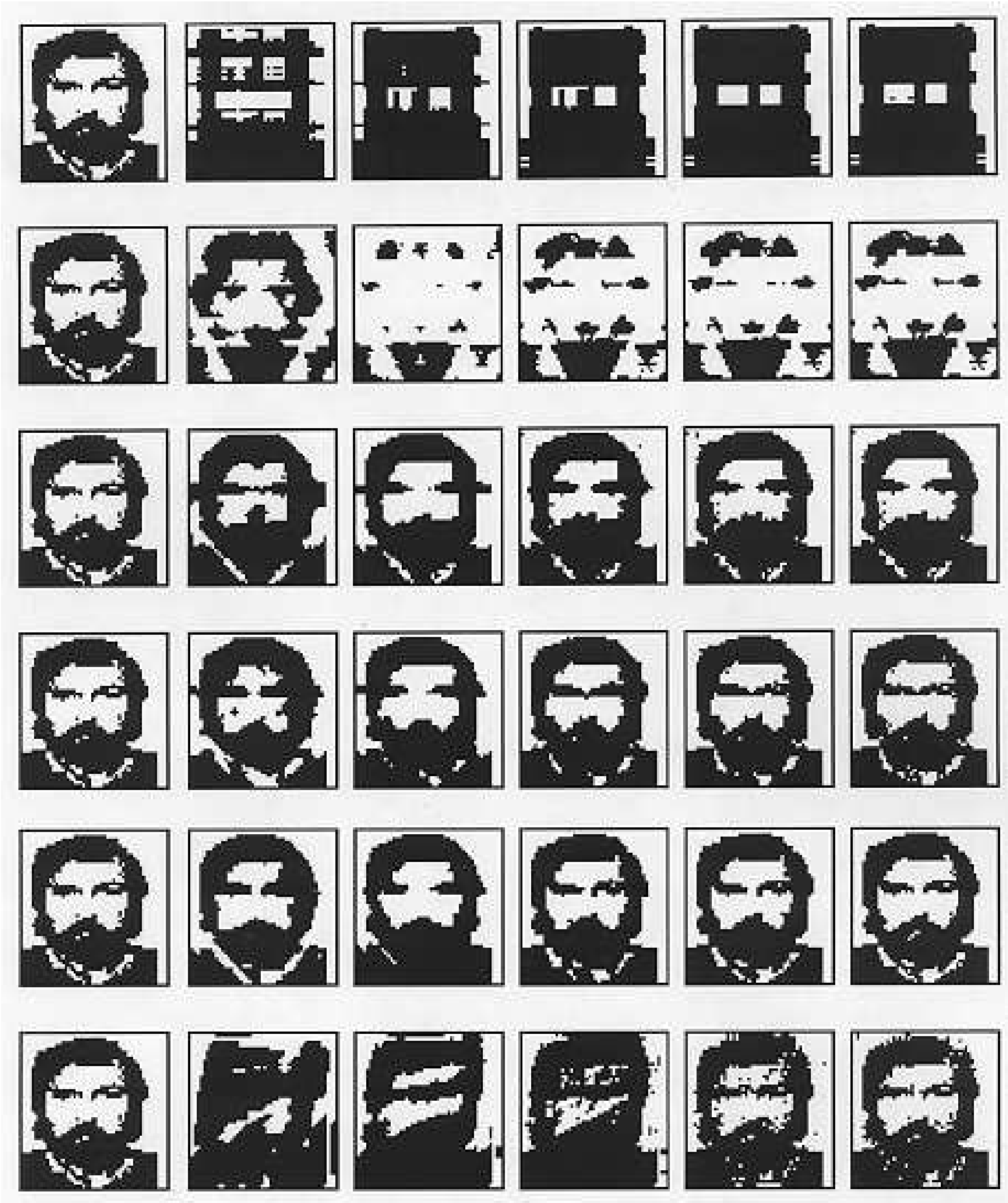
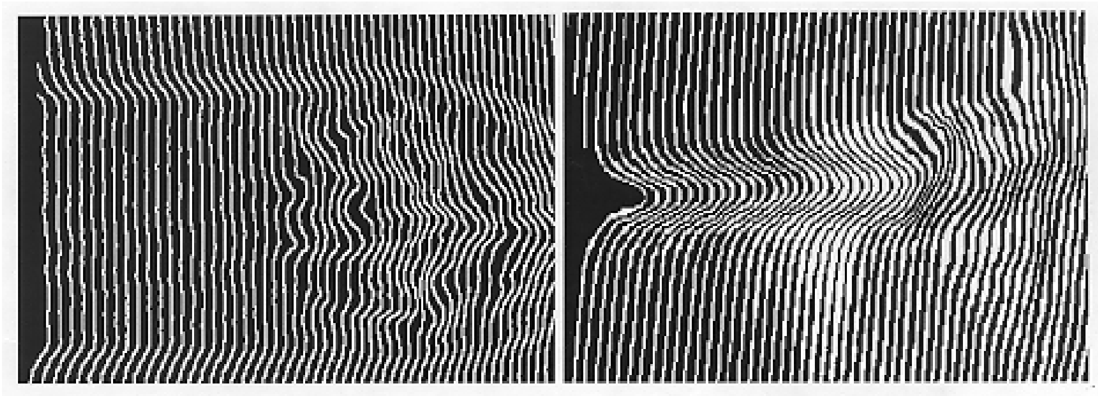


