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Numerical Studies for the Instability of Tangent Velocity Discontinuity in Compressible Gases

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1 Introduction

One of the characteristic hydrodynamical instabilities is the Kelvin–Helmholtz instability (KHI), or tangential velocity discontinuity. Most studies of this problem pertain to the case of incompressible fluids for which the problem formulation is simpler while capabilities of the methods are broader. Results of solution to the problem for incompressible fluids and review of the literature can be found in [1]–[4]. In the small perturbation approximation the problem of tangential velocity discontinuity instability in compressible gases for 2D (plane) perturbations was studied by Landau [5].

Analytical description of finite perturbations where amplitude is comparable with wavelength is hardly possible, therefore the principal method of studying is the numerical simulation method. This paper gives the results of numerical simulation using the Eulerian technique EGAK [6],[7] of the non-linear stage of the problem under consideration for compressible gases (see also [8],[9]).

Despite the efficiency of using finite-difference methods for solving complex hydrodynamic instabilities there are some difficulties related to computational grid discreteness and presence of approximation viscosity.

When studying unstable flows one, as a rule, considers evolution of determinate perturbation specified at the initial time in the form of one or several harmonics. With time owing to process non- linearity perturbations of shorter wavelengths develop, however, the approximation viscosity present in the difference schemes leads to distorted description of small-scale flows. Therefore, quantitative description of the non-liner stage is limited to the time starting from which the harmonics of wavelengths comparable with computational cell size develop.

2 Non-linear stage of KHI evolution for sinusoidal perturbation

The following problem is considered

There are two ideal gases in the upper and lower half-spaces, having the same initial pressures and adiabatic indices, equal to $P_1 = P_2 = 0.6$ and $\gamma_1 = \gamma_2 = 5/3$, respectively. For same initial time, there has been specified an x-periodical sinusoidal perturbation on the interface

$$y(t = 0, x) = a_1 \sin(2\pi x), \tag{1}$$

where $a_1 = 0.1$, and perturbation wavelength is $\lambda = 1$.

Tangential velocity discontinuity $\Delta u_x = 1$ satisfied the plane flow conditions for relatively small 2D perturbations [6].

$$\Delta u_x < (c_1^{2/3} + c_2^{2/3})^{3/2}, \tag{2}$$

where c_1 and c_2 are the sound velocities in the upper and lower gases, respectively. Initial density was $\rho_2 = 1$, and ρ_1 was varied as $\rho_1 = 1, 5, 10$.

Computational domain size along x axis was $L_n = 1$. The computational grid is square, there were 50 cells per perturbation wavelength period. On side boundaries periodic boundary conditions were used, and on the bottom and top boundaries there were rigid walls.

The interface (1) was approximated by piecewise-conditions function. This made the velocity discontinuity spread in y $d = (1 \div 2)h$, where h — is the mesh size.

As shown by the analyses of initial perturbations spectrum, the first harmonic a_1 has its actual amplitude differing from 0.1 by no more than 1 %, with the amplitudes of harmonics $a_n (1 < n < 10)$ not higher than $0.01a_1$. Therefore, the approximation assumed for initial perturbation was reasonable accurate.

Figures 1,3 illustrate some calculations. Figure 1 shows interface shapes at different times for the case where the gases have the same initial density. What is observed is the interface spiraling. This tendency is less marked for density ratio equal to 10 (Figure 2).

Figure 4 shows the interpenetration zone thickness $L(t) = y_{\max} - y_{\min}$, where $y_{\max}$ and $y_{\min}$ are the maximum and minimum y-coordinates of the interface. At an earlier stage, the curves $L(t)$ are linear, with the rise rate decreasing with larger density difference, and this is qualitatively consistent with the date from linear theory of small perturbations growing in incompressible liquids. For gases having the same densities, L is no longer observed at the level $L \approx 0.7$.