Figure 3: The reconstruction results for the mesh with 48×48 cells.



(a) **(b)**

Figure 4: Propane jet interferograms. The jet is outcoming into the air through a square hole $10 \times 1\text{mm}^2$. (a) viewed from the broad end, (b) viewed from the narrow end. The interferential strip period is 0.3 mm.

they are obtained using the full angle $\theta = \pi$. Decreasing this angle will require more aspects.

To validate the optical interferential method for gas flow recording, an experimental system was assembled with the Mach-Zehnder interferometer as the main component. The AYG:Nd^{3+} laser radiation (the second harmonic with the wavelength 0.5μm, pulse duration 15 ns) was used to pass through a propane jet in air. Figure 4 presents the interferograms of the propane jet injected into the air through a square hole $10 \times 1\text{mm}^2$. (Figure 4a viewed from the broad end of the jet, Figure 4b from the narrow one, and Figure 5a through a round hole 1.4 mm in diameter.

All photos show the jet perturbation. Their quality allows us to use them for the validation of our method of reconstruction tomography. The shadow image of a propane jet coming out of a round hole was obtained using computer the procedure (Figure 5b). This pattern corresponds to higher absorption of propane since the shift of interference lines is proportional to its density. The shadow "photo" demonstrates how the laminar jet becomes turbulent, and one can see the structure of arising turbulent flow.

4 Conclusions

In this paper we analyzed the possible applications of optical diagnostics to experimental study of gas-gas turbulent mixing. It is shown that 3D spatial concentration distributions of materials in TMZ may be obtained from experimental data with the help of interferometry techniques. These techniques may be used for measurement of

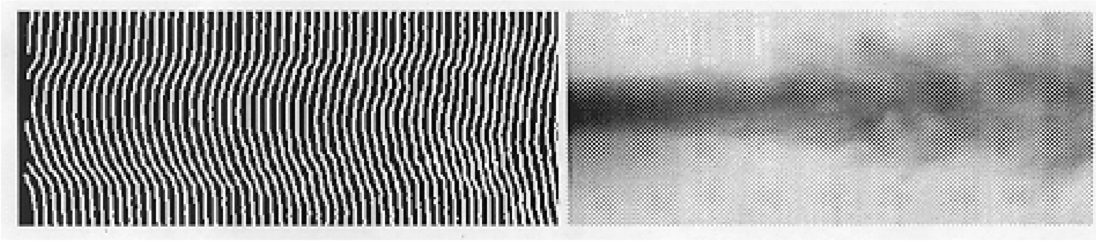


Figure 5: Propane jet interferogram. The jet is coming into air through a round hole 1.4 mm in diameter. (a) interferogram, (b) computed shadow image. The interferential strip period is 0.1 mm.

wavefront variations (integrated along the light trajectory), that are associated with turbulence development, primarily variations in partial material concentrations in TMZ. As concluded from numerical simulation of the reconstruction process, with the method suggested, gas density distributions in TMZ may be reconstructed with the total number of aspects $p = (4 \div 10)$, obtaining major details whose scale is about several per cent of the representative linear size of the object. The possibility of obtaining interference patterns with desired quality is shown experimentally.

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