While density variations n the flows of interest were within 30 % the contribution of the velocity discontinuity smoothing can be estimated by making use of the data from shear flow instability studies of incompressible liquids. That the incompressible liquid approximation is suitable for this purpose, is also supported by the similar calculations

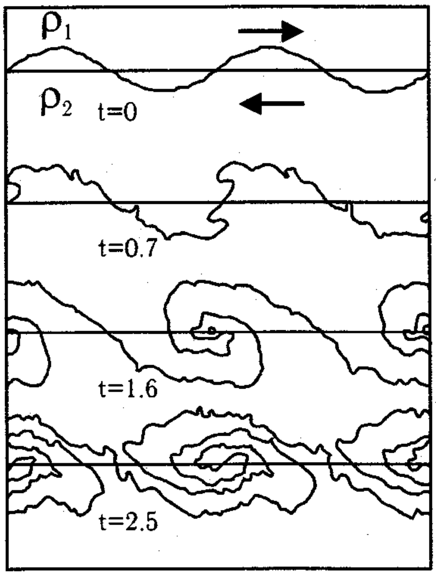

Figure 1: Interface shape evolution ($\delta = 1$).

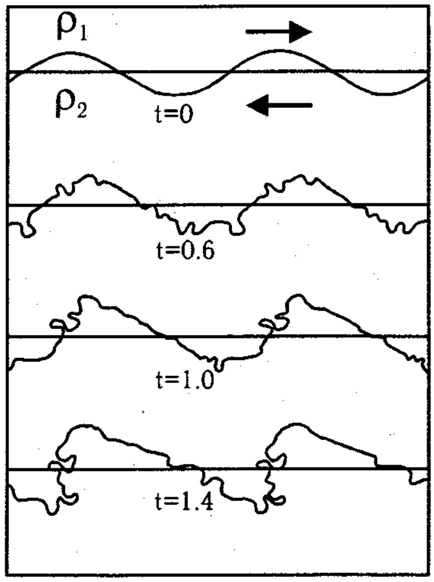

Figure 2: Interface shape evolution ($\delta = 10$).

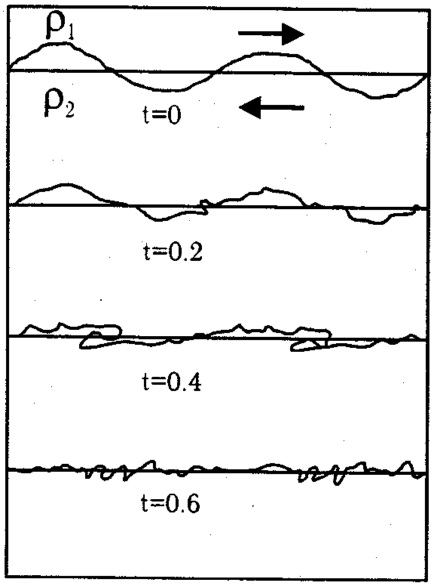

Figure 3: Interface shape evolution in the stable case ($\delta = 1$).

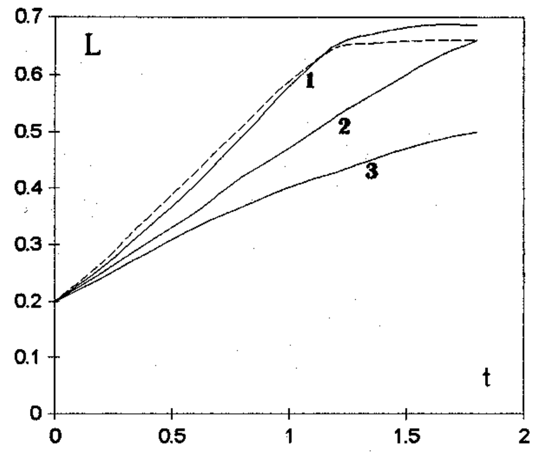

Figure 4: Dependence $L(t)$ for sinusoidal perturbation. 1 — $\delta = 1$; 2 — $\delta = 5$; 3 — $\delta = 10$; dashed line — the problem analogous to the problem 1 ($\delta = 1$), but sound speed was doubled.

where sound velocities in gases were increased two-fold (dashed line in Figure 4). The interface shapes obtained by this calculation differ from the previous calculated data by no more than a mesh space.

When the velocity discontinuity smoothing region ($0 < y < d$) has u_x linearly dependent on y, the instable perturbations spectrum is upper limited by the wave number value $k^* \approx 1.3/d$ [2]. For $k = 2p/\lambda < 0.4k^*$, the perturbations increments differ from those in the velocity discontinuity problem by less than 10 %. The calculations were performed with the spacing scale for velocity discontinuity smoothing $d = 0.02$, the harmonic referring to k^* is that of the number $n^* \approx 11$. Harmonics numbered $n > 11$ would not grow. Bat perturbations with harmonic numbers $n \approx$6 to 11 will grow significantly slower than for velocity discontinuity case.

Small additional harmonics caused by stepwise shape approximation of the initial perturbation, and the above considerations both indicate reasonably accurate numerical description of the nonlinear stage of perturbations growth during a limited time. The mathematical viscosity effects analysis made using analytical assessments and by numerically solving supplementary problems [10], shows the calculation results for $t > 2$ are more likely to be qualitative. Good accuracy of the data obtained for $t < 2$ has been proved by the calculation with the mesh size taken twice as small.

The numerical solutions were verifies for accuracy and representativeness using a calculation involving the velocity discontinuity $\Delta u_x = 5$. From criterion (2), the flow like this should be steady with respect to comparatively small 2D perturbations.

The interface shape as resulting from this calculation is given in Figure 3. Unlike the non-steady case, there is no perturbations amplitude growth observed here, thus showing qualitative agreement with the conditions of reference [5].

Thus, the following may be concluded from the analyses of the calculation results.

A harmonically perturbed interface will transform with time into a periodic set of spirals. Spiraling is the more rapid, the smaller the density difference between the gases. What was to be observed, that the lateral spiral growth is limited by the value $L \cong 0.7l\lambda$, this being due to the periodic nature of initial perturbations. The shear flow instability criterion referring to 2D perturbations (2) has also proved valid for finite amplitude perturbations.

3 Evolution of perturbations specified superposed harmonics

A computation series was performed where harmonics interaction and its effect on gas interpenetration were studied.

The calculations have been made for gases having one and same density $\rho_0 = 1$ and the same pressures $P_0 = 0.6$. The velocitiy discontinuity was $\Delta u_x = 1$. There were more computational cells used per unit length (60) than in the above-described calculations.

Initially, the interface perturbation was specified as:

– superposition of two harmonics:
$z(t = 0, x) = a_1 \cos(2\pi x/\lambda_1) + a_k \cos(2\pi x/\lambda_k)$, where $\lambda_k = l/k(k = 1, 3)$, and

– saw-tooth shaped.

Table 1 shows some parameters and results of the calculations.

The following notation is used here: $A_k = a_k/\lambda_k$ — harmonic amplitude ratio (for saw-tooth type perturbation Fourier coefficients are given), $L = y_{\max}(t, x) - y_{\min}(t, x)$ — gas interpenetration thickness, $L_0 = y_{\max}(t = 0, x) - y_{\min}(t = 0, x)$.

Figure 5 illustrates the interface shape evolution as obtained in calculations 2 through 4, respectively.

The calculations all observe the gas interpenetration zone stops growing thicker. The final column in table 1 includes asymptotic values L.

N	type	A_1	A_2	A_3	A_5	L_0	L
1	—	0.02	—	0.1	—	0.106	0.45
2	—	0.1	—	0.1	—	0.268	0.5
3	—	0.67	0.1	—	—	0.175	0.43
4	saw-tooth	0.082	—	0.03	0.2	0.02	0.43

Table 1: Input parameters and results of the calculations

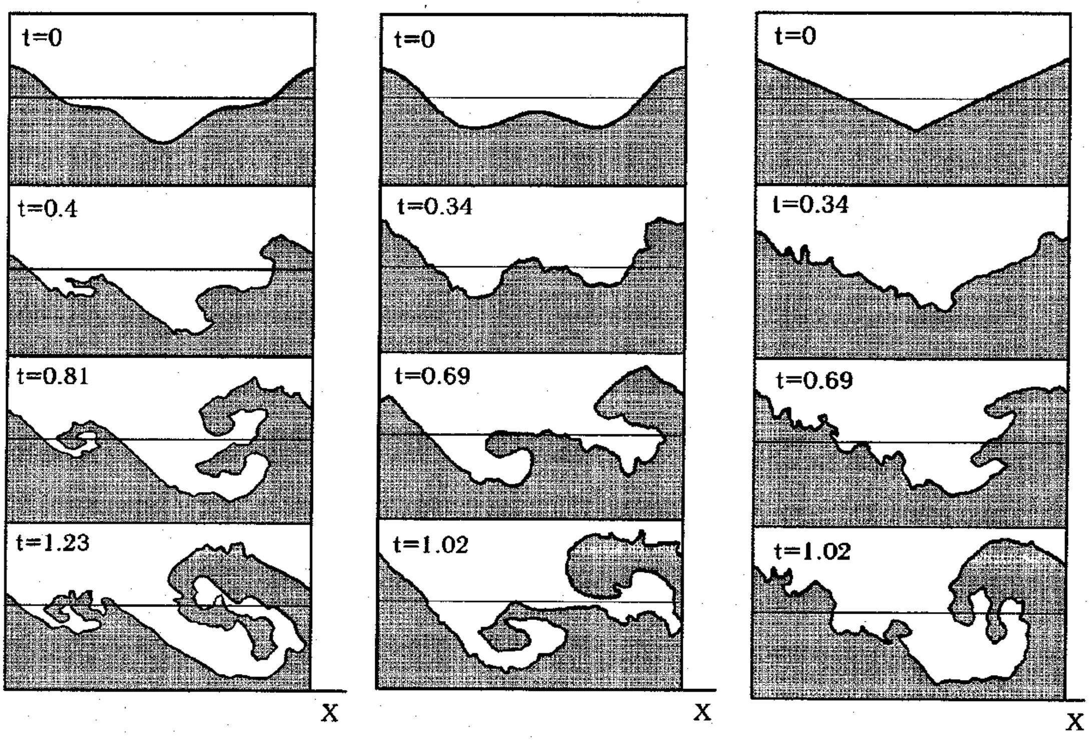

Figure 5: Interface shape, (a) — calculation 2; (b) — calculation 3; (c) — calculation 4

Given the first harmonic has smaller amplitude than the third one (calculation 1), a two-stage flow pattern will be then observed. Initially, there will be the first harmonic rapidly increasing in amplitude, thus making the gas interpenetration zone grow thicker. Over time, the growth rate of L will be decreasing. Then, the increase in L becomes dependent on the amplitude growth of the harmonic of $n = 3$, and thus the zone thickness grows more rapidly. At later times, L growth rate again will decrease. However, when the first harmonic's initial amplitude is larger than that for the harmonic of $n = 3$, no two-stage flow pattern will be observed.

Why the gas interpenetration zone stops growing thicker, can be understood from the following qualitative analogy. By virtue of its periodic nature, the perturbations spectrum is limited and contains for any t only harmonics with the wave number $k \geq 2\pi/L_n$, where L_n — is the computational field size in x axis. Shear flow is a rather complex pattern during its nonlinear stage (with a set of spirals forming). If one considers it as averaged flow with continuously distributed velocity, then it should be stable against perturbations having wave number $k > k^* \approx (1-2)/L$ [11]. By times where $L > (1-2)L_n/2\pi$, this flow should become stable against perturbations having any wave numbers possible.

